An Empirical Investigation of the Performance of Korean Leveraged **Exchange Traded Funds**

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31 May, 2014

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ABSTRACT

In this paper, we study the tracking error and the compounding effect of leveraged ETFs actively traded in the Korea Exchange (also known as KRX). The data are collected on a daily basis and range from the inception dates to May 28, 2013.

The main findings are as follows. Firstly, on average, the actual returns on leveraged ETFs are statistically close to their promised leveraged returns over the holding period of up to a week. However, the actual returns on leveraged ETFs deviate from the target returns when the length of the holding period is beyond a week. This poor performance worsens as the holding period increases. We find that the poor performance of leveraged ETFs is mainly attributed to the compounding effect. Secondly, the results of the regression analyses suggest that investors can buy and hold 2x bull ETFs for a week or a month at most, while (-1x) bear ETFs for three months. When the holding period is a year, there are significant losses of initial investments (2.3 percent ~ 5.8 percent) if the nominal amounts are not rebalanced. Thirdly, as for tracking errors measured by volatility of the return deviation, for a holding period of 3 months or less, tracking errors of leveraged ETFs are mainly explained by price inefficiency and tracking errors due to fund managers while, for the holding period of over 3 months, they are explained by the compounding effect.

For a long-term investment horizon, the compounding effect is the major factor of the return deviation and the tracking error of the leveraged ETFs. Investments in leveraged ETFs for longer horizons are not recommendable due to the negative compounding effect and volatility.

Keywords: ETF, Leveraged ETF, NAV, Tracking Error, Compounding Effect, Pricing Error JEL *Classification*: G11, G14, G15, G23

1. Introduction

An exchange-traded fund (hereafter ETF) is an investment fund traded on stock exchanges which tracks the return on its benchmark index such as equity index and asset's price.³ Among the ETFs traded in the Korea Exchange (hereafter KRX), equity ETFs⁴ can be classified into two categories: traditional ETFs and leveraged ETFs. As for traditional ETFs, ETF funds incept and redeem baskets of securities (units of ETFs) in order to achieve the target return. Unlike traditional ETFs, leveraged ETFs use stocks, futures, options, and borrowings to achieve the promised leveraged returns.

In October 2005, Societe General Asset Management (SGAM) launched leveraged ETFs in France. In the US markets, the Proshares Series were launched in the middle of 2006. In South Korea, Kodex Inverse was introduced in September 2009. As of May 28, 2013, total of eight equity leveraged ETFs were traded in the KRX. Among the eight equity leveraged ETFs, four equity ETFs are bull ETFs which track twice (2x) the returns on the benchmark indices⁵ on a daily basis, whereas the other four equity ETFs are bear ETFs which track one times the opposite (-1x) of the returns on the benchmark indices on a daily basis. While total net assets of leveraged ETFs are small in the entire ETF market, their trading volumes are disproportionately large. As of May 28, 2013, the net assets of 2x bull ETFs and (-1x) bear ETFs accounted for approximately 17.5 percent of the total net asset values, but their trading volumes accounted for approximately 63.4 percent of the entire ETFs. As such, investors are considered to trade more actively bull and bear ETFs than traditional ETFs in the Korean market. At the end of May 2013, leveraged ETFs traded in the KRX are ranked fourth in the world followed by those of US, UK, and Japan in terms of trading volume.⁶

The tracking error of a leveraged ETF occurs when the actual return on a leveraged ETF cannot exactly track the designed leveraged return on its benchmark index. As for a traditional ETF, the factors of tracking errors include pricing inefficiency, tracking error due to fund managers, related expenses, and dividends (Shin & Soydemir, 2010). These factors of tracking error similarly occur in a leveraged ETF. However, as for dividends factor, it only affects the tracking error of bull ETFs at or around the ex-dividend date, whereas it affects the tracking errors of traditional ETFs during the period of dividend accumulation. With regard to a leveraged ETF, tracking error not only depends on those factors of traditional ETFs but also on the compounding effect. This compounding effect occurs when an investor buys and holds (Buy-and-Hold) leveraged ETFs for longer than one day (Lu et al, 2009; Shum & Kang, 2012). The compounding effect would cause the cumulative return on a leveraged ETF to either increase more than or decrease less than the designed leveraged return. In

³ The website of the Korea Exchange (www.krx.co.kr)

⁴ In this paper, an equity ETF is classified into a traditional ETF and a leveraged ETF according to whether leverage is embedded or not. Leverage is not embedded in a traditional ETF, while it is embedded in a leveraged ETF.

⁵ The benchmark indices include equity indices, currencies, and interests

⁶ KRX ETF monthly, June 2013

other cases, however, the compounding effect could cause the cumulative return on a leveraged ETF to be negative even when the return on its underlying index fluctuates but changes little over the same holding period.⁷ In this case, the compounding effect with volatility could negatively impacts on performance of bull and bear ETFs. This means that the compounding effect prevents leveraged ETFs from properly tracking their target returns. On a theoretical basis, an investor who invests in bull and bear ETFs could avoid this negative compounding effect by rebalancing the exposures on a daily basis over the holding period. However, Trainor & Baryla (2008) warns so-called constant leverage trap.⁸

For this study, the data of all leveraged ETFs traded in the KRX are collected on a daily basis and the data periods are from the listing dates to May 28, 2013. Main methodologies in this paper are the decomposition of return deviation suggested by Shum & Kang (2012), the tracking error from the regression, and the tracking error by the volatility of return deviation suggested by Shin & Soydemir (2010).

Our main findings are as follows. Firstly, with regard to the return deviation, on average, the performances of leveraged ETFs are statistically close to their promised returns within a holding period of a week, but start to deteriorate over a holding period of longer than a week. As for the factors of return deviation, in case of bull leveraged ETFs, tracking error due to fund managers, dividends, and related expenses mainly explain the return deviation over the holding period of a month or less, while the compounding effect does over the holding period of 3 month or longer. In case of bear ETFs, the results are generally similar to those of bull ETFs but may differ in some parts. For example, incomes such as interests and dividends are prone to reduce the negative return deviation in bear ETFs. Secondly, the results of the regression analyses suggest that investors can buy and hold bull ETFs for a week or a month at most, while investors can hold bear ETFs for three months. However, if initial investments are not rebalanced, there are 2.3 percent \sim 5.8 percent losses of invested amounts over a year holding period. Note that these losses occur irrelevant to the daily return variation and they are economically significant. Hence, investing in leveraged ETFs over a longer investment horizon is not desirable. Thirdly, the results of tracking error measured by volatility of the return deviation suggest that over short holding periods factors related to pricing efficiency⁹ are those which explain the tracking errors of leveraged ETFs. However, over the holding periods of three month or longer, the compounding effect is the main factor of tracking errors. Fourthly, the results of price errors ("PE")¹⁰ suggest that some leveraged ETFs are traded at a slight premium to their NAVs,

⁷ This is known as volatility drag.

⁸ Trainor & Baryla (2008) argue that rebalancing of nominal amounts could deteriorate the cumulative returns on leveraged ETFs due to so-called "constant leveraged trap". In other words, an investor should rebalance his or her exposures on a daily basis in order to exactly track the target return. However, an investor could inevitably buy leveraged ETFs at an offer price and/or sell them at a bid price in some circumstances. In those cases, rebalancing of exposures results in deteriorating the performance of leveraged ETFs.

⁹ Factors related to pricing efficiency include tracking errors due to fund managers, related expenses and incomes.

¹⁰ Price error ("PE") is defined as PE = (leveraged ETF's price - its NAV)/its NAV. NAV refers to net asset value.

while the others are at a discount. Some ETFs are traded at a nontrivial discount so that arbitrage transaction could be executed even when bid-ask spreads and related expenses are considered.

The remainder of this paper is organized as follows. In Section 2, we present the data and descriptive statistics. In Section 3, we describe the characteristics of bull and bear ETFs traded in the KRX and explain the compounding effect. In Section 4, we review related literatures and establish the research models. In Section 5, we report the estimation results. Finally, Section 6 concludes the paper.

2. Data and Descriptive Statistics

2.1 Data

For this paper, the data are acquired from the KRX website.¹ The sample leveraged ETFs include all equity leveraged ETFs traded in the KRX as of May 28, 2013. Eight leveraged ETF are traded in the KRX and these eight leveraged ETFs are classified into four bull ETFs and four bear ETFs. Kodex Leverage, Tiger Leverage, Kstar Leverage, and Kindex Leverage are bull leveraged ETFs which track twice (2x) the returns on the benchmark indices, whereas Kodex Inverse, Tiger Inverse, Kosef Inverse¹¹, and Kindex Inverse are bear ETFs which track one times the opposite (-1x) of the returns on the benchmark indices. From the sample leveraged ETFs, daily closing prices, daily trading volumes, total net assets at the date of May 28, 2013, daily net asset values ("NAV") are collected, while from its benchmark index, daily closing prices are collected. Kindex Leverage and Kindex Inverse are not analyzed in this paper. This is because the two ETF time series are not long enough to make a meaningful comparison with the other leveraged ETFs. Kindex Leverage and Kindex Inverse were introduced in January 2012 and September 2011, respectively.

All the 2x bull ETFs such as Kodex Leverage, Tiger Leverage, Kstar Leverage, and Kindex Leverage use the KOSPI200¹² as their underlying indices, whereas all the (-1x) bear ETFs such as Kodex Inverse, Tiger Inverse, Kosef Inverse, and Kindex Inverse use the F-KOSPI200¹¹ as their underlying indices.

2.2 Descriptive Statistics

¹¹ Kosef Inverse will be delisted in June 26, 2013 according to the regulation. The trading value of a leveraged ETF traded in the KRX should maintain at least approximately US\$4.5million as its net worth for a month on a daily basis (Woori Asset Management (www.kosef.co.kr; KRX ETF monthly, June 2013). US\$1=1,100 KRW (Korean Won)

¹² The KOSPI200 is an equity index that is designed for stock index futures and stock index options. It consists of 200 largecap stocks traded in the KRX in light of market & industry representation and liquidity. The F-KOSPI200 refers to KOSPI200 futures.

As of May 28, 2013, the total net assets of 2x bull ETFs traded in the KRX were approximately US\$2,285 million which accounted for 14.9 percent of the entire ETF market values, while those of (-1x) bear ETFs were approximately US\$395 million which accounted for 2.66 percent. In terms of trading volume, 2x bull ETFs, (-1x) bear ETFs, and major traditional ETFs accounted for 44.2 percent, 19.2 percent, and 27.1 percent respectively. This represents 90.5 percent of the whole volumes of ETFs⁶. Further, if the bull and bear ETFs are only considered, total net assets of both bull and bear ETFs accounted for 17.5 percent, while the trading volumes accounted for 63.4 percent in the same period. In other words, while the market capitalization of leveraged ETFs is relatively small compared to that of traditional ETFs, the trading volume of leveraged ETFs is disproportionately large. This means that investors trade leveraged ETFs far more actively than traditional ETFs.

Here we focus on each leveraged ETF. In terms of total net assets, Kodex Leverage is $17 \sim 198$ times greater than the other bull ETFs, while Kodex Inverse is $30 \sim 166$ times greater than the other bear ETFs. In terms of trading volume, Kodex Leverage and Kodex Inverse also dominate the market (Table 1). Investors are inclined to concentrate on trading Kodex Leverage and Kodex Inverse. The concentrated trading could not only cause a sort of liquidity risks to the other leveraged ETFs but also trigger the liquidity provider ("LP") to intervene the market. This intervention might distort the market. Tiger Leverage and Kstar Leverage were listed on the KRX at the same date. Hence, it is easy to make a comparison with each other. In terms of total net asset and trading volume, Kodex Leverage among 2x bull ETFs dominates the bull ETF market, while Kodex Inverse among (-1x) bear ETFs dominates the bear ETF market. As such, we explain the empirical results mainly with Kodex Leverage and Kodex Inverse.

[Table 1 about here]

As the skewness statistics in bull and bear ETFs are positive, outliers more often occur over the mean prices. For kurtosis, the magnitudes of kurtosis are under three for all bull ETFs. However, for bear ETFs, the kurtosis of Kodex Inverse is under three, while those of the other bear ETFs are over three. Overall, both bull and bear ETFs traded in the KRX are not normally distributed at the 5 percent significance level (Table 2). However, Kindex Leverage has a normal distribution at the 1 percent significance level, presumably due to the relatively short trading period. Kindex Leverage has been traded in the KRX since January, 2012.

[Table 2 about here]

3. The characteristics of the leveraged ETF and the Compounding Effect

3.1 The characteristics of the leveraged ETFs

Traditional ETFs incept and redeem the units of ETFs (baskets of securities) in order to achieve the target returns. On the other hand, leveraged ETFs track the stated leveraged returns by using stocks, futures, options, and borrowings.

