Insurance, Debt Covenant, and Overinvestment

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Abstract

This paper builds a theoretical model that, in the absence of market frictions, an investment strategy is determined by insurance. In the model, management finances a new opportunity using a blend of equity and debt with a covenant that requires insurance coverage to make the debt riskless. In maximizing the total expected value of the investment, the management takes account of the insurance priced in an actuarially fair market. Using an optimal-stopping framework, the model shows that management has an incentive to move an investment decision forward with insured debt. The overinvestment problem arises from a covenant in the debt contract requiring insurance coverage.

JEL classification: Keywords: Property insurance; Investment Strategy; Capital Structure

1 Introduction

Mayers and Smith (1982) propose that an investment strategy is expected not to be determined by a covenant requiring insurance coverage if the insurance is fairly priced in a competitive market with no frictions. Mayers and Smith (1987) later add the insight that a *debt* covenant requiring insurance coverage can mitigate an underinvestment problem in virtue of which, with debt in place, management is likely to forego a growth opportunity if it acts in the best interests of its equity holders (Myers, 1977: hereafter,

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Myers). By incorporating appropriate insurance requirements into debt contracts, management can resolve the resulting conflict between debt holders and equity holders. From this type of arrangement, equity holders both bear the cost of and benefits from a risky investment; including an insurance covenant in a debt contract protects debt holders in the event of strategic default on the part of equity holders. Subsequent studies, such as Schnabel and Roumi (1989) and Garven and MacMinn (1994), confirm that by issuing debt with an insurance covenant attached, management can avoid moral hazard.

In the insurance literature, on the other hand, studies suggest that management can engage in risk-taking behavior in the presence of a debt covenant requiring insurance. Empirically, banks are likely to take risks with deposit insurance (Grossman, 1992; Lee et al., 1997; Ioannidou and Penas, 2010; and Cull et al., 2005). Investors can undertake an ex-ante unprofitable project with government loan guarantees (Lewis et al., 1986). Angoua et al. (2008; hereafter, Angoua et al.) suggest that equity holders are likely to implement high-risk investment strategies with privately guaranteed debt financing. In spite of all this attention in the literature, however, these studies miss a theoretical mechanism in virtue of which an insurance covenant on debt can risk moral hazard in the form of conflicts between an insurer and management. We fill the gap in the insurance literature by proposing a model of moral hazard in debt with an insurance covenant attached.

Using an optimal-stopping framework, we show that management can move an investment decision forward with insured debt; the overinvestment problem emerges because management takes account of the insurance. The investment strategy is determined by an insurance covenant on the debt. In our model, note that insurance is actuarially fair in the absence of market frictions. Hau (2007; hereafter Hau) suggests that an overinvestment problem, which can occur only when it is risky to replace damaged assets in case of casualty loss, can be mitigated by an appropriate insurance requirement on debt. He argues that "with riskless asset reconstitution, overinvestment is impossible..." However, we show that an overinvestment problem can arise due to an insurance requirement on debt, even though replacing damaged assets is riskless.¹

Our paper adds an additional insight to Mayers and Smith (1982) because insurance

¹Management in our case has an incentive to engage in a risk-taking behavior because it knows that in bad states of the world the damaged assets will be non-stochastically reconstituted by insurance.

can determine an investment decision made by management even when it maximizes the total expected value of an investment with a debt covenant requiring insurance. The investment strategy is remarkably similar in this case, even though management maximizes the value of equity; this result is consistent with that of previous studies that examine management acting on behalf of equity holders. We consider a potential incentive conflict between *management* and an insurer; note that Angoua et al. study an incentive conflict between *equity holders* and an insurer.² Insofar as management in our study is required to purchase insurance to protect its bond holders from limited liability, it exhibits opportunistic behavior as equity investors in Angoua et al. do.³ It is surprising that an investment strategy that seeks to maximize the total value is determined by an insurance covenant.

In order to show an overinvestment incentive in the presence of insurance, we take Merton's (1977) approach to estimate the value of an insurance policy, abstracting from an insurer's management activities such monitoring (Merton and Bodie, 1992). To build the simplest possible model, we consider a new investment opportunity rather than a growth opportunity; here management has an incentive to overinvest in an independent project. As a consequence, an insurance policy is issued at the time of an investment when the debt-equity capital structure is formed; we take no account of wealth transfers to and from existing claimholders as in Selby et al. (1988). In addition, an insurance policy incurs no safety loading (Schnabel and Roumi, 1989), does not alter agents' financing conditions (Garven and MacMinn, 1994), and is not issued by a vulnerable insurer (Gendron et al., 2002). Contracting costs such as taxes and bankruptcy costs are not considered.

The structure of the paper is as follows. Section 2 reviews theoretical developments in investment strategies with insurance. Our theory that investment strategies can be a function of insurance is developed in both section 3 and section 4. Following backward induction, we assume that an investment decision is already made with an insurance

²In Angoua et al., both management and the insurer maximize the value of their equity holders. Management has an incentive to take risks even though both management and its insurer work on problem-solving; the level of risk that both choose is lower than the one that management chooses unilaterally.

³Mayers and Smith (1982 : 284) provide a positive argument: "insurance contracts allow a [the] firm to shift risk to an [the] insurance company, achieving an efficient allocation of risk for the firm's other claimholders..."

policy in section 3. Taking a contingent-claims approach, we divide the market value of an investment into a) the value of insurance, b) the value of debt, and c) the value of equity. Section 4 analyzes the effects of a debt covenant requiring insurance coverage on an investment strategy. Section 5 concludes.

2 Literature Review

This section reviews previous studies that analyze relationships between insurance and investment. Management can purchase insurance as part of its financing policy (Mayers and Smith, 1982). In an actuarially fair insurance market, an investment decision for management is expected not to be affected by insurance when the decision is made on behalf of both equity and debt holders.⁴ We propose that a new investment decision can be determined by insurance, even though the decision is made in a case in which management maximizes the value of the investment based on debt with an insurance covenant attached.

Mayers and Smith (1987) propose that an investment decision made on behalf of equity holders can be determined by insurance. A debt covenant requiring insurance can eliminate Myers's underinvestment problem, in which management that acts in the best interests of its equity holders is not likely to invest in a growth opportunity. With leverage, equity holders are likely not to invest in a growth opportunity because both equity holders and debt holders share the benefits from the investment, whereas only equity holders bear its cost. By purchasing required insurance, equity holders both bear the cost of and benefit from an investment. As a result, property insurance that makes debt riskless can resolve a potential conflict between equity holders and debt holders. The benefit of property insurance nonetheless can be diminished in the presence of contracting costs that imply safety loading fees (Schnabel and Roumi, 1989).⁵ Garven and MacMinn (1994) suggest that issuing debt covered by an insurance covenant can

⁴Nonetheless, the value of an investment can be determined by insurance through transaction costs related to bankruptcy and capital structure, heterogeneous agents in terms of risk-bearing, and the insurance provider's management activities, such as monitoring (Mayers and Smith, 1982; and Merton and Bodie, 1992), of which this paper takes no account.

