The Volatility Information Implied in the Term Structure of VIX

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Abstract

This study examines whether the volatility information implied in the term structure of VIX can improve the prediction of realized volatility. We first propose several approaches to compile maturity independent proxies of volatility from the VIX term structure and then investigate the information content of these proxies for future realized volatility. The empirical results on the S&P 500 index show that in terms of both in-sample estimation and out-of-sample forecasting, the proxies representing the information on the VIX term structure are more informative than the single VIX with a particular time to maturity. Our empirical results are robust to alternative model specifications and various forms of volatility.

Keywords: Options; VIX; Term Structure; Volatility; Forecasting.

JEL Classification: G17; G13.

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1. Introduction

Although the CBOE has proposed a new approach to compute the VIX volatility index in 2003, it is still maturity dependent. While most previous studies adopt the CBOE standard to use the 30-day VIX, the information on the term structure is still missing. Since, different from a general asset price, volatility behaves some special stylized facts like clustering and mean-reverting, the term structure of VIX may just reflect market participants' expectation on how volatility will change. Therefore, this study investigates the role of the VIX term structure in volatility forecasting. In general, the empirical results support the significant incremental contribution of the information on the VIX term structure for forecasting realized volatility.

Asset return volatility plays an important role in assessing derivatives prices and managing financial risks. Due to its empirical properties such as persistency and mean-reversion, volatility is much more predictable than returns.[1] Ever since the advent of the Black-Scholes option pricing formula (Black and Scholes (1973)), option prices are a source of valuable information when forecasting volatility. Poon and Granger (2003) provide a comprehensive survey of volatility forecasting studies and conclude that, although biased, the option-implied volatility is the best predictor

[1] Volatility forecasting inaccuracy in many studies is attributable to the use of a very noisy proxy for volatility, such as squared daily returns (see Andersen and Bollerslev (1998)). Useful forecasts of volatility can be made when realized volatility is measured accurately using intraday prices (for instance, Blair, Poon and Taylor (2001), Li (2002), Martens and Zein (2004), and Pong, Shackleton , Taylor and Xu (2004)).

of realized volatility. The superiority of the option-implied volatility has been well documented by numerous previous studies, such as Xu and Taylor (1995), Fleming (1998), Blair, Poon, and Taylor (2001), Ederington and Guan (2002), and Jiang and Tian (2005).

The implied volatility is usually referred to the volatility value that equates the Black-Scholes option price and the corresponding market price. However, it depends on the strike and the expiration of the option contract. Among many different implied volatility, which one is more appropriate to use becomes an empirical issue. In early years, most studies adopt the implied volatility recovered from the nearest-month ay-the-money (ATM) contract. In 1993, the Chicago Board Options Exchange (CBOE) introduced a volatility index called VIX, which was compiled from eight S&P 100 index (OEX) options comprising of near at-the-money, nearby and second nearby calls and puts. This index was to reflect the implied volatility of a 30-calender-day ATM option. About ten years later, CBOE introduced a revamped new VIX computed by a model-free formula, which is based on the theoretical work of Carr and Madan (1998), Demeterfi, Derman, Kamal and Zou (1999), and Britton-Jones and Neuberger (2000), instead of Black-Scholes formula. This new VIX is compiled from the prices of S&P 500 index (SPX) options with a wide range of strike prices, as opposed to only eight near-ATM ones. As expected, the new VIX essentially contains more information than the old one and its advantage in volatility forecasting has been supported by Jiang and Tian (2005).

Nonetheless, although incorporating the information of a volatility smile, the model-free implied volatility index is still maturity dependent. VIX is usually regarded as a 30-day implied volatility index, though it is obtained from the interpolation of two nearby model-free implied volatilities. Therefore, the studies that use the new formula to investigate the predictive power of implied volatility have to choose a particular maturity. Almost all of the studies in the fast growing literature on implied volatility only consider the 30-day maturity.[2] The information of the volatility term structure remains missing when studying volatility forecasting with VIX. Since volatility exhibits some unique properties such as clustering and mean-reverting, its term structure should contain market participants' expectation on the change of volatility. Thus, it is natural to explore whether it is possible to compile a proxy for the VIX term structure and whether this proxy can improve the performance of VIX in volatility forecasting

In this study, we propose some approaches to extract the information implied in the VIX term structure. In particular, we adopt a two-factor model to decompose VIX

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[2] Numerous studies have investigated implied volatility smile. See, for example, Rubinstein (1994), Pena, Rubio and Serna (1999), Foresi and Wu (2005), Zhang and Xiang (2008), Chang, Ren and Shi (2009), and Xing, Zhang and Zhao (2010), among others. However, little attention has been paid to the term structure of implied volatility. See, for example, Xu and Taylor (1994) and Campa and Chang (1995).

and compile the proxies for the VIX term structure. In addition, we use the variables representing the level and slope of the VIX term structure as the proxies. We empirically examine the performance of these proxies on volatility forecasting of the S&P 500 index returns. The in-sample and out-of-sample results jointly indicate that the information content and forecasting ability of the proxies compiled from the VIX term structure are superior to those drawn from the VIX with a particular time to maturity. When using an alternative two-factor model or defining volatility in terms of various forms, the conclusions drawn from the empirical results are unaltered.

The remainder of the paper is organized as follows. Section 2 gives a brief description of our data and filtration rules. Section 3 introduces the methods to extract the information from the VIX term structure and the empirical specifications. Section 4 provides empirical results and discusses their implications from both in-sample estimation and out-of-sample forecasting. We conduct several robustness tests in Section 5, and conclude this study in Section 6.

