Stability of financial system and connection distribution in bipartite network

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Hyoung goo, Kang

Hyeon jong, Jeong

Abstract

To understand the relationship between stability of financial system and connection distribution of banks and assets, we develop and use simplified model in bipartite network and suggest implications for regulators. Major findings are as follows: First, the more the proportion of banks with higher degree of connection between banks and assets, the more robust the financial system is. The system is more stress-resistant but simultaneous risk gets higher (robust yet fragile). Second, banks with lower degree over 1 function as weak shock transmitter. Therefore, to mitigate risk contagion or system failures, less proportion of these banks in the network is beneficial for the whole system. These banks are weak links of risk spillover during financial crisis and systemically important intermediaries for regulatory body. Third, to contain or mitigate risk contagion, financial system with well diversified and specialized banks (barbell structure) is more desirable than uniform structure. The more proportion to the 2 extremes in connection distribution, the more stable the financial system is to the asset shock in bipartite network. More capital buffer or limit to leverage ratio to banks with lower degrees could enhance the resilience of the whole system.

I. Introduction

The 2008 financial crisis led to the massive failures of banks in the United States. According to Federal Deposit Insurance Corporation (FDIC), 465 banks failed for the period from 2008 to 2012, which is a stark contrast to 10 failed banks in the five years prior to 2008.[1] A cascading failure in financial system is a failure of interconnected banks in which the failure of a bank can trigger the default of successive banks. As financial networks are becoming increasingly dependent on one another, financial risk contagion or cascading failure using network model has been investigated for a recent decade.

There is not a clear legal or formal definition of what constitutes "systemic" in risk management context. Broadly, an institution, market or instrument is defined as systemic by regulatory body if its failure causes widespread distress, either as a direct impact or as a trigger for broader contagion. (FSB, IMF and BIS, 2009) And many existing literatures focus on relationship between network properties and system risk. For example, previous researches analyze homogenous random network which assumes each bank has the same probability of being connected to another bank or asset. And they use simulation approach to test the effect of parameters such as connection probability, capitalization of banks and leverage ratio on system risk with a given level of shock.

But the effect of various structure of network on the system failures has not been investigated well. Financial network is made of complicated links between banks and assets. And each network has specific connection distribution which is different from random network model. If all the banks in the network are specialized bank, degree of connection would be one. On the extreme of the other side, if all the banks are fully diversified universal banks, degree of connection would equal to number of assets in the market. And many other banks will be located in somewhere between them. Financial network is a combination of banks with various spectrums of connection degrees, therefore each network has typical connection distribution of its own.

The research question in the paper is "what is the effective network structure in connection distribution for stability of financial system". We are interested in finding connection distribution of banks and assets in financial network which minimize default probability induced by asset price shock. First, using uniform connection distribution, this paper sheds insight to the risk propagation process in the bipartite network which consists of banks and assets. Then other connection distribution is tested to find out more stress-resistant system. Also we investigate how to efficiently allocate limited capital to banks in the system to mitigate the loss from system failures. To understand the relationship between connection distribution in network and stability of financial system, we develop and use simplified model and suggest implications for regulators.

This paper is organized as follows. We begin by reviewing literatures regarding system risks and network structure. Next we postulate simplifying assumptions and model specification including network structure, structure of bank's balance sheet and default propagation process to simulate default rate as function of shock to asset. After explaining detailed test, we close with conclusions and limitations.

II. Literature Review

As economic entities become increasingly interconnected, a shock in a financial system can provoke significant cascading failures throughout the system. Financial risk contagion comes through different channels including counterparty risk, common asset holdings, general loss of confidences and others (Osiński et al., 2013). There are two approaches to investigate the risk contagion in theoretical literature. First, many of the existing researches are investigating risk contagion due to direct interbank lending which occurs when bankrupt institution is unable to pay its debt and consequently causes other institutions to fail. Counterparty risk and roll-over risk caused by interconnected banks are being extensively studied in Gai and Capadia (2010), May and Arinaminpathy (2010) and others.

[1] Failed Bank List, Federal Deposit Insurance Corporation (May, 2014).

Second approach focuses on indirect balance-sheet linkages. In this paper, our attention is devoted to indirect common asset holdings (asset commonality) risk. Common asset holdings risk or risk due to overlapping portfolios occurs in the event that asset price fluctuation causes an institution to fail and the resulting endogenous 'fire-sale' of assets by that institution further decreases asset price. Common asset holdings risk is important in system risk because it can be easily observed during debt deflation in financial crisis and even makes the situation worse. Cont and Wagalath (2011) illustrate feedback effects due to distressed selling lead to endogenous correlations between assets which were previously uncorrelated.

Regarding the role of diversification on system risk, existing literature demonstrates that diversification is optimal for individual institutions, but not for system (Ibragimov et al., 2011, Beal et al., 2011, Caccioli et al., 2012). Ibragimov et al. (2011) demonstrate that diversification is optimal for individual institutions, but may be suboptimal for system. Individual bank may succeed in eliminating the idiosyncratic risks embedded in the portfolios by sharing risks, but interconnectedness increases the risk of massive systemic failure. Also Beale et al. (2011) argues that diversification is good for individual institutions but diversification in similar ways increases multiple failures. They demonstrate that higher portfolio diversity among banks, not just diversification is conducive to reduce system risk. As portfolio diversity reduces system risk and diversification plays a role of decreasing individual risk, these two strategies have a trade-off relationship.

According to Allen and Gale (2000) as well as Gai and Kapadia (2010), network connectivity can have opposite effects on risk contagion. When the network is sparsely connected, increasing degrees of connection will increase risk contagion and weaken the network. On the other hand, when degree of connection is sufficiently large, diversification benefits overweight risk contagion through network and strengthen the network. Also Nier et al. (2007) investigates how systemic risk is affected by the structure of the financial system and argues that non-monotonic effect of the degree of connectivity on contagion effect in simulated interbank market.

Relatively small attention was given to the indirect effect of risk propagation through common asset holdings in financial crisis. Recently several studies investigated the relationship between connection probability and default rate in bipartite network of bank and asset. According to Cifuentes et al. (2005), when combined with mark-to-market practice, changes in asset prices interact with externally imposed solvency requirements and generate amplified endogenous responses that are disproportionately large relative to initial shock. With an analysis of financial risk contagion due to overlapping portfolios, Caccioli et al. (2012) find that financial intermediary system has ‘robust yet fragile’ characters. There are regions in parameter space where cascading failures are very unlikely, but when it occurs almost the entire system is affected. While it shows average default rate as function of probability of connection between bank and asset, the analysis does not deal with the effect of various structure of network on system failures. For example, Erdős–Rényi model which is used in the literature generates homogeneous random graphs that sets an edge between each pair of nodes with equal probability, independently of the other edges, but it does not have heavy tails as in real networks and has low clustering.

It would be more reasonable to consider actual financial network as collection of diversified universal banks and many other specialized banks whose business and expertise are focused on specific financial assets (e.g. commercial mortgages, distressed assets and small business loans etc). As a natural consequence, real probability of degrees of connection might differ across financial intermediaries and do not follow ordinary poisson distribution[2] which is implicitly assumed in Erdős–Rényi probability. Also from regulatory perspective, it is critical to find weak link of the system through which financial contagion occurs, not just system risk’s sensitivity to connection probability because connection distribution or network structure is hard to control. In

[2] In random graph model of Erdős–Rényi probability, a graph is constructed by connecting nodes randomly. Each edge in the graph has probability p independent from every other edge. The distribution of the degree of any particular node is binomial and follows poisson distribution for degree k, large number of nodes, n and constant np. $P(k)=(np)^k e^{-np}/k!$. But Barabasi and albert (1999) demonstrates that in random graphs, highly connected nodes decrease exponentially with degree of connection, thus nodes with large connectivity are practically absent. Meanwhile, connection degree for real-world network such as world wide web or cited papers follows power-law distribution with highly connected nodes having a large chance of occurring.

this study, we try to find insights from relationship between the network structure with varying connection distribution and system risk using simulation approach in bipartite network.

Our approach complements the results of the Caccioli et al. (2013) and Huang et al. (2013) which analyze the relationship between network structure and financial contagion due to overlapping portfolios. Huang et al. (2013) empirically test the bank-asset network model with 2007 US commercial banks balance sheet data and identify significant portion of actual failed banks using bipartite model of banks and assets. We use similar approach in bipartite network model which contains two types of nodes, banks and assets with links between them.

