Options and Swaps on Motor Insurance Loss Rate

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Abstract

To manage the risk of mismatch between the actual and anticipated claim amounts in a motor insurance pool, we introduce new concepts such as motor loss rate options and motor loss rate swaps. These hybrid derivatives can transfer the motor insurance loss rate risks to the capital markets. For the valuation of the motor loss rate-linked securities, we assume that motor insurance aggregate claims follow a compound Poisson distribution. Using the Fourier transform, we derive integral expressions for the price of a ratchet option and a fixed-for-floating plain vanilla swap on the motor loss rate.

Key words:

Securitization, Risk transfer, Ratchet option, Swaps, Motor insurance loss rates.

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1. Introduction

Motor loss rate risk has long been a major concern for motor insurers. Usually the amount of future claims may not be predicted completely. The motor insurers want to manage the risk of mismatch between the actual claims and the anticipated claim amounts. If the actual motor loss rates increase more than the expected loss rates, motor insurers will have to make additional payments during the contract period. This will lead to losses on their motor insurance business. The insurers may need additional, low-cost tools to manage motor loss exposure. The insurer will profit if the realized losses from the motors insured decrease below the expected level. The recent concept of insurance securitization provides additional creative options for insurers to transfer insurance risks within the insurance/reinsurance industry or to the capital market. Motor insurance companies attempt to transfer the loss rate risks to the capital market where there is greater capacity to absorb these risks compared to the reinsurance market. Insurance companies may use tradable financial securities that have little or no correlation with the original risk itself.

The traditional methods of reinsurance are still dominant, but some innovative applications of securitization have been gradually introduced into the market. Chang et al. (2011) calculate the prices of catastrophe equity put options using a Markov Modulated Poisson processes. More discussions on catastrophe bonds can be found in Lakdawalla and Zanjani (2011), Härdle and Cabrera (2010), Barrieu and Loubergé (2009), and Lee and Yu (2002). Cummins and Weiss (2009) provide a survey on the recent developments of various types of insurance/financial instruments. Klein and Wang (2009) examine and compare regulatory and other government policies on the financing of catastrophe risks in the United States and the

EU. For a motor insurance example, AXA introduced motor insurance securitization, selling EUR 200 million of bonds in 2005.[1] Bae et al. (2009) illustrate the motor securitization methods based on a concept similar to CDOs. Tranches of bonds are constructed on the basis of the expected loss ratio from groups of motor insurance policyholders. They develop motor loss rate bonds using the structure of synthetic CDOs such that the coupon payments of each tranche depend on the level of the loss rates of the underlying motor insurance pool. They show the pricing methods of the tranches and the pricing formulas where the loss distribution is modeled with a discounted compound Poisson process.

There are several motivating factors for the securitization of motor insurance portfolio risks. These include the issuer gaining an alternative source of financing, a channel of risk transfer, and a method of capital management that helps improve the solvency of the company. Securitization also allows the insurer to eliminate counterparty risks by accessing traditional asset-backed securities investors. Also, it allows the insurer to access those tools that are used by banks for risk management and anticipate the expected evolution of solvency rules. These transactions will also optimize the insurer’s business and balance sheet with respect to volume, pricing, and terms. Motor insurance securitization creates new investment opportunities for the investors by providing greater diversification of traditional portfolios. Such groups of investors are the insurer’s policyholders, the government, the companies and

[1] For more detailed information and discussion see Deringer (2006), De Mey (2007), AXA Financial Protection (2005), and Towers Perrin, Tillinghast (2006).
Also, empirical evidences of movements toward further securitization have been observed from the market. In June 2007, AXA has launched its second securitization of motor insurance risks based on individual motor policies underwritten by its German, Belgian, Italian, and Spanish operations. The first one, based on the French motor insurance portfolio, was a great success. AXA's second one went one step further by combining individual motor portfolios from four countries into a global portfolio providing diversification of motor insurance loss risks. This movement presents significant implications for the future of motor insurance-linked securities (MILS) toward the creation of standardized markets, where risk concentration should be avoided.

organizations related to the motoring industry, and the general investors who seek high-yield securities.

In this paper, we show the stochastic loss rate models and introduce some new concepts in motor loss rate-linked securities such as the motor insurance loss rate options and swaps. The characteristics of these securities are described and the pricing methods are derived using the martingale method.

We first describe the characteristics of a motor loss rate ratchet option and swap and the stochastic motor loss rate models. Then, the pricing methods are shown, and a few numerical examples are enumerated. The conclusion includes discussions and suggestions on the issues of these new securities.

2. Characteristics of Motor Insurance Loss Rate Swap and Option

A motor loss rate swap is a contract to exchange cash flows in the future based on the outcome of at least one presumably random motor loss rates. The objective is for insurers to hedge motor loss rate risks by exchanging one or more future cash flows, at least one of which is random.