⁵An insurance policy, the premiums of which include a safety loading, is less likely to control the incentive problem (Schnabel and Roumi, 1989); in a competitive market, insurance premiums in Mayers and Smith (1987) are priced at actuarially fair rates.

control the underinvestment problem more effectively when the insurance increases an investor's borrowing capacity than otherwise.

Similar to Hau and Angoua et al., we consider an overinvesment problem originating from limited liability, which in previous studies causes an underinvestment problem. In Hau, an insurance policy can mitigate an overinvestment problem that occurs only when reconstituting damaged assets is risky. 6 However, the paper suggests that management is likely to take risks with insurance, even though damaged assets are sure to be reconstituted by an insurer in some bad states of the world. Therefore, we seem to be in a similar position to that of Angoua et al. because both studies examine a conflict of interest between management and an insurance provider. Shifting risk to an insurance provider, management in Angoua et al. serves the interests of equity holders, whereas in our study management serves the interests of both equity holders and debt holders.

Management can overinvest in a new project financed by bonds, a covenant of which requires the management to purchase sufficient insurance; such overinvestment is possible with riskless asset reconstitution or no safety loading. In the overinvestment problem we study here, we take no account of Selby et al.'s (1988) wealth transfers between bond holders and debt holders. Moreover, following previous studies such as Merton (1977), Sosin (1980), Merton and Bodie (1992), and Mody and Patro (1996), this study estimates the value of insurance on debt in a contingent-claims approach.

3 The Values of Insurance, Debt, and Equity

We show that an investment strategy is determined by insurance. Before analyzing the strategy, this section derives the value of an investment that finances both debt, the holders of which require insurance coverage to make the debt riskless, and equity. To derive values related to the investment, we assume by backward induction that the investment decision is already made

Our model has four agents: management, an insurer, a debt holder and an equity

⁶Management can overinvest if the value of an investment is higher than the promised bond repayment in some asset reconstitution states. Realizing the existence of some asset reconstitution states in which the value of an investment is higher than the value of debt, management can attempt to devalue both the investment and the debt. The problem can be mitigated by an insurance policy, in virtue of which debt investors receive a promised repayment amount back by a well-designed indemnity schedule.

holder; unlike management in Mayers and Smith (1987) and in Angoua et al., in our study management maximizes the value of an investment project. Consistent with models in the extant literature, the model assumes that all the risk-neutral agents have the same expectation about future economic environments and have no information asymmetries. The markets for financial products including insurance are free of frictions such as bankruptcy-related costs. In highly competitive environments, moreover, all the financial products are priced at fair market values. Taking a contingent-claims approach as in Mella-Barral and Perraudin (1997), our model divides the market value of an investment into three pieces: the value of insurance, the value of debt, and the value of equity.

This section confirms both i) Merton's (1977) isomorphic correspondence between insurance and a put option on equity and ii) Mayers and Smith's (1982) argument that the value of a project is not determined by insurance.⁷ Note that this section contributes to the literature by proposing the value of insurance in an American options framework, which fully covers default risks from limited liability.

3.1 Basic Assumptions

We build a model with a standard assumption of a contingent-claims analysis. Per period, a project produces a unit of output, the price p_t of which follows a geometric Browinian motion

$$
dp_t = \mu p_t dt + \sigma p_t dB_t, \qquad (1)
$$

where μ and σ are the instantaneous constant return and volatility, respectively. Over time t , the constant riskless interest rate is r . The net earning flows from the project is

$$
p_t - \omega,\tag{2}
$$

where costs per period $\omega > 0$.

⁷By doing so, I can propose a simple closed-form expression for an investment strategy with insurance in section 4. By explicitly considering market friction such as bankruptcy cost and debt renegotiation as in Mella-Barral and Perraudin (1997), we cannot derive a closed-form expression for an investment strategy; the economic implications would not be invariant, even though we consider the frictions.

3.2 The Pure Equity Investment

Under risk neutrality, the project value under pure equity financing W_t follows

$$
rW_t = p_t - \omega + \frac{d}{d\Delta} E_t(W_{t+\Delta})|_{\Delta=0}
$$
\n(3)

based on financial market equilibrium; the project generates the earning flows according to equation (2). The value holds as long as p_t does not reach a low level of p_c , which is to be derived. By Ito's lemma, the project value evolves according to the ordinary differential equation

$$
rW(p) = p - \omega + \mu p W'(p) + \frac{\sigma^2}{2} p^2 W''(p),
$$
\n(4)

because we assume $W_t = W(p_t)$ is a twice continuously differentiable function of p_t .⁸

Equation (4) requires three boundary conditions. Under a no-bubble condition, the investment value equals the value of the sum of expected discounted cash flows $E_t(\int_t^{\infty} (p_v - \omega) \times exp[-r(v - t)] dv) = p_t/(r - \mu) - \omega/r$. Under a no-arbitrage condition, at p_c , there is no difference between holding and abandoning the project $W(P_c) = 0$ because no bankruptcy-related costs are involved. The trigger point p_c is determined by management seeking to maximize the project value: $W'(P_c) = 0$.

The value of the investment under pure equity financing is

$$
W(p) = \frac{p}{r - \mu} - \frac{\omega}{r} - \left(\frac{p_c}{r - \mu} - \frac{\omega}{r}\right)\left(\frac{p}{p_c}\right)^{\lambda} \text{ for } p \ge p_c,
$$
\n⁽⁵⁾

where λ is the negative root of the equation $\zeta(\zeta - 1)\sigma^2/2 + \zeta\mu = r$ and

$$
p_c = -\frac{\lambda}{(1-\lambda)}\frac{\omega}{r}(r-\mu). \tag{6}
$$

For the derivation, see the appendix of Mella-Barral and Perraudin (1997)

The model is distinguished from that of Mella-Barral and Perraudin (1997) because bankruptcy does not compromise the efficiency of the project and no debt renegotiation is allowed;⁹ it is also distinguished from that of You (2013) because insurance is to be explicitly considered when the investment decision is made. The next section considers property insurance with debt.

⁸The time argument is to be suppressed if it is clear from the context.

⁹The model abstracts from direct and indirect bankruptcy-related costs; we can use proposition 1 in Mella-Barral and Perraudin (1997) with $\xi_0 = 1$, $\xi_1 = 1$, $\gamma = 0$, and $X_t = W_t$

3.3 Insurance, Debt and Equity Valuation

Suppose now that an investment decision is made in the presence of debt and equity. Launching an investment project, management has already purchased property insurance in compliance with a requirement in the bond contract. For simplicity, a lump sum (single-payment) premium for the insurance has been paid up front.