III. Model Specification

Banks are interconnected in many ways and this paper will consider only the risk of common asset holdings[3]. When a bank is not able to meet its liability and fails, it needs to sell its assets to meet the liability. If the market can't absorb the sale, then the value of this asset will decrease, which in turn may cause other institutions to fail. If the financial system is stable, shocks will not propagate. But if it is unstable a shock can be amplified and trigger cascading bankruptcies. Thus we can create a bipartite banking network composed of banks and bank assets. And we propose a cascading failure model to describe the risk propagation process during crises. This study relies on several simplified assumptions on the balance sheet of individual bank, how the network is structured as well as risk propagation process which will be described in the next section. We relax these assumptions by varying network structures.

3.1. Network Structure

In this paper, our focus is on how connection distribution affects the systemic failures in crisis. Thus hypothetical structure of network system is tested with shocks given to a random asset. While these simplified assumptions are rather arbitrary, we can test varied structure and gain insight from it, as discussed later.

First, random bipartite network is drawn between banks and assets. Probability of linkage between bank and asset follows Erdős–Rényi probability with i.i.d. assumption. In graph theory, the Erdős–Rényi model generates random graphs that set an edge between each pair of nodes with equal probability, independently of the other edges. Let's assume a bipartite graph of N banks and M assets, in which each bank has u assets and each asset is invested by v banks. Then total degree of banks is the same as total degree of assets and equation $u/M = v/N$ holds. Edge is drawn with probability of u/M and with limit of N, random networks have poisson degree distribution for both banks and assets. But Erdős–Rényi graph has low clustering and does not have heavy tails as in real networks. So other simplified discrete degree distributions such as uniform, bullet and barbell distribution are tested.

Second, uniform degree distribution is simply generated. Uniform degree distribution is symmetrically discrete or categorical distribution where each of categorized bank group has specific connection degree. If there are 10 bank groups with varying degree of connection in uniform degree distribution, banks in each bank group share the same connection degree. For example, each bank group which is 10% of total banks has connection with random assets according to its degrees. 10% of banks have degree of 1 and another 10% of banks have degree of 2 and so on. Uniform degree distribution is unlikely to have resemblance with real network, but helps to investigate the risk propagation process and find the weak link of system failures.

And lastly, we construct bullet and barbell degree distribution. In bullet degree distribution, all banks have the same degree of connection. Bullet degree distribution is a special case of Erdős–Rényi model with probability of 1. Whereas, in barbell degree distribution, banks are divided between banks with lower and higher degrees.

[3] No link between banks or between assets is assumed in the test. Therefore, in bipartite network, there is no interbank market or credit risk contagion. Also the model does not account for asset return correlation which usually becomes higher during financial stress for simplicity.

Barbell degree distribution is constructed to measure the role of banks with specific connection degree in system failures.

3.2. Structure of Balance Sheet

The structure of balance sheet has several assumptions. All the banks have the same size of total assets, A_0 at time 0 and assets are evenly split in balance sheet. Thus investments per assets, B^m are simply total assets divided by degree of connection. And the balance sheets of the banks with the same degree of connection are homogenous across banks. The number of assets, $m \in \{1, 2, \cdots, M\}$[4] and the individual bank, $i \in \{1, 2, \cdots, N\}$. The capital, E_0 and liability, L_0 is the same across all i and initial leverage ratio, λ_0 is total asset divided by equity capital at time 0.

$A_0 = A_{i,0} = \sum_{m=1}^{M} B_{i,0}^{m}$ for $m=1, 2, \cdots, M$ and $i=1, 2, 3, \cdots N$

$A_0 = E_0 + L_0 = E_{i,0} + L_{i,0}$ for all i

$E_0 = E_{i,0}$ for all i

λ_0 *(Leverage ratio)* $= A_0 / E_0$ for all i

Important assumption of structure of balance sheet is that total asset and equity are the same for all i irrespective of network structure. That means total asset of banks with higher connection degrees is equal to that of banks with lower degrees. This assumption is made to test the effect of degree distribution on risk contagion irrespective of bank size or centrality properties of network structure. Number of assets and leverage ratio are chosen for the maximum shock to a random asset not to cause defaults of banks with fully diversified banks[5].

3.3. Default propagation process

Usually default propagation process starts with significant loss of assets and the following market crash. In this paper, using simplified assumptions, we assume default propagation or risk contagion in financial network proceeds as follows;

3.3.1 First shock and default condition

All the shocks to an asset represent idiosyncratic risk and asset returns are assumed to have no correlation with other asset's returns. With first shock to a random asset, all banks function as shock absorber up to their capital buffer. Less diversified banks are more prone to default due to more exposure to infected asset, while more diversified banks are less exposed to default risk because, in uniform degree distribution, invested assets decrease as degree of connection increases.

Asset side of bank i's balance sheet at time 0, $A_{i,0}$ is the sum of investment amount, $B^{m}_{i,0}$ for $m=1, 2, \cdots, M$. There are M assets in the market and a random asset, B^1 suffers from first shock of p at time 0. If investment amount, $B^{1}_{i,0}$ in balance sheet of bank i at time 1 is affected linearly by shock p, It can be shown that,

Total asset of bank i at time 0 is $A_{i,0} = \sum_{m=1}^{M} B_{i,0}^{m}$ for $m=1, 2, \cdots, M$

After shock, the investment amount in balance sheet of bank i at time 1, $B^{1}_{i,1}$ *is*

$B^{1}_{i,1} = B^{1}_{i,0} \times (1-p)$ *for* $i=1, 2, 3, \cdots, N$

And total asset of bank i at time 1 is $A_{i,1} = \sum_{m=1}^{M} B_{i,1}^{m}$ for $m=1, 2, \cdots, M$

[4] Huang et al (2013) empirically test the bi-partite banking network model with 2007 US commercial banks balance sheet data. They use the Commercial Banks - Balance Sheet Data (CBBSD) from Wharton Research Data Services from 1976 to 2008, which contains the amounts of 13 specific assets including real estate loans, agricultural loans and others. In this paper we set the number of assets to 10 to simplify the problem. Also Caccioli et al. (2013) test the effect of the crowding on the contagion probability. As crowing parameter, $n=N/M$ increases, the default probability graph shifts to the left in Figure 2-A.

[5] For example, let's assume that initial leverage ratio is 10 and equity is 10% of total asset across all i. 100% shock to a random asset will erase all the value of investment of the asset in bank balance sheet. But fully diversified banks which invest in 10 assets will not default because maximum loss is limited to 10% of total assets which is equal to equity/asset.

If liability of bank j at time t is greater than total asset of the bank j at time t (less than zero equity), default condition is met. Then, bank j defaults at time t, if $L_{j,t} > A_{j,t} = \sum_{m=1}^{M} B_{j,t}^{m}$ for $m=1, 2, \cdots, M$

We assume that defaulted banks liquidates all its assets to meet the liability (fire-sale) and total asset decreases to zero at time $t+1$, $\sum_{m=1}^{M} B_{j,t+1}^{m}=0$ for $m=1, 2, \cdots, M$

3.3.2 Successive shock and risk contagion

Risk contagion starts with second and successive shocks which captures pure spillover effect. When second shock is generated by fire-sale of defaulted bank j due to first shock, bank with degree of 1 will not directly propagate shock because there is no link between banks and its connection is limited to infected asset. Meanwhile bank j with degree of connection over 1 functions as shock transmitter.

When the shock spills over to another asset through bank-asset linkage of banks with degree over 1 in bipartite network, more banks are affected. As a result, shocks are transmitted from bank with lower connection degrees to banks with higher degrees through common asset channels.

In the course of fire-sale from defaulted banks, unit price of asset m is adjusted by deterministic market impact function as defined below. Market impact is assumed to exponentially increase when fraction of asset m of defaulted bank j, $X^{m}_{j,t+1}$ is liquidated at time t+1. The function means linear market impact for log of liquidated fraction. Unlike normal conditions, under fire-sale due to series of defaults and limited liquidity to absorb in short-term period, market impact is expected to be linear or superlinear (J. Gatheral, 2010).

Unit price of asset m at time t+1 = $f(X^{m}_{j,t+1}) \times$ Unit price of asset m at time t
Market impact function is $f(X^{m}_{j,t+1})=e^{-\alpha \times X^{m}_{j,t+1}}$
where $\alpha > 0$ is a positive constant[6] and $X^{m}_{j,t+1}$ is fraction of asset m of defaulted bank j liquidated up to time t+1

Banks with medium connection degree are 'shock absorber' up to their capital at first, but 'shock transmitter' later. When these groups of banks do not have enough capital to absorb transmitted shocks, risk contagion occurs through connections these banks have. Therefore the shock is transmitted by banks with lower end of degrees over 1 (i.e., bank with degree 2) at early stage of system failure. They have lower probability of connection with infected assets than banks with higher degrees, but their exposure to infected asset is greater. And risk spreads out gradually from bank with lower degrees to one with higher degrees through common asset channel.