Definition 1. *A motor loss rate swap is an agreement between two parties to exchange payments involving at least one random motor loss rate dependent payment for a certain period of time.*

In a simple case, a motor loss rate swap involves the exchange of a single fixed payment for a single random motor loss rate-dependent payment. Suppose that at time 0, two parties enter into an agreement to exchange a pre-fixed value k_t for a random value S_t that is dependent on the realized motor loss rate at some future time t. The fixed amount k_t may depend on the past empirical history on loss experiences. S_t is the actual loss amount realized until time t. Therefore, it is a random variable at time 0. It can be related to the number of accidents from time 0 to time t and the realized loss amounts for each accident from a specified reference motor insurance pool. It is reasonable for the two parties to make an agreement that they would exchange only the net difference between the two payment amounts. For example, party A pays party B a value of $k_t - S_t$ if $k_t > S_t$, or party B pays party A a value of $S_t - k_t$ if $S_t > k_t$. In this case, party A benefits when the realized loss amount S_t is higher than the pre-fixed amount k_t. It makes losses if S_t turns out to be lower than k_t. Therefore, Party A has a long position to S_t, whereas party B has a short position to S_t.

Insurance companies may use motor loss rate swaps to exploit natural hedging across their motor insurance businesses.

When an insurer is particularly interested in managing extreme loss rate risks, a swap contract seems to be inappropriate. In this case, the insurer has a higher chance to make a payment rather than to be paid at each settlement point. Even though the corresponding swap rate may be determined based on the historical data, both parties would like to enter into a swap contract when they expect a fair chance of getting paid. Alternatively, a motor loss rate ratchet option can be used to protect the motor insurance providers against the risks of higher motor loss rates but without giving up the possible benefits of lower motor loss rates. In other words, a motor loss rate ratchet option "insures" the motor insurance providers. The cost of the motor loss rate option is the "insurance premium."

Definition 2. *A motor loss rate ratchet option is a series of call options on motor loss rates where strike thresholds are reset periodically*.

A motor loss rate ratchet option is a right, not an obligation, to exchange cash flows related to motor loss rates. On each exercise date, the option holder receives the excess amount of loss above the pre-specified strike threshold. It is important to reset strike thresholds periodically based on the realized loss rate of the previous period. By doing this, both counterparties are eager to remain in the contract even when they experience extreme loss events in some periods.

These hybrid derivatives have several advantages over motor loss rate bonds. They can be arranged at lower transaction costs than a bond issue. The proposed swap and option cover multiple time periods. Thus, it is more efficient than buying a series of stop loss reinsurances. They are more flexible and can be tailor-made. Most of the arrangements are private placements. They do not need a liquid market. It involves willing counterparties who exploit their comparative advantages or trade views on the development of motor loss rate over time. Their flexibility and low costs provide motor insurers with advantages over the traditional reinsurance treaties.

3. Motor Insurance Loss Rate Models

The following section describes the derivation involved in the securities pricing via the pricing formulas for the stop loss premiums. We assume that the aggregate claims for motor insurance follow a marked point process.[2]

Denote the Poisson process with the parameter λ_t by N_t, for any time $t \geq 0$. On a probability space (Ω, F, P), we assume that the aggregate loss S_t is

$$S_t = \int_{(0,t]} \int_{R^+} g(u,x) N(du,dx) = \sum_{i=1}^{N_t} g(T_i, X_i), \tag{1}$$

where T_i's are jump times of the Poisson process N_t and the magnitudes X_i's of positive random shocks are independently and identically distributed with the distribution function $F_X(x)$. The random shock X_i arrived at time t results in a claim measured by a continuous function $g(t,x)$ defined on $(0, T] \times R_+$, which is increasing in x. Here, we further assume that N_t and X_i are independent for model simplicity. Note that $N(du, dx)$ is a Poisson random measure with the mean measure

$$m(du, dx) = du\, \nu_u(dx),$$

where $\nu_u(dx) = \lambda_u dF_X(x)$ is the Lévy measure.

The following theorem can be shown by using standard machinery in the probability theory. See also Lemma 4.3 of Resnik (1986).

Theorem 1. *For any continuous bivariate function $g \in C((0,T] \times R_+)$ such that $\int_{(0,T]} \int_{R_+} g(u, x) N(du,dx) < \infty$, the following holds:*

$$E\left[\exp\left(\int_{(0,T]} \int_{R_+} g(u,x) N(du,dx)\right)\right]$$

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[2] For modelling the aggregate losses, losses will be accumulated from the date of issue to maturity.

$$= \exp\left[\int_{(0,T]}\int_{R_+} \lambda_u \left(e^{g(u,x)} - 1\right) dF_X(x)\, du\right]. \quad (2)$$

We assume that the Poisson process N_t is a time homogeneous process characterized by the constant rate parameter λ, and the claim size distribution is a discounted random shock $g(u, X) = e^{-ru} X$. Then, the aggregate loss process (1) is also referred to as a discounted compound Poisson process. See Delbaen and Haezendonck (1987), Paulsen (1993), and Nilsen and Paulsen (1996) for more details on the distribution of a discounted compound Poisson process.