As mentioned above, the goal of leveraged ETFs is to deliver the stated leveraged returns on their underlying indices. Thus, leveraged ETFs have some different characteristics compared to those of traditional ETFs. The main differences are as follows. Firstly, the yearly fees of leveraged ETFs are approximately twice those of traditional ETFs which use KOSPI200 and KRX100 as their underlying indices; the fees of traditional ETFs range from 0.15 percent to 0.26 percent per annum and those of leveraged ETFs range from 0.15 percent to 0.64 percent per annum.¹² The reason that the fees of leverage ETFs are higher than those of traditional ETFs is that additional expenses such as interest expenses and hedging costs are inevitable to track the stated leveraged returns. Secondly, bull ETFs, like traditional ETFs, include stocks and ETFs in their baskets of securities and so get dividends. For traditional ETFs, the dividends are distributed to investors after expenses are accounted for. However, for bull ETFs, dividends are usually used to cover the costs of leverage and hedging. As a result, dividends are not generally distributed to investors. On the other hand, for bear ETFs, dividends are not the issues. Bear ETFs mostly use the F-KOSPI200 and short positions of traditional ETFs in order to achieve the target returns. Thus, there are no stocks in their baskets and so no dividends. Thirdly, taxes are not imposed on traditional ETFs. However, taxes are imposed on leveraged ETFs over a holding period but actual taxes are not usually imposed to investors.¹³ Fourthly, the compounding effect occurs when an investor buys and holds (Buy-and-Hold) leveraged ETFs for two days or longer.

Among these differences, the most unique difference between a traditional ETF and a leveraged ETF is the compounding effect. This compounding effect could cause the cumulative return on a leveraged ETF to increase more or decrease less than the promised return (positive compounding effect). On the other hand, the compounding effect with volatility could cause the cumulative return on a leveraged ETF to be negative even when the price of underlying index fluctuates but changes little over the holding period (negative compounding effect).

3.2 Compounding Effect

A return on a stock can generate a compounding effect. For example, let us assume that (1) an

¹³ The fund managers generally track the designed leveraged returns with a basket of securities on the exchange, and thus the standard of assessment for tax is not increased (Samsung Asset Management, www.kodex.com)

investor buys and holds a stock for two days, and (2) the daily return at date t and date t+1 is 10 percent and -10 percent, respectively. Under the assumptions, the two-day cumulative return on the stock will be -1 percent even when the arithmetic sum of two day returns is zero percent. This is because $r_t r_{t+1}$ of Equation (1) additionally impacts on the buy-and-hold's return:

$$(r_{t}+1)(r_{t+1}+1) - 1 = r_{t}r_{t+1} + r_{t} + r_{t+1}$$
(1)
(*r_t* = the return at date t)

In other words, a compounding effect is generated in this case.

A different style of compounding effect occurs when an investor buys and holds bull ETFs or bear ETFs for any longer than one day. A leveraged ETF fund rebalances daily the amounts of leverage to achieve the target return. In this circumstance, if an investor buys and holds leveraged ETFs for 2 days or longer without rebalancing the nominal amounts, a compounding effect occurs. In other words, the daily rebalancing causes the returns on leveraged ETFs compounded over a holding period of two days or longer. We here examine the compounding effect of a 2x bull leveraged ETF traded in the KRX. If r_t is the return at date t, Equation (1) times two becomes Equation (2). Equation (2) shows twice the return on the benchmark index over a holding period of two days that the 2x bull ETF should achieve on a daily basis:

$$2[(r_t+1)(r_{t+1}+1) - 1] = 2r_t r_{t+1} + 2r_t + 2r_{t+1}$$
(2)
(r_t = the return at date t)

Equation (3) shows the return on a bull leveraged ETF when an investor buys and holds it over a holding period of two days:

$$(2r_t+1)(2r_{t+1}+1) - 1 = 4r_tr_{t+1} + 2r_t + 2r_{t+1}$$
(3)

Equation (3) minus Equation (2) equals Equation (4):

$$2r_t r_{t+1} \tag{4}$$

 $2r_tr_{t+1}$ of Equation (4) shows the difference between the return on a 2x bull ETF that an investor gets and the return on its underlying index that the fund manager has to provide over two-days holding periods. This is called the compounding effect of a 2x bull ETF (Lu et al, 2009; Shum & Kang, 2012).

 $2r_tr_{t+1}$ of Equation (4) has a combination of 9 cases. The cases of (r_t, r_{t+1}) are (+,+), (+,0), (+,-), (0,+), (0,0), (0,-), (-,+), (-,0), and (-,-). For the case of (+,+) and (-,-), a positive compounding effect is likely to be generated. In this case, either the cumulative return on a 2x bull ETF achieves more than the target return, or the negative cumulative return realizes less than the target return. For the case of (+,0), (0,+), (0,0), (0,-), and (-,0), there is no compounding effect. For the case of (+,-) and (-,+), a negative compounding effect could be generated. In other words, the cumulative return on a bull ETF could be negative even when the daily return on its underlying index fluctuates but changes little, similar to the example of Equation (1). In general, the compounding effect with volatility drag¹⁴ negatively impacts on the performance of leveraged ETFs. As the length of the holding period lengthens, the negative compounding effect causes the return on leveraged ETF to deviate far away from the target return (Wang, 2009; Lu et al. 2009). Charupat & Miu (2011) argue that the performances of leveraged ETFs could be poor especially when the benchmark indices fluctuate, but do not change by much. Meanwhile, when the holding period of a leveraged ETF is three days or longer, the combination of cases will increase sharply. Equation (5) generalizes an equation for the compounding effect of a 2x bull ETF:

Compounding Effect =
$$[\prod_{i=1}^{n} (2r_t + 1) - 1] - 2[\prod_{i=1}^{n} (r_t + 1) - 1]$$
 (5)

$$(r_t = \text{the return at date t})$$

If an investor buys and holds a leveraged ETF that mimics the negative multiple of the returns on its benchmark index over a holding period of two-days or longer, the magnitude of the compounding effect is much larger than that of a bull ETF. Equation (6) shows the compounding effect when an investor buys and holds a (-2x) bear ETF for two-day holding periods:

$$6r_t r_{t+1} \tag{6}$$

When Equation (6) is compared to Equation (4), the magnitude of the compounding effect of a -2x bear ETF is 3 times that of a 2x bull ETF. As such, investors who want to invest bear ETFs over a long investment horizon have to carefully consider the compounding effect.

All bear ETFs traded in the KRX are designed to provide (-1x) returns on the benchmark indices. When the holding period of (-1x) bear ETF is two days, the compounding effect is $2r_tr_{t+1}$ which is the same value as that of a 2x bull ETF. Equation (7) generalizes the compounding effect of a (-1X)

¹⁴ Shum & Kang (2012) calls this phenomenon flat-return effect where the performances of leveraged ETFs are negative even when arithmetic sum of the cumulative returns converges to zero percent. In general it is called volatility drag.

bear leveraged ETF:

Compounding Effect =
$$[\prod_{i=1}^{n} (-r_t + 1) - 1] + [\prod_{i=1}^{n} (r_t + 1) - 1]$$
 (7)

 $(r_t = \text{the return at date t})$

The compounding effect is a kind of the risk of an investor who buys and holds a leveraged ETF for any holding period of longer than one day, but not the risk of the fund manager who rebalances daily leverages. An investor investing in a leveraged ETF has to rebalance the value of investments in order to minimize the risk related to the compounding effect on a daily basis. On a theoretical basis, an investor can minimize the negative compounding effect by properly rebalancing the exposures, but he or she could go into so called "a constant leverage trap" (Trainor & Baryla, 2008). In other words, if an investor rebalances the exposures in order to achieve the target return, he or she would buy leveraged ETFs at an offer price or sell them at a bid price. This could cause the investor's expected return to deteriorate further.

Because of the compounding effect with volatility drag and tracking errors, leveraged ETFs can hardly achieve the stated leveraged returns over a holding period of any longer than one day (Charupat & Miu, 2011). Carver (2009) argues that a leveraged ETF can perform poorly over longer investment horizons due to ill-timed rebalancing and the geometric nature of return compounding. He examines the poor performance of a leveraged ETF with Equation (8):

$$g = \mu(w_{riskv}) + r(1 - w_{riskv}) - (w_{riskv} \cdot \sigma)^2/2$$
(8)

(g: the expected growth. μ : the rate of return from the risky assets. w_{risky} : the proportion of the portfolio in the risky asset. σ : the standard deviation of the risky asset's log return. r: the borrowing and lending rate)

For an example, for a 2x bull leveraged ETF, the portfolio weights are 200 percent in the risky asset and -100 percent in the risk free asset. According to Equation (8), the value of growth rate (g) could be negative if the standard deviation (σ) of the risk asset is fairly high. As the length of the holding period increases, the standard deviation (σ) of the risky asset is likely to rise, which would cause the value of growth rate (g) to become negative and increasingly deteriorate. Carver (2009) argues that the volatility of the risky assets will cause the value of investments to eventually converge to zero when an investor invests in leveraged ETFs over a longer investment horizon.

Wang (2009) and Lu et al. (2009) also show that if value of investments is not rebalanced, it will converge to zero when an investor holds leveraged ETFs infinitely. Equation (9) is the natural log value of the equation suggested by Wang (2009):

$$\ln \frac{A_T}{A_0} = x \ln \frac{S_T}{S_0} + (x - x^2) \sigma^2 T/2$$
(9)

 $(\ln \frac{A_T}{A_0})$: the log return on a leveraged ETF. $\ln \frac{S_T}{S_0}$: the log return on the benchmark index. x : leverage. σ : volatility of the benchmark index. T: the holding period)

Let us examine the scalar value of $(x-x^2)\sigma^2 T/2$ by dividing into $(x-x^2)$ and $\sigma^2 T/2$. The value of $(x-x^2)$ is always less than zero if the value of leverage (x) is greater than 1 or negative. In $\sigma^2 T/2$, the volatility (σ) is formed as quadratic variation, which makes the volatility substantially larger. Overall, what $(x-x^2)\sigma^2 T/2$ means is that the leverage and the volatility formed with quadratic variation will deteriorate the return on a leveraged ETF very quickly as the length of the period increases. As a result, Equation (9) shows that the longer term holding of a leveraged ETF deteriorates the expected return even when a leveraged ETF properly tracks the stated leverage. As such, investments of leveraged ETFs over a long horizon are not desirable.

4. Literature Review and Methodology

Practically, leveraged ETFs cannot exactly deliver the promised leveraged returns. Accordingly, tracking error is inevitable. When we examine tracking error of a leveraged ETF, we need to review it from a fund manager's perspective and an investor's perspective, respectively.

From a fund manager's perspective, the NAV of leveraged ETF is to provide properly the stated leveraged return day by day. To achieve this, a leveraged ETF has to rebalance daily the exposures to maintain the designed leverage. However, it is not easy to track the designed ratio returns. In traditional ETFs, the volatility of underlying indices, changes of a basket of securities, related expenses, the cash flow of a fund, and dividends are major factor that the return on traditional ETFs deviates from the promised returns (Shin & Soydemir, 2010; Charupat & Miu, 2011). These are also factors to affect the tracking error of leveraged ETFs. Jarrow (2010) argues that a k-times leveraged ETF will not track k times the return on its benchmark index due to benchmark index's volatility and interest expenses over the investment horizon. Asset management firms managing some sample leveraged ETFs argue in its investment paper that the volatility of futures price and the extension & contraction of the basis¹² would cause the NAVs of the leveraged ETFs¹⁵ to deviate from their desired leveraged returns.

From an investor's perspective, tracking errors depend not only on the factors of traditional ETFs but also on the length of holding period (Charupat & Miu, 2011). Traders are prone to buy and hold

¹⁵ The basis is defined as the difference between the KOSPI200 and its futures price.

leveraged ETFs for two days or longer. As a result, the compounding effect prevents leveraged ETFs from exactly tracking the desired leverage ratios.

In summary, if the definition of tracking error of leveraged ETFs includes pricing efficiency factor, the factors of tracking error can be classified as pricing inefficiency, tracking error due to fund management (including expenses, incomes, and dividends), and the compounding effect.

4.1 Decomposition of Return Deviation of a Leveraged ETF

If a leveraged ETF exactly deliver the target return over a specific holding period, the return deviation would not be generated. Accordingly there is no tracking error. If we examine the factors to affect the return deviation of a leveraged ETF, we can identify which factor affects the tracking error of a leveraged ETF, and by how much. In this paper, at first we investigate the return deviation of the leveraged ETF and then examine the tracking error with several methods mentioned in the chapter 4.2^{16} .