Lastly, banks with higher degree of connection are 'shock absorber' as weights of each asset holdings are minimal in balance-sheet and effectively expose themselves less to infective assets. With diversification benefits, they withstand risk contagion to the extent of capital buffer and survive the massive failures until no more capital is left to absorb the shocks amplified in the network. But when they fail, the risk decimates the whole system through their diversified asset-bank links. Default propagation process or negative feedback loops persist until no further defaults occur in the system.

Figure 1 is graphical representation of system propagation process in bipartite network. Detailed risk propagation path along degree of connections will be discussed in simulation results below.

IV. Simulation Results

[6] High value of α implies an inelastic demand for the illiquid asset, so that price changes will be higher for a given fraction of assets from defaulted banks in the market. Following Caccioli et al. (2012), the parameter α is chosen to be 1.0536 such that the price drops by 10% when 10% of the asset is liquidated.

4.1. Simplifying assumptions and parameters

This paper assumes simplified financial network to simulate risk contagion process in bipartite financial network. In numerical simulations N and M are finite numbers. Number of banks, N is 100 and number of assets, M is 10. Each result is tested with number of 1,000 simulations, unless stated otherwise. As our focus is on the relationship between the network structure with varying connection distribution and default probability, parameters of our interests in the test are initial shock to random asset (p), connection probability between bank and asset (Erdős–Rényi probability), connection distribution for decile bank groups (uniform, bullet and barbell distribution) and initial leverage ratio ($\lambda_0=A_0/E_0$).

4.2. Results of Tests

4.2.1. Default rate as function of probability of connection

We first begin by random graph with Erdős–Rényi probability. Figure 2 describes default rate as function of probability of connection with initial shock of 30% given to a random asset out of 10. Probability of connection between bank and asset follows Erdős–Rényi probability with i.i.d. assumption. The simulation results help to understand whether higher probability of connection promotes system stability or exacerbate financial contagion by increasing channels of risk propagation. Allen and Gale (2000), Gai and Kapadia (2010) explains that increasing network connectivity have opposite effects depending on mean degree. Figure 2-A shows there exist different effect of diversification benefits in the network system and nonmonotonicity of the contagion probability. Increased connectivity increases banks' risk through network when probability of connection between banks and assets are lower than critical level, 0.35. But, if the banks' portfolios encompass sufficient number of assets, diversification allows banks to reduce risk because each bank owns smaller fraction of the infected asset. Probability of connection above 0.75 decreases default probability close to zero.

In Figure 2-B, we define system failure as case in which default probability exceeds predetermined threshold, 10%. Magnitude of system failure is the fraction of failed banks averaged only over simulations with system failure case. It shows robust yet fragile properties of financial system which provides the same results with Caccioli et al. (2012). The more the proportion of banks with higher degree of connection, the more robust the financial system is. The system is more stress-resistant but simultaneous risk gets higher (robust yet fragile).

Not surprisingly, over maximum default probability, shocks from system failures are absorbed by the market and default probability decrease to zero. But catastrophic default cases make the whole system behaves like one individual bank[7]. Probability of connection over critical level in financial system lowers the probability of system failures, but at the same time it makes the system exposed to black swan events. Once system failures occur in this phase almost all banks are destined to declare default through interwined connections.

4.2.2. Default probability dynamics

Next we investigate the path of risk contagion in bipartite network. Table 1 in Panel A denotes the default probability due to shock in network with discrete uniform degree distribution. In uniform distribution, there are decile bank groups and banks in each group have the same degree of connection. The network is composed of 10 bank groups of which the degrees of connection ranges from 1 to 10. And all the banks in each bank group are randomly connected to assets. The reason for the assumption of uniform distribution is that, even though it is unlikely to resemble the real financial network, it helps to understand how the risk contagion proceeds clearly as default probability changes in time.

Number of banks and assets are 100 and 10 respectively and initial shock to a random asset is 30%. All the numbers are averages over 1,000 simulations. Assets are evenly split in balance sheet and investment per assets is simply total assets divided by degree of connections. The results shows that 3.1% defaults induced by first

[7] Regulator cares less about solvency of specific bank as long as the scale of bank failures is under threshold or tolerance level. And it has incentive to preempt propagation of system risk by isolating individual banks in financial network, while individual bank are incentivized to be "Too Interconnected To Fall (TICTF)" in financial crisis.

shock at time 1 lead to total system failures in consecutive waves of shock. First shock is common shock and it only affects the balance-sheet of the bank which invests in the specific asset (A1 in panel B). Banks with degree 1 and 2 are affected due to insufficient capital buffer, 10% of its total assets. And pure spillover begins by banks with higher degrees and escalates to more diversified banks. Financial fiasco is maximized when the most diversified banks collapse and risk spread to entire system in 3rd and 4th shock at time 3 and 4 respectively. With first 4 shocks, almost all banks defaults and average asset price declines to less than 10% at time 5. And with final shock, banks which do not invest in infected asset of A1 are unable to avoid default as soon as bank with highest degrees falls in previous shock.

Important point to note is that risk contagion does not arbitrarily occur and follows certain chains or path along individual bank's degree of connection. Exposure to the affected asset in uniform degree distribution with assumption of evenly split assets in the portfolio is linearly dependent on the degree of connection. Also the loss induced by shock is linearly dependent on it too. Thus defaults by first shock are limited to bank groups with certain degree. With the shock of 30%, the loss incurred to banks with degree of 1 and 2 are -30% and -15% respectively, leading to sure default. But banks with higher degrees of connection survive the first shock.

And fire-sale by defaulted banks and second shock affects banks with adjacent degrees of connection in a decreasing order at time 2. Bank of 3 degrees defaults with probability of 28.5% and bank of 4 degrees fails with probability of 26.3% and so on. These risk contagion path implies not only that banks with undiversified portfolio are first victims of system failures, but also that connection degree order is critical for the risk to propagate. As far as degree of connection is concerned in bipartite network, system risk contagion does not leap as we will show later in barbell distribution.

This simulation does not assume correlation of asset returns as is usual case in reality[8]. As a result, undiversified banks with minimal probability of connection to infected asset will remain as last survivor of risk contagion. Condition of system failures or breakdown of all intermediaries in the network is met when risk spreads over to most diversified banks in time 3 and spiral of failures reaches undiversified banks in time 4 and 5. Taken these points together, systemically important are not just the most connected banks in the system as is usually believed. On the contrary, less diversified banks such as bank with lower degree over 1 play a key role in transmitting risk before full-fledged system failures. The former amplifies and spreads the risk to the whole system and the latter links or conveys the risk to the former which in turn paralyze the whole system.

In Panel B of Table 2, the loss to the asset is represented in percentage of initial amount in uniform degree distribution. A1 is the infected asset and initial shock is 30% of asset price. Magnitude of negative feedback increases in the course of risk contagion by banks with higher degrees. When banks with degree of 1 and 2 default at time 1, loss to the asset except A1 is 2% at time 2. But when banks with higher degrees default at time 2, loss to the asset except A1 amounts to 11~12% at time 3. As risk spills over to bank with higher degrees, it is harder to contain the contagion and capital injection or bail-out cost by treasury increases exponentially. To preclude massive failures of financial network in uniform degree distribution, it is critical to stop the progression of risk contagion before banks with high degrees collapse.

And in simulation of shock propagation, banks with degree of 1 inflicts only indirect impact to other assets because bank-asset link is one and risk is slowly spills over to other assets. Most of risk contagion comes only through other banks with higher degrees over 1. Specifically all the assets except A1 suffer gradual loss from shock compared to A1. And only after banks with higher degree defaults with full-fledged 3rd shock, other assets suffer significant losses at time 4. On the other hand, banks with degree over 1 inflicts direct impact to other assets and play a key role to convey risks to banks with higher degrees causing entire collapse of the system in the end. The function of bank groups with different connection degrees in risk propagation process can be summarized as below.

[8] Counteractive immunization using negative correlated assets in the portfolio can efficiently reduce system risk (Kobayashi & Hasui, 2014).

- **Full specialization banks**
  Full specialization banks with connection degree of 1 are first victim of initial shock. And with high exposure to certain risky assets, they are the origins of risk contagion. Specialized bank with high leverage ratio makes the case worse. Therefore, microprudential policy needs to monitor market conditions and take measures to preclude vulnerable full specialization banks (e.g., Liquidity and capital buffer, limit to leverage ratio).

- **Fully diversified banks**
  The more the proportion of banks with higher degree of connection, the more robust the financial system is. The system is more stress resistant but simultaneous risk gets higher as was shown in Figure 2. If all the banks are fully diversified with uniform weight to each asset they hold, it coincides with individual banks optimal portfolio (Beale et al., 2011). In that case, the financial system is single representative bank. With more diversification and less diversity, banks have incentive to be on the same boat of financial system. Fully diversified banks, if defaulted, amplifies and spreads the risk to the whole system leading to system failures.