2. Data

The primary dataset in this study consists of the daily best bid and ask prices of the S&P 500 index options, which are obtained from OptionMetrics along with the details of the contracts including types (call or put), time-to-maturities and strike prices. We use the mid-quotes (averages of bid and ask prices) to serve as the proxies of market prices. From the same database, we obtain the daily zero-curve rates, with which we use the interpolation technique to generate the risk-free interest rates that match the time-to-maturities of all option contracts. The sample period covers from Jan 2, 1998 through August 31, 2012.

Several standard exclusion filters are applied to select our final sample of the S&P 500 index options. First, options with less than one week to expiration are excluded due to liquidity-related biases and market microstructure concerns. Second, we exclude options with time to maturity beyond one year, also because of liquidity-related biases. Third, options with quotes lower than $0.5 are excluded from the sample because of the impact of price discreteness on option valuation. Fourth, we exclude option quotes not satisfying the arbitrage restrictions from the sample.

The VIX with maturity τ_j at time *t*, VIX_{t,τ_j}^M, is calculated by the CBOE VIX formula (CBOE (2009)). In particular,

$$VIX_{t,\tau_j}^{M\ 2} = \frac{2}{\tau_j}\sum_i \frac{\Delta K_i}{K_i^2} e^{r\tau_j} Q(K_i) - \frac{1}{\tau_j}\left[\frac{F}{K_0} - 1\right]^2, \quad (1)$$

where F is the forward index level derived from index option prices, K_i is the strike price of i^{th} out-of-the-money option, $\triangle K_i$ is the interval between strike prices defined as $\Delta K_i = (K_{i+1} - K_{i-1})/2$, K_0 is the first strike below the forward index level, r is the risk-free interest rate to expiration, and $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike K_i.

Since future realized variance is the target to forecast, we obtain the high-frequency levels of the S&P 500 indices at the 5-minute interval from Olsen Data to calculate realized variance. Following the theory of Andersen et al. (2001), we calculate the daily realized variance as the sum of squared intraday returns within the day, and then times the daily realized variance by 252 (the number of trading days in one year) to get the annualized variance. The summary statistics of VIXs and realized variance are reported on Table 1.

[Insert Table 1 about Here]

3. The Methodologies and Estimation Approaches

We propose two types of methods to extract information from the term structure of VIX. One is parametric, and the other is non-parametric.

3.1. The Parametric Method

In the stochastic volatility framework of Heston (1993), squared VIX is a weighted average between the instantaneous variance and the risk-neutral long-term mean level of variance. In other words, squared VIX can be decomposed into the instantaneous variance and its long-term mean level. To calibrate out the instantaneous variance and the long-term mean level of variance, we need several VIX levels with different horizons. Therefore, the estimates of these two types of variance will serve as the gradients used to compile the proxies for the information implied in the VIX term structure. Since this decomposition is a sort of aggregate information from several VIX values across maturities, it may improve the volatility forecasting performance of VIX.

Nevertheless, in the stochastic volatility model of Heston (1993), the instantaneous variance is the only random factor. Recent studies show that two risk factors are necessary to properly capture the variance dynamics of financial asset returns[3]. Thus, in this paper, we adopt the 2-factor model of Egloff, Leippold, and Wu (2010) in which the long-run mean level of volatility is another independent mean-reverting process (ELW hereafter).

We first introduce how to decompose the squared VIX by means of the ELW model. Consider 2-factor variance risk dynamics controlled by the following stochastic differential equations under the risk-neutral measure:

$$dv_t = k_v(m_t - v_t)dt + \sigma_v\sqrt{v_t}dB_t^v, \tag{2}$$

$$dm_t = k_m(\theta_m - m_t) + \sigma_m\sqrt{m_t}dB_t^m, \tag{3}$$

where B_t^v and B_t^m are Brownian motions with $dB_t^v dB_t^m = 0$, and v_t is the instantaneous variance reverting to a stochastic mean level m_t, which follows another square-root mean-reverting process. Egloff, Leippold, and Wu (2010) show that under this specification, the squared VIX with a horizon of τ at time *t*, $VIX_{t,\tau}^2$,

[3] These studies include Zhang and Huang (2010), Zhang, Shu and Brenner (2011), and Duan and Yeh (2011).

can be decomposed as

$$VIX_{t,\tau}^2 = \varphi_v(\tau)v_t + \varphi_m(\tau)m_t + (1 - \varphi_v(\tau) - \varphi_m(\tau))\theta_m, \quad (4)$$

where $\varphi_v(\tau) = \frac{1-e^{-k_v\tau}}{k_v\tau}$ and $\varphi_m(\tau) = \frac{1+\frac{k_m}{k_v-k_m}e^{-k_v\tau}-\frac{k_v}{k_v-k_m}e^{-k_m\tau}}{k_m\tau}$. It is noticeable that the squared VIX only depends on the constant parameters k_v, k_m, and θ_m, and time-varying variables v_t and m_t, but is irrelative to σ_v and σ_m.

Now that in the model some parameters are constant, but the others are time-varying, a two-step estimation procedure is required. Following the efficient iterative two-step procedure proposed by Bates (2000) and Huang and Wu (2004), and modified by Christoffersen, Heston and Jacobs (2009), we will estimate the parameters with a two-step procedure similar that adopted by Luo and Zhang (2012). We delineate the procedure as follows.

Step 1: Given initial values of $\{k_v,\ k_m,\ \theta_m\}$ for the ELW model, we solve the following T optimization problems to obtain the time series of $\{v_t,\ m_t\}$ with *t=1,…,T.* That is,

$$\{\hat{v}_t, \hat{m}_t\} = argmin \sum_{j=1}^{N_t} (VIX_{t,\tau_j}^M - VIX_{t,\tau_j})^2,\ t=1,\dots,T, \quad (5)$$

where VIX_{t,τ_j}^M and VIX_{t,τ_j} are the market value and the corresponding theoretical value of VIX (formula (4)) with maturity τ_j at time *t*, respectively. N_t is the number of maturities available at time *t*.