Tang & Xu (2013) argue that the total return deviation of a leveraged ETF could be attributed to the compounding effect, tracking error due to fund managers, and market inefficiency. Shum & Kang (2012) defines the return deviation of leveraged ETF as the difference between the return on the NAV of a leveraged ETF and the designed leveraged return on its benchmark index over a specific holding period. They also investigate the return deviation by decomposing it into two buckets: return deviation due to fund managers (the return on the NAV of a leveraged ETF - the compounding effect) and return deviation due to the compounding effect (the compounding effect - the designed leveraged return on its benchmark index).

In this paper, for decomposition of the return deviation of a leveraged ETF, we define the total return deviation as the difference between the return on a leveraged ETF and the designed leveraged (constant leverage, "CL") return on its benchmark index over a specific holding period. As for leveraged ETFs traded in the KRX, the designed leverage of bull ETF is two (2x), while that of bear leveraged ETFs is minus one (-1x). Equation (10) shows the decomposition of return deviation of the leveraged ETF based on CL of the return on its benchmark index:

$$TTE_{it} = ER_{i,t} - CL \cdot IR_{i,t}$$
⁽¹⁰⁾

¹⁶ In measuring tracking error, researchers generally use the regression analysis between an ETF and its benchmark index, volatility of return deviation, and average absolute difference of return deviation (Shin & Soydemir, 2010; Wang, 2009; Lu et al, 2009; Charupat & Miu, 2011). However, some researchers employ return deviation itself as tracking error. In this paper, we employ the definition of tracking error which is generally used. A main reason to do this is to clarify confusion when we conduct comparative analysis among the return deviation, the regression, and the volatility of return deviation.

$$= (ER_{i,t} - NR_{i,t}) + (NR_{i,t} - CR_{i,t}) + (CR_{i,t} - CL \cdot IR_{i,t})$$
$$= PTE_{i,t} + MTE_{i,t} + CTE_{i,t}$$

(*CL*: constant leverage (2 or -1). $ER_{i,t}$: the return on a leveraged ETF_i over a holding period. $NR_{i,t}$: the return on the NAV_i of the leveraged ETF_i over a holding period. $CR_{i,t}$: the compounding effect over a holding period. $IR_{i,t}$: the return of the underlying index over a holding period. $PTE_{i,t}$: $ER_{i,t} - NR_{i,t}$. $MTE_{i,t}$: $NR_{i,t} - CR_{i,t}$. $CTE_{i,t}$: $CR_{i,t} - CL \cdot IR_{i,t}$)

TTE, the total return deviation, is defined as the difference between the return on a leveraged ETF and the designed leveraged return on its benchmark index. TTE will not be different from zero if the performance of a leveraged ETF is successful in delivering the designed leverage ratio (2x or -1x) over a specific holding period. According to Equation (10), TTE can be divided into PTE, MTE, and CTE. By decomposing it into three factors, we can investigate more thoroughly what factor affects more on TTE and by how much.

PTE is defined as the difference between the return on a leveraged ETF and the return on its NAV. Traders and liquidity providers ("LP") would be major roles to influence on PTE. If the market works efficiently, the magnitude of PTE is statistically and economically meaningless. Also, it is reasonable to presume that PTE is not closely correlated with the length of the holding period considering that PTE depends on the market efficiency.

MTE is defined as the difference between the return on the NAV of the leveraged ETF and the compounding effect. In other words, the MTE is the return deviation between the return on the NAV of the leveraged ETF and its target return excluding the compounding effect. The factors of MTE include tracking error due to fund managers, related expenses, dividends, and incomes. The tracking error due to fund managers can be reduced if the fund managers manage the funds properly. Expenses such as fees are a kind of the regulation factor and also are usually paid quarterly. Meanwhile, related incomes and expenses impact on MTE of the bull ETF and MTE of the bear ETF differently. As for the bull ETF, incomes such as dividends and interest incomes are usually used to cover the costs of leverage and hedging. In some senses, the reinvestment effect of dividends can be effective in bull ETFs. As for traditional ETFs, ETFs accumulate dividends and pay them out after accounting for related expenses to investors. As for the bear ETFs, the bear ETFs track the designed ratio returns by using the KOSPI200 futures and short positions of other ETFs. As a result, there is no dividend issue. However, there are sufficient cash flows in the bear ETFs. As mentioned above, the bear ETFs track the target return by the futures and short position of other ETFs. Accordingly, bear ETFs tend to have plenty of cash flows, which are usually invested in stocks, bonds, and deposits. The incomes from the investments are likely to reduce MTE, namely the return deviation. To sum up, the value of MTE of the bull ETF is negative mainly due to related expenses, but that of the bear ETF could be positive over a short term holding period due to related incomes. All factors mentioned above may increase or

decrease MTE of bull and bear ETFs, similarly or differently.

As for the dividend factor, we here investigate it in more detail as to the difference between a traditional ETF and a bull ETF. The NAV of the bull ETF could go up only at or around the exdividend date¹⁷ (e.g., the end of December and the end of March). This increased NAV tends to push MTE up at the same date. However, as for traditional ETFs, dividends impact on MTE during the period of dividend accumulation. DeFusco et al. (2011) find that dividends in traditional ETFs affect pricing deviation during the period of dividend accumulation¹⁸. Frino & Gallagher (2001) also finds that dividends in mutual funds affect the tracking error. To sum up, the dividend factor is likely to affect MTE of traditional ETFs during the period of dividend accumulation whereas it affects MTE of bull ETFs only at or around the ex-dividend date. As for a bear ETF, there are any issues about dividends on the face, but it could be affected indirectly by volatility of underlying index and futures at or around the ex-dividend date.

CTE is defined as the difference between the compounding effect and the designed leveraged return on the benchmark index. By CTE, we can investigate the compounding effect of leveraged ETFs separately. Also, to be sure that related expenses and incomes could be compounded if CTE is highly correlated with MTE. Shum & Kang (2012) investigate the return deviation between actual returns on the NAV of leveraged ETFs and their target returns. They decompose the return deviation into two factors; the tracking error due to fund managers (MTE) and the compounding effect (CTE). They find that the compounding effect increases as the holding periods of leveraged ETFs increases. Tang & Xu (2013) also find that the compounding effect is one of factors to impact on the return deviation of leveraged ETFs.

4.2 Regression Analyses of Leveraged ETF Tracking Errors

Tracking errors are measured in various ways. Tracking error also is a bit differently defined and employed according to the method of the measurement and researchers' needs. Common methods for measuring tracking errors are regression analysis, average absolute difference of the return deviation or standard deviation of the return deviation (Shin & Soydemir, 2010; Wang, 2009; Lu et al, 2009; Charupat & Miu, 2011). For example, Shin & Soydemir (2010) measure the tracking error of a traditional ETF by standard deviation of return deviation and average absolute difference of the return deviation where the return deviation is defined as the difference between the return on the NAV of a traditional ETF and the return on its benchmark index. In some papers, return deviation itself is

¹⁷ According to the regulation, an ex-dividend date is the previous day of a reference date which determines the person who will get dividends. Hence, the price of a stock scheduled to pay dividends will fall. But as for ETFs, dividends are accumulated into the NAV as a type of dividends payable at the same date. Therefore, the value of NAV changes little at or around the ex-dividend date.

¹⁸ Pricing deviation is defined as the difference between the price of a traditional ETF and the price of its benchmark index.

defined as tracking error. For instance, Roll (1992) measures a tracking error by mean and variance of the return deviation itself. He defines tracking error as the difference between the manager's return and the benchmark return.

In this paper, with respect to the bound of return deviation, it includes the return on the leveraged ETF and the return on its benchmark index. Also, we employ regression analysis and standard deviation of the return deviation when we measure the tracking error of leveraged ETFs¹⁶. The results of average absolute difference of the return deviation are similar to those of standard deviation of the return deviation. Hence, we will not report those results.

4.2.1 Measurement of Tracking Error by the Regression

In this section, we employ regression analysis to estimate the tracking error of a leveraged ETF over holding periods of various lengths. When the tracking error of a leveraged ETF is measured by regression method, we can set up the return on a leveraged ETF as a dependent variable and the return on its benchmark index as an explanatory variable. After conducting the regression, we will get the intercept, α , and the regression coefficient, β and then examine whether the values of the intercept and the coefficient are statistically appropriate (Wang, 2009; Lu et al, 2009; Charupat & Miu, 2011). In our sample, the data could be overlapped when we calculate the returns over a specific holding period. This overlapping could cause heteroskedasticity or autocorrelation (Shum & Kang, 2012). As such, we carry out the regression using the Newey-West estimator, suggested by Newey & West (1987), to overcome this potential bias:

$$ER_{i,t} = \alpha_i + \beta_i IR_{i,t} + \varepsilon_{i,t} \tag{11}$$

 $(ER_{i,t})$: the return on the leveraged ETF_i over a holding period. $IR_{i,t}$: the return on its benchmark index. α_i : intercept. β_i : regression coefficient. $\varepsilon_{i,t}$: error term)

The intercept, α , is a kind of Jensen's alpha (Jensen, 1967). Jensen(1967) measures the excess return of a fund manager who manages mutual funds by using alpha (α) of equation (8)¹⁹. The concept of Jensen's alpha can be applied to ETF's (Shin & Soydemir, 2010). The performance of fund managers managing traditional and leveraged ETFs depends on how much they deliver the target returns. In this circumstance, for traditional ETFs, the alpha is a kind of expenses rather than excess returns (Shin & Soydemir, 2010). Therefore, in traditional ETFs, the alpha is most likely to be

¹⁹ Jensen (1967)'s equation (8): $(R_{i,t} - R_{ft}) = \alpha_i + \beta_i (R_{m,t} - R_{ft}) + \tilde{\mu}_{i,t}$. Shin & Soydemir (2010) employ Jensen (1967)'s model to their ETF models. In this paper, Jensen's model can be employed to the equation (11) as follows. $(ER_{i,t} - R_{ft}) = \alpha_i + \beta_i (IR_{i,t} - R_{ft}) + \varepsilon_{i,t}$, R_{ft} : risk free rate.

negative. In bull ETFs, alpha is a kind of expense as well. However, as for bear ETFs, the value of alpha can be positive over a short term holding period (Wang, 2009; Shum & Kang, 2012). As mentioned earlier, bear ETFs are prone to leave plenty of cash flows, which generate excess of incomes over expenses.

The regression coefficient, β , is the target leverage which managers of leveraged ETFs have to track. If value of β is higher than the leverage, the return on a leveraged ETF is more sensitive than the return on its benchmark index. If value of β is lower than the leverage, the return on a leveraged ETF is less sensitive than the return on its benchmark index. While managers of leveraged ETFs are likely to pay attention to β than α , investors of leveraged ETFs pay attention to α because the intercept is expenses irrelevant to the return variation

Examining the tracking errors of leveraged ETFs with the simple regression, we need to test whether either the intercept, α , is significantly different from 0 (H₀=0) or the regression coefficient, β , is significantly from the promised leverage (H₀=2 for bull ETFs or (-1) for bear ETFs). If the null hypothesis for α is statistically rejected, the magnitude of α is needed to test whether it is economically meaningful. If β is statistically rejected, the magnitude of β is needed to test how far it deviates from the designed ratio.

Managers of leveraged ETFs must provide daily the stated leveraged return. Accordingly, managers of leveraged ETFs are in different positions from general investors who are likely to hold without rebalancing their exposures over a specific period longer than one day. In this regard, we need to examine the intercept, α , and the regression coefficient, β , of a leveraged ETF over various holding periods. Furthermore, we can look into how long investors could carry their investments without big losses. For this purpose, we carry out the simple regression for the holding periods of one day, two days, one week, one month, three months, and one year, respectively.

Wang (2009) estimates the intercept, α , and the regression coefficient, β , of bull ETFs and bear ETFs traded in the US markets. The results suggest that, for one-day horizon, the α is not statistically different from zero while the β is statistically different from the designed leverage so that the ETFs do not achieve the designed leverage. In his study, the estimated leverage ratios in bull ETFs are typically less than their designed ones, while the estimated leverage ratios in bear leveraged ETFs are greater than their designed ones. However, he concludes that the estimated daily returns on leveraged ETFs do not exactly but fairly well deliver the stated leverage returns.²⁰

Lu et al. (2009) examine the long-term performance of bull ETFs and bear ETFs traded in the US markets over the holding periods of 2 days, 5 days, 21 days, 63 days, and 252 days. They find that the actual returns on leveraged ETFs do not move away from the target returns over a holding period of

²⁰ The sample used by Wang (2009) are SSO (underlying index: S&P 500, 2X), QLD (underlying index: NASDAQ 100, 2X), BGU (underlying index: RUSSELL 1000, 3X), and DXD (underlying index: DOW 30, -2X).

up to a month. But the performance of leveraged ETFs deviates far from the target return over a year holding period²¹.