- **Less than fully diversified banks**
  Banks with lower degree over 1 which do not have enough capital to survive risk propagation, function as the critical shock transmitter or risk conveyor. Therefore, to forestall risk contagion and system failures, less proportion of these banks in the network is beneficial for the whole system. These banks are weak link of risk spillover in crisis and have systemically important implications for regulatory body. They are weak link because they have more exposure to risky assets than fully diversified banks and larger market impact to connected asset is generated from fire-sale when defaulted.

Therefore, in order to assess the systemic importance of financial intermediaries, it is not enough to assess the initial impact a bank could have on other banks in crisis. The important implication from the default dynamics in bipartite network is that financial intermediaries' defaults have limited first round effects on the network system, but in subsequent rounds of contagion, these cumulative effects could lead to significant failures of other intermediaries in the network.

4.2.3. Default probability in bullet distribution

Now we investigate the role of bank groups with specific connection degrees in risk contagion process. Figure 3-A describes average default probability as function of shock in bullet distribution. Bullet distribution is the distribution where all the banks are randomly connected to assets and have the same connection degree. And in the graph, default probability in bullet distribution with degree of connection, D=1, 3, 5, 7, 9 are shown. As all the banks share common degree of connection, default probability show jump with specific level of shock. Maximum default probability of bullet distribution (D=1) is 10% and risk contagion does not occur because there is no direct link between banks and connection degree of infected banks is 1. Notably, bullet distribution with degree over 1, suffers phase transition and shock threshold which leads to system failure coincides with connection degree divided by leverage ratio.

Given the shock to a random asset of bank j at time t, $B^{m}_{j,t}$, bank j defaults, if $E_{j,t} = A_{j,t} - L_{j,t} < Loss_{j,t} = Shock(p) \times B^{m}_{j,t}$

With the assumption of evenly split portfolio in bank balance sheet, $B^{m}_{j,t} = \dfrac{A_{j,t}}{connection\ degree\ of\ bank\ j}$

Shock threshold, $p*$ equals to $\dfrac{E_{j,t}}{B^{m}_{j,t}} = \dfrac{E_{j,t}}{A_{j,t}} \times connection\ degree\ of\ bank\ j = \dfrac{Connection\ degree\ of\ bank\ j}{\lambda}$

Figure 3-B denotes phase transition in network with bullet distribution. Phase is divided into stable and unstable one by extent of default rate in financial network. Unstable phase is defined as the phase where all the banks default by varying degrees of shocks to a random asset and stable phase means the other cases. Phase transition occurs starting from bullet distribution with degree 2 and we can find the linear relationship between degrees and shocks causing phase transition (shock threshold, p^*).

4.2.4. Default probability in barbell distribution

Now we investigate the weak link in risk propagation process in more details. In Figure 4, default probability as function of shock in barbell distribution is presented. Barbell distribution is the same as uniform distribution except that it omits specific bank groups in between the both ends. And in the graph, default probability in barbell distribution excluding bank groups with degree of 2, 3 and 4 is shown.

The graph represents average default probability as function of shock in uniform and barbell distribution. N, the number of banks[9] is 100 and M, the number of assets is 10. Simulations are run 1,000 times. When excluding bank group with degree of 2, threshold of shock to cause massive default goes to the right and default probability decreases compared to uniform distribution. As more bank groups with degree over 1 are excluded, shock transmission process breaks. In uniform distribution, 11% shock to a random asset results in 20% default rate. But barbell distribution excluding bank group of connection degree 2, is more robust and 19% shock causes the same results. Barbell distribution excluding bank group of connection degree 2, 3 withstand the shock up to 31% for the same default probability.

Therefore, the longer the distance between banks with degree 1 and higher degrees, the more stable and stress-resistant the financial system is to the asset shock in bipartite network. Asset price shocks are first absorbed by bank with degree 1 and even with the default of bank with degree 1, shocks from fire-sale do not spread out to more diversified banks because there are no risk conveyors to reach banks with higher degrees.

Practically, it is not feasible for the regulator to control network topology or limit the connection distributions of private financial intermediaries in the system. It is more realistic to impose more capital to the weak link of the network and increase monitoring activities.

4.2.5. Capitalization and default probability

So we test the effect of capitalization on default rate. Table 2 denotes the effect of enhanced capital buffer on default probability in uniform distribution. For each decile bank group categorized by degree of connection from 1 to 10, capital enhancement to equity in balance sheet is made to test the effect of increased capitalization on default probability. Initial shock to a random asset is assumed to be 30%. In the table, total default probability row denotes number of defaulted banks out of total banks with assumption of the same equity capital across the banks. Equity is 10% of total asset and leverage ratio is 10 for all banks. All the numbers in the table are the averages over 1,000 simulations. Total default probability with enhancement represents default probability when specific bank group has received capital enhancement, all others being equal. Then these two probabilities differ only in equity capital of chosen bank group (from 10% to 15% of total asset). Difference is represented in the last row.

As shown in Table 1, average default probability to shock of 30% in uniform distribution is 96.2%. Meanwhile, with additional capital buffer in specific bank group, default rate decreases. Our interest is, as from regulator's perspective, how to allocate limited capital to specific bank group to contain system failures with efficacy. The difference is the biggest when additional capital is allocated to bank group with degree of 2 and gradually decrease as additional capital flows to higher degrees.

[9] In uniform distribution, total number of banks is 100. In barbell distribution excluding D=2, total number of banks is adjusted to 99 and number of banks in each bank group equals to 11. The same procedure for barbell distribution excluding D=2, 3 and D=2, 3, 4.

Implication is that for system stability or more robust system, it is critical to strengthen the weak link of intermediaries in risk propagation process and regulatory body needs reinforced monitoring and regulations of them for macroprudential policy. And the result shows that most of the weak links are centered in bank groups with lower degrees over 1. Interestingly, only 5% increase of total capital in the network results in 33% reduction of default probability when it is allocated to the weakest link of risk contagion process. As for the bank group with higher degrees, the difference is marginal. For bank groups with degrees over 4, the difference is less than 3% which implies that once risk spills over to sufficiently diversified banks, it is hard to mitigate risk contagion even with more capitalization.

V. Empirical test

Stylized model and simulation approach in this paper are based on several unrealistic assumptions. But in real banking network, we do not expect the same asset size, equal leverage ratio and evenly split loan portfolio across banks. Thus we use U.S. defaulted banks data in 2007 global financial crisis to empirically test the relationship between risk contagion process and connection degrees. We use the two data sets in this paper. The first is the Failed Bank List[10] from Federal Deposit Insurance Corporation (FDIC). FDIC provides all FDIC-insured institutions failed since October 1, 2000 and Figure 5-A describes number of monthly defaulted banks from October, 2000 to April, 2014. Total number of defaulted banks is 522 for the period and we investigate 370 banks failed during 1/1/2008 –7/1/2011 period.

The second dataset is the balance sheet data of banks at the end of 2007. FDIC provides the statistics[11] on banking in quarterly publication with detailed aggregate financial information for all FDIC-insured institutions. Balance sheet data of all institutions including commercial banks and savings banks were retrieved from dataset at 2007 year end. Number of institutions reporting is 8,534. In aggregate balance sheet, risky assets out of total assets are mostly loan assets (60%) and securities (15%). Securities assets include U.S. Government securities such as pass throughs issued by F.N.M.A. and F.H.L.M.C, non-mortgage backed securities issued by U.S. government enterprises (GSEs). To measure the connection degree, we use the amount of gross loan asset data in each bank's balance sheet at the end of 2007. Description of gross loan data from aggregate balance sheet in 2007 is shown in Table 3.

To control for different leverage ratio and asset size, we excluded banks with high leverage ratio and small assets. Leverage ratio is measured as total assets divided by total equity. Average leverage ratio of failed banks is 11.28 whereas average of all banks is 9.6. Highly leveraged banks are more exposed to credit risk and might fail earlier than banks with less gearing irrespective of their loan portfolio diversification. Thus we excluded banks with leverage ratio over 20. Also it is possible that failed banks with too small assets do not play meaningful roles in the risk contagion process and inclusion of these banks inflates the default probability of banks with certain connection degrees. We excluded banks with assets less than $ 100 million. After screening, number of sample failed banks for 1/1/2008 – 7/1/2011 period is 295.

To empirically test the relationship between risk contagion process and connection degrees, we need to define degree of connection of banks in real network. Instead of the assumptions of evenly split asset weight in banks' balance sheet, we need to incorporate each asset's actual weight to calculate connection degree. Banks tend to specialize in market segments where they have an expertise. Specialization is usually accompanied by concentration of regions, financial products or specific sectors in their business at the expense of diversification. Herfindhal-Hirschman index[12] or Shannon entropy[13] is a concentration measure and has been widely used in the loan portfolio concentration literature (Á vila et al, 2012). As a proxy for connection degree, we use normalized Herfindhal-Hirschman index(HHI). HHI ranges from 1/n to one, whereas normalized HHI ranges from 0 to 1.

