

JLS method for endogenous bubbles: financial derivative price analysis in short time horizons

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Abstract

We take a novel approach to modify and apply the Johansen-Ledoit-Sornette (JLS) bubble-detection model to short-term KOSPI financial derivatives, which has greater relative noise level but conforms better with a central model assumption, which assumes no major exogenous information during the bubble formation. In the past decade, there has been much debate in all areas of trend detection models, specifically the JLS model, which arisen from the areas of physics statistics and complex systems. It examines long-term price series with super-exponential growth and oscillation with accelerating frequency. The model claims to have great efficacy in detecting an endogenous bubble and the critical end time; our goal is to improve upon this method, examine various JLS methodologies in empirical settings and provide a novel tool in the fields of asset management and financial econometrics.

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We take a novel approach to modify and apply the Johansen-Ledoit-Sornette (JLS) bubble-detection model to short-term KOSPI financial derivatives, which has greater relative noise level but conforms better with a central model assumption, which assumes no major exogenous information during the bubble formation. In the past decade, there has been much debate in all areas of trend detection models, specifically the JLS model, which arisen from the areas of physics statistics and complex systems. It examines long-term price series with super-exponential growth and oscillation with accelerating frequency. The model claims to have great efficacy in detecting an endogenous bubble and the critical end time; our goal is to improve upon this method, examine various JLS methodologies in empirical settings and provide a novel tool in the fields of asset management and financial econometrics.

JEL Classification: O18, O16, R50, G34, G38

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1. Introduction

In the past decade, physics statistics and complex system theories have been partially translated into the field of finance. There have been many attempts ranging from chaos theory to artificial intelligence to predict financial time series or describe it. However, many have focused on the long-term systematic market reversal (i.e. "bubble bursts"), which mostly neglect the value of exogenous information without formally assuming market efficiency, or have focused entirely on indiscriminating application of pattern recognition methods on ultra short-term price movements, often taking the form of market microstructure analysis. And all have focused on index-based or equity-based analysis.

To our knowledge, there have been no attempts to utilize a formally and clearly-realized rational model with these latest tools of physics statistics to financial derivatives in the short-term, which is critical to produce risk-adjusted returns in the asset management industry. What we seek to accomplish is to not only apply the JLS model to financial derivatives but to also treat the estimated "critical time" as part of the risk management and investment horizon determination tools. This should be critical especially to the finite nature of financial derivatives.

The benefits of studying with financial derivatives in a short time-horizon are

three-fold. First, financial derivatives bear less liquidity and are more susceptible to herding effects. Second, taking a practical stance, being able to correctly predict short-term movements allows an asset manager to manage a portfolio more effectively while minimizing risk, thus decreasing potential volatility and raising risk-adjusted returns. Lastly, the argument that the late-stage bubble can be characterized by the competition between value-investor and noise traders can be even better-realized due to the limited life-span of financial derivatives, meaning that both kinds of traders would exhibit contradicting values framed in similar time horizons, making the effects more pronounced.

This paper is the first to 1) examine the JLS model for short-term bubbles of financial derivatives, and 2) apply this model to derivative products. We take no normative view on the best way to prevent a financial crisis or an impending crash; we only seek to understand the effectiveness of JLS models in predicting an impending turning of a bubble.

Predicting an abrupt changes in price trends in the financial market has been an important topic; specifically, given the formation of a clearly defined financial bubble, it is imperative that regulatory bodies are equipped with proper tools to detect an impending crash. In the past decade, many such new tools have been developed for that

purpose. A particular method, Log-Periodic Power Law (JLS), developed by Sornette, has recently come into spotlight as a grounded, rational-expectation based model that aims at detecting crashes through the formation of herding effects.

JLS has been examined and modified by several researchers, including xxxxxxxx, to varying degrees of success. However, to the author's knowledge, it has never been applied to derivative products, which are generally less liquid and thus more susceptible to herding effects and sudden price swings. Moreover, such method has never been applied to ultra short-term data before. Much of the JLS application has centered on the price movements (bubble formation and crash) as consequences of herding effects devoid of exogenous news or information. We find that, based on such logic, the JLS model might be more suitable when applying to short-term data and treat the price movements in isolation.