Charupat & Miu (2011) look into the performance of bull ETFs and bear ETFs traded in the Canadian markets when the length of the holding period is one day, one week, one month, one quarter, and one year. They find that bull and bear ETFs successfully deliver the target returns over holding periods of up to a week. But, beyond one month, the actual returns move away from the designed returns.²²

Shum & Kang (2012) also investigate the intercept, α , and the regression coefficient, β , in bull and bear ETFs traded in the US and Canadian markets when the length of the holding period is one day, one week, one month, and one year. They report similar results to those of the other studies mentioned above²³

Assume that the regression coefficient, β , in Equation (11) exactly mimics the designed leverage of its benchmark index. And then, we multiply both sides of Equation (11) by expected value. Then, we obtain the following equation:

$$E(ER_{i,t}) - CL \times E(IR_{i,t}) = \alpha_i + E(\varepsilon_{i,t})$$
(12)

 $(ER_{i,t})$: the return on leveraged ETF_i s over a holding period. $IR_{i,t}$: the return on the benchmark index over a holding period. α_i : intercept β_i : regression coefficient. $\varepsilon_{i,t}$: error term. CL: designed leverage.)

Equation (12) means that if a leveraged ETF exactly delivers the designed return, the difference between the actual average returns on a leveraged ETF and the average target returns (mean of TTE) equals the sum of both the intercept, α , and the expected value of error term in Equation (11). According to the assumption of error term in Equation (11), the expected value of error term will converge to zero. As a result, the mean of return deviation (mean of TTE) has a similar characteristic of the intercept in Equation (11). Also, we can estimate further what affects the intercept, α , over various holding periods with PTE, MTE, and CTE because TTE is sum of PTE, MTE, and CTE.

 ²¹ The sample used by Lu et al (2009) are DDM (underlying index: DOW 30, 2X), DXD (underlying index: DOW 30, -2X), SSO (underlying index: S&P 500. 2X), SDS (underlying index: S&P 500. -2X), QLD (underlying index: NASDAQ 100, 2X), QID (underlying index: NASDAQ 100, -2X), UWM (underlying index: RUSSELL 2000, 2X), and TWM (underlying index: RUSSELL 2000, -2X).
 ²² The sample used by Charupat&Miu (2011) are HXU (underlying index: S&P/TSX 60, -2X), HXD (underlying index:

²² The sample used by Charupat&Miu (2011) are HXU (underlying index: S&P/TSX 60, -2X), HXD (underlying index: S&P/TSX 60, -2X), HFU (underlying index: S&P/TSX Capped Financials, 2X), HFD (underlying index: S&P/TSX Capped Financials, -2X), HEU (underlying index: S&P/TSX Capped Energy, 2X), HED (underlying index: S&P/TSX Global Gold, 2X), and HGD (underlying index: S&P/TSX Global Gold, - 2X).

²X).
²³ The sample used by Shum&Kang (2012) are HXU (underlying index: S&P/TSX 60, 2X), HXD (underlying index: S&P/TSX 60, -2X), HGU (underlying index: S&P/TSX Global Gold, 2X), HGD (underlying index: S&P/TSX Global Gold, -2X), DIG(underlying index: DJ/US Oil & Gas, 2X), DUG(underlying index: DJ/US Oil & Gas, -2X), EFU (underlying index: MSCI EAFW, -2X), EEV (underlying index: MSCI Emerging Markets, -2X), SSO (underlying index: S&P 500, 2X), and SDS (underlying index: S&P 500, -2X).

4.2.2 Measurement of Tracking Error Using Volatility of Return Deviation

Shin & Soydemir (2010) suggest the standard deviation (volatility) of return deviation as the measurement of the tracking error when they examine the tracking error of traditional ETFs. In this paper, their method will be employed to measure the tracking error of leveraged ETFs traded in the KRX. However, we measure the tracking errors of leveraged ETFs in more detail. In other words, we investigate the tracking error by each basket (TTE, PTE, MTE, and CTE) discussed in the Chapter 4.1 over various holding periods:

$$TTE_{i,t} \quad \text{Tracking Error} = \sqrt{\frac{\sum_{t=1}^{n} (TTE_{i,t} - \overline{TTE_{i,t}})^2}{n-1}}$$
(13)

$$PTE_{i,t} \quad \text{Tracking Error} = \sqrt{\frac{\sum_{t=1}^{n} (PTE_{i,t} - \overline{PTE_{i,t}})^2}{n-1}}$$

$$MTE_{i,t} \quad \text{Tracking Error} = \sqrt{\frac{\sum_{t=1}^{n} (MTE_{i,t} - \overline{MTE_{i,t}})^2}{n-1}}$$

$$CTE_{i,t} \quad \text{Tracking Error} = \sqrt{\frac{\sum_{t=1}^{n} (CTE_{i,t} - \overline{CTE_{i,t}})^2}{n-1}}$$

 $(TTE_{i,t}: ER_{i,t} - CL \cdot IR_{i,t}. PTE_{i,t}: ER_{i,t} - NR_{i,t}. MTE_{i,t}: NR_{i,t} - CR_{i,t}. CTE_{i,t}: CR_{i,t} - CL \cdot IR_{i,t}. ER_{i,t}:$ the return on leveraged ETF_i over a holding period. NR_{i,t}: the return on ETF_i's NAV over a holding period. CR_{i,t}: the compounding effect over a holding period. IR_{i,t}: the return on the benchmark index, CL: constant leverage)

Tracking error of TTE measures the volatility of TTE. It shows how well actual return on a leveraged ETF tracks the designed leveraged return on its underlying index. We, also, can examine each tracking error of PTE, tracking error of MTE, and tracking error of CTE with method of the decomposition of return deviation. As mentioned before, TTE can be decomposed into PTE, MTE, and CTE, but tracking error of TTE cannot be decomposed. We just presume which tracking error impacts more on tracking error of TTE.

Tracking error of PTE measures the volatility of PTE. Tracking error of PTE has similar characteristics of PTE. Thus, it is likely to be neutral to the holding periods.

Tracking error of MTE measures the volatility of MTE. Tracking error of MTE has also similar characteristics of MTE. Thus, tracking error of MTE depends on volatility of underlying index, tracking error due to fund managers, related incomes and expenses, dividends, and the compounding effect. We reviewed those factors in Chapter 4.1 and thus skip the reviews. But, we here need to examine related incomes and expenses in more detail. These incomes and expenses tend to be

received or paid periodically (e.g., every three months). Accordingly, MTE or tracking error of MTE would not move in proportion to the length of the period.

Tracking error of CTE measures the volatility of CTE. Tracking error of CTE like CTE is likely to be highly correlated with the length of the period. As Carver (2009), Lu et al. (2009) and Wang (2009) argue, a long horizon investment of leveraged ETFs is likely to result in big loss of the investments. The main factor would be the compounding effect in consideration of the relationship between the length of holding period and the compounding effect. Thus, the compounding effect is likely to cause tracking error of TTE to become higher as the length of the period increases. Charupat & Miu (2011) argue that the compounding effect in leveraged ETFs is the main factor of tracking error.

4.3 Price Error of the Leveraged ETF

Pricing efficiency in ETFs measures how the market price of an ETF is close to its NAV. Price error ("PE") of Equation (14) is generally used to measure pricing efficiency of traditional and leveraged ETFs:

Price Error:
$$PE_{i,t} = \frac{ETF_{i,t} - NAV_{i,t}}{NAV_{i,t}}$$
 (14)

(ETF_{i,t}: the closing price of leveraged ETF_i on date t, NAV_{i,t}: the price of the NAV of ETF_i on the same date)

Charupat & Miu (2011) employ Equation (14) to examine pricing efficiency of the leveraged ETFs. If the price of a leveraged ETF is the same as the price of its NAV, there is no price error in a leveraged ETF. Also, if there is no price error in a leveraged ETF, the value of PTE or PTE tracking error in a leveraged ETF will not be different from zero. In other words, PE is highly correlated with both PTE and PTE tracking error.

Elton et al. (2002) find that the Spider where its benchmark index is the S&P500 slightly deviates from its NAV on average, but almost all of the differences are transient and disappear within one day. Engle et al. (2006) find that the premiums (discounts) for the domestic ETFs traded in the US markets are generally small compared to their NAVs. However, they find that as for the international ETFs, premiums (discounts) are much larger and more persistent. They also find that the volatility of price errors in ETFs is related to that of their NAVs. Charupat & Miu (2011) also argue that the embedded leverage in a leveraged ETF causes the NAV of a leveraged ETF to be more volatile than that of a traditional ETF on the same benchmark index. Petajisto (2013) finds that most of traditional ETFs tend to trade at a premium to NAVs. He also argues that this mispricing would transfer a

considerable wealth from less sophisticated individual investors to institutional investors.²⁴ Charupat & Miu (2011) report that bull ETFs trade at a discount or a slight premium to NAVs, while bear ETFs trade at a larger premium on average.

When traders execute arbitrage transactions in leveraged ETFs, they have to consider the spread²⁵, transaction costs, and time interval for the indicative NAV price.²⁶ If the magnitude of price error is large enough to cover the related expenses and risks, the arbitrage transactions could occur. In traditional ETF markets, the arbitrage transaction can be executed by incepting and redeeming the units of ETFs (baskets of securities). Madura & Richie (2004) argue that ETFs can provide traders to deal with mispricing and overreaction properly because there is no restriction of short selling and its portfolio is consisted of securities. In leveraged ETFs, inception and redemption are done in cash rather than in unit. This makes it easier and less costly to execute an arbitrage transaction (Charupat & Miu, 2011).

5. Empirical Results

5.1 Decomposition of Return Deviation of Leveraged ETFs

By decomposing total return deviation across various investment horizons, we can observe how TTE, PTE, MTE, and CTE vary. By doing this, we can also find what kind of return deviations affect the total return deviation (TTE) the most and the least. To find preliminary evidence, we estimate the correlation coefficients (1) between TTE and PTE, (2) between TTE and MTE, (3) between TTE and CTE, and (4) between MTE and CTE respectively.

In the case of bull ETFs, correlation between TTE and PTE and correlation between TTE and MTE decrease as the holding period increases while correlation between TTE and CTE increases as the holding period increases (Table 3(a)).²⁷ This is because price inefficiency, fund managers' tracking errors, related expenses, and dividends explain total return deviation when the buy-and-hold period is short. When the period is long, however, the compounding effect dominates in total return deviation. In other words, CTE dominates PTE and MTE in total return deviation when bull ETFs are held for long-term.

The correlations between MTE and CTE in bull ETFs were negative overall although the magnitude is not economically significant (Table 3(a)). It means that there is no synergy effect

²⁴ In his sample, Petajisto (2013) only include ETFs which meet over \$10 million in terms of assets and \$100,000 in terms of daily trading volume.

The spread of leveraged ETFs traded in the KRX is 5 KRW.

²⁶ KRX publicizes the indicative NAV ("INAV") every 2 seconds. KRX changed the time interval of the INAV from 10 seconds to 2 seconds in November 18, 2011.

When holding period is one year, correlation coefficient is near one.

between factors affecting tracking errors of traditional ETFs and the compounding effect of bull ETFs. Therefore, we conclude that the compounding effects of expenses and incomes are not significant to investors.

In KODEX leverage, for example, correlation between TTE and PTE is 0.332 for one-day holding period, and 0.290 for one-year holding period. For one-year holding period, it shows similar number to that for one-day holding period (Table 3(a)). PTE of a bull ETF, the difference between return on a bull ETF and return on its NAV, is neutral to the holding period. It means that PTE does not change much as long as leveraged ETF markets are efficient. Summing up, total return deviation increases but PTE does not vary much as the holding period gets longer.

For KODEX leverage, correlation between TTE and MTE is 0.692 for one-day holding period and 0.107 for one-year holding period (Table 3(a)). MTE consists of fund managers' tracking error, related expenses, and dividends. Fund managers' tracking errors are not much affected by the holding period because bull ETFs rebalances daily the amounts of leverage to achieve the target return. Expenses and dividends are periodical factors so that their effects on MTE are limited. Accordingly, MTE does not have to increase in proportion to the length of the holding period. Therefore, correlation between TTE and MTE can be low even though the holding periods for bull ETFs are longer.