Management pays contractual coupon flows b and the remaining net earning flows $p - \omega - b$ go to the equity holders. Financial market equilibrium requires that the value of insurance I_t , the value of debt L_t and the value of equity V_t satisfy

$$
rI(p) = \mu pI'(p) + \frac{\sigma^2}{2} p^2 I''(p).
$$
\n(7)

$$
rL(p) = b + \mu pL'(p) + \frac{\sigma^2}{2} p^2 L''(p),
$$
\n(8)

$$
rV(p) = p - \omega - b + \mu p V'(p) + \frac{\sigma^2}{2} p^2 V''(p),
$$
\n(9)

respectively, as $I(p)$, $L(p)$, and $V(p)$ are a twice continuously differentiable function of p. As our approach adopts an analytical framework for American-style options, it is distinguished from those of Merton (1977) and Merton and Bodie (1992), who adopt a framework for European-style options.¹⁰

Equity holders under limited liability default rationally in the sense that they default when the value of a project reaches the value of the debt. When p_t first hits a constant low level p_b , $V(p_b) = 0$. The decision is optimal for an equity value maximizer $V'(p_b) = 0$, which is known as a smooth-pasting condition. Under a no-bubble condition, moreover, the equity value equals the value of the sum of expected discounted cash flows $E_t(\int_t^{\infty} (p_v - \omega - b) \times exp[-r(v - t)] dv) = p_t/(r - \mu) - (\omega + b)/r$. The equity value is

$$
V(p) = \frac{p}{r - \mu} - \frac{\omega + b}{r} - \left(\frac{p_b}{r - \mu} - \frac{\omega + b}{r}\right)\left(\frac{p}{p_b}\right)^{\lambda},\tag{10}
$$

¹⁰In their framework, nonetheless, L and V at the maturity date satisfy $L(p) = \min\{W(p), b/r\}$ and $V(p) = \max\{0, W(p) - b/r\}$, respectively. In the presence of insurance, in addition, V at the maturity date is not affected by the insurance: $V(p) = \max\{0, W(p) - b/r\}$. When management falls short of promised payments for debt, these payments are covered by the insurance. Therefore, $I(p) = -\min\{0, W(p) - \frac{b}{r}\} = \max\{0, \frac{b}{r} - W(p)\}\$ and the debt with the insurance becomes riskless: $L_I(p) = b/r$.

where p_b is given by

$$
p_b = -\frac{\lambda}{(1-\lambda)} \frac{\omega + b}{r} (r - \mu). \tag{11}
$$

See appendix A.1 for the derivation of equations (10) and (11).

To derive the value of debt, we assume that when p_t is high enough equity holders want to avoid default and the debt becomes riskless; $Lim_{p\to\infty}L(p) = b/r$. When defaulting, the equity holders transfer the defaulted project to bond holders with no default-related costs; $L(p_b) = W(p_b)$ in (5) in a frictionless market. The value of the debt is

$$
L(p) = \frac{b}{r} + (W(p_b) - \frac{b}{r})(\frac{p}{p_b})^{\lambda}.
$$
 (12)

See appendix A.2 for the derivation.

Our study analyzes the effects of an insurance requirement on debt contracts on investment. When p_t is high enough, an insurance policy has no value because the equity holders are not likely to default: $Lim_{p\to\infty}I(p)=0$; note that, like other studies of capital structure, the present study remains silent on liquidity risks. With no efficiency losses from default, at p_b a project transferred from equity holders to debt holders is to be transferred again to an insurer: $I(P_b) = L(p_b) = W(p_b)$.

Proposition 1. Let $W(p_t)$ denote the value of an investment under pure equity financing and p_b denote the default trigger for an equity holder who pays fixed debt services b . In a highly competitive insurance market of no frictions and of no private information, the value of property insurance that makes debt riskless is

$$
I(p) = -(W(p_b) - \frac{b}{r})(\frac{p}{p_b})^{\lambda}.
$$
\n
$$
(13)
$$

See appendix A.3 for the proof.

Proposition 1 confirms Merton's (1977) isomorphic correspondence between insurance and a put option on equity. The value of insurance is replicated by a long position in a default put option. The value of equity in (10) is replicated by a portfolio involving a long position in the project, a short position in riskless debt, and a long position in a default put option; the value of debt in (12) is also replicated by a portfolio involving a long position in riskless debt, and a short position in a default put option.

Note that the value of an abandonment option $A(p) = -\left(\frac{p_c}{r-\mu} - \frac{\omega}{r}\right)$ $(\frac{p}{r})(\frac{p}{p_c})^{\lambda} (> 0)$ is included in equations in (10), (12), and (13); the debt value includes a long position in the abandonment option. Equation (13) can be restated as $I(p) = -\left(\frac{p_b}{r-\mu} - \frac{\omega+b}{r}\right)$ $(\frac{p}{r}) (\frac{p}{p_b})^{\lambda} + (\frac{p_c}{r-\mu}$ ω $\frac{\omega}{r}$)($\frac{p}{p_c}$)^{λ} with (5). The value of insurance in (13) includes the value of an abandonment option because an insurer is allowed to abandon a project. With no abandonment option for a project, $I(p)^{na} = -\left(\frac{p_b}{r-\mu} - \frac{\omega+b}{r}\right)$ $(\frac{p}{r})(\frac{p}{p_b})^{\lambda}$, which is the same as the put option on equity in (5).

Corollary 1. The value of insurance increases as the underlying asset price decreases; the value of insurance increases as the value of debt increases.

See appendix A.4 for the derivation.

Corollary 1 is known from previous studies in the insurance literature. With high underlying asset prices, management is less likely to be covered by insurance because the project is less exposed to risks. With high coupons, equity holders are more likely to default and the insurance becomes more valuable.

3.4 Firm Value with Insurance and Risky Debt

In our model, Modigliani and Miller's (1958) theorem holds because the total value of a leveraged project is $W(p) = V(p) + L(p) = \frac{p}{r-\mu} - \frac{\omega}{r} - (\frac{p_c}{r-\mu} - \frac{\omega}{r})$ $(\frac{p}{r})(\frac{p}{p_c})^{\lambda}$, from equation (10) and equation (12), which is converted to $L(p) = \frac{b}{r} + (\frac{p_b}{r-\mu} - \frac{\omega+b}{r})$ $(\frac{p}{r})^{\lambda} - (\frac{p_c}{r-\mu} - \frac{\omega}{r})^{\lambda}$ $(\frac{p}{p_c})^{\lambda}$.

Management acts on behalf of both debt and equity holders and it has an option to transfer a project in some bad states of the world. Note that in case of casuality loss a project is to be transferred to debt holders (Mayers and Smith, 1987). Immediately after taking control of the project, the debt holders again transfer it to an insurer with no bankruptcy-related costs.¹¹ The value of a leveraged project with insurance is

$$
W_I(p) = V(p) + L(p) + I(p)
$$

= $W(p) + I(p)$
= $\frac{p}{r - \mu} - \frac{\omega}{r} - \left(\frac{p_b}{r - \mu} - \frac{\omega + b}{r}\right)\left(\frac{p}{p_b}\right)^{\lambda}$. (14)

¹¹These strong but common assumptions can be relaxed, even though the relaxation reduces the explanatory power of our theoretical model.

Our model also confirms Mayers and Smith's (1982) argument that after an investment decision the value of a project is not determined by an insurance requirement in debt contracts. A project's value with insurance is higher than the project's value with no insurance; $W_I(p) - W(p) > 0$, as shown in appendix A.5.¹² The premium for the insurance is $\pi(p) = I(p)$ in (13) with zero contracting costs, which implies no safety loading fees in an actuarially fair insurance market. From equation (14), $W_I(p) - \pi(p) = W(p)$; an insurance requirement on debt increases the value of an investment by its premium.