Section 2 discusses the JLS methodology, recent development and parameter vulnerabilities. Section 3 discusses the improved methodologies taken in association with Li (2010). Section 4 develops the test methodologies, expectations and provides thoughts for further developments.

2. The Basic JLS Model

Much debate has centered on the proper definition of a financial bubble (Gurkaynak, 2008), though literature has primarily focused on the statistical methodologies and testing. In reality, the formation of a bubble was only apparent after it bursts, a "not knowing it's there until it's gone" sentiment. But in theory, a financial bubble can be characterized as a persisting and positive deviation of actual asset price from the fundamental value that is assumed to exist.

A pivotal, if not particularly rigorous, concept in the field of behavioral finance is the concept of "bigger fools." This means that an investor would purchase an asset despite a clear deviation or a complete absence of fundamental value because she believes that there will be a bigger fool to pay a steeper price for such an asset.

In essence, the JLS approach characterizes the financial market in the following manner: participants of the financial market are hierarchical, meaning there are different sizes of investors, and they trade with each other, often competing and taking different views. This is especially true in derivatives markets. When the herding effect begins to form for any reason (nonlinear interaction) and becomes apparent in the system, a larger proportion of investors would choose to follow the trend, and the financial market will engage in a large-scale, collective buying spree that would form a self-organized and

self-reinforcing bubble regime. When a majority of the market participants becomes imitators, the change of price regime then occurs at a critical time. Here, we are interested in the continuation of the trend and the timing of the reversal.

Note that there need not be a reason for a bubble formation. We merely seek to understand that, when the rain happens, the ground is bound to get wet; we do not predict the precise timing of precipitation because, from a risk manager's perspective, it is not practical and utterly impossible. However, what is unique about JLS is that it does indeed provide the potential range of price trend reversals, and this serves as an important tool in forming a short-term portfolio of financial derivatives.

In terms of data analysis, we are examining the two main features leading up to a trend reversal. First, JLS looks for the super-exponential acceleration during a bubble formation. Second, there should be oscillations with accelerating frequency.

It is understandable that the very definition of a bubble is subject to debate. Some have also argued over the definition of a crash. Due to the nascent nature of the JLS model, almost every single aspect has been debated over. From the model extension, to the time horizon selection and the starting date, to the model fitting method, ranging from the traditional approach of minimizing mean-square error to genetic algorithm, bootstrapping and pattern recognition, to even the usage of prices or logged prices. Even

the generation of synthetic data to test the robustness of the parameters has caused controversy. We foresee the application of the JLS method to financial derivatives as a long yet fruitful process, and we have prepared to attempt various methods taking the point of view as a risk manager and algorithmic trader.

Derivation of a Basic JLS Model

The basic model is built on a rational expectation setting. Note that the following is based on a review paper by Sornette (2013) which examines and compares various criticisms and methodologies of the model. Given that there are numerous methods involved in this novel approach in finance literature, it is critical that we clarify the underlying framework and examine the applicability of JLS to short-term variations of derivative prices.

First, an asset price is consisted of the fundamental value and the bubble component. It is difficult to separate the fundamental and bubble component. In terms of derivatives, the case can be different. We deem derivatives a better tool to examine JLS because the bubble detection method aims at uncovering finite-time singularity. This coincides with the finite nature of financial derivatives; they operate within finite time horizons. Mathematically, the JLS model treats the bubble component as independent from the fundamental value dynamics. The latter can follow a geometric Brownian motion as

standard practices in literature, based on certain valuation models. The former, however, is entirely subject to beliefs, the dissolution of such beliefs and the associated liquidity concerns following the dissolution (margin call, contract expiration and etc.).

The JLS model is developed from standard assumption such that the price dynamic of the bubble component can be described by a stochastic differential equation with drift and jump.