When KODEX Leverage is held for two days, correlation between CTE and TTE is 0.144. When the holding period is 1 year, correlation between CTE and TTE becomes 0.969 (Table 3(a)). This result shows that the compounding effects dominantly affect total return deviations when bull ETFs are bought and held for long-term.

[Table 3(a) about here]

When we see correlation structures of bear ETFs in Table 3(b), correlations between TTE and PTE are high in shorter holding periods. Correlations between TTE and MTE are, however, low in shorter holding periods. These are due to the fact that MTEs are positive in bear ETFs. When KODEX inverse is held for one year, for example, correlation coefficients between TTE and MTE, and between MTE and CTE are -0.867 and -0.904 respectively. Bear ETFs track the underlying index of given multiple using KOSPI 200 futures or short position of other ETFs. Besides putting on margins for futures contracts or paying fees, most of net assets are invested in deposits or short term investments. Due to this, interests earned and dividends exceed costs, eventually MTEs become positive. When bear ETFs are held for one year, mean of TTE, mean of MTE, and mean of CTE are -1.181 percent, 1.542 percent, and -2.709 percent respectively (See Table 4(b)) When bear ETFs are held for one year, compounding effects(-) are greater than MTE(+), and this makes TTEs negative. Therefore, correlations between TTE and MTE, and correlations between MTE and CTE are both negative. Summing up, the compounding effects are shaping negative total return deviation of bear

ETFs when the holding periods get longer. And crossing out interests and dividends and costs has an effect to decrease the size of negative total return deviations.

[Table 3(b) about here]

Table 4(a) shows means of TTE, PTE, MTE, and CTE over various holding periods. When holding periods are greater than or equal to 1 month, a mean of bull ETF return and a mean of 2 x underlying index return are statistically significantly different at 1 percent confidence level. Especially, TTE of KODEX leverage is significantly different from zero at 1 percent confidence level even when the holding period is one week. The means of all bull ETF returns are less than 2 x underlying index returns.

In case of KODEX leverage, annualized means of TTE are -2.71 percent, -2.66 percent, -2.68 percent, -2.94 percent, -3.71 percent, -5.10 percent for 1 day, 2 days, 1 week, 1 month, 3 months, and 1 year respectively. Annualized means of TTEs increase from 1 day to 1 week buy-and-hold horizon and decrease for horizons longer than a week. Considering the results in Table 3(a), the influences of PTEs and MTEs are smaller, but those of CTEs are larger as the holding periods get longer. In other words, the negative compounding effects dominate as the holding periods become longer.

Analyzing annualized TTEs, PTEs, MTEs, CTEs over various holding periods, we find that the negative compounding effects of bull ETFs are distinct. KODEX leverage is held for 2 days and 1 year, for example, means of annualized TTEs are -2.66 percent and -5.10 percent, and means of annualized PTEs are -0.22 percent and 0.11 percent, and means of annualized MTEs are -2.48 percent and -2.18 percent, and means of annualized CTEs are 0.05 percent and -3.03 percent, respectively. The mean of TTE is decreasing as the holding periods get longer. The mean of PTE increases from positive to negative, but the magnitude is small. This is because PTE are not affected by the holding periods. MTE does not change much and CTE dramatically changes from positive to negative. When the holding period is 2 days, the means of TTEs of bull ETFs are mostly explained by the means of MTEs. When the holding period is one year, CTEs, as well as MTEs, explain variations in TTE. Overall, return deviations of bull ETFs tend to increase as the holding period gets longer. It is mainly due to the negative compounding effect. Table 3(a) also explains this phenomenon. Equation (8) (Carver, 2009) and equation (9) (Wang, 2009) warn that investing in leveraged ETFs for long-term horizon is harmful to investors. Our empirical results show that the main reason of this is the negative compounding effects.

[Table 4(a) about here]

Examining the decomposition of return deviations of bear ETFs, we find that MTEs of bear

ETFs are quite different from those of bull ETFs. That is, MTEs of bear ETFs are positive while those of bull ETFs are negative. For example, we saw that the mean of MTE is -2.183 percent when KODEX leverage is held for one year. When KODEX inverse is held for one year, however, the mean of MTE is 1.542 percent (Table 4(a) & 4(b)). This is because bear ETFs use KOSPI 200 futures or short position of other ETFs to track a negative multiple of the index. Unlike the bear ETFs, bull ETFs use stocks, ETFs, and index futures to track a multiple of the related index and this increases costs to manage the fund. In other words, managing bear ETFs left plenty of cash flows and incomes are generated from them. Therefore, MTEs of bear ETFs become positive. Correlation structures among return deviations of bear ETFs in Table 3(b) confirm this finding. Positive MTEs of bear ETFs offset the negative compounding effects more or less for short term horizon, but the negative compounding effects dominate positive MTEs for long-term horizon, so total return deviations have negative values.

To sum up, bear ETFs deviate from the target returns over a long investment horizon mainly due to the negative compounding effect. This result is similar to that of bull ETFs mentioned above.

[Table 4(b) about here]

If a leveraged ETF perfectly replicates twice of index return or (-1) x index return, mean of TTE equals mean of an intercept and residuals in (12). Table 6 shows that β is very close to the chasing multiple (2 or -1) over a week to a month in case of bull ETFs, over two days to three months in case of bear ETFs. In other investment horizons, the results are not different much. Therefore, most of means of TTEs are α in (11). It means that return deviations are the costs that investors should bear and are not relevant to return variations. Means of TTEs of KODEX leverage and KODEX inverse are -5.102 percent and -1.181 percent, respectively. They are the estimates of α in (11). That of KODEX leverage, in particular, is too large to be ignored.

Conducting the Phillips-Perron unit root test to return deviations of leveraged ETFs over various holding periods, we cannot find evidence that TTEs, CTEs and MTEs are stationary over one year investment horizon (Table 5). Over three month horizons, CTEs show some degree of nonstationarity but the hypothesis of nonstationarity was rejected at 1 percent confidence level. When investment horizon extends to one year, we surely find nonstationarity not only from CTEs but also from TTEs. Thus, nonstationarity of TTEs comes from that of CTEs (Table 5 & Figure 1). However, the MTE of KODEX Inverse is nonstationary. Overall, this seems to be due to the uniqueness of MTEs of bear ETFs.

[Table 5 about here]

In Figure 1, we see that TTEs and CTEs of KODEX leverage and KODEX inverse do not seem to be stationary when the holding period is one year. Also, TTEs and CTEs of KODEX leverage and KODEX inverse of one-year holding period move together. Large correlation coefficients of TTEs or CTEs in Table 3(b) confirm our visual evidence.

[Figure 1 about here]

5.2 Regression Analyses of Leveraged ETF Tracking Errors

Table 6 shows the results of regression of bull ETFs' returns on underlying index returns over one-day, two-day, one-week, one-month, three-month and one-year horizons. When investment horizon is one week, α of KODEX leverage is not equal to zero at 5 percent confidence level but the size of the coefficient is not large enough to be economically important. In cases of TIGER leverage and KSTAR leverage, α is not different from zero. It means that the costs irrelevant to return variations and the compounding effects are negligible if the bull ETFs are held within a week. When holding period is one month, α is different from zero in KODEX leverage and KSTAR leverage. Intercepts of all bull ETFs are different from zero at 1 percent confidence level and show negative signs when the investment horizon is longer than or equal to three months. At first glance, it seems to be due to costs. However, as you may observe correlations in Table 3(a) and decompositions of return deviations in Table 4(a), CTEs start to affect TTEs from three-month investment horizon. Therefore, it is the compounding effect that makes α negative. When KODEX leverage, TIGER leverage and KSTAR leverage are held over a year, the intercepts(α), irrelevant to return variations, are -5.82 percent, -4.62 percent and -5.31 percent. These numbers are statistically and economically significant.

Table 6 reports that beta coefficients (β) of KODEX leverage, TIGER leverage and KSTAR leverage are not different from the stated leverage (2) over two-day, one-week, one-month and threemonth horizons. When holding period is one-day and two-day, β is less than two. It means that leverage ETF returns are less sensitive than 2x underlying index return. When the holding period is one-year, β is greater than two. It means that leverage ETF returns are more sensitive than 2x underlying index return.

Considering α and β both, holding period shorter than or equal to a week does not do harm to KODEX leverage investors. However, considering all bull ETFs, investing in bull ETFs for longer than or equal to a month are not recommendable to any investor even if they are risk lovers. This finding is consistent with research findings of the U.S. and Canadian bull and bear ETF markets (Wang, 2009; Lu et al, 2009; Charupat & Miu, 2011; Xu & Kang, 2012).

In case of bear ETFs, intercept is not different from zero if the holding period is shorter than three months. When the holding period is one year, however, α s range from -2.54 percent to -2.30

percent, which is not irrelevant to return variations. Compared to bull ETFs, α s are quite small (Table 6). This is because incomes earned on cash reduce expenses in bear ETFs. However, just like bull ETFs, investments in bear ETFs for a long term are harmful to investors.

 β s of bear ETFs are not much different from the stated leverage (-1) as long as the holding period is from 2 days to 3 months. Considering α and β both, investment in bear ETFs for up to 3 months is not harmful to investors. Compared to bull ETFs, bear ETFs can be held longer term. This is because incomes earned on cash reduce most of costs.

By using similarity of TTE in (12) to the intercept in (11), we analyzed the relations between them in the chapter 5.1. Here we investigate further the relations between the components of TTE and the intercept in (11). When KODEX leverage is held for a year, mean of TTE is -5.10 percent. Means of PTE, MTE, and CTE are 0.11 percent, -2.18 percent, and -3.03 percent, respectively. Also, the alpha in regression is -5.82 percent when the holding period is a year. It means that the intercept reflects the compounding effect and fund management costs. When the holding period for KODEX inverse is a year, means of TTE, PTE, MTE, CTE, and alpha are -1.18 percent, -0.01 percent, 1.54 percent, -2.71 percent and -2.54 percent, respectively. Similar to bull ETFs, alphas of bear ETFs reflect the compounding effects and fund management costs and incomes. Unlike bull ETFs, means of MTEs are positive in bear ETFs.

Carver (2009) and Wang (2009) show that when bull ETFs and bear ETFs are held for long term such as five years or ten years, value of investment can decrease substantially due to the compounding effect and volatility. We also find that the returns on leveraged ETFs deviate from the target return as the length of holding period increases. When bull or bear ETFs are held for longer than a year, return deviation has a large negative number. It means that alpha becomes smaller. Negative alpha is a cost irrelevant to return variations. It means that investments in leveraged ETFs for a long-term horizon are not beneficial to investors of at all. This is why investors have to pay more attention to α than to β investing in leveraged ETFs. Finally, large adjusted R² demonstrates good fitness of our results.

[Table 6 about here]

5.3 Analyses of Tracking Errors Using Volatility of Return Deviations

Table 7 reports tracking errors of TTE, PTE, MTE and CTE over various horizons. According to the table, tracking errors of TTEs increase as investment horizons get longer. Also, we can find what kinds of factors affect tracking errors of TTEs and how they vary across various buy-and-hold horizons. When bull ETFs are held for short-term (1-day, 2-day and 1-week), most of tracking errors of TTEs are explained by tracking errors of PTE and tracking errors of MTE. It means that total tracking errors are explained mostly by pricing inefficiency, tracking errors by fund management,

trading fees, and dividends when the holding period is short-term. When bull ETFs are held for one month, total tracking errors are explained evenly by tracking errors of PTE, tracking errors of MTE, and tracking errors of CTE. When the holding period is three months, tracking errors of TTE are explained mostly by tracking errors of MTE and tracking errors of CTE. However, tracking errors of TTE are explained mostly by tracking errors of CTE when the holding period is one year. It means that the compounding effect is the main source of total tracking errors in bull ETFs when investment horizons are long enough.

As for KODEX leverage, let's take a look at the tracking errors in connection with the mean of return deviation. When KODEX leverage is held for two days, means of TTE, PTE, MTE and CTE are -0.021 percent, -0.002 percent, -0.020 percent, and 0.000 percent, respectively (Table 4(a)) and tracking errors of TTE, PTE, MTE and CTE are 0.506 percent, 0.371 percent, 0.525 percent and 0.040 percent, respectively (Table 7). When KODEX leverage is held for two days, the mean of MTE mostly explains the mean of TTE, but the mean of PTE does little. In case of the tracking errors, however, tracking errors of PTE and MTE evenly explain tracking error of TTE. As we observe in Table 5 and Figure 2, the mean of TTE are not affected by the mean of PTE while the tracking error of TTE are quite affected by the tracking error of PTE because PTE are not variant around the mean due to arbitrage transactions.