Step 2: With $\{v_t,\ m_t\}$ estimates obtained in Step 1, we minimize the following loss function to estimate $\{k_v,\ k_m,\ \theta_m\}$ for the ELW model. Namely,

$$\{\hat{k}_v,\ \hat{k}_m,\ \hat{\theta}_m\} = argmin \sum_{t=1}^{T} \sum_{j=1}^{N_t} (VIX_{t,\tau_j}^{M} - VIX_{t,\tau_j})^2. \qquad (6)$$

An iteration procedure between Step 1 and Step 2 is executed until the convergence criterion in the objective function of step 2 is reached.

As the squared VIX can be decomposed into two time-varying components, the instantaneous variance and the long-term mean level of variance, a natural way to incorporate the information of VIX term structure is to replace $VIX_{t,\tau}$ with v_t and m_t. Alternatively, since it is well documented that volatility exhibits mean-reverting, we can examine the incremental contribution of the relative position of the instantaneous variance to its long-term mean level, $v_t - m_t$, which may be informative to the following volatility change. Table 2 shows the descriptive statistics of estimated parameters. In the ELW model, the estimated mean-reverting speed of instantaneous variance is equal to 6.4716. The estimated long-term mean level of the instantaneous variance long-term mean is about 0.0330, near 0.0576, the mean of estimated long-term mean levels. The instantaneous variance is highly correlated to the squared VIX, and so is the relative position of the instantaneous variance to its long-term mean level, $v_t - m_t$.

[Insert Table 2 about Here]

Therefore, we specify the empirical model as the following regression:

$$RV_{t+1}^2 = a + \sum_{i=0}^4 b_i RV_{t-i}^2 + cVIX_{t,\tau}^2 + c_1 v_t + c_2(v_t - m_t) + e_t, \quad (7)$$

where RV_t denotes the annualized realized volatility and e_t is the residual term at time t. The inclusion of 5 lagged realized variances is to control for the effect of volatility clustering. With alternative sets of coefficient constraints, we run the following sub-models:

Model 1: c_1=c_2=0

Model 2: c=c_2=0

Model 3: c=0

Model 4: c_2=0

Model 5: the full model

3.2. The Non-parametric Method

Since we can obtain several VIX values with different maturities at the same time, there are two straightforward ways to extract the volatility information from the VIX term structure. First, in addition to the 30-day squared VIX, we can also include the difference between the squared values of the 60- and 30- day VIX levels to represent the slope of the VIX term structure. Due to the mean-reverting property of volatility, the magnitude of the slope may indicate how likely the reverting will occur and therefore contain some information about future volatility. Second, we can generate the first two components of the term structure of squared VIX values using the principal component analysis, as these two components can essentially serve as the proxies of the level and slope of the VIX term structure. These two alternative approaches are intuitive and simple to transform maturity-dependent squared VIX values into maturity-independent components.

We provide descriptive statistics for the VIX term structure data with fixed maturities of 30, 60, 90, 180, 270, and 360 days in Panel A of Table 3. Some stylized facts can be found. For example, the average squared VIXs are not monotonic; they rise from 0.0530 for the 30-day squared VIX to 0.0553 for the 180-day squared VIX and then decrease; the variance of squared VIXs decreases as maturity increases; and all squared VIXs are highly skewed and leptokurtic as previously documented, especially for the short-day squared VIXs. The summary statistics of the first two principal components of the VIX term structure are reported in Panel B of Table 3. The main principal component explains around 94% of the total variation in the data, and the first two components together explain more than 99%. In addition, the eigenvectors imply that the first and second principal components are related to the level and slope factors in the term structure curve of the squared VIX, respectively. Thus, the first principal component shares some features with the squared VIX such as highly skewed and leptokurtic, and the second component has elements with different signs.

[Insert Table 3 about Here]

We denote $PCA1_t$ and $PCA2_t$ respectively the first two principal components of the squared VIX term structure, and specify two additional empirical models as:

$$\text{Model 6:} RV_{t+1}^2 = a + \sum_{i=0}^{4} b_i RV_{t-i}^2 + f_1 PCA1_t + f_2 PCA2_t + e_t \quad (8)$$

$$\text{Model 7:} RV_{t+1}^2 = a + \sum_{i=0}^{4} b_i RV_{t-i}^2 + g_1 VIX30_t^2 + g_2 (VIX60_t^2 - VIX30_t^2) + e_t \quad (9)$$

The contribution of VIX term structure on volatility forecasting is evaluated by comparing the performance of Models 2, 3, 6 and 7 against Model 1 (the benchmark model), respectively, and observing the significance of parameters in Models 4 and 5.

4. Empirical Results

Table 4 summarizes the OLS regression results for Models 1−7 with the parametric results from the ELW model. Figures in brackets are the standard errors of the corresponding coefficient estimates.

[Insert Table 4 about Here]

We first notice that the adjusted R^2 for Models 2, 3, 6, and 7 are all higher than that for Model 1. This evidence suggests that the variables representing the term structure of squared VIX explain more of the variation in future realized volatility than squared VIX itself. In particular, as indicated by the comparison between Models 1 and 3, the instantaneous variance and its relative relation with the long-term mean jointly outperform the model with the 30-day VIX. Next, we consider the encompassing regression results from Models 4 and 5. The coefficients of v_t and v_t-m_t remain significantly positive, but that of squared VIX_t becomes negative, though significant. This evidence suggests the superior informational efficiency for the variables converted from the term structure of squared VIX.