$$\frac{dp}{p} = \mu(t)dt + \sigma dW - \kappa dj, \quad (1)$$

where p is the asset's "bubble price", $\mu(t)$ is the drift/ trend and dW is the increment of a standard Wiener process zero mean and unit variance. The term dj represents a discrete jump such that $j = 0$ before the crash and $j = 1$ after the crash. This means that j is a condition variable that describes the market state. The loss magnifier is then determined by the parameter k . The dynamics of the jumps is described by a crash hazard rate $h(t)$. $h(t)dt$ is the probability that the crash will occur during t and $t+dt$, conditional on that a crash *has not* happened. Thus, we have:

$$E_t[dj] = 1 \times h(t)dt + 0 \times (1 - h(t)dt) \quad (2)$$

And therefore, the expectation of dj is:

$$E_t[dj] = h(t)dt. \quad (3)$$

Under the assumption of the JLS model, noise traders collectively exhibit a herding behavior through imitating others' trading patterns, thus, in this case, may destabilize an established bubble, causing an abrupt reversal. And the model describes such herding effect by the dynamics of the crash hazard rate:

$$h(t) = B'(t_c - t)^{m-1} + C'(t_c - t)^{m-1} \cos(\omega \ln(t_c - t) - \phi'). \quad (4)$$

The cosine part incorporates the existence of possible hierarchical cascades (Johnson and Sornette, 1997) of accelerating dissonance among traders, gradually decelerating the bubble growth. It can be thought of as resulting from a preexisting hierarchy in noise trader sizes (Zhou, Sornette, Hill and Dunbar, 2005). Based on Sornette's review paper (2013), the above expression also contains a hyperbolic power law growth ending at a finite-time singularity, which embodies the positive feedbacks framework.

The non-arbitrage based on unconditional expectation $E[dp]$ of the price increment must be zero. This indicates that the noise traders have no informational advantage over each other, thus the theoretical price step must be zero given that a crash has not occurred.

By taking the expectation of (1):

$$\mu(t) \equiv E \left[\frac{dp/dt}{p} \right]_{\text{no crash}} = \kappa h(t) , \quad (5)$$

To elaborate on the above hazard rate expression (4), we consider the hazard rate as a

characteristic feature of a specific complex system, and it captures a system's stability before a critical time. Due to the nonlinear nature of the interactions among traders, an external disturbance (change of belief in a hierarchal structure) can cause different structural changes during different system states. Thus, the hazard rate does not describe the external force but the susceptibility of the entire system to external disturbances. The bigger the hazard rate, the more susceptible a complex system is to external disturbance.

Then expression in (4) is the standard "Hierarchical Diamond Lattice" model selected for simulate the network structure in the financial market (Johansen, Ledoit, Sornette, 2000). Its better assumption is that the market has different levels of investors based on connections, and the least-connected investors have $2^{(p-1)}$ times fewer neighbors than the most connected ones, where p is the number of hierarchy.

Using (5) and substituting (4) and integrating the JLS equation, we arrive at:

$$\ln E[p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) - \phi) \quad (6)$$

where

$$B = -\kappa B' / m$$

and

$$C = -\kappa C' / \sqrt{m^2 + \omega^2}$$

JLS has a critical limitation such that the expression (4) only describes a single

hierarchical structure, thus only applicable up to the end of the bubble. Consequently, the JLS model makes no prediction beyond the critical time t_c .

After the establishment of the model, the essential part of the task is to estimate the parameters of the model for prediction purposes. This part has been subjected to various debates, and different methods were developed. The original paper employed the method of minimizing the sum of Squared Residuals.

3. Methodologies

Taking a pragmatic stance, in this paper, we seek to answer the following questions:

- What is the likelihood that we can predict short-term price reversals in real time?
- If a given prediction is false, what is the potential loss as a contract holder, and how to adjust for such prediction failures?

These questions must first be answered with a clearly defined, trend reversal detection framework. Due to the transitory nature of price movements, an unambiguous price bubble definition remains an unsolved task. Its difficulty has two parts: (i) the rise of fundamental value is impossible to measure as an aggregate due to different investment horizons and methodologies and (ii) it is impossible to distinguish between an exponentially growing fundamental price and growing bubble price.

Here, due to our novel approach in applying the base JLS model to short-term

derivative prices, we posit that the overnight jump of derivative prices are typically resulted by the trading activities of informed traders. If there is an overnight rise in price, we treat it as a result of information dissimulation.

A recent development by Lin and Sornette (2013) has included a stochastic mean reversion dynamics of the critical time, capturing the uncertain anticipation and the resulting effects by trader interactions.