[10] http://www.fdic.gov/bank/individual/failed/banklist.html
[11] http://www2.fdic.gov/idasp/warp_download_all.asp
[12] Hirschman (1945), Herfindahl (1950)
[13] Shannon (1948)

$$HHI = \sum_{i=1}^{n} \xi_i^2$$

$$\text{Normalized } HHI = 1 - \frac{(HHI - 1/n)}{1 - 1/n}$$

Where i is the number of loans in the asset side of balance sheet and ξ is the exposure of loan relative to the total loan asset values. Normalized HHI is 1 when loan portfolios are most diversified and 0 when loan portfolios are most concentrated. Figure 5-B is comparison of connection degrees between failed banks and all banks measured with the 2007 balance sheet data. Failed banks have more concentrated loan portfolios than all banks. Average connection degree of failed banks and all banks are 0.74 and 0.76 respectively.

In the bipartite model discussed in the previous section, we demonstrated risk contagion does not arbitrarily occur and follows path along individual bank's degree of connection. Therefore we aligned connection degrees of failed banks along with the date of defaults with actual data in 2007 financial crisis. Figure 6 is a scatter plot of failed banks for the period from January 2008 to June 2011. As is indicated in Figure 5-A, 2007 financial crisis ignited series of default starting from early 2008 and climbed to the climax of crisis in 2011. But due to the U.S. government's bail-out of systemically important financial institutions and massive stimulus packages to stabilize financial system, massive system failures in the model did not occur. But Figure 6 helps us to understand the risk contagion process before full-fledged shocks are dispersed by the most diversified banks in the financial system. As time passes and crisis culminates, there are more defaults of banks with higher connection degrees which imply risk contagion from concentrated banks to diversified ones.

Table 4 shows month to default of failed banks sorted by normalized HHI. Sample period for failed banks is from January 2008 to June 2011. During the period, 370 commercial and savings banks failed. And failed banks were screened by leverage ratio and asset size into 295 banks. All the data are from Failed Bank List and balance sheet database of Federal Deposit Insurance Corporation. Then normalized HHI of failed banks are calculated based on balance sheet data in 2007 and failed banks are sorted by normalized HHI into quartile groups. Average normalized HHI and average default date of each sorted bank group are shown in third and fourth columns. Month to default is the number of months from 01/01/2008 to average default date. The most concentrated failed banks have average normalized HHI of 0.551 and average default date is 03/11/2009. The month to default of the most concentrated banks is shortest and more diversified banks failed later. Even though default dates of quartile failed bank groups 3 and 4 are quite close, the implication from empirical results is the same as the conclusion drawn from simulation using models.

VI. Conclusions and limitations

This paper is based on the idea that risk contagion process follows certain path. Knowing the default path can help the regulator to mitigate the cascading failures in financial system and break negative feedback loops in bank-asset network. We model a highly stylized banking system and take simulation approach with assumptions. We keep a number of parameters constant and investigate the effects on systemic stability of market shock in various network structures.

Major findings are as follows: First, the more the proportion of banks with higher degree of connection between banks and assets, the more robust the financial system is. The system is more stress-resistant but simultaneous risk gets higher (robust yet fragile). Second, banks with lower degree over 1 function as critical shock transmitter. Therefore, to mitigate risk contagion or system failures, less proportion of these banks in the network is beneficial for the whole system. These banks are weak links of risk spillover during financial crisis and systemically important intermediaries for regulatory body. Third, to contain or mitigate risk contagion, financial system with well diversified and specialized banks is more desirable than uniform structure. The more proportion to the two extremes in connection distribution, the more stable the financial system is to the asset shock in bipartite network. More capital buffer or limit of leverage ratio to banks with lower degrees could enhance the resilience of the whole system.

From the regulator perspective, it is essential for macroprudential policy makers to oversight the structure of the whole system and monitor which intermediaries function as links to financial risk propagation. To contain or preclude risk contagion, barbell structure is more desirable than bullet structure with less than full diversification. Specifically barbell network structure which is constructed with banks with full specialization and full diversification might effectively mitigate the risk transmission in financial crisis by eliminating weak link of contagion in financial crisis. More feasible policy could be selective capital requirement for financial institution according to its degree of connection in the network. More capital buffer or limit of leverage ratio to bank with lower degree over 1 could enhance the resilience of the whole system. Liquidity requirements can mitigate contagion and play a similar role to capital buffers in curtailing systemic failure. (Cifuentes et al., 2005)

Stylized model in the paper can help regulators to find out which banks are weak link or systemically important in asset price shock. But simulation methodologies involve many assumptions, some of which might bias the results. The model has simplified assumptions such as the same bank size, equal capital, uniform asset allocation in balance-sheet, no counterparty risk and zero correlation of asset returns. Also existence of shadow banking system might amplify the risk contagion intensity in complicated network. But we do not expect different results from qualitative point of view.

In further study, the mechanistic model considered in the paper could be tested against real data in period of financial crisis. By relaxing assumptions in this paper such as homogeneity of bank balance-sheet, deterministic market impact function and discrete connection distributions, empirical test could reveal more realistic relations between system stability and connection distributions. For example, by matching the order of connection degrees of defaulted banks and actual timing of defaults with actual data, we could test whether banks with lower degrees defaulted earlier than diversified banks. Also additional test could be made to find the order of default rate in categorized bank groups assorted by its connection degrees before failure of fully diversified banks. Lastly we could apply the model to perform stress tests on current financial systems with selective capital enhancement to bank groups of different connection degrees. These could make it available to test the effectiveness of policies aimed at reducing system risk.

Figure 1. Default propagation process in bipartite network

Figure 1 is graphical representation of default propagation process in bipartite network. There are 3 banks (N1, N2 and N3) which invest in 3 assets (M1, M2 and M3) and connection is denoted as line between them. With price drop of random asset, M1, all banks which have connection with the infected asset function as shock absorber up to their capital buffer. Less diversified bank (i.e. N1) is more prone to default due to more exposure to infected asset, while more diversified bank (i.e. N3) is less exposed to default risk. When bank N1 defaults, we assume that fire-sale of all the assets in affected banks follows and risk contagion process begins with second and successive shocks. When second shock is generated by fire-sale of defaulted bank, it functions as shock transmitter. Shocks are transmitted from bank with lower connection degrees to banks with higher degrees through common asset channels. Default propagation process or negative feedback loops persist until no further defaults occur in the system.

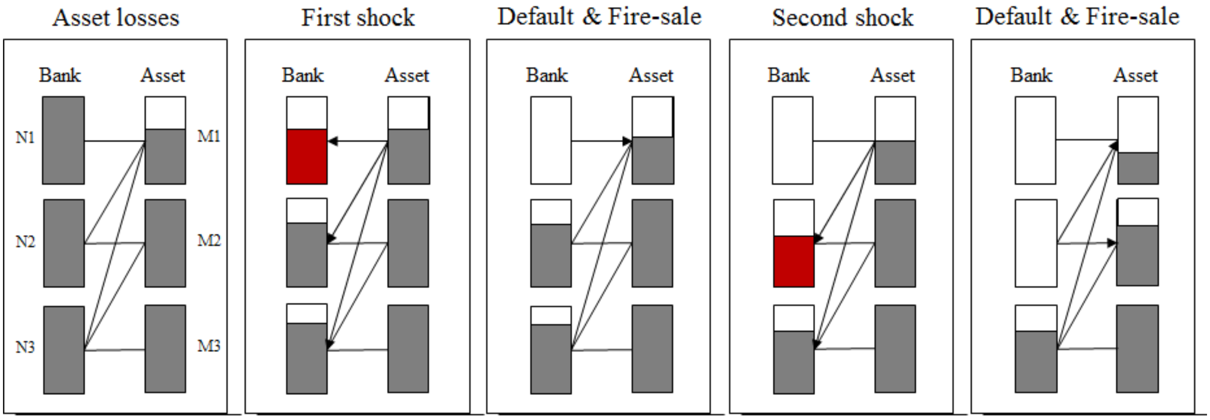

Figure 2. Default probability as function of connection probability

Figure 2-A represents average default rate as function of probability of connection with initial shock of 30% given to a random asset. The average default rate was calculated from 1,000 simulations. Probability of connection between bank and asset follows Erdős–Rényi probability with i.i.d. assumption. It shows there exist different effect of diversification benefits in the network system and nonmonotonicity of the contagion probability. Figure 2-B represents the magnitude of system failure as function of connection probability with initial shock of 30%. In the graph, we define system failure as case in which default probability exceeds predetermined threshold, 10%. Magnitude of system failure is the fraction of failed banks averaged only over simulations with system failure case. The graph shows robust yet fragile properties of financial system. The more the proportion of banks with higher degree of connection, the more robust the financial system is. The system is more stress-resistant but simultaneous risk gets higher (robust yet fragile).