In addition to the one-day prediction, we also conduct the regressions for longer forecast horizons. Table 5 shows the OLS regression results from Models 1−7 for the 1- and 2-month forecast horizons, respectively. Figures in brackets are the standard errors of the corresponding coefficient estimates, which are estimated following a robust procedure taking into account of serial correlation (Newey and West, 1987).

[Insert Table 5 about Here]

As shown in Table 5, for both regressions for 1- and 2-month forecast horizons, the adjusted R^2 of Models 2, 3, and 7 are all still higher than that of Model 1. Some coefficients in Models 4 and 5 become insignificant, but the signs of them remain the same, that is, the coefficients of squared VIX_t are negative and those of v_t are positive. In general, the one-day and one- and two-month predictions jointly suggest that the forecasting power of implied information decays with the horizon length and that the information from the VIX term structure is more informative than a single VIX value in terms of volatility forecasting.

In addition to comparing in-sample performance of several models, we also implement the out-of-sample forecasting. Time series of forecasts are obtained by estimating Models 1−7 with rolling samples. Each model is estimated initially over the 2000 trading days of the in-sample period. Forecasts of realized variance are made for the next day, say, day T+1, using the in-sample parameter estimates. The data are then rolled forward one day, deleting the observation at time T-1999 and adding on the observation at time T+1, and re-estimated the regression model to generate the forecast for time T+2. We repeat this rolling method until the end of the out-of-sample forecast period. With these time series of forecasts, we regress realized variance on the variance forecasts. Namely, we run the following regression:

$$RV_t^2 = \alpha + \beta \widehat{RV}_t^2 + \varepsilon_t, \tag{10}$$

where $\widehat{RV}_t$ stands for the volatility forecasts. Furthermore, we calculate mean-squared errors between RV_t^2 and $\widehat{RV}_t^2$ for each model. The results of out-of-sample forecasting are reported in Table 6.

[Insert Table 6 about Here]

As shown in Table 6, all slope coefficients are significantly positive. However, the mean-squared error for Model 1 is the second largest, and the adjusted R^2 for Model 1 is the second lowest. Actually, the mean-squared error for Model 2 is significant lower than that for Model 1 according to the t-test statistics. The superior forecasting effectiveness of the variables from the VIX term structure, v_t and v_t-m_t, are also supported by the highest adjusted R^2 of Models 2 and 3.

5. Robustness Analysis

We conduct three types of robustness tests for our empirical results. First, we examine the information content of VIX with alternative time to maturity and compare it with that implied in the term structure of VIX. Second, we use an alternative 2-factor model to decompose the VIX term structure. Third, we run the regression models of alternative forms of volatility.

5.1. The VIXs with Alternative Maturities

Instead of the standard 30-day VIX, squared VIXs with alternative maturities are considered. Due to liquidity-related biases, we replace the squared 30-day VIX only by squared 60-, 90-, and 180-day VIX, respectively. The regression results for Model 1 and 5 are reported on Table 7.

[Insert Table 7 about Here]

From Table 7, we find that each adjusted R^2 for Model 1 is lower than that for Model 2 on Table 4. Concerning Model 5, the results on Table 7 are similar to those on Table 4: the coefficient of v_t is significantly positive, but that of squared VIX_t is negative. This evidence suggests that the information from the VIX term structure is more informative than a single VIX with a particular maturity.

5.2. The Alternative 2-factor Model

Luo and Zhang (2012) propose an alternative 2-factor model, in which they directly specify the stochastic long-term mean level as a martingale, that is,

$$dm_t = dM_t, \tag{11}$$

where dM_t is the increment of a martingale process. Similarly, the decomposition of the squared VIX turns to

$$VIX_{t,\tau}^2 = \varphi_v(\tau)v_t + \varphi_m(\tau)m_t, \tag{12}$$

where $\varphi_v(\tau) = \frac{1-e^{-k_v\tau}}{k_v\tau}$ and $\varphi_m(\tau) = 1 - \varphi_v(\tau)$. This model is abbreviated as the LZ model. We follow the same 2-stage estimation procedure to generate the constant parameter, κ_v, and the time-varying parameters, v_t and m_t, which are then used to run the same regression models. The in-sample estimation results and the out-of-sample evaluation of volatility forecasting for the LZ model are reported in Tables 8 and 9, respectively.

[Insert Table 8 about Here]

[Insert Table 9 about Here]

Overall, the empirical results from the LZ model are qualitatively similar to those from the ELW model. For example, the estimated mean-reverting speeds of instantaneous variance for the two models are close (6.4716 and 6.7216 for the ELW and LZ models, respectively). In other words, the variables representing the information of the VIX term structure outperform the 30-day VIX in terms of both in-sample and out-of-sample volatility forecasting.

5.3. Alternative Forms of Volatility

In our main results we have shown the strong support for the contribution of the VIX term structure on volatility forecasting under the variance-version models. Following the empirical specifications in many early studies, moreover, we also investigate the issues of interest with the volatility-version and the logarithmic-volatility-version models to ensure the robustness of our analysis.