In a complete insurance market of no information asymmetries, decision-making for agents is predictable (Mayers and Smith, 1987); under a competitive market, an assumption of public information is not incentive incompatible, as shown in Chan et al. (1992). Conditional on the predictable decision-making, this section has assigned unique expected values for insurance, debt, and equity. The expected values including the value of an investment in the next section are outcomes of strategic behaviors.

4 Investment Decision in the Presence of a Debt Covenant Requiring Insurance

This section analyzes the effects of insurance on an investment strategy. Management determines the timing of a new investment, when management issues debt with an insurance covenant attached and equity. To analyze the strategy, we assume that when management initiates the investment, investment cost k is weakly larger than the value of debt, which is of a simple, homogeneous class.¹³

Let $F(p)$ denote the value of an investment option, which is a twice continuously differentiable function of p . As the financial market is in equilibrium, F satisfies

$$
rF(p) = \mu pF'(p) + \frac{\sigma^2}{2} p^2 F''(p), \qquad (15)
$$

as the option does not generate any cash flows. The general solution to (15) is known to be $F(p) = D_1 p^{\alpha} + D_2 p^{\lambda}$ where D_1 , and D_2 are to be determined.

 $\frac{12}{12}$ The increase is the difference between the value of a default option and the value of an abandonment option.

¹³This innocuous assumption leads to a unique solution for the investment timing strategy. Note that, with no assets in place before the investment, management takes no account of wealth transfers between existing claim holders, as in Selby et al. (1988).

When p_t first hits some constant level p^* , management invests in a project with no lag, as in Bar-Ilan and Strange (1996). At the optimal timing for the investment, the management achieves the net present value of the investment by financing equity and debt, with a covenant that requires property insurance; $F(p^*) = W_I(p^*) - [k +$ $\pi(p^*)$. Note that management takes account of the property insurance to make the investment decision. In a competitive market of no private information, the premium on the insurance is fairly priced; $\pi(p^*) = I(p^*)$ in equation (13). As a result, $F(p^*) =$ $W_I(p^*) - [k + \pi(p^*)] = W(p^*) + I(p^*) - [k + \pi(p^*)] = W(p^*) - k$; Mayers and Smith's (1982) argument holds. Modigliani and Miller's (1958) theorem also holds, as $F(p^*)$ = $W(p^*) - k = V(p^*) + L(p^*) - k$ (or $= V(p^*) + L(p^*) + I(p^*) - [k + \pi(p^*)]$). At p^* , the marginal cost of waiting for an investment is the same as the marginal benefit of waiting for the investment; $F'(p^*) = W'_I(p^*)$, which is known as a smooth-pasting condition. Note that the management takes account of a covenant requiring insurance coverage on the debt contract. In addition, the option is of no value when p is low enough; $\lim_{p\to 0} F(p) = 0.$

Proposition 2. In a highly competitive market of no private information, the value of an investment financed by debt with a property insurance covenant attached and equity is

$$
F(p) = \left[\frac{p^*}{r - \mu} - \frac{\omega}{r} - \left(\frac{p_c}{r - \mu} - \frac{\omega}{r}\right)\left(\frac{p^*}{p_c}\right)^{\lambda} - k\right]\left(\frac{p}{p^*}\right)^{\alpha},\tag{16}
$$

and the trigger value for investment is

$$
p^* = \frac{\alpha}{\alpha - 1}(r - \mu)(k + \frac{\omega}{r}) + \frac{\alpha}{\alpha - 1}\frac{1}{\lambda - 1}(r - \mu)\frac{\omega}{r}\left(\frac{p^*}{p_c}\right)^{\lambda} - \frac{1}{\alpha - 1}\frac{\lambda}{\lambda - 1}(r - \mu)\frac{\omega + b}{r}\left(\frac{p^*}{p_b}\right)^{\lambda},\tag{17}
$$

where p_c and p_b are from equations (6) and (11), respectively.

See appendix (A.6) for the proof.

The first term on the right-hand side of (17) represents the effects of an uncertain business environment, the second term represents the effect of an abandonment option, and the third term represents the effects of insurance. In order to enrich our understanding of the implications of Proposition 2, we analyze each term separately. The third term disappears if management takes no account of insurance. With no insurance, equation(16) is converted to $F(p) = \left[\frac{p^*}{r-\mu} - \frac{\omega}{r} - \left(\frac{p_c}{r-\mu} - \frac{\omega}{r}\right)\right]$ $(\frac{p}{r})\left(\frac{p^*}{p_c}\right)$ $\frac{p^*}{p_c}\big)^\lambda - k \big](\frac{p}{p^*})^\alpha,$

which is distinguished from equation (16) because equation (17) is converted to $p^* =$ α $\frac{\alpha}{\alpha-1}(r-\mu)(k+\frac{\omega}{r})$ $\frac{\omega}{r}) + \frac{\alpha - \lambda}{\alpha - 1}$ 1 $\frac{1}{\lambda-1}(r-\mu)\frac{\omega}{r}$ $\frac{\omega}{r}(\frac{p^*}{p_c}$ $\frac{p^*}{p_c}$ ^{λ}.¹⁴ Both the new option value and the new trigger are independent of financial structure, as $F(p)$ and p^* are not a function of b (or p_b).¹⁵ With no insurance, in our model, capital structure is irrelevant to the investment timing decision. However, the decision becomes a function of capital structure with a debt covenant requiring insurance coverage.

The second term disappears if management has no abandonment option and no insurance. With no abandonment option, the value of an investment in (5) is $W(p)$ = $\frac{p}{r-\mu} - \frac{\omega}{r}$ $\frac{\omega}{r}$.¹⁶ Equation (16) is converted to $F(p) = (\frac{p^*}{r-\mu} - \frac{\omega}{r} - k)(\frac{p}{p^*})^{\alpha}$ and equation (17) is converted to $p^* = \frac{\alpha}{\alpha - \alpha}$ $\frac{\alpha}{\alpha-1}(r-\mu)(k+\frac{\omega}{r})$ $(\frac{\omega}{r})$, as in Dixit (1989). By comparing the trigger with no abandonment and the trigger with abandonment, we find that with an abandonment option an investment trigger includes an additional term of $\frac{\alpha-\lambda}{\alpha-1}$ 1 $\frac{1}{\lambda-1}(r-\mu)\frac{\omega}{r}$ $\frac{\omega}{r}$ $\left(\frac{p^*}{p_c}\right)$ $(\frac{p^*}{p_c})^{\lambda} < 0.$ With the option, management has an incentive to take risks because it can abandon the project in some bad states of the world.

Uncertainty delays an investment decision because an investment trigger increases with uncertainty; the Marshallian trigger under no uncertainty is $p^* = (r - \mu)(k + \frac{\omega}{r})$ $\frac{\omega}{r}).$ Nonetheless, with an option to abandon, management is more likely to invest in a project because it is protected from some bad states of the world; the investment trigger decreases with an abandonment option. ¹⁷

Proposition 3. A unique solution to p^* in (17) exists.

See appendix (A.7) for the proof.