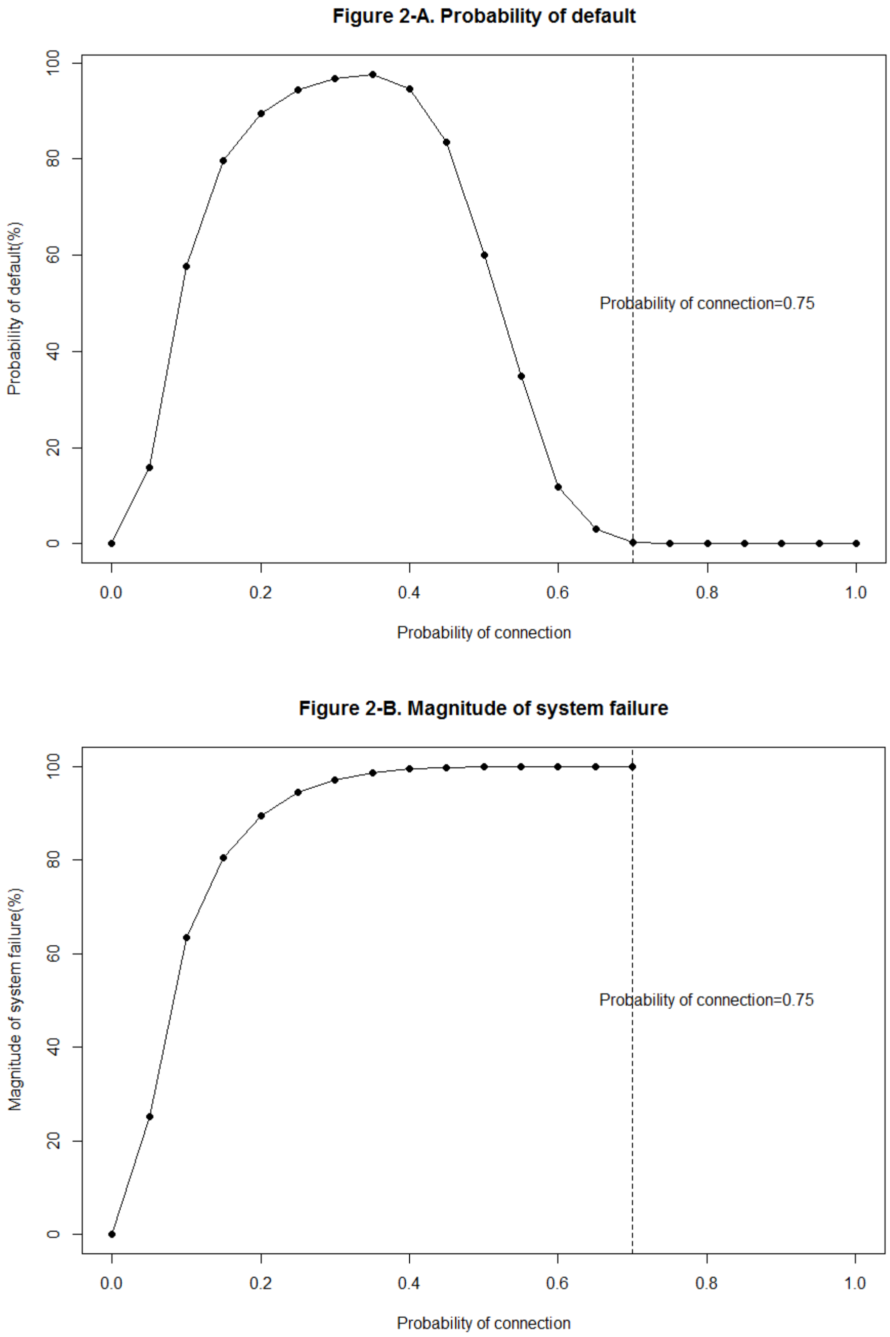

Table 1. Default probability dynamics and loss to the asset in uniform degree distribution

Panel A denotes the default probability due to shock in uniform degree distribution. There are decile bank groups and banks in each group have the same decile degree of connection. The network is composed of 10 bank groups of which the degrees of connection range from 1 to 10. All the banks in each bank group are randomly connected to assets. Number of banks and assets are 100 and 10 respectively and initial shock to a random asset is 30%. All the numbers are averages over 1,000 simulations. Assets are evenly split in balance sheet and investment per assets is simply total assets divided by degree of connections. First shock affects only the balance-sheet of the bank which invests in the specific asset and there is no risk contagion at time 1. Pure spillover begins with banks of higher degrees and escalates to more diversified banks. Financial fiasco is maximized when the most diversified banks collapse and risk spread to entire system at time 3 and 4. Magnitude of negative feedback increases due to risk contagion by banks with higher degrees. As risk spills over to bank with higher degrees, it is harder to contain the risk contagion and capital injection or bail-out cost by treasury increases exponentially. In Panel B, the loss to the asset is represented in percentage of initial amount by shock in network with uniform degree distribution. A1 is the infected asset and first shock is 30% of asset price. With first 4 shocks, almost all banks defaults and average asset price declines to less than 10% at time 5.

Panel 1-A. Default probability due to shock in uniform degree distribution

Bank group with degree of connection	1	2	3	4	5	6	7	8	9	10	Average
Amount of investment per asset	100	50	33.3	25	20	16.7	14.3	12.5	11.1	10	
t=1	10.6	20.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.1
t=2	0.3	0.0	28.5	26.3	8.9	1.5	0.2	0.0	0.0	0.0	6.6
t=3	15.6	11.6	10.2	19.9	42.2	51.3	54.1	54.7	54.5	54	36.8
t=4	57.1	52.9	48.2	42	38.9	37.9	37.2	36.9	37.2	37.6	42.6
t=5	11.4	10.4	8.4	7.1	5.6	5.0	4.3	4.1	4.0	4.1	6.4
t=6	1.0	0.9	0.8	0.7	0.5	0.4	0.4	0.5	0.5	0.5	0.6
t=7	0.1	0.1	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0
t=8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total											96.2

Panel 1-B. Loss to the asset by shock in uniform degree distribution

Asset	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	Average
t=1	-30.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-3.00
t=2	-51.91	-2.14	-2.21	-2.28	-2.18	-2.08	-2.38	-2.09	-2.25	-2.42	-7.19
t=3	-67.79	-12.15	-11.84	-12.06	-11.61	-11.74	-12.48	-11.87	-12.22	-12.46	-17.62
t=4	-90.97	-56.09	-55.93	-56.03	-55.42	-55.76	-56.24	-55.86	-56.20	-56.18	-59.47
t=5	-97.35	-90.79	-90.97	-91.05	-90.46	-90.62	-90.68	-90.59	-90.73	-90.98	-91.42
t=6	-97.91	-95.63	-95.54	-95.74	-95.53	-95.64	-95.60	-95.49	-95.55	-95.64	-95.83
t=7	-97.98	-96.08	-96.05	-96.18	-96.11	-96.09	-96.07	-96.05	-96.08	-96.09	-96.28
t=8	-97.98	-96.12	-96.09	-96.21	-96.14	-96.12	-96.13	-96.09	-96.11	-96.11	-96.31

Figure 3. Default probability as function of shock and phase transition in bullet distribution

Figure 3-A describes average default probability as function of shock in bullet distribution. Bullet distribution is the distribution where all the banks are randomly connected to assets and have the same connection degree. And in the graph, default probability in bullet distribution with degree of connection, D=1, 3, 5, 7, 9 are shown. As all the banks share common degree of connection, default probability show jump with specific level of shock. Maximum default probability of bullet distribution (D=1) is 10%. Risk contagion does not occur because there is no direct link between banks and connection degree of infected banks is 1. Figure 3-B denotes phase transition in network with bullet distribution. Phase is divided into stable and unstable one by extent of default rate in financial network. Unstable phase is defined as the phase where all the banks default by varying degrees of shocks to a random asset and stable phase means the other cases. Phase transition occurs starting from bullet distribution with degree 2 and we can find the linear relationship between degrees and shocks causing phase transition.