Similar to those specifications in the previous section, the volatility-version and logarithmic-volatility-version Models 1−5 are specified with various coefficient constraints, respectively, from the models specified as:

$$RV_{t+1} = a + \sum_{i=0}^{4} b_i RV_{t-i} + cVIX_{t,\tau} + c_1\sqrt{v_t} + c_2(\sqrt{v_t} - \sqrt{m_t}) + e_t; \quad (13)$$

$$log(RV_{t+1}) = a + \sum_{i=0}^{4} b_i \, log(RV_{t-i}) + clogVIX_{t,\tau} + c_1 log\sqrt{v_t} + c_2\big(log\sqrt{v_t} -$$

logmt+et. (14)

As to Model 6, we repeat the principal component analysis for the term structure of VIX or log(VIX), instead of squared VIX. Then we specify Model 6 as

$$RV_{t+1} = a + \sum_{i=0}^{4} b_i RV_{t-i} + f_1 PCA1_t + f_2 PCA2_t + e_t; \quad (15)$$

$$\log\,(RV_{t+1}) = a + \sum_{i=0}^{4} b_i log\,(RV_{t-i}) + f_1 PCA1_t + f_2 PCA2_t + e_t. \quad (16)$$

Model 7 is specified in a similar way as:

$$RV_{t+1} = a + \sum_{i=0}^{4} b_i RV_{t-i} + g_1 VIX30_t + g_2(VIX60_t - VIX30_t) + e_t; \quad (17)$$

$$log\ (RV_{t+1}) = a + \sum_{i=0}^{4} b_i log\ (RV_{t-i}) + g_1 log\ (VIX30_t) + g_2(log\ (VIX60_t) - log\ (VIX30_t)) + e_t. \quad (18)$$

Tables 10 and 11 demonstrate the results of in-sample estimation for volatility-version and logarithmic-volatility-version models, respectively.

[Insert Table 10 about Here]

[Insert Table 11 about Here]

For these regression results, we find no material change in statistical inferences between corresponding models of different versions, except for Model 5 of volatility-version, the coefficient of $\sqrt{v_t}$ becomes negative and VIX_t positive, though that of $\sqrt{v_t} - \sqrt{m_t}$ remains significantly positive. What is changed is the increased regression R^2. Moreover, in both volatility-version and logarithmic-volatility version, the adjusted R^2 for Models 2 and 4 are almost the same. This fact suggests that the term structure variables $\sqrt{v_t}$ and $log\sqrt{v_t}$ subsume all information contained in VIX_t and $log(VIX_t)$, respectively.

6. Concluding Remarks

In this study, we propose some approaches to investigate the volatility information implied in the term structure of VIX, and empirically examine its information content for future realized volatility. In particular, we transform the VIX term structure to maturity independent proxies of volatility expectation. Using the S&P 500 index as the underlying asset, the in-sample estimation results indicate the informational efficiency of the VIX term structure and the out-of-sample forecasting results show that the variables compiled from the term VIX structure provide more accurate forecasts than the VIX with a particular time to maturity. We conclude that the information from the VIX term structure provides promising incremental contribution for volatility forecasting.

Our empirical results are robust to alternative model specifications and various forms of volatility. In addition to pointing out the usefulness of the information on the VIX term structure for volatility forecasting, this study contributes to literature by providing some approaches to transfer maturity dependent VIX to maturity independent proxies of volatility expectation.

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Table 1 Summary Statistics of VIXs and Realized Variance

In this table we provide summary statistics of squared VIX (classified according to the days to maturity) and annualized realized variance. The sample period is from January 2, 1998 to August 31, 2012.

	Mean	Std. Dev.	Skewness	Kurtosis	Numbers of Observations
VIX^2(7-60 days)	0.0519	0.0539	4.8212	40.7796	7365
VIX^2(61-180 days)	0.0557	0.0431	3.1344	17.4395	8729
VIX^2(beyond 180 days)	0.0541	0.0333	2.4562	13.3670	8602
Realized Variance	0.0371	0.0761	11.0496	211.4441	3639

Table 2 Summary Statistics and Correlation Matrix of VIX^2, v_t, m_t, and v_t-m_t

In this table we provide descriptive statistics for the 30-day squared VIX and the estimated vt, mt, and vt-mt as well as $\hat{k}_v, \hat{k}_m$ and $\hat{\theta}_m$ from the ELW model in Panel A, and their correlation matrix in Panel B.

Panel A. Summary Statistics

	Mean	Std. Dev.	Skewness	Kurtosis
VIX^2	0.0530	0.0540	4.3385	27.9331
m_t	0.0576	0.0353	1.8584	5.4569
v_t	0.0514	0.0642	4.7839	33.1385
v_t-m_t	-0.0063	0.0543	5.5978	50.9400

$\hat{k}_v = 6.4716, \hat{k}_m = 0.2937, \hat{\theta}_m = 0.0330$

Panel B. Correlation Matrix

	VIX^2	m_t	v_t	v_t-m_t
VIX^2	1	--	--	--
m_t	0.6338	1	--	--
v_t	0.9890	0.5343	1	--
v_t-m_t	0.7573	-0.0188	0.8351	1

Table 3 Descriptive Statistics of VIXs with Fixed Maturities and the First Two Components from the PCA Analysis on the VIX Term Structure

In this table we provide descriptive statistics for the VIX term structure data with 30, 60, 90, 180, 270, and 360 days maturities in Panel A, and for the first two principal components (PCA1 and PCA2) of the VIX term structure in Panel B.

Panel A. Descriptive Statistics of VIXs

	Mean	Variance	Skewness	Kurtosis	Minimum	Maximum
VIX_{30}^2	0.0530	0.0029	4.3414	27.9739	0.0084	0.6219
VIX_{60}^2	0.0541	0.0023	3.7689	20.9810	0.0077	0.5515
VIX_{90}^2	0.0544	0.0019	3.2225	15.2465	0.0105	0.4273
VIX_{180}^2	0.0553	0.0014	2.6927	11.1542	0.0132	0.3638
VIX_{270}^2	0.0548	0.0012	2.3980	9.0639	0.0134	0.3072
VIX_{360}^2	0.0543	0.0011	2.0734	7.1551	0.0044	0.2787

Panel B. Descriptive Statistics of the First Two Components

	Mean	Variance	Skewness	Kurtosis	Minimum	Maximum
PCA1	0.1298	0.0103	3.2971	16.3697	0.0187	1.0757
PCA2	0.0224	0.0007	-2.1884	12.6206	-0.2209	0.1033