According to Sornette's review paper (2013), there has been a debate regarding the reasonable value of m . Bree & Joseph (2013) proposes a relaxation of a so-called reasonable value because if the best-fitted parameters have exponents beyond a reasonable range, then there can be an added information to the price series. We will follow Bree & Joseph's reasoning in order to capture as much price information as possible. Moreover, aside from limiting arbitrarily the value of m , methods such as bootstrap and ensemble have been developed. We seek to examine all methods in its application in financial derivative time series.

Super exponential movements based on the JLS model

A conceptual concept of JLS is that, different from standard financial bubble models, is that a bubble follows super exponential growth. Logically, a bubble price occurs when price rises above an asset's fundamental value. But given that it is difficult to ascertain

the fundamental price, the benefit of JLS is that it describes a bubble as a price growth that is so drastic that it is unlikely to be based on any fundamental changes of underlying assets but based on a feedback mechanism among traders, creating a regime that is intrinsically transient.

There have been debates and justifications regarding the type of bubbles that a JLS model can detect. Sornette (2013) claims that the JLS model is used for endogenous bubbles, meaning a bubble that is generated by positive feedback mechanism. However, due to the long horizon that Sornette examines, it is difficult to determine what information constitutes exogenous shocks and what kind of news can be disregarded as irrelevant. Here we disagree with their claim that the JLS model is only used for endogenous bubbles because it is difficult and arbitrary to determine the bubble type. Our approach to apply this method to short-term derivative data would be a more appropriate setting for JLS model because, during a super exponential price growth within a single trading day, during the trading hours, it is less likely that such growth is caused by an universal exogenous news or fundamental changes.

JLS model extensions

Because the expression (6) is a standard JLS model, which is a special case of a more general, second-order JLS Landau formula, it has been shown that the more general

form of JLS model (also the extension) provides better and more timely fitting results (Sornette 2013). Here, we will use both the first-order form and the second order JLS Landau-type JLS model as described by Sornette and Johansen (1997) as well as a non-parametric method proposed by Sornette and Zhou (2003).

Parameter Estimation

Two major challenges of fitting the JLS model are the non-linearity nature of the model and the existence of several local minima. In this paper, we will employ a taboo search (Cvijovic, Klinowski, 1995) to solve the multiple minima problem and a traditional Levenberg-Marquart search algorithm for find solution for non-linear problems to obtain approximation of model parameters. The taboo search is useful in a way that it provides a preliminary screening and gives a space of possible solutions.

In order to test the robustness of the parameters, we follow the test methods by Johansen (1999, 2000) to test on GARCH-generated synthetic time series. Zhou and Sornette (2002) recommended tests using synthetic time series with different types of noises, including the ones generated based on the power law distributions. For our purpose, we deem it critical to conduct the robustness test, akin to the Monte Carlo analysis but with data generated specifically for power-law cases.

The most special feature of JLS model is the ability to predict a critical time at which a bubble stops growing. It is an interesting feature in the way that, from a capital allocation point of view, theoretically one can take advantage of the continued rise of a bubble and then exit when the critical time arrives. If the JLS model parameter estimations are robust, then whether the ending of the bubble takes the form of a crash or gradual decline, asset managers can still take advantage of such prediction whether to reverse their trade thesis.

From the model, a distribution of t_c is obtained. We will use a nonparametric method detailed by Li and Racine (2006) to obtain the most probable time t_c for the end of the bubble. Sornette (2013) also recommends employing the bootstrap method to generate bootstraps in which the residuals of the first model calibration on the time series are used to generate synthetic time series through reshuffled blocks of residuals.

4. Empirical Approach and Expectation

Though there has been much debate over model fitting methodologies and even the very definitions of bubbles and crashes, here we aim at examining the validity of such methods in short-term financial derivatives. Despite the various methods and methodologies, a major assumption is that an endogenous bubble can be detected based on the price time series and a critical time can be extrapolated and estimated. Based on

Sornette's claim (2013), if JLS model is specifically used for an endogenous bubble, implying that a price growth is purely due to herding behavior without exogenous news, then by limiting the examination to a shorter horizon during trading hours, we should have better prediction rates.

For the next revision, we will provide the empirical results when the JLS and the chosen methodologies above are applied to the KOSPI products, including stock options and futures. We will also provide alternative methodologies to complement the shorter time horizon and address the price jumps and overnight price movements.

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