Figure 3-A. Default probability in bullet distribution

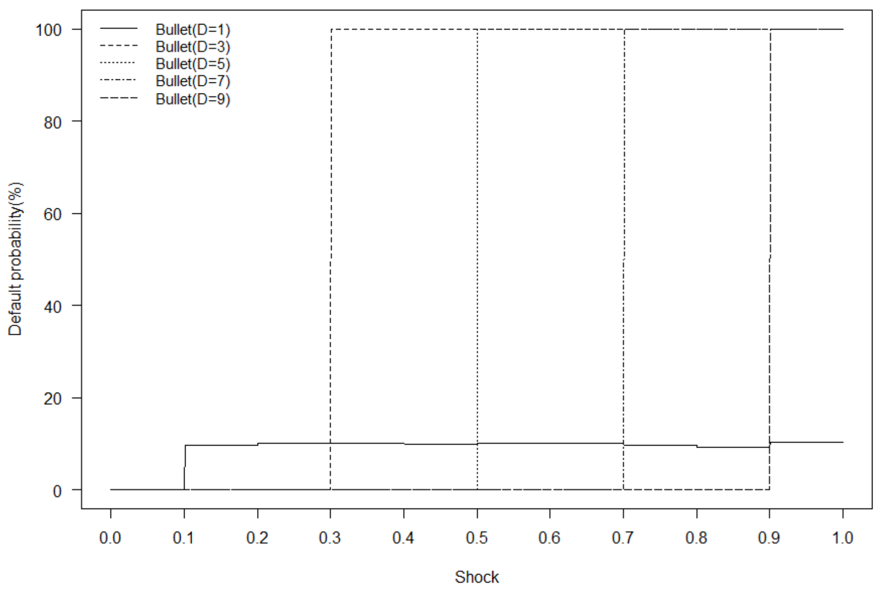

Figure 3-B. Phase transition in bullet distribution

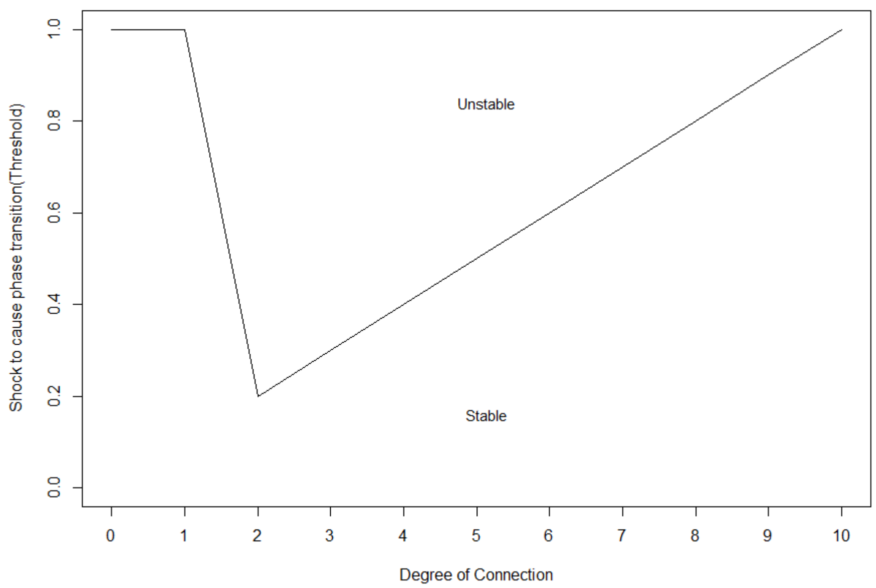

Figure 4. Default probability as function of shock in Barbell distribution

Figure 4 represents average default probability as function of shock in uniform and barbell distribution. Barbell distribution is the same as uniform distribution except that it omits specific bank groups in between the both ends. And in the graph, default probability in barbell distribution excluding bank groups with degree of 2, 3, 4 is shown. N, the number of banks is 100 and M, the number of assets is 10. Number of simulations are 1,000. When excluding bank group with degree of 2, threshold of shock to cause massive default goes to the right and default probability decreases compared to uniform distribution. As more bank groups with degree over 1 are excluded, shock transmission process breaks. The longer the distance between banks with degree 1 and higher degrees, the more stable and stress-resistant the financial system is to the asset shock in bipartite network. Asset price shocks are first absorbed by bank with degree 1 and even with the default of bank with degree 1, shocks from fire-sale do not spread out to more diversified banks because there are no risk conveyors to reach banks with higher degrees.

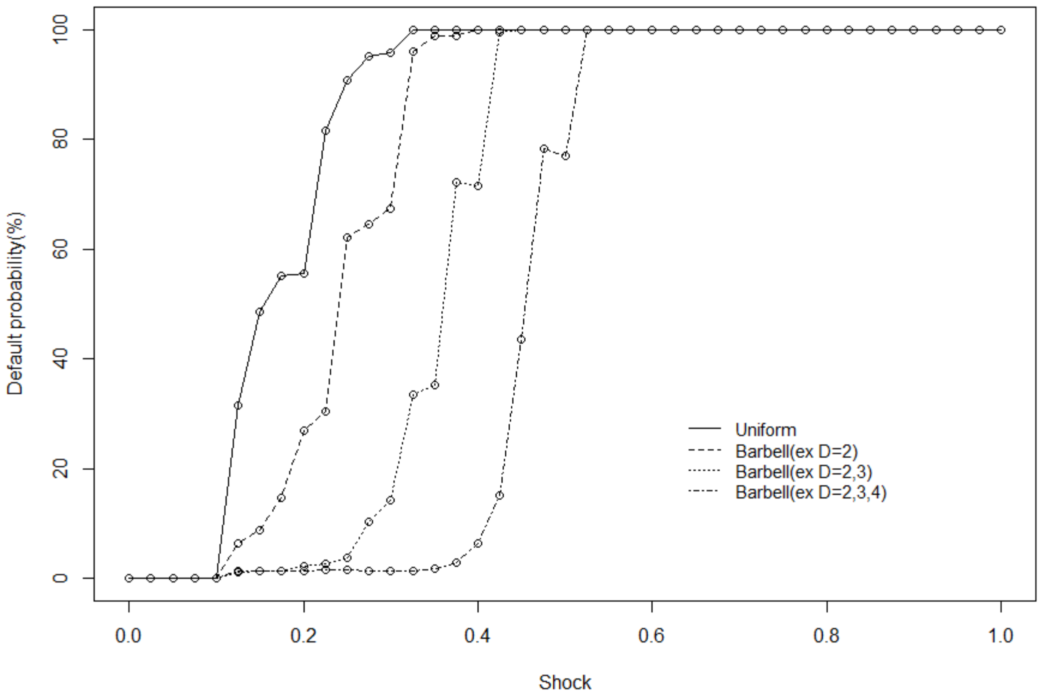

Table 2. Effect of enhanced capital buffer on default probability in uniform distribution

Table 2 denotes the effect of enhanced capital buffer on default probability in uniform distribution. For each decile bank group categorized by degree of connection from 1 to 10, capital enhancement to equity in balance sheet is made to test the effect of increased capitalization on default probability. Initial shock to a random asset is assumed to be 30%. In the table total default probability row denotes number of defaulted banks out of total banks with assumption of the same equity capital across the banks. Equity is 10% of total asset and leverage ratio is 10 for all banks. All the numbers in the table is the average over 1,000 simulations. Total default probability with enhancement represents default probability when specific bank group has received capital enhancement, all others being equal. Then these two probabilities differ only in equity capital of chosen bank group (from 10% to 15% of total asset). Difference is represented in the last row. As shown in Table 1, average default probability to shock of 30% in uniform distribution is 96.2%. Meanwhile, with additional capital buffer in specific bank group, default rate decreases. The difference is the biggest when additional capital is allocated to bank group with degree of 2 and gradually decrease as additional capital flows to higher degrees. As for the bank group with higher degrees, the difference is marginal. For bank groups with degrees over 4, the difference is less than 3% which implies that once risk spills over to sufficiently diversified banks, it is hard to mitigate risk contagion with more capitalization.

Bank group with capital enhancement	1	2	3	4	5	6	7	8	9	10
Total default probability (%)	96.2	96.2	96.2	96.2	96.2	96.2	96.2	96.2	96.2	96.2
Total default probability with enhancement (%)	95.21	63.21	78.45	89.39	94.64	94.21	95.82	95.61	95.52	94.03
Difference (%)	0.99	32.99	17.75	6.81	1.56	1.99	0.38	0.59	0.68	2.17

Table 3. Description of gross loan data from aggregate balance sheet in 2007

The table shows aggregate gross loan data from Federal Deposit Insurance Corporation. FDIC provides the statistics on banking in quarterly publication with detailed aggregate financial information for all FDIC-insured institutions. Balance sheet data of all institutions including commercial banks and savings banks were retrieved from dataset at 2007 year end. Number of institutions reporting is 8,534.