Table 4 In-sample Estimation

The empirical models (1) through (5) are specified as $RV_{t+1}^2 = a + \sum_{i=0}^{4} b_i RV_{t-i}^2 + cVIX_{t,\tau}^2 + c_1 v_t + c_2(v_t - m_t) + e_t$ with various coefficient constraints, where RV_t denotes the annualized realized volatility and e_t is the residual term at time t. The model (6) is specified as $RV_{t+1}^2 = a + \sum_{i=0}^{4} b_i RV_{t-i}^2 + f_1 PCA1_t + f_2 PCA2_t + e_t$, where $PCA1$ and $PCA2$ stand for the first and second principal component, respectively. The model (7) is specified as $RV_{t+1}^2 = a + \sum_{i=0}^{4} b_i RV_{t-i}^2 + g_1 VIX30_t^2 + g_2(VIX60_t^2 - VIX30_t^2) + e_t$, where $VIX30$ and $VIX60$ are 30-day and 60-day VIX, respectively. Numbers in brackets under the parameter estimates are the standard errors. *, **, and *** indicate that the coefficient is significantly different from zero at the 10%, 5%, and 1% level, respectively.

	Model						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
a	-0.009***	-0.005***	0.006***	0.004***	0.006***	-0.001	-0.001
	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)
b_0	0.26***	0.20***	0.16***	0.17***	0.16***	0.25***	0.19***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_1	0.10***	0.09***	0.08***	0.09***	0.08***	0.11***	0.08***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_2	0.04**	0.02	0.01	0.01	0.01	0.03*	0.02
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_3	0.003	-0.006	-0.01	-0.002	-0.01	0.004	-0.01
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_4	-0.01	-0.03*	-0.03**	-0.03*	-0.03**	-0.01	-0.03*
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
c	0.59***			-0.93***	-0.35*		
	(0.03)			(0.11)	(0.19)		
c_1		0.62***	0.49***	1.43***	0.83***		
		(0.03)	(0.03)	(0.10)	(0.19)		
c_2			0.27***		0.19***		
			(0.03)		(0.05)		
f_1						0.28***	
						(0.02)	
f_2						-0.59***	
						(0.04)	
g_1							0.56***
							(0.03)
g_2							-1.36***
							(0.11)
$Adj\text{-}R^2$	0.5441	0.5614	0.5714	0.5703	0.5717	0.5456	0.5627
D-W	1.96	1.95	1.96	1.96	1.96	1.95	1.98

Table 5 In-sample Estimation for Alternative Forecast Horizons

The empirical models (1) through (5) are specified as $RV_{t,N}^2 = a + \sum_{i=0}^{4} b_i RV_{t-i}^2 + cVIX_{t,\tau}^2 + c_1 v_t + c_2(v_t - m_t) + e_t$ with various coefficient constraints, where RV_t denotes the annualized realized volatility and e_t is the residual term at time t, and $RV_{t,N}$ is the N-day annualized realized volatility at time t. The model (6) is specified as $RV_{t,N}^2 = a + \sum_{i=0}^{4} b_i RV_{t-i}^2 + f_1 PCA1_t + f_2 PCA2_t + e_t$, where *PCA1* and *PCA2* stand for the first and second principal component, respectively. The model (7) is specified as $RV_{t,N}^2 = a + \sum_{i=0}^{4} b_i RV_{t-i}^2 + g_1 VIX30_t^2 + g_2(VIX60_t^2 - VIX30_t^2) + e_t$, where *VIX30* and *VIX60* are 30-day and 60-day VIX, respectively. Numbers in brackets under the parameter estimates are the standard errors, which are estimated following a robust procedure taking into account of serial correlation [Newey and West (1987)]. *, **, and *** indicate that the coefficient is significantly different from zero at the 10%, 5%, and 1% level, respectively.

Panel A: 1-month

	Model						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
a	0.006*	0.008***	0.012***	0.012***	0.012***	0.009***	0.009***
	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)
b_0	0.13***	0.10**	0.08	0.08	0.08	0.13***	0.10**
	(0.05)	(0.05)	(0.05)	(0.06)	(0.05)	(0.05)	(0.05)
b_1	0.08***	0.07***	0.07***	0.07***	0.07***	0.08***	0.07***
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
b_2	0.06**	0.05**	0.05*	0.05*	0.05*	0.06**	0.05**
	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)
b_3	0.06***	0.06***	0.06***	0.06***	0.06***	0.06***	0.06***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_4	0.08**	0.07**	0.07**	0.07**	0.07**	0.08**	0.07**
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
c	0.30***			-0.44	-0.44		
	(0.11)			(0.37)	(0.48)		
c_1		0.31***	0.26**	0.70*	0.69		
		(0.11)	(0.11)	(0.37)	(0.48)		
c_2			0.10		0.002		
			(0.09)		(0.08)		
f_1						0.14**	
						(0.06)	
f_2						-0.23**	
						(0.11)	
g_1							0.29***
							(0.11)
g_2							-0.60*
							(0.32)
Adj-R^2	0.5480	0.5562	0.5589	0.5601	0.5600	0.5455	0.5549
D-W	0.11	0.11	0.12	0.12	0.12	0.12	0.13

Table 5 (continued)