When KODEX leverage is held for one year, the means of TTE, PTE, MTE and CTE are -5.102 percent, 0.106 percent, -2.183 percent, and -3.025 percent, respectively (Table 4(a)) and the tracking errors of TTE, PTE, MTE and CTE are 3.378 percent, 0.512 percent, 0.787 percent and 3.246 percent, respectively (Table 7). As you see, the mean of MTE and the mean of CTE explain the mean of TTE of KODEX leverage. However, as for the tracking error, tracking error of CTE explains mostly tracking error of TTE of KODEX leverage. The reason that the tracking error of MTE of KODEX leverage has a very limited effect on the tracking error of TTE in one-year investment horizon is that the ETF properly tracks daily index returns, and related expenses and incomes are paid or received periodically.

Tracking errors of bear ETFs are not different from those of bull ETFs. (Table 7) However, the main difference between the two types of ETFs is that tracking errors of PTEs explain most tracking errors of TTEs for short investment horizon. Tracking errors of MTEs are not large because incomes and costs are crossed out over time. Similar to bull ETFs, tracking errors of CTEs explain most of tracking errors of TTEs for long investment horizon. We mentioned earlier that the difference between return deviations of bear ETFs and those of bull ETFs comes from the MTE. As for tracking error, however, tracking errors of bear ETFs are similar to those of bull ETFs. This similarity is because expenses and incomes are paid or received periodically.

Table 8 demonstrates annualized tracking errors of leveraged ETFs. Overall, the annualized

tracking errors of leveraged ETFs are large when investment horizon is from 1 day to 1 week and becomes small when investment horizon is longer than a week. However, the tracking errors get larger again when investment horizon is one year. When holding periods are short, PTE and MTE are the main sources of variations in bull ETF and PTE is the source of variations in inverse ETF. CTE explains most variations in both bull and bear ETFs when investment horizon is long. To sum up, the compounding effect is the key contributor to total tracking errors of bull and bear ETFs when investment horizon is longer.

[Figure 2 about here]

[Table 7 about here]

[Table 8 about here]

5.4 Fund managers' management of tracking errors

ETF fund managers tracks 2x or (-1x) returns on underlying indices on a daily basis. The fund manager's tracking error is a main component of MTE. To examine fund manager's ability to keep track of underlying indices, we examine MTEs when the holding period is one day.

Main contributors of MTE include fund managers' tracking errors, related expenses, and incomes including dividends. These affect MTEs or tracking errors of MTEs. MTEs of bull ETF due to dividends diminish as expenses are settled. Expenses and incomes will not be significantly contributed to MTE because incomes will offset costs. While incomes from bull ETFs are not enough to pay off costs, those from bear ETFs are high enough to pay off their costs. Considering all these, the effect of costs and incomes on MTEs is thought to be very limited. Therefore, we can presume whether fund managers track the promised leveraged return on the underlying indices properly with MTEs or tracking errors of MTEs when the holding period is one day.

When bull ETFs are held for a day, the difference between means of NAV returns and the means of underlying indices 2x returns is not statistically different from zero (Table 9). It means that bull ETFs are well managed even when costs and dividends are taken into account. Figure 3 shows MTE time series of KODEX leverage, TIGER leverage, and KSTAR leverage. The return deviation moves around zero and Phillips-Perron unit root tests show that MTEs are stationary (Table 5). Also, annualized tracking errors of MTEs of bull ETFs range from 6.8 percent to 7.6 percent and these are not large (Table 8). Overall, although dividends and costs are considered, fund managers keep track of underlying indexes of bull ETF well.

In case of bear ETFs, NAV return on KODEX inverse is statistically different from the (-1x) underlying index only but the difference is minimal (Table 9). The tracking errors of MTEs of bear

ETFs look quite stable because the annualized tracking errors of MTE of bear ETFs are 1.09 percent~1.54 percent, which is less than those of bull ETF. This seems due to that costs, a main contributor of the tracking error, are crossed out by incomes. Also, MTEs of bear ETFs are all stationary (Figure 3, Table 5). Summing up, bear ETFs, like bull ETFs, are well managed by fund managers.

[Table 9 about here]

[Figure 3 about here]

5.5 Price Error

In our samples of bull and bear ETFs, price errors are reported. In all bull ETFs and KODEX inverse, especially, price errors are significantly different from zero at 1 percent confidence level, which means that price errors cannot to be ignored. KODEX leverage, TIGER leverage, KODEX inverse and TIGER inverse trade at a discount compared to their NAVs while KSTAR leverage and KOSEF inverse trade at a premium. Discount rate of KODEX leverage is large enough to execute an arbitrage trade but others are quite small. Annualized NAV return volatilities of bull ETFs are about 40 percent. It means that NAV volatilities of bull ETFs significantly affect price errors. However, annualized volatilities of bear ETFs are around 20 percent, which is quite small compared to those of bull ETFs (Table 10). This is consistent with the finding of Engle et al. (2006) and Charupat & Miu (2011) that the volatility of price errors in ETFs is related to that of their NAVs.

To conduct arbitrage transactions, price errors should be larger than bid-ask spread and transactions costs. Charupat & Miu (2011) report that arbitrage opportunities exist when price errors should be less than -0.1 percent or more than +0.1 percent of NAVs if the spread and transactions costs are considered. In case of KODEX leverage, the mean of price error is -0.243 percent, which implies that arbitrage transactions can be executed. Even if means of price errors are around zero, arbitrage opportunities exist if volatilities of price errors are high enough. In our sample, price errors range from 4.88 percent to 5.46 percent. The difference between 5th percentile and 95th percentile is less than 1 percent. Therefore, we conclude that arbitrage opportunities are generally unavailable to arbitrageurs.

In case of KODEX leverage and KODEX inverse, autocorrelation coefficients of the first and the second lags are significant. It means that price errors today are correlated with price errors of one day and two days ago. In other sample, autocorrelation coefficients of the first lags are only significant (Table 10). Phillips-Perron unit root test results report that price errors of ETFs in our sample are

stationary.²⁸ Considering the results in Table 10, we conclude that the market price quickly converges to the equilibrium price through arbitrage trades.

[Table 10 about here]

6. Conclusion

Investigating the data of Korean leveraged ETFs, we find that returns on leveraged ETFs are not much different from those on the indicated multiple of underlying indices for short buy-and-hold periods. However, the return deviations get larger as buy-and-hold periods are longer. When the holding period of leveraged ETFs is short, fund manager's tracking errors, related expenses and incomes explain most the return deviations. However, when the holding period of leveraged ETFs is longer than three months, especially a year, the negative compounding effect is the main contributor to the return deviations. In bear ETFs, incomes earned on cash reduce the return deviations.

As for results of the regression, we find that the slopes (β) of bull ETFs are not statistically different from zero when the holding periods are from one week to three months. The slopes of bear ETFs are not statistically different from zero when the holding periods are from two days to three months. In case of intercept (α), we find that the intercepts of bull ETFs are not statistically different from zero when the holding period is within one week. However, the intercept of bear ETF is not statistically different from zero when the holding period is within three months. Considering α and β altogether, we conclude that investments in bull ETFs for up to one week is not harmful to investors, but for longer than one month does harm to them. We also conclude that investments in bear ETFs for up to three months are fine to investors. However, when investors buy and hold the leveraged ETFs for a year without rebalancing the exposures, estimated α s range from -5.8 percent to -2.3 percent. These losses come mostly from the negative compounding effect.

In this paper, tracking errors using volatilities of the return deviations are also examined. We find that when the holding period is up to one week, price inefficiency, fund managers' tracking errors, and related expenses explain a large portion of total tracking errors of bull ETFs, whereas price inefficiency explains most of total tracking errors of bear ETFs. As the holding period gets longer, however, the compounding effects explain most of total tracking errors of bull ETFs and bear ETFs both. We examine MTEs of one-day holding period in order to determine whether fund managers appropriately deliver the promised ratio returns. We find that fund managers keep track of underlying indices properly. We find that some Korean leveraged ETFs trade at a discount and others trade at a premium on average. This is evidence that arbitrage opportunities are found in some leveraged ETFs.

²⁸ Test results will be given on request.

However, price errors of the leveraged ETFs are found to be stationary processes and the prices of leveraged ETFs quickly converge to the mean prices. In sum, we conclude that the leveraged ETF markets are efficient.

For a long-term investment horizon, the compounding effect is the main factor of the return deviation and the tracking error of the leveraged ETFs. Therefore, the buy-and-hold strategy for leveraged ETFs for long horizons is not recommendable.

Considering that historical data on an intraday and long term basis become available, we expect more academic research on the leveraged ETFs to come. Investigating arbitrage opportunities in leveraged ETF markets, for example, will be very interesting.

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Table 1. General Information on Leveraged ETFs²⁹

This table displays general information on the leveraged ETFs. The data periods for leveraged ETFs are from their inception to May 28, 2013. Volume: daily mean. Total net assets: value as of May 28, 2013. 2x bull ETF: Kodex Leverage, Tiger Leverage, Kstar Leverage, and Kindex Leverage. -1x bear ETF: Kodex Inverse, Tiger Inverse, Kosef Inverse, and Kindex Inverse. F-KOSPI200 refers to KOSPI200 futures.

Fund Name	Listing date	Benchmark Index	Volume*	Net Asset**
Kodex Leverage	Feb. 22, 2010	KOSPI200	15,147	2,129
Tiger Leverage	Apr. 09, 2010	KOSPI200	473	124
Kstar Leverage	Apr. 09, 2010	KOSPI200	20	22
Kindex Leverage	Jan. 27, 2012	KOSPI200	197	11
Kodex Inverse	Sep. 16, 2009	F-KOSPI200	12,261	377
Tiger Inverse	Mar. 29, 2010	F-KOSPI200	290	12
Kosef Inverse	Apr. 15, 2010	F-KOSPI200	23	2
Kindex Inverse	Sep. 08, 2011	F-KOSPI200	112	4

*: unit in thousands. **: unit in US\$ millions (KRW/US\$: 1,100).

²⁹ Korea Exchange (www.krx.co.kr). KRX ETF monthly, June, 2013. Samsung Asset Management (www.kodex.com). Mirae Asset Management (www.tigeretf.com). KB Asset Management (www.kbam.co.kr). Woori Asset Management (www.kosef.co.kr). Korea Investment Management (www.kindexetf.com). Management fees vary with the fund companies, and range from 0.15% to 0.64% per year. As of June 25, 2013, the fees of the leveraged ETFs on a yearly basis are as follows: Kodex Leverage (0.64%), Tiger Leverage (0.59%), Kstar Leverage (0.50%), Kindex Leverage (0.30%), Kodex Inverse (0.64%), Tiger Inverse (0.59%), Kosef Inverse (0.40%), and Kindex Inverse (0.15%).

Table 2. Descriptive Statistics of Leveraged ETFs

This table provides summary statistics of bull and bear ETFs. The data periods for the leveraged ETFs are from their inception to May 28, 2013. 2x bull ETF: Kodex Leverage, Tiger Leverage, Kstar Leverage, and Kindex Leverage. -1x bear ETF: Kodex Inverse, Tiger Inverse, Kosef Inverse, and Kindex Inverse. J-B refers to the Jacque-Bera test which tests a normal distribution of data. *: significance level at the 5%. **: significance level at the 1%. KRW/US\$=1,100.

Fund Name	Mean	Max.	Min.	Std.	Skewness	Kurtosis	J-B
Kodex Leverage	11.59	16.89	8.29	1.7594	0.8560	2.9659	0.00**
Tiger Leverage	10.34	14.76	7.20	1.4971	0.7010	2.8403	0.00**
Kstar Leverage	10.27	14.85	7.21	1.5234	0.7982	2.9341	0.00**
Kindex Leverage	4.28	5.00	3.56	0.3162	0.1613	2.2937	0.02*
Kodex Inverse	7.52	9.62	6.32	0.8262	0.7460	2.2619	0.00**
Tiger Inverse	7.76	9.86	6.77	0.6314	0.9627	3.0783	0.00**
Kosef Inverse	7.93	10.14	6.90	0.6501	1.0274	3.2933	0.00**
Kindex Inverse	8.40	10.05	7.72	0.4044	0.9427	4.1577	0.00**

Table 3(a). Correlation: Bull ETFs

This table displays the correlation between TTE and PTE, MTE & CTE and the correlation between MTE and CTE in the bull ETFs. TTE: return on a bull ETF– 2x return on its underlying index. PTE: return on a bull ETF– the compounding effect. CTE: the compounding effect–2 x return in its underlying index. The holding period of 1 day, 2 days, 1 week, 1 month, 3 months, and 1 year are 1, 5, 21, 63, and 252 trading days, respectively.