	$ in 000's	Weight
Total loans (gross)	7,778,009,501	100.00%
All real estate loans	4,781,777,684	61.48%
Construction and land development	629,494,230	8.09%
Commercial real estate	968,667,629	12.45%
Multifamily residential real estate	202,817,014	2.61%
1-4 family residential	2,852,857,559	36.68%
Farmland	57,409,735	0.74%
Real estate loans in foreign offices:	70,531,517	0.91%
Farm loans	56,786,230	0.73%
Commercial and industrial loans	1,439,126,568	18.50%
Loans to individuals	1,058,458,465	13.61%
Total other loans	441,860,489	5.68%

Figure 5. Number of defaults and comparison of connection degrees

Figure 5-A describes number of monthly defaulted banks from October, 2000 to April, 2014. Total number of defaulted banks is 522 for the period. The data is from the Failed Bank List of Federal Deposit Insurance Corporation (FDIC). FDIC provides list of all FDIC-insured institutions failed since October 1, 2000. Figure 5-B is comparison of connection degrees between failed banks and all banks measured with the 2007 balance sheet data. The data is from the statistics on aggregate financial information for all FDIC-insured institutions from FDIC. Balance sheet data of all institutions including commercial banks and savings banks were retrieved form balance sheet dataset in 2007. As a proxy for connection degree in gross loan portfolio, we use normalized Herfindhal-Hirschman index (HHI). Normalized HHI is 1 when loan portfolios are most diversified, whereas 0 when loan portfolios are most concentrated.

Figure 5-A. Number of defaults

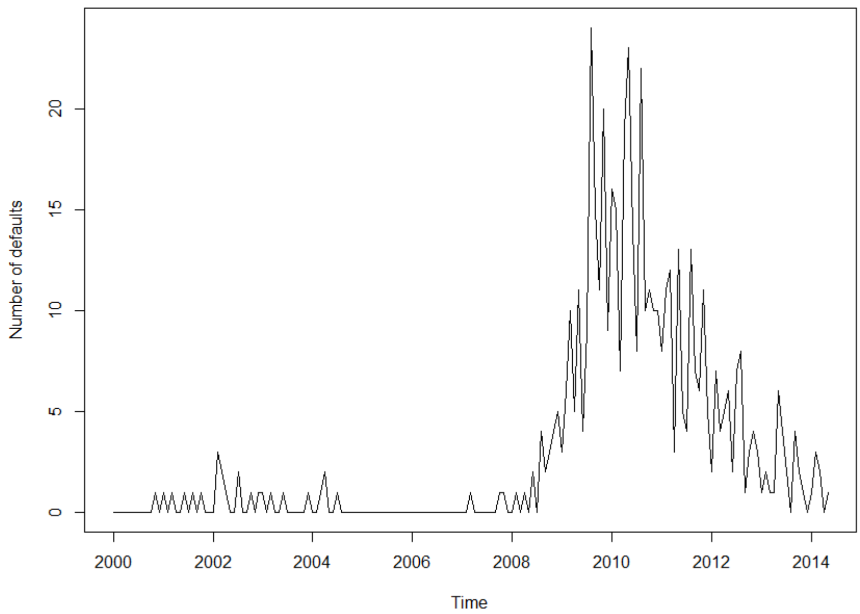

Figure 5-B. Comparison of connection degrees

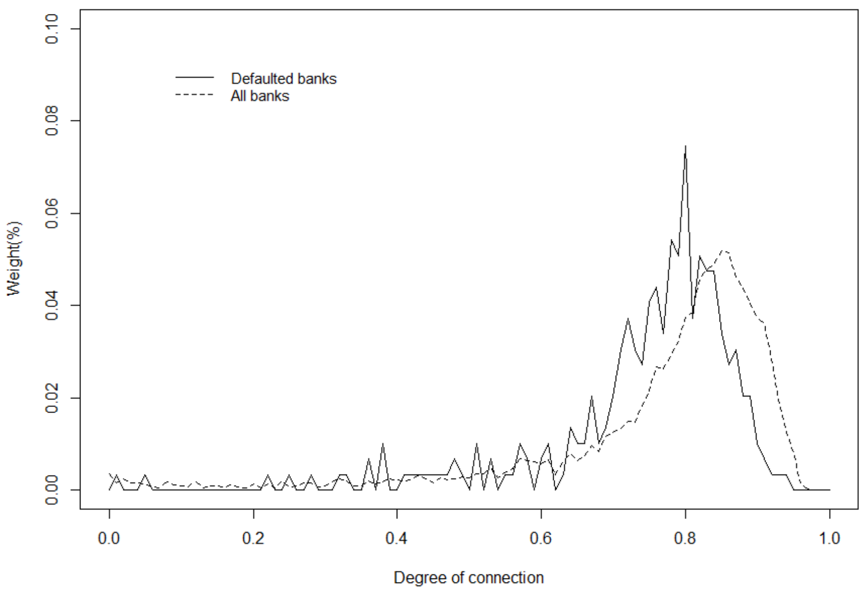

Figure 6. Scatter plot of failed banks for the period from 2008 to 2011

Figure 6 is a scatter plot of failed banks for the period from January 2008 to June 2011. During the period, 370 commercial and savings banks failed. And failed banks were screened by leverage ratio and asset size into 295 banks. As a proxy for connection degree in gross loan portfolio, we use normalized Herfindhal-Hirschman index (HHI). Normalized HHI is 1 when loan portfolios are most diversified and 0 when loan portfolios are most concentrated. It shows the connection degrees of failed banks along with the date of defaults.

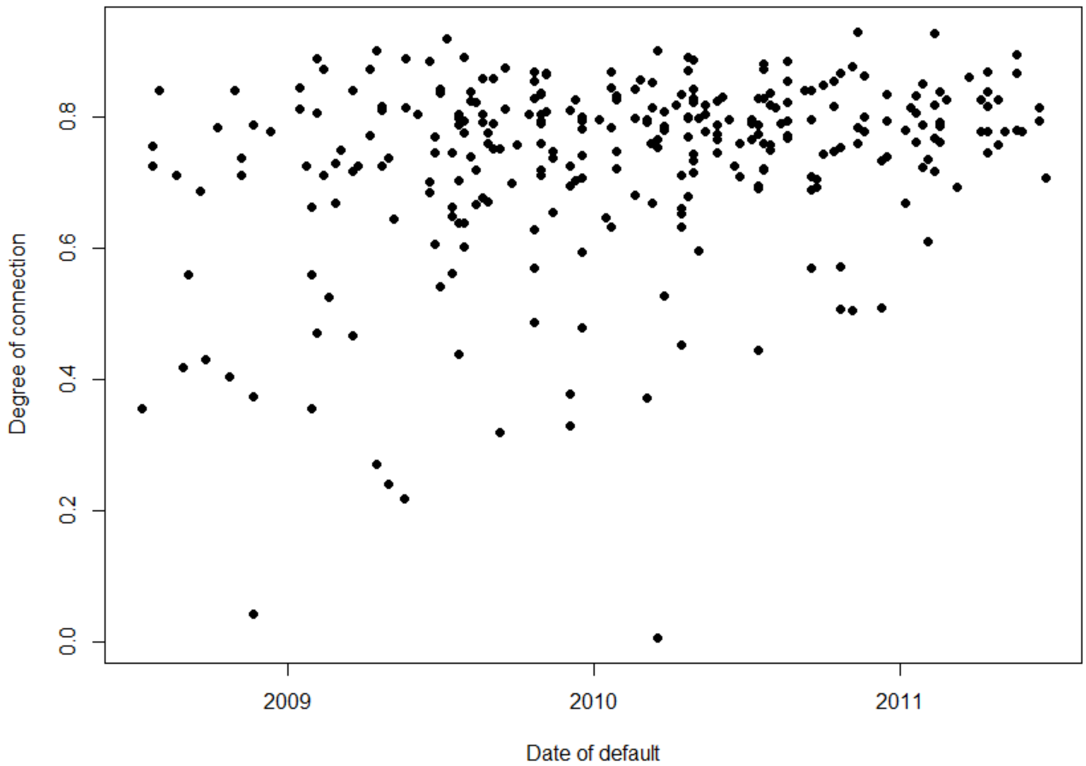

Table 4. Month to default of failed banks sorted by normalized HHI

The table shows month to default of failed banks sorted by normalized HHI. Sample period for failed banks is from January 2008 to June 2011. During the period, 370 commercial and savings banks failed. And failed banks were screened by leverage ratio and asset size into 295 banks. All the data are from Failed Bank List and balance sheet database of Federal Deposit Insurance Corporation. Normalized HHI of failed banks are calculated based on balance sheet data in 2007 and failed banks are sorted by normalized HHI into quartile groups. Average normalized HHI and average default date of each sorted bank group are shown in third and fourth columns. Month to default is the number of months from 01/01/2008 to average default date.

Quartile	Number of banks	Average Normalized HHI	Average default date	Month to default
1(Concentrated)	74	0.551	2009-11-03	22.4
2	74	0.743	2010-02-07	25.6
3	73	0.797	2010-04-14	27.8
4(Diversified)	74	0.855	2010-03-20	27.0
Total	295			

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