¹⁴The general solution to the financial market equilibrium condition in (15) requires new boundary conditions; the value-matching condition is $F(p^*) = W(p^*) - k$, the smooth-pasting condition is $F'(p^*) = W'(p^*)$, and the initial condition is $\lim_{p\to 0} F(p) = 0$.

 15 For management, the marginal value of waiting changes with insurance. The precise effects of insurance on the optimal trigger is not the third term on the right-hand side of (17) but $-\frac{1}{\alpha-1}\frac{\lambda}{\lambda-1}(r-\mu)\frac{\omega+b}{r}(\frac{p^*}{p_b})$ $\frac{p^*}{p_b}\big)^{\lambda}+\frac{1}{\alpha-1}\frac{\lambda}{\lambda-1}(r-\mu)\frac{\omega}{r}(\frac{p^*}{p_c})$ $\frac{p^*}{p_c}$) $\lambda \equiv \Phi$. The optimal trigger value in (17) reduced by insurance is $\Phi = -\frac{1}{\alpha-1} \frac{\lambda}{\lambda-1} (r-\mu) \frac{b}{r} (\frac{p^*}{p_b})$ $\frac{(p^*)}{p_b}\big)^{\lambda} - \frac{1}{\alpha-1}\frac{\lambda}{\lambda-1}(r-\mu)\frac{w}{r}(\frac{p^*}{p_b})$ $\frac{(p^*)}{p_b}\big)^{\lambda}$ + $\frac{1}{\alpha-1}\frac{\lambda}{\lambda-1}(r-\mu)\frac{\omega}{r}(\frac{p^*}{p_c})$ $\frac{p^*}{p_c}$) $^{\lambda}$ = $-\frac{1}{\sqrt{2}}$ $\alpha - 1$ λ $\frac{\lambda}{\lambda-1}(r-\mu)\frac{b}{r}$ $rac{b}{r}(\frac{p^*}{p_b}$ $(\frac{p}{p_b})^{\lambda}$ | {z } − $-\frac{1}{\sqrt{2}}$ $\alpha - 1$ λ $\frac{\lambda}{\lambda-1}(r-\mu)\frac{w}{r}$ $\frac{w}{r} p^{*\lambda} (p_b - \lambda - p_c - \lambda)$ | {z } + $< 0.$

¹⁶The general solution to the financial market equilibrium condition in (15) requires the new boundary conditions: $F(p^*) = W(p^*) - k$, $F'(p^*) = W'(p^*)$, and $\lim_{p \to 0} F(p) = 0$.

¹⁷Let p_{na}^* denote the trigger with no abandonment option and p_a^* denote the trigger with an abandonment option, then p_{na} ^{*} = $\frac{\alpha}{\alpha-1}(r-\mu)(k+\frac{\omega}{r})$ and $p_a^* = \frac{\alpha}{\alpha-1}(r-\mu)(k+\frac{\omega}{r}) + \frac{\alpha-\lambda}{\alpha-1}\frac{1}{\lambda-1}(r-\mu)\frac{\omega}{r}(\frac{p^*}{p_c})$ $\frac{p^{*}}{p_{c}}\big)^{\lambda}.$ As a result, $p_a^* - p_{na}^* = \frac{\alpha - \lambda}{\alpha - 1} \frac{1}{\lambda - 1} (r - \mu) \frac{\omega}{r} (\frac{p^*}{p_c})$ $\frac{p^*}{p_c}$)^{λ} < 0.

With property insurance, an optimal timing strategy exists. At the optimal timing, the market value for the insurance is defined as $I(p^*) = -(W(p_b) - \frac{b}{r_b})$ $(\frac{b}{r})\left(\frac{p^*}{p_b}\right)$ $\frac{p^*}{p_b}$ ^{λ}, from equations (13) and (17).¹⁸ Corollary 1 indicates that the expected value of insurance is a function of debt service. With no market frictions, an exogenously determined b is required for the values of debt and equity, as in previous studies.¹⁹ The value of insurance that makes debt riskless is directly related to b , as in previous studies by Mayers and Smith (1987), Garven and MacMinn (1994), and Hau (2007); see Proposition 1.

Now, we can analyze the effects of insurance on investment. In particular, an investment timing strategy can be determined by insurance.

Proposition 4. If management fulfills an insurance requirement on debt covenants, p^* decreases with b: $\frac{\partial p^*}{\partial b} < 0$.

For the proof, see appendix (A.8).

Proposition 4 argues that an investment strategy is a function of insurance. We confirm that when management takes account of insurance it has an incentive to overinvest in an immature project.

At the time when management invests in a project, management pays upfront premiums to purchase property insurance required by debt. As the insurance protects the debt holders from limited liability, the value of the insurance is determined by the debt. Management knows that the risks that it faces are reduced because, in some bad states of the world, the project will be covered by the insurer. As a result, with insurance the management has an incentive to pursue a lower marginal value of waiting to make the investment. Moral hazard therefore arises because management can be expected to hold an option to transfer the project to an insurer, even though replacing damaged assets in case of casualty loss is non-risky.

Our study suggests the presence of moral hazard with insurance, which mitigates conflicts of interest shown in previous studies such as Myers (1977), Schnabel and Roumi (1989), and Garven and MacMinn (1994). Unlike Hau's study, ours proposes overinvestment with non-stochastic asset reconstitution. In addition, unlike Angoua et al.'s

¹⁸In a perfectly competitive insurance market, the value of incentive-compatible insurance can be derived as agents do not have access to private information.

¹⁹Previous studies in the corporate finance literature that assume fixed debt services include Myers (1977), Mella-Barral and Perraudin (1997), and You (2013).

study, ours proposes an American options model. In addition, our results are robust even though management maximizes the value of equity, which is a common assumption in previous studies.

Proposition 5. If management maximizes the value of equity, the optimal trigger decreases as the debt service increases: $\frac{\partial p^*}{\partial b} < 0$.

For the proof, see appendix (A.9).

The optimal triggers for both a value maximizer and an equity value maximizer are a function of b; it is interesting to see that both trigger values decrease with debt. Even though management maximizes the project value by following a rational strategy, the project will be transferred to an insurer at no costs if the equity holders make a strategic default decision; the debt holders are protected by the insurance that makes the debt riskless. In order to show risk-taking behaviors with insurance, we propose an investor who maximizes the total expected value of investment rather than the value of the equity, as shown in Angoua et al.. We propose that insurance can encourage a new investment decision due to a conflict of interest between management and an insurance provider.

5 Conclusion

This article shows that an investment decision can be determined by insurance in the absence of market frictions. Using an optimal-stopping framework, we argue that management can move an investment decision forward with a property insurance requirement on debt covenants. The overinvestment problem can emerge in the presence of insurance because if the investment value falls short of the debt value, the damaged assets will be fully replaced by property insurance.