KODEX leverage	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(ρ)	1.000	1.000	1.000	1.000	1.000	1.000
ΡΤΕ(ρ)	0.332	0.311	0.307	0.262	0.240	0.290
ΜΤΕ(ρ)	0.692	0.734	0.741	0.710	0.672	0.107
CTE(ρ)		0.144	0.249	0.538	0.690	0.969
ΜΤΕ(ρ)		1.000	1.000	1.000	1.000	1.000
CTE(ρ)		-0.008	-0.058	-0.035	0.008	-0.095
TIGER leverage	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(ρ)	1.000	1.000	1.000	1.000	1.000	1.000
ΡΤΕ(ρ)	0.638	0.586	0.590	0.395	0.311	0.228
ΜΤΕ(ρ)	0.637	0.652	0.700	0.670	0.597	-0.132
CTE(ρ)		0.131	0.232	0.425	0.586	0.939
ΜΤΕ(ρ)		1.000	1.000	1.000	1.000	1.000
CTE(ρ)		-0.010	-0.121	-0.166	-0.209	-0.423
KSTAR leverage	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(ρ)	1.000	1.000	1.000	1.000	1.000	1.000
ΡΤΕ(ρ)	0.511	0.524	0.520	0.357	0.176	0.147
ΜΤΕ(ρ)	0.606	0.670	0.694	0.682	0.670	0.387
CTE(ρ)		0.082	0.230	0.553	0.675	0.961
ΜΤΕ(ρ)		1.000	1.000	1.000	1.000	1.000
CTE(ρ)		-0.015	-0.079	-0.029	-0.004	0.166

Table 3(b). Correlation: Bear ETFs

This table displays the correlation between TTE and PTE, MTE & CTE and the correlation between MTE and CTE in the bear ETFs. TTE: return on a bear ETF– (-1x) return on its underlying index. PTE: return on a bear ETF– return on its NAV. MTE: return on the NAV of a bear ETF–the compounding effect. CTE: the compounding effect–(-1x) return in its underlying index. The holding period of 1 day, 2 days, 1 week, 1 month, 3 months, and 1 year are 1, 5, 21, 63, and 252 trading days, respectively.

KODEX inverse	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(ρ)	1.000	1.000	1.000	1.000	1.000	1.000
ΡΤΕ(ρ)	0.969	0.968	0.843	0.484	0.385	-0.037
ΜΤΕ(ρ)	0.353	0.263	0.211	-0.074	-0.207	-0.867
CTE(ρ)		0.051	0.340	0.824	0.918	0.994
ΜΤΕ(ρ)		1.000	1.000	1.000	1.000	1.000
CTE(ρ)		-0.185	-0.222	-0.364	-0.478	-0.904
TIGER inverse	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(ρ)	1.000	1.000	1.000	1.000	1.000	1.000
ΡΤΕ(ρ)	0.974	0.971	0.875	0.559	0.324	-0.052
MTE(p)	0.161	0.084	0.186	0.339	0.428	-0.152
CTE(ρ)		0.067	0.332	0.800	0.926	0.993
ΜΤΕ(ρ)		1.000	1.000	1.000	1.000	1.000
CTE(ρ)		-0.164	0.003	0.242	0.323	-0.199
KOSEF inverse	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(ρ)	1.000	1.000	1.000	1.000	1.000	1.000
ΡΤΕ(ρ)	0.972	0.972	0.901	0.571	0.371	0.076
MTE(p)	0.078	-0.019	0.020	-0.045	-0.191	-0.715
CTE(ρ)		0.052	0.269	0.773	0.907	0.989
ΜΤΕ(ρ)		1.000	1.000	1.000	1.000	1.000
CTE(ρ)		-0.173	-0.132	-0.143	-0.332	-0.767

Table 4(a). Decomposition of Return Deviation: Bull ETFs

This table displays the decomposition of return deviation of the bull ETFs. TTE: return on a bull ETF– 2x return on its underlying index. PTE: return on a bull ETF– return on its NAV. MTE: return on the NAV of a bull ETF– the compounding effect. CTE: the compounding effect–2 x return in its underlying index. The holding period of 1 day, 2 days, 1 week, 1 month, 3 months, and 1 year are 1, 5, 21, 63, and 252 trading days, respectively.

KODEX leverage	1 day	2 days	1 week	1 month	3 months	1 year
TTE(m)	-0.011%	-0.021%	-0.053%	-0.245%	-0.926%	-5.102%
(p-value)	(0.500)	(0.235)	(0.009)	(0.000)	(0.000)	(0.000)
PTE(m)	-0.001%	-0.002%	-0.005%	-0.004%	0.009%	0.106%
MTE(m)	-0.010%	-0.020%	-0.043%	-0.184%	-0.517%	-2.183%
CTE(m)		0.000%	-0.006%	-0.057%	-0.418%	-3.025%
TIGER leverage	1 day	2 days	1 week	1 month	3 months	1 year
TTE(m)	-0.005%	-0.008%	-0.025%	-0.125%	-0.582%	-4.203%
(p-value)	(0.816)	(0.709)	(0.305)	(0.000)	(0.000)	(0.000)
PTE(m)	-0.001%	0.000%	-0.003%	-0.006%	0.002%	-0.041%
MTE(m)	-0.003%	-0.008%	-0.016%	-0.058%	-0.180%	-0.849%
CTE(m)		0.000%	-0.006%	-0.060%	-0.404%	-3.313%
KSTAR leverage	1 day	2 days	1 week	1 month	3 months	1 year
TTE(m)	-0.008%	-0.015%	-0.037%	-0.170%	-0.717%	-4.805%
(p-value)	(0.668)	(0.485)	(0.133)	(0.000)	(0.000)	(0.000)
PTE(m)	-0.002%	-0.002%	-0.004%	-0.005%	-0.013%	-0.038%
MTE(m)	-0.006%	-0.013%	-0.027%	-0.105%	-0.300%	-1.455%
CTE(m)		0.000%	-0.006%	-0.060%	-0.404%	-3.313%

Table 4(b). Decomposition of Return Deviation: Bear ETFs

This table displays the decomposition of return deviation of the bear ETFs. TTE: return on a bear ETF– (-1x) return on its underlying index. PTE: return on a bear ETF– return on its NAV. MTE: return on the NAV of a bear ETF–the compounding effect. CTE: the compounding effect–(-1x) return in its underlying index. The holding period of 1 day, 2 days, 1 week, 1 month, 3 months, and 1 year are 1, 5, 21, 63, and 252 trading days, respectively.

KODEX inverse	1 day	2 days	1 week	1 month	3 months	1 year
TTE(m)	0.006%	0.012%	0.023%	0.056%	-0.132%	-1.181%
(p-value)	(0.609)	(0.483)	(0.265)	(0.008)	(0.000)	(0.000)
PTE(m)	0.000%	0.000%	-0.002%	-0.007%	-0.010%	-0.014%
MTE(m)	0.006%	0.012%	0.030%	0.127%	0.386%	1.542%
CTE(m)		0.000%	-0.005%	-0.064%	-0.507%	-2.709%
TIGER inverse	1 day	2 days	1 week	1 month	3 months	1 year
TTE(m)	0.008%	0.008%	0.026%	0.092%	0.025%	-1.734%
(p-value)	(0.636)	(0.480)	(0.296)	(0.050)	(0.000)	(0.000)
PTE(m)	0.000%	-0.007%	-0.007%	-0.007%	-0.003%	0.065%
MTE(m)	0.008%	0.015%	0.039%	0.166%	0.504%	1.968%
CTE(m)		0.000%	-0.006%	-0.066%	-0.476%	-3.766%
KOSEF inverse	1 day	2 days	1 week	1 month	3 months	1 year
TTE(m)	0.007%	0.015%	0.032%	0.089%	-0.004%	-1.972%
(p-value)	(0.705)	(0.588)	(0.351)	(0.034)	(0.000)	(0.000)
PTE(m)	0.000%	0.001%	0.001%	0.002%	0.001%	0.013%
MTE(m)	0.007%	0.014%	0.036%	0.152%	0.464%	1.938%
CTE(m)		-0.001%	-0.006%	-0.065%	-0.469%	-3.924%

Table 5. Phillips-Perron Unit Root Test

This table displays Phillips-Perron unit root test for TTE, PTE, MTE, and CTE. TTE: return on a leveraged ETF -2x or (-1x) return on its underlying index. PTE: return on a leveraged ETF– return on its NAV. MTE: return on the NAV of a leveraged ETF–the compounding effect. CTE: the compounding effect–2x or (-1x) return on the underlying index. Numbers in this table are p-values. The holding period of 1 day, 2 days, 1 week, 1 month, 3 months, and 1 year are 1, 5, 21, 63, and 252 trading days, respectively.

KODEX leverage	1 day	2 days	1 week	1 month	3 months	1 year
TTE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0868
РТЕ	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MTE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
CTE		0.0000	0.0000	0.0000	0.0025	0.2430
TIGER leverage	1 day	2 days	1 week	1 month	3 months	1 year
TTE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0112
РТЕ	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MTE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
СТЕ		0.0000	0.0000	0.0000	0.0031	0.1833
KSTAR leverage	1 day	2 days	1 week	1 month	3 months	1 year
TTE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0456
РТЕ	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MTE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
CTE		0.0000	0.0000	0.0000	0.0031	0.1833
KODEX inverse	1 day	2 days	1 week	1 month	3 months	1 year
TTE	0.0000	0.0000	0.0000	0.0000	0.0000	0.3173
РТЕ	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MTE	0.0000	0.0000	0.0000	0.0000	0.0000	0.4005
MTE CTE	0.0000	0.0000 0.0000	0.0000 0.0000	0.0000 0.0000	0.0000 0.0002	0.4005 0.5071
MTE CTE TIGER inverse	0.0000 1 day	0.0000 0.0000 2 days	0.0000 0.0000 1 week	0.0000 0.0000 1 month	0.0000 0.0002 3 months	0.4005 0.5071 1 year
MTE CTE TIGER inverse TTE	0.0000 1 day 0.0000	0.0000 0.0000 2 days 0.0000	0.0000 0.0000 1 week 0.0000	0.0000 0.0000 1 month 0.0000	0.0000 0.0002 3 months 0.0000	0.4005 0.5071 1 year 0.3448
MTE CTE TIGER inverse TTE PTE	0.0000 1 day 0.0000 0.0000	0.0000 0.0000 2 days 0.0000 0.0000	0.0000 0.0000 1 week 0.0000 0.0000	0.0000 0.0000 1 month 0.0000 0.0000	0.0000 0.0002 3 months 0.0000 0.0000	0.4005 0.5071 1 year 0.3448 0.0000
MTE CTE TIGER inverse TTE PTE MTE	0.0000 1 day 0.0000 0.0000 0.0000	0.0000 0.0000 2 days 0.0000 0.0000 0.0000	0.0000 0.0000 1 week 0.0000 0.0000 0.0000	0.0000 0.0000 1 month 0.0000 0.0000 0.0000	0.0000 0.0002 3 months 0.0000 0.0000 0.0000	0.4005 0.5071 1 year 0.3448 0.0000 0.0000
MTE CTE TIGER inverse TTE PTE MTE CTE	0.0000 1 day 0.0000 0.0000 0.0000	0.0000 0.0000 2 days 0.0000 0.0000 0.0000 0.0000	0.0000 0.0000 1 week 0.0000 0.0000 0.0000 0.0000	0.0000 0.0000 1 month 0.0000 0.0000 0.0000 0.0000	0.0000 0.0002 3 months 0.0000 0.0000 0.0000 0.0015	0.4005 0.5071 1 year 0.3448 0.0000 0.0000 0.5243
MTE CTE TIGER inverse TTE PTE MTE CTE KOSEF inverse	0.0000 1 day 0.0000 0.0000 0.0000 1 day	0.0000 0.0000 2 days 0.0000 0.0000 0.0000 2 days	0.0000 0.0000 1 week 0.0000 0.0000 0.0000 1 week	0.0000 0.0000 1 month 0.0000 0.0000 0.0000 1 month	0.0000 0.0002 3 months 0.0000 0.0000 0.0000 0.0015 3 months	0.4005 0.5071 1 year 0.3448 0.0000 0.0000 0.5243 1 year
MTE CTE TIGER inverse TTE PTE MTE CTE KOSEF inverse TTE	0.0000 1 day 0.0000 0.0000 0.0000 1 day 0.0000	0.0000 0.0000 2 days 0.0000 0.0000 0.0000 2 days 0.0000	0.0000 0.0000 1 week 0.0000 0.0000 0.0000 1 week 0.0000	0.0000 0.0000 1 month 0.0000 0.0000 0.0000 1 month 0.0000	0.0000 0.0002 3 months 0.0000 0.0000 0.0000 0.0015 3 months 0.0000	0.4005 0.5071 1 year 0.3448 0.0000 0.0000 0.5243 1 year 0.2074
MTE CTE TIGER inverse TTE PTE MTE CTE KOSEF inverse TTE PTE	0.0000 1 day 0.0000 0.0000 1 day 0.0000 0.0000	0.0000 0.0000 2 days 0.0000 0.0000 0.0000 2 days 0.0000 0.0000	0.0000 0.0000 1 week 0.0000 0.0000 0.0000 1 week 0.0000 0.0000	0.0000 0.0000 1 month 0.0000 0.0000 0.0000 1 month 0.0000 0.0000	0.0000 0.0002 3 months 0.0000 0.0000 0.0000 0.0015 3 months 0.0000 0.0000	0.4005 0.5071 1 year 0.3448 0.0000 0.0000 0.5243 1 year 0.2074 0.0000
MTE CTE TIGER inverse TTE PTE MTE CTE KOSEF inverse TTE PTE MTE	0.0000 1 day 0.0000 0.0000 0.0000 1 day 0.0000 0.0000 0.0000	0.0000 0.0000 2 days 0.0000 0.0000 0.0000 2 days 0.0000 0.0000 0.0000	0.0000 0.0000 1 week 0.0000 0.0000 0.0000 1 week 0.0000 0.0000 0.0000	0.0000 0.0000 1 month 0.0000 0.0000 0.0000 1 month 0.0000 0.0000 0.0000	0.0000 0.0002 3 months 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000	0.4005 0.5071 1 year 0.3448 0.0000 0.0000 0.5243 1 year 0.2074 0.0000 0.0137