Panel B: 2-month

	Model (1)	(2)	(3)	(4)	(5)	(6)	(7)
a	0.013***	0.015***	0.016***	0.017***	0.016***	0.014***	0.015***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
b_0	0.11**	0.09*	0.09*	0.08	0.09*	0.12***	0.10**
	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.04)	(0.05)
b_1	0.07***	0.07***	0.07***	0.07***	0.07***	0.07***	0.07***
	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)	(0.03)	(0.02)
b_2	0.05**	0.05**	0.05**	0.05**	0.05**	0.05**	0.05**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_3	0.05***	0.05***	0.05***	0.05***	0.05***	0.05***	0.05***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_4	0.06**	0.06**	0.05**	0.06**	0.06**	0.06**	0.06**
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
c	0.21**			-0.23	-0.55		
	(0.10)			(0.33)	(0.42)		
c_1		0.21**	0.20**	0.41	0.73*		
		(0.10)	(0.10)	(0.33)	(0.42)		
c_2			0.02		-0.10		
			(0.08)		(0.07)		
f_1						0.10*	
						(0.05)	
f_2						-0.12	
						(0.10)	
g_1							0.20**
							(0.10)
g_2							-0.21
							(0.29)
$Adj\text{-}R^2$	0.3993	0.4031	0.4031	0.4042	0.4050	0.3979	0.4002
D-W	0.06	0.05	0.05	0.05	0.06	0.06	0.06

Table 6 Out-of-sample Forecasting Evaluation

The regression model is specified as $RV_t^2 = \alpha + \beta \widehat{RV}_t^2 + \varepsilon_t$, where RV_t denotes the annualized realized volatility, $\widehat{RV}_t$ is the fitted value of the annualized volatility, and ε_t is the residual term at time t. Numbers in brackets under the parameter estimates are the standard errors. *, **, and *** indicate that the coefficient is significantly different from zero at the 10%, 5%, and 1% level, respectively.

	Model						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept(α)	0.007***	0.009***	0.015***	0.013***	0.015***	0.011***	0.011***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Slope(β)	0.799***	0.816***	0.836***	0.826***	0.835***	0.798***	0.830***
	(0.020)	(0.020)	(0.021)	(0.021)	(0.021)	(0.021)	(0.021)
Adj-R^2	0.4852	0.4971	0.4981	0.4949	0.4951	0.4711	0.4950
Mean-squared error	0.00619	0.00598	0.00597	0.00600	0.00601	0.00634	0.00597

Table 7 In-sample Estimation for Models (1) and (5) with Alternative Maturities of VIXs

The empirical models are specified as $RV_{t+1}^2 = a + \sum_{i=0}^{4} b_i RV_{t-i}^2 + cVIX_{t,\tau}^2 + c_1 v_t + c_2(v_t - m_t) + e_t$ with τ =60, 90 and 180 days, respectively, where RV_t denotes the annualized realized volatility and e_t is the residual term at time t. Numbers in brackets under the parameter estimates are the standard errors. *, **, and *** indicate that the coefficient is significantly different from zero at the 10%, 5%, and 1% level, respectively.

	Model (1)			Model(5)		
	τ =60	90	180	60	90	180
a	-0.008***	-0.007***	-0.006***	0.004**	0.006***	0.007***
	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
b_0	0.32***	0.35***	0.38***	0.11***	0.10***	0.16***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_1	0.13***	0.14***	0.15***	0.09***	0.08***	0.08***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_2	0.06***	0.07***	0.08***	0.01	-0.01	0.01
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_3	0.02	0.03*	0.04**	-0.001	-0.01	-0.01
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_4	0.02	0.03*	0.05***	-0.03*	-0.04**	-0.03**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
c	0.46***	0.38***	0.30***	-2.60***	-2.26***	-0.38*
	(0.03)	(0.03)	(0.03)	(0.24)	(0.20)	(0.23)
c_1				3.15***	2.80***	0.85***
				(0.25)	(0.21)	(0.22)
c_2				-0.81***	-0.89***	0.02
				(0.10)	(0.11)	(0.15)
$Adj\text{-}R^2$	0.5268	0.5178	0.5098	0.5844	0.5856	0.5716
D-W	1.97	1.98	1.99	1.98	1.98	1.96

Table 8 In-sample Estimation with an Alternative 2-factor Model

The empirical models (1) through (5) are specified as $RV_{t+1}^2 = a + \sum_{i=0}^{4} b_i RV_{t-i}^2 + cVIX_{t,\tau}^2 + c_1 v_t + c_2(v_t - m_t) + e_t$ with various coefficient constraints, where RV_t denotes the annualized realized volatility and e_t is the residual term at time t. Numbers in brackets under the parameter estimates are the standard errors. *, **, and *** indicate that the coefficient is significantly different from zero at the 10%, 5%, and 1% level, respectively.

	Model (1)	(2)	(3)	(4)	(5)
a	-0.009***	-0.005***	0.008***	0.005***	0.008***
	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)
b_0	0.26***	0.20***	0.16***	0.17***	0.16***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_1	0.10***	0.09***	0.08***	0.09***	0.09***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_2	0.04**	0.02	0.01	0.01	0.01
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_3	0.003	-0.006	-0.01	-0.001	-0.01
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_4	-0.01	-0.03*	-0.03**	-0.03*	-0.03**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
c	0.59***			-1.01***	-0.36*
	(0.03)			(0.11)	(0.19)
c_1		0.61***	0.44***	1.46***	0.79***
		(0.03)	(0.03)	(0.10)	(0.19)
c_2			0.31***		0.23***
			(0.03)		(0.05)
Adj-R^2	0.5441	0.5608	0.5720	0.5704	0.5723
D-W	1.96	1.95	1.96	1.96	1.96

Table 9 Out-of-sample Forecasting Evaluation with an Alternative 2-factor Model

The regression model is specified as $RV_t^2 = \alpha + \beta\widehat{RV}_t^2 + \varepsilon_t$, where RV_t denotes the annualized realized volatility, $\widehat{RV}_t$ is the fitted value of the annualized volatility, and ε_t is the residual term at time t. Numbers in brackets under the parameter estimates are the standard errors. *, **, and *** indicate that the coefficient is significantly different from zero at the 10%, 5%, and 1% level, respectively.