A Proofs of section 3.3

A.1 Equations (10) and (11)

The general solution to equation (9) is $V(p) = A_0 + A_1 p^{\alpha} + A_2 p^{\lambda}$ where α and λ are the positive and negative roots of the equation $\zeta(\zeta - 1)\sigma^2/2 + \zeta\mu = r$, respectively, and A_0 ,

A₁, and A₂ are to be determined. As we have $Lim_{p\to\infty}V(p) = p_t/(r - \mu) - (\omega + b)/r$, $V(p) = p_t/(r - \mu) - (\omega + b)/r + A_2 p^{\lambda}$. As an equity holder makes a strategic default decision, we know $V(p_b) = p_b/(r - \mu) - (\omega + b)/r + A_2 p_b^{\lambda} = 0$ and therefore $A_2 =$ $-\left(\frac{p_b}{r-\mu}-\frac{\omega+b}{r}\right)$ $(\frac{h+b}{r})p_b^{-\lambda}$. We can solve the equity holder's maximization problem, as $V'(p_b) = 0$ yields p_b . Q.E.D.

A.2 Equation (12)

The general solution to equation (8) is $L(p) = B_0 + B_1p^{\alpha} + B_2p^{\lambda}$, where B_0 , B_1 , and B_2 are to be determined. Under a no-bubble condition, $L(p) = b/r + B_2p^{\lambda}$. With no market frictions, $L(p_b) = W(p_b)$ and $B_2 = (W(p_b) - \frac{b}{r_b})$ $\frac{b}{r}$) p_b ^{- λ}. Q.E.D.

A.3 Proposition 1

The general solution to equation (9) is $I(p) = C_1 p^{\alpha} + C_2 p^{\lambda}$ where C_1 , and C_2 are to be determined. As we have $Lim_{p\to\infty}I(p) = 0$, $I(p) = C_2P^{\lambda}$. In addition, we have $I(P_b) = W(p_b)$. Q.E.D.

A.4 Corollary 1

From equation (13), we have $I(p) = -\left(\frac{p_b}{r-\mu} - \frac{\omega+b}{r}\right)$ $(\frac{p}{r})^{\lambda}+(\frac{p_c}{r-\mu}-\frac{\omega}{r})$ $(\frac{p}{r})(\frac{p}{p_c})^{\lambda}=\frac{1}{1-\lambda}$ $1-\lambda$ $\omega + b$ $rac{+b}{r}(\frac{p}{p_0})$ $\frac{p}{p_b}$ ^{\rangle} $-$ 1 $1-\lambda$ ω $\frac{\omega}{r} \Big(\frac{p}{p_c}$ $(p_c)^{\lambda}$ because p_c and p_b are in equations (6) and (11), respectively. $\frac{\partial I(p)}{\partial p}=\frac{\lambda}{1-\lambda}$ $1-\lambda$ $\omega+b$ $rac{+b}{r}(\frac{p}{p_l})$ $\frac{p}{p_b}$ $\lambda p^{-1} - \frac{\lambda}{1-\lambda}$ $1-\lambda$ ω $\frac{\omega}{r} \Big(\frac{p}{p_o}$ $(\frac{p}{p_c})^{\lambda} p^{-1} = \frac{\lambda}{1 - \lambda}$ $1 - \lambda$ ω r $\left[\right(\frac{p}{p}\right]$ \bar{p}_b $)^{\lambda}-(\frac{p}{\tau})$ p_c $)^{\lambda}]p^{-1}$ $\qquad \qquad \qquad$ − − $+$ λ $1 - \lambda$ b r (p p_b $)^{\lambda} p^{-1}$ $\overbrace{}$ \lt

0, as $\lambda < 0$ and $p_c < p_b$ with $b > 0$. Note that with $b = 0$, $p_c = p_b$ and $\frac{\partial I}{\partial P} = 0$; with no debt the insurance is of no value.

 $\frac{\partial I(p)}{\partial b} = \frac{1}{1-p}$ $1-\lambda$ 1 $rac{1}{r}(\frac{p}{p_l})$ $(\frac{p}{p_b})^{\lambda} - \frac{\lambda}{1-\lambda}$ $1-\lambda$ $\omega+b$ $rac{+b}{r}(\frac{p}{p_l}$ $(\frac{p}{p_b})^{\lambda} p_b^{-1} \frac{\partial p_b}{\partial b} = \frac{1}{r}$ $rac{1}{r}(\frac{p}{p_0})$ $(\frac{p}{p_b})^{\lambda} > 0$. As expected, the value of debt increases with b: $\frac{\partial L(p)}{\partial b} > 0$. The proof is as follows. From equation (12); $\frac{\partial L(p)}{\partial b}=\frac{1}{r}-\frac{1}{r}$ $rac{1}{r}(\frac{p}{p_l})$ $\frac{p}{p_b})^{\lambda} = \frac{1}{r}$ $\frac{1}{r}(1-(\frac{p}{p_l}$ $(\frac{p}{p_b})^{\lambda}$ > 0 as $p > p_b$.

A.5 $W_I(n) - W(n) > 0$

 $W_I(p) - W(p) = -\left(\frac{p_b}{r-\mu} - \frac{\omega + b}{r}\right)$ $(\frac{p}{r})^{\lambda}+(\frac{p_c}{r-\mu}-\frac{\omega}{r})^{\lambda}$ $(\frac{p}{p_c})^{\lambda} = \frac{1}{1-\lambda}$ $1-\lambda$ b $rac{b}{r}(\frac{p}{p_0})$ $\frac{p}{p_b}\big)^{\lambda}+\frac{1}{1-}$ $1-\lambda$ w $\frac{w}{r}\left[\left(\frac{p}{p_b}\right)^{\lambda}-\left(\frac{p}{p_b}\right)\right]$ $(\frac{p}{p_c})^{\lambda}$] = 1 $1-\lambda$ b $rac{b}{r}(\frac{p}{p_l}$ $\frac{p}{p_b}\big)^{\lambda} + \frac{1}{1-}$ $1-\lambda$ ω $\frac{w}{r}p^{\lambda}[p_b^{-\lambda}-p_c^{-\lambda}]$ with equations (6) and (11). $W_I(p)-W(p) > 0$, as $\lambda < 0$ and $p_b > p_c$.

A.6 Proposition 2

As the option is of no value when p is low enough, $F(p) = D_1 p^{\alpha}$. $F(p^*) = D_1 (p^*)^{\alpha} =$ $W_I(p^*) - [k + \pi(p^*)] = W(p^*) - k$. With equation (5), $D_1 = \left[\frac{p^*}{r-\mu} - \frac{\omega}{r} - \left(\frac{p_c}{r-\mu} - \frac{\omega}{r}\right)\right]$ $\frac{\omega}{r}$) $(\frac{p^*}{p_c})$ $\frac{p^*}{p_c}\big)^\lambda$ $k](p^*)^{-\alpha}$. As a result, we have $F(p) = \left[\frac{p^*}{r-\mu} - \frac{\omega}{r} - \left(\frac{p_c}{r-\mu} - \frac{\omega}{r}\right)\right]$ $(\frac{p}{r})\left(\frac{p^*}{p_c}\right)$ $(\frac{p^*}{p_c})^{\lambda} - k \, (\frac{p}{p^*})^{\alpha}.$