Table 6. Regression Analyses

This table displays the results of regression of return on a leveraged ETF on return on its underlying index. The data periods for the leveraged ETFs are from their inception to May 28, 2013. The holding period of 1 day, 2 days, 1 week, 1 month, 3 months, and 1 year are 1, 5, 21, 63, and 252 trading days, respectively. *: significance level at the 5%. **: significance level at the 1%.

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KODEX leverage	1 day	2 days	1 week	1 month	3 months	1 year
# of obs.	813	812	809	793	751	562
intercept(a)	-0.0001	-0.0002	-0.0005*	-0.0025**	-0.0098**	-0.0582**
$coefficient(\beta)$	1.9250**	1.9382**	1.9908	2.0034	2.0331	2.2172**
adjusted R ²	0.9670	0.9799	0.9886	0.9922	0.9908	0.9924
TIGER leverage	1 day	2 days	1 week	1 month	3 months	1 year
# of obs.	780	779	776	760	718	529
intercept(a)	0.0000	-0.0001	-0.0002	-0.0013	-0.0062**	-0.0462**
$coefficient(\beta)$	1.9333**	1.9585*	1.9911	2.0017	2.0248	2.1574**
adjusted R ²	0.9539	0.9726	0.9924	0.9918	0.9924	0.9928
KSTAR leverage	1 day	2 days	1 week	1 month	3 months	1 year
# of obs.	780	779	776	760	718	529
intercept(a)	-0.0001	-0.0001	-0.0004	-0.0017**	-0.0077**	-0.0531**
$coefficient(\beta)$	1.9032**	1.9389**	1.9941	2.0046	2.0312	2.1900**
adjusted R ²	0.9594	0.9728	0.9844	0.9914	0.9910	0.9910
KODEX inverse	1 day	2 days	1 week	1 month	3 months	1 year
# of obs.	921	920	917	901	859	670
intercept(a)	0.0001	0.0001	0.0002*	0.0006	-0.0014	-0.0254**
$coefficient(\beta)$	-0.9785*	-0.9890	-0.9927	-1.0083	-0.9954	-0.7773**
adjusted R ²	0.9539	0.9743	0.9880	0.9888	0.9844	0.9751
TIGER inverse	1 day	2 days	1 week	1 month	3 months	1 year
# of obs.	789	788	785	769	727	538
intercept(a)	0.0001	0.0001	0.0002	0.0010	-0.0001	-0.0230**
coefficient(β)	-0.9428**	-0.9820	-0.9900	-1.0078	-0.9769	-0.7625**
adjusted R ²	0.9001	0.9545	0.9783	0.9817	0.9789	0.9558
KOSEF inverse	1 day	2 days	1 week	1 month	3 months	1 year
# of obs.	776	775	772	756	714	525
intercept(a)	0.0001	0.0001**	0.0003*	0.0010	-0.0001	-0.0237**
$coefficient(\beta)$	-0.9705**	-1.0015	-1.0044	-1.0217	-0.9941	-0.7873**
adjusted R ²	0.9064	0.9474	0.9775	0.9854	0.9828	0.9582

Table 7. Tracking Errors of Leveraged ETFs

This table displays tracking errors of bull and bear ETFs. TTE: return on a leveraged ETF–2x or (-1x) return on its underlying index. PTE: return on a leveraged ETF–return on its NAV. MTE: return on the NAV of a leveraged ETF–the compounding effect. CTE: the compounding effect–2x or (-1x) return on the underlying index. Numbers in this table are p-values. The holding period of 1 day, 2 days, 1 week, 1 month, 3 months, and 1 year are 1, 5, 21, 63, and 252 trading days, respectively.

KODEX leverage	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	0.454%	0.506%	0.573%	0.919%	1.494%	3.378%
ΡΤΕ(σ)	0.368%	0.371%	0.373%	0.401%	0.433%	0.512%
ΜΤΕ(σ)	0.480%	0.525%	0.570%	0.767%	1.070%	0.787%
CTE(σ)		0.040%	0.146%	0.501%	0.974%	3.246%
TIGER leverage	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	0.544%	0.594%	0.677%	0.942%	1.376%	2.913%
ΡΤΕ(σ)	0.427%	0.457%	0.441%	0.483%	0.473%	0.532%
MTE(σ)	0.426%	0.492%	0.546%	0.797%	1.085%	1.054%
$CTE(\sigma)$		0.041%	0.149%	0.511%	0.993%	3.119%
KSTAR leverage	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	0.510%	0.592%	0.677%	0.977%	1.495%	3.394%
ΡΤΕ(σ)	0.438%	0.454%	0.460%	0.457%	0.468%	0.519%
ΜΤΕ(σ)	0.473%	0.524%	0.581%	0.779%	1.108%	0.828%
$CTE(\sigma)$		0.041%	0.149%	0.511%	0.993%	3.119%
KODEX inverse	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	0.277%	0.291%	0.303%	0.564%	0.903%	3.436%
ΡΤΕ(σ)	0.261%	0.280%	0.273%	0.290%	0.272%	0.278%
MTE(σ)	0.068%	0.069%	0.084%	0.124%	0.222%	0.562%
CTE(σ)		0.039%	0.161%	0.525%	0.920%	3.957%
TIGER inverse	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	0.416%	0.393%	0.413%	0.739%	1.130%	3.531%
ΡΤΕ(σ)	0.412%	0.394%	0.385%	0.430%	0.399%	0.381%
MTE(σ)	0.095%	0.093%	0.107%	0.148%	0.188%	0.213%
CTE(σ)		0.041%	0.170%	0.560%	0.994%	3.607%
KOSEF inverse	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	0.411%	0.433%	0.429%	0.680%	1.029%	3.195%
ΡΤΕ(σ)	0.415%	0.445%	0.423%	0.437%	0.420%	0.418%
MTE(σ)	0.097%	0.103%	0.107%	0.125%	0.182%	0.427%
$CTE(\sigma)$		0.042%	0.171%	0.565%	1 001%	3 506%

Table 8. Annualized Tracking Errors of Leveraged ETFs

This table displays annualized tracking errors of leveraged ETFs. TTE: return on a leveraged ETF-2x or (-1x) return on its underlying index. PTE: return on a leveraged ETF-return on its NAV. MTE: return on the NAV of a leveraged ETF- the compounding effect. CTE: the compounding effect-2x or (-1x) return on the underlying index. Numbers in this table are p-values. The holding period of 1 day, 2 days, 1 week, 1 month, 3 months, and 1 year are 1, 5, 21, 63, and 252 trading days, respectively.

KODEX leverage	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	7.214%	5.681%	4.071%	3.184%	2.989%	3.378%
ΡΤΕ(σ)	5.836%	4.160%	2.649%	1.389%	0.866%	0.512%
$MTE(\sigma)$	7.625%	5.893%	4.046%	2.658%	2.141%	0.787%
CTE(σ)		0.448%	1.038%	1.735%	1.947%	3.246%
TIGER leverage	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	8.637%	6.666%	4.804%	3.264%	2.753%	2.913%
ΡΤΕ(σ)	6.780%	5.129%	3.131%	1.673%	0.946%	0.532%
$MTE(\sigma)$	6.769%	5.525%	3.876%	2.762%	2.170%	1.054%
CTE(σ)		0.456%	1.058%	1.769%	1.987%	3.119%
KSTAR leverage	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	8.103%	6.649%	4.807%	3.385%	2.990%	3.394%
ΡΤΕ(σ)	6.951%	5.101%	3.268%	1.583%	0.935%	0.519%
$MTE(\sigma)$	7.513%	5.878%	4.123%	2.700%	2.217%	0.828%
CTE(σ)		0.456%	1.058%	1.769%	1.987%	3.119%
KODEX inverse	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	4.396%	3.266%	2.150%	1.952%	1.807%	3.436%
ΡΤΕ(σ)	4.140%	3.140%	1.941%	1.004%	0.545%	0.278%
$MTE(\sigma)$	1.086%	0.777%	0.596%	0.430%	0.444%	0.562%
CTE(σ)		0.443%	1.140%	1.818%	1.841%	3.957%
TIGER inverse	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	6.609%	4.415%	2.933%	2.560%	2.259%	3.531%
ΡΤΕ(σ)	6.539%	4.425%	2.732%	1.490%	0.797%	0.381%
$MTE(\sigma)$	1.511%	1.044%	0.763%	0.512%	0.376%	0.213%
CTE(σ)		0.463%	1.207%	1.941%	1.987%	3.607%
KOSEF inverse	1 day	2 days	1 week	1 month	3 months	1 year
ΤΤΕ(σ)	6.520%	4.862%	3.045%	2.357%	2.058%	3.195%
ΡΤΕ(σ)	6.581%	4.999%	3.000%	1.512%	0.840%	0.418%
$MTE(\sigma)$	1.541%	1.162%	0.763%	0.434%	0.364%	0.427%
$CTE(\sigma)$		0.467%	1.217%	1.957%	2.002%	3.506%

Table 9. MTEs and Tracking Errors of MTEs

This table displays MTEs and tracking errors of MTEs. P-value tests whether return on the NAV of a leveraged ETF equals 2x or (-1x) return on underlying index. Annualized tracking error is calculated on a 252 trading day basis.

FUND	MTE mean	p-value	MTE tracking error	MTE tracking error (annualized)
KODEX leverage	-0.010%	0.570	0.480%	7.625%
TIGER leverage	-0.003%	0.819	0.426%	6.769%
KSTAR leverage	-0.006%	0.724	0.473%	7.513%
KODEX inverse	0.006%	0.009	0.068%	1.086%
TIGER inverse	0.008%	0.024	0.095%	1.511%
KOSEF inverse	0.007%	0.043	0.097%	1.541%

Table 10. Price Errors of Leveraged ETFs

P-value tests whether price of a leveraged ETF equals its NAV. The asterisk (*) in autocorrelation columns shows that the coefficient is out of 95% confidence band of Bartlett's formula for MA(q). Annualized volatility of NAV of a leveraged ETF is calculated on a 252 trading day basis. *: significance level at the 5%. **: significance level at the 1%.

	Mean	Percentile		Volatility	Autocorrelation		Volatility (annualized)
		5%	95%	PE	lag1	lag2	NAV
KODEX leverage	-0.243%**	-0.733%	0.219%	0.344%	0.244*	0.121*	39.67%
TIGER leverage	-0.042%**	-0.480%	0.377%	0.339%	0.189*	0.070	40.30%
KSTAR leverage	0.054%**	-0.385%	0.592%	0.335%	0.117*	0.033	39.98%
KODEX inverse	-0.093%**	-0.379%	0.224%	0.206%	0.199*	0.081*	20.16%
TIGER inverse	-0.025%*	-0.457%	0.273%	0.326%	0.179*	0.042	20.40%
KOSEF inverse	0.023%*	-0.315%	0.318%	0.307%	0.067	-0.074	20.65%



Figure 1. TTEs and CTEs of KODEX leverage and KODEX inverse: one-year holding period



Figure 2. PTE of KODEX Leverage: two-day holding period