	Model				
	(1)	(2)	(3)	(4)	(5)
Intercept(α)	0.007***	0.008***	0.016***	0.013***	0.016***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Slope(β)	0.799***	0.814***	0.840***	0.826***	0.840***
	(0.020)	(0.020)	(0.021)	(0.021)	(0.021)
Adj-R^2	0.4852	0.4992	0.5037	0.4979	0.4991
Mean-squared error	0.00619	0.00597	0.00592	0.00597	0.00597

Table 10 In-sample Estimation with the Volatility-version Models

The empirical model (1) through model (5) are specified as $RV_{t+1} = a + \sum_{i=0}^{4} b_i RV_{t-i} + cVIX_{t,\tau} + c_1\sqrt{v_t} + c_2(\sqrt{v_t} - mt)+et$ with various coefficient constraints, where RV_t denotes the annualized realized volatility and e_t is the residual term at time t. The model (6) is specified as $RV_{t+1} = a + \sum_{i=0}^{4} b_i RV_{t-i} + f_1 PCA1_t + f_2 PCA2_t + e_t$, where *PCA1* and *PCA2* stand for the first and second principal component, respectively. The model (7) is specified as $RV_{t+1} = a + \sum_{i=0}^{4} b_i RV_{t-i} + g_1 VIX30_t + g_2(VIX60_t - VIX30_t) + e_t$, where *VIX30* and *VIX60* are 30-day and 60-day VIX, respectively. Numbers in brackets under the parameter estimates are the standard errors. *, **, and *** indicate that the coefficient is significantly different from zero at the 10%, 5%, and 1% level, respectively.

	Model						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
a	-0.018***	-0.004**	0.009***	-0.001	0.007**	0.005	-0.005*
	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.004)	(0.003)
b_0	0.28***	0.25***	0.23***	0.24***	0.22***	0.26***	0.24***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_1	0.14***	0.12***	0.12***	0.12***	0.11***	0.13***	0.13***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_2	0.06***	0.05***	0.05***	0.05***	0.04**	0.05***	0.05***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_3	0.0002	-0.006	-0.004	-0.01	-0.01	-0.001	-0.004
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_4	0.04**	0.03**	0.04**	0.03**	0.03*	0.04**	0.03**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
c	0.45***			-0.07	0.69***		
	(0.03)			(0.06)	(0.11)		
c_1		0.47***	0.42***	0.53***	-0.24**		
		(0.02)	(0.02)	(0.06)	(0.11)		
c_2			0.09***		0.25***		
			(0.02)		(0.03)		
f_1						0.22***	
						(0.01)	
f_2						-0.49***	
						(0.03)	
g_1							0.45***
							(0.02)
g_2							-0.66***
							(0.07)
$Adj\text{-}R^2$	0.6881	0.6951	0.6973	0.6951	0.7003	0.6918	0.6947
D-W	1.95	1.95	1.96	1.96	1.95	1.96	1.96

Table 11 In-sample Estimation for the Logarithmic-volatility-version Models

The empirical model (1) through model (5) are specified as $log(RV_{t+1}) = a + \sum_{i=0}^{4} b_i\, log(RV_{t-i}) + clogVIX_{t,\tau} + c_1 log\sqrt{v_t} + c_2\left(log\sqrt{v_t} - log\sqrt{m_t}\right) + e_t$ with various coefficient constraints, where RV_t denotes the annualized realized volatility and e_t is the residual term at time t. The model (6) is specified as $log\ (RV_{t+1}) = a + \sum_{i=0}^{4} b_i log\ (RV_{t-i}) + f_1 PCA1_t + f_2 PCA2_t + e_t$, where *PCA1* and *PCA2* stand for the first and second principal component, respectively. The model (7) is specified as $log\ (RV_{t+1}) = a + \sum_{i=0}^{4} b_i log\ (RV_{t-i}) + g_1 log\ (VIX30_t\ ) + g_2(log\ (VIX60_t\ ) - log\ (VIX30_t\ )) + e_t$, where *VIX30* and *VIX60* are 30-day and 60-day VIX, respectively. Numbers in brackets under the parameter estimates are the standard errors. *, **, and *** indicate that the coefficient is significantly different from zero at the 10%, 5%, and 1% level, respectively.

	Model						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
a	-0.07***	-0.18***	-0.21***	-0.17***	-0.19***	-0.17***	-0.14***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.02)
b_0	0.21***	0.18***	0.18***	0.18***	0.17***	0.21***	0.19***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_1	0.16***	0.14***	0.14***	0.14***	0.14***	0.16***	0.15***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_2	0.07***	0.06***	0.06***	0.06***	0.06***	0.07***	0.06***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_3	0.03*	0.02	0.02	0.02	0.02	0.03*	0.03
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
b_4	0.06***	0.06***	0.06***	0.05***	0.05***	0.07***	0.06***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
c	0.52***			0.03	0.16**		
	(0.03)			(0.06)	(0.07)		
c_1		0.53***	0.50***	0.51***	0.37***		
		(0.03)	(0.03)	(0.05)	(0.06)		
c_2			0.04***		0.06***		
			(0.01)		(0.01)		
f_1						0.23***	
						(0.01)	
f_2						-0.46***	
						(0.03)	
g_1							0.52***
							(0.03)
g_2							-0.58***
							(0.08)
$Adj\text{-}R^2$	0.7014	0.7091	0.7098	0.7091	0.7102	0.7023	0.7055
D-W	1.95	1.95	1.95	1.95	1.95	1.96	1.95