According to equations (5) and (13), we restate $I(p) = -\left(W(p_b) - \frac{b}{r}\right)$ $(\frac{b}{r})(\frac{p}{p_b})^{\lambda}=-[\frac{p_b}{r-\mu} \frac{\omega}{r} - \left(\frac{p_c}{r-\mu} - \frac{\omega}{r}\right)$ $\frac{\omega}{r}\big)\big(\frac{p_b}{p_c}\big)^\lambda - \frac{b}{r}$ $\frac{b}{r}](\frac{p}{p_b})^{\lambda} = -(\frac{p_b}{r-\mu} - \frac{\omega+b}{r})$ $(\frac{p}{r}) (\frac{p}{p_b})^{\lambda} + (\frac{p_c}{r-\mu} - \frac{\omega}{r})$ $(\frac{p}{r})(\frac{p}{p_c})^{\lambda}$. Now, we know $W_I(p) = W(p) + I(p) = \frac{p}{r-\mu} - \frac{\omega}{r} - \left(\frac{p_b}{r-\mu} - \frac{\omega}{r}\right)$ $(\frac{p}{p_b})^{\lambda}$. According to $F'(p^*) = W'_I(p^*),$

$$
\alpha \left[\frac{p^*}{r - \mu} - \frac{\omega}{r} - \left(\frac{p_c}{r - \mu} - \frac{\omega}{r} \right) \left(\frac{p^*}{p_c} \right)^{\lambda} - k \right] p^{*-1} = \frac{1}{r - \mu} - \lambda \left(\frac{p_b}{r - \mu} - \frac{\omega + b}{r} \right) \left(\frac{p^*}{p_b} \right)^{\lambda} p^{*-1} \Leftrightarrow
$$
\n
$$
\alpha \left[\frac{p^*}{r - \mu} - \frac{\omega}{r} - \left(\frac{1}{\lambda - 1} \frac{\omega}{r} \right) \left(\frac{p^*}{p_c} \right)^{\lambda} - k \right] = \frac{p^*}{r - \mu} - \frac{\lambda}{\lambda - 1} \frac{\omega + b}{r} \left(\frac{p^*}{p_b} \right)^{\lambda}, \text{ from (6) and (11) } \Leftrightarrow
$$
\n
$$
\frac{p^*}{r - \mu} - \frac{\omega}{r} - \left(\frac{1}{\lambda - 1} \frac{\omega}{r} \right) \left(\frac{p^*}{p_c} \right)^{\lambda} - k = \frac{p^*}{r - \mu} \frac{1}{\alpha} - \frac{\lambda}{\lambda - 1} \frac{1}{\alpha} \frac{\omega + b}{r} \left(\frac{p^*}{p_b} \right)^{\lambda} \Leftrightarrow
$$
\n
$$
\frac{\alpha - 1}{\alpha} \frac{p^*}{r - \mu} = (k + \frac{\omega}{r}) - \frac{\lambda}{\lambda - 1} \frac{1}{\alpha} \frac{\omega + b}{r} \left(\frac{p^*}{p_b} \right)^{\lambda} + \left(\frac{1}{\lambda - 1} \frac{\omega}{r} \right) \left(\frac{p^*}{p_c} \right)^{\lambda} \Leftrightarrow
$$
\n
$$
p^* = \frac{\alpha}{\alpha - 1} (r - \mu) (k + \frac{\omega}{r}) + \frac{\alpha}{\alpha - 1} \frac{1}{\lambda - 1} (r - \mu) \frac{\omega}{r} \left(\frac{p^*}{p_c} \right)^{\lambda} - \frac{1}{\alpha - 1} \frac{\lambda}{\lambda - 1} (r - \mu) \frac{\omega + b}{r} \left(\frac{p^*}{p_b} \right)^{\lambda}
$$

A.7 Proposition 3

According to equation (17), define $f(p) = p - \frac{\alpha}{\alpha - \alpha}$ $\frac{\alpha}{\alpha-1}(r-\mu)(k+\frac{\omega}{r})$ $\frac{\omega}{r}$) and $g(p) = \frac{\alpha}{\alpha - 1}$ 1 $\frac{1}{\lambda-1}(r-\mu)\frac{\omega}{r}$ $\frac{\omega}{r}(\frac{p}{p_{a}}$ $(\frac{p}{p_c})^{\lambda}$ $-\frac{1}{\alpha}$ $\alpha-1$ λ $\frac{\lambda}{\lambda-1}(r-\mu)\frac{\omega+b}{r}$ $rac{+b}{r}$ $\left(\frac{p}{p_l}\right)$ $(\frac{p}{p_b})^{\lambda}$. The proof is completed if we show $f(p_b) \leq g(p_b)$ at the lower bound from (11). With equations (6) and (11), $f(p_b) = -\frac{\lambda}{1-\lambda}$ $\frac{\lambda}{1-\lambda}(r-\mu)\frac{\omega+b}{r}-\frac{\alpha}{\alpha-1}$ $\frac{\alpha}{\alpha-1}(r-\mu)(k+$ ω $(\frac{\omega}{r})$ and $g(p_b) = \frac{\alpha}{\alpha - 1}$ 1 $\frac{1}{\lambda-1}(r-\mu)\frac{\omega}{r}$ $\frac{\omega}{r}(\frac{\omega+b}{b}$ $\frac{+b}{b}$)^{$\lambda - \frac{1}{\alpha -}$} $\alpha-1$ λ $\frac{\lambda}{\lambda-1}(r-\mu)\frac{\omega+b}{r}$ $\frac{+b}{r}$. We can prove that $-\frac{\lambda}{1-\lambda}$ $\frac{\lambda}{1-\lambda}(r-\mu)\frac{\omega+b}{r}-\frac{\alpha}{\alpha-1}$ $\frac{\alpha}{\alpha-1}(r-\mu)(k+\frac{\omega}{r})$ $\frac{\omega}{r}) \leq \frac{\alpha}{\alpha - 1}$ $\alpha-1$ 1 $\frac{1}{\lambda-1}(r-\mu)\frac{\omega}{r}$ $\frac{\omega}{r}(\frac{\omega+b}{b}$ $\frac{+b}{b}$)^{λ} – 1 α−1 λ $\frac{\lambda}{\lambda-1}(r-\mu)\frac{\omega+b}{r}$ $\frac{+b}{r}$, as $p^* > p_b$.

$$
\frac{\alpha}{\alpha-1} \frac{1}{\lambda-1} (r - \mu) \frac{\omega}{r} (\frac{\omega+b}{b})^{\lambda} - \frac{1}{\alpha-1} \frac{\lambda}{\lambda-1} (r - \mu) \frac{\omega+b}{r} \ge \frac{\lambda}{\lambda-1} (r - \mu) \frac{\omega+b}{r} - \frac{\alpha}{\alpha-1} (r - \mu)(k + \frac{\omega}{r}) \Leftrightarrow
$$

$$
\frac{\alpha}{\alpha-1} \frac{1}{\lambda-1} (r - \mu) \frac{\omega}{r} (\frac{\omega+b}{b})^{\lambda} - \frac{\alpha}{\alpha-1} \frac{\lambda}{\lambda-1} (r - \mu) \frac{\omega+b}{r} \ge -\frac{\alpha}{\alpha-1} (r - \mu)(k + \frac{\omega}{r}) \Leftrightarrow
$$

$$
\frac{1}{\lambda-1} \frac{\omega}{r} (\frac{\omega+b}{b})^{\lambda} - \frac{\lambda}{\lambda-1} \frac{\omega+b}{r} \ge -(k + \frac{\omega}{r}) \Leftrightarrow
$$

$$
\frac{1}{\lambda-1} \omega (\frac{\omega+b}{b})^{\lambda} - \frac{\lambda}{\lambda-1} (\omega+b) \ge -rk - \omega \Leftrightarrow
$$

$$
-\omega - \frac{\omega}{\lambda-1} (\frac{\omega+b}{b})^{\lambda} + \frac{\lambda}{\lambda-1} (\omega+b) \le rk.
$$
 (18)

According to the assumption, investment cost k at the optimal timing is weakly greater than the value of debt $L(p) = \frac{b}{r} + (\frac{p_b}{r-\mu} - \frac{\omega+b}{r})$ $(\frac{p}{r}) (\frac{p}{p_b})^{\lambda} - (\frac{p_c}{r-\mu} - \frac{\omega}{r})$ $(\frac{p}{r})(\frac{p}{p_c})^{\lambda}$ in (12), where $p \geq p_b$.

$$
k \ge L(p_b) \Leftrightarrow
$$

\n
$$
k \ge \frac{b}{r} + \left(\frac{p_b}{r - \mu} - \frac{\omega + b}{r}\right) - \left(\frac{p_c}{r - \mu} - \frac{\omega}{r}\right)\left(\frac{p_b}{p_c}\right)^{\lambda} \Leftrightarrow
$$

\n
$$
k \ge \frac{b}{r} + \frac{1}{\lambda - 1} \frac{\omega + b}{r} - \frac{1}{\lambda - 1} \frac{\omega}{r} \left(\frac{\omega + b}{\omega}\right)^{\lambda}, \text{ from (6) and (11)} \Leftrightarrow
$$

\n
$$
rk \ge b + \frac{\omega + b}{\lambda - 1} - \frac{\omega}{\lambda - 1} \left(\frac{\omega + b}{\omega}\right)^{\lambda}.
$$

\n(19)

We want to show (18) with the assumption of (19). The proof is complete because

$$
b + \frac{\omega + b}{\lambda - 1} - \frac{\omega}{\lambda - 1} (\frac{\omega + b}{\omega})^{\lambda} \ge -\omega - \frac{\omega}{\lambda - 1} (\frac{\omega + b}{b})^{\lambda} + \frac{\lambda}{\lambda - 1} (\omega + b) \Leftrightarrow
$$

$$
\omega + b = \frac{\lambda}{\lambda - 1} (\omega + b) - \frac{\omega + b}{\lambda - 1}.
$$

A.8 Proposition 4

As we know $\alpha > 1$ and $\lambda < 0$, $\frac{\alpha}{\alpha - 1}(r - \mu)(k + \frac{\omega}{r})$ $\frac{d\omega}{dr}$ > 0, $g(p) < 0$, $g'(p) > 0$, and $g''(p) < 0$. Now, we can prove $\frac{\partial p^*}{\partial b} < 0$ by showing $\frac{\partial g(p)}{\partial b} < 0$, where $g(p) = \frac{\alpha}{\alpha - 1}$ 1 $\frac{1}{\lambda-1}(r-\mu)\frac{\omega}{r}$ $\frac{\omega}{r}$ $\left(\frac{p}{p}\right)$ $(\frac{p}{p_c})^{\lambda}$ — 1 α−1 λ $\frac{\lambda}{\lambda-1}(r-\mu)\frac{\omega+b}{r}$ $\frac{+b}{r}(\frac{p}{p_0}$ $(\frac{p}{p_b})^{\lambda}$.

$$
\frac{\partial g(p)}{\partial b} = -\frac{1}{\alpha - 1} \frac{\lambda}{\lambda - 1} \frac{r - \mu}{r} \left(\frac{p}{p_b}\right)^{\lambda} + \frac{\lambda}{\alpha - 1} \frac{\lambda}{\lambda - 1} (r - \mu) \frac{\omega + b}{r} \left(\frac{p}{p_b}\right)^{\lambda} (p_b)^{-1} \frac{\partial p_b}{\partial b}
$$
\n
$$
= -\frac{1}{\alpha - 1} \frac{\lambda}{\lambda - 1} \frac{r - \mu}{r} \left(\frac{p}{p_b}\right)^{\lambda}
$$
\n
$$
+ \frac{\lambda}{\alpha - 1} \frac{\lambda}{\lambda - 1} (r - \mu) \frac{\omega + b}{r} \left(\frac{p}{p_b}\right)^{\lambda} \frac{\lambda - 1}{\lambda} \frac{r}{\omega + b} \frac{1}{r - \mu} \frac{\lambda}{\lambda - 1} \frac{r - \mu}{r} < 0.
$$
\n
$$
\Leftrightarrow -\frac{1}{\alpha - 1} \frac{\lambda}{\lambda - 1} \frac{r - \mu}{r} + \frac{\lambda}{\alpha - 1} \frac{\lambda}{\lambda - 1} \frac{r - \mu}{r} < 0
$$
\n
$$
\Leftrightarrow -\frac{1}{\alpha - 1} + \frac{\lambda}{\alpha - 1} < 0
$$
\n
$$
\Leftrightarrow \frac{\lambda - 1}{\alpha - 1} < 0.
$$

A.9 Proposition 5 for an Equity Value Maximizer

The option value F satisfies equation (15), the general solution of which is also $F(p)$ = $E_1p^{\alpha} + E_2p^{\lambda}$ where E_1 , and E_2 are to be determined. As the option is of no value when p is low enough, $\lim_{p\to 0} F(p) = 0$, $F(p) = E_1 P^{\alpha}$.

At the trigger p^* , as management maximizes the equity value, $F(p^*) = V(p^*) - [k +$ $\pi(p^*) - (L(p^*) + I(p^*))$, where $V(p)$ is from (10), $L(p^*)$ is from (12), and $I(p^*)$ is from (13). Note that the debt is funded only for the investment and the insurance is fairly priced. As a consequence, $F(p^*) = V(p^*) + L(p^*) - [k + \pi(p^*) - I(p^*)] = W(p^*) - k$, which demonstrates that, at p^* , Modigliani and Miller's (1958) theorem holds.

For the equity value maximizer, a smooth-pasting condition $F'(p^*) = V'(p^*)$ leads to $p^* = \frac{\alpha}{\alpha - \alpha}$ $\frac{\alpha}{\alpha-1}(r-\mu)(k+\frac{\omega}{r})$ $(\frac{\omega}{r}) + \frac{\alpha}{\alpha - 1}$ 1 $\frac{1}{\lambda-1}(r-\mu)\frac{\omega}{r}$ $\frac{\omega}{r}(\frac{p^*}{p_c}$ $\frac{p^*}{p_c}\big)^\lambda - \frac{1}{\alpha - 1}$ $\alpha-1$ λ $\frac{\lambda}{\lambda-1}(r-\mu)\frac{\omega+b}{r}$ $rac{+b}{r}$ $\left(\frac{p^*}{p_b}\right)$ $(\frac{p^*}{p_b})^{\lambda}$, which is the same as the trigger in (17) and the option value is the same as equation (16).

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