Mapping techniques, such as Fourier transform and its inverse transform, will be employed in calculating the market prices of stop loss premiums and other relevant securities.

Corollary 2. *Let us denote the distribution function of S_t by $F_S(x,t)$. The Fourier transform of the distribution of S_t for a given t is expressed as*

$$\hat{f}_S(u,t) = \int_{R^+} e^{iux} dF_S(x,t) = E[e^{iuS_t}]$$

$$= \exp\left\{\int_0^t \lambda_s \left(\hat{f}_X(ue^{-rs}) - 1\right) ds\right\}, \quad (3)$$

where $\hat{f}_X(u)$ is the Fourier transform of the distribution of a claim size random variable X.

Numerous risk neutral probability measures are present because the market is incomplete, and each probability measure does not result in any arbitrage price for insurance risks. The Esscher transform is suitable for such changes in probability measures because of certain specific characteristics. The Esscher transform is known as the minimal entropy martingale probability measure in a geometric Levy process model. The Esscher transform maximizes the expected power utility function.[3] The response of the market to insurance risks can be interpreted by the Esscher parameter h of the Esscher transform, which can be obtained under the martingale state. The real interest rate of zero reflects the constant risk adjustment parameter h (as determined by martingale condition). In this study, the risk adjustment parameter is a time invariant function due to the discounting effect. For each maturity $t > 0$ and under a filtered probability space $\{F_t \mid t > 0\}$, we define a probability measure Q whose Radon-Nikodym derivative is

$$\left.\frac{dQ}{dP}\right|_{F_t} = \frac{e^{h_t S_t}}{E^P[e^{h_t S_t}]} . \tag{4}$$

Equivalently,

$$dF_S^Q(x,t) = \frac{e^{h_t x}}{E^P[e^{h_t S_t}]} dF_S^P(x,t) ,$$

when $E^P[e^{h_t S_t}]$ exists. Note that h_t is non-negative deterministic function that satisfies the martingale condition described below in Eq. (*).[4]

By corollary 2, we have,

$$\hat{f}_S^Q(u,t) = \int_0^\infty e^{iux} \frac{e^{h^* x}}{E^P[e^{h^* S_t}]} f_S^P(x,t)dx = \frac{\hat{f}_S^P(u-ih^*,t)}{\hat{f}_S^P(-ih^*,t)}$$

$$= \exp\left\{\lambda\int_0^t \left\{\hat{f}_X^P((u-ih^*)e^{-rs}) - \hat{f}_X^P(-ih^* e^{-rs})\right\}ds\right\}. \tag{5}$$

3 See Gerber and Shiu (1994) or Miyahara and Fujiwara (2003) for details.

4 Once maturity t is fixed, h_t is determined and assumed to be constant over the period $(0,t]$. For notational convenience, it is denoted by h^*.

Equation (5) can be represented as follows. This enables us to identify the distribution of S_t under the changed measure Q.

$$\hat{f}_S^Q(u,t) = \exp\left\{\lambda\int_0^t\left\{\hat{f}_X^P((u-ih^*)e^{-rs}) - \hat{f}_X^P(-ih^*e^{-rs})\right\}ds\right\}$$

$$= \exp\left\{\int_0^t \lambda\hat{f}_X^P(-ih^*e^{-rs})\left\{\frac{\hat{f}_X^P((u-ih^*)e^{-rs})}{\hat{f}_X^P(-ih^*e^{-rs})} - 1\right\}ds\right\}.$$

Remark 1. By comparing the above with (3), for each fixed maturity t, we can conclude the following:

(i) The Poisson parameter λ has changed to $\lambda_s^Q = \lambda\hat{f}_X^P(-ih^*e^{-rs})$, $s \leq t$;

(ii) The distribution of the claim size, $dF_X^P(x)$, has changed to

$$dF_X^Q(x,s) = \frac{\exp\{h^*e^{-rs}x\}}{\hat{f}_X^P(-ih^*e^{-rs})}dF_X^P(x) \quad \text{and} \quad \hat{f}_X^Q(ue^{-rs}) = \frac{\hat{f}_X^P((u-ih^*)e^{-rs})}{\hat{f}_X^P(-ih^*e^{-rs})}, \ s \leq t.$$

Note that the Lévy measure under Q is

$$\nu_s^Q(dx) = \lambda_s^Q\, dF_X^Q(x,s) = \lambda\, \exp\{h^*e^{-rs}x\}\, dF_X^P(x).$$ ■

For each t,

$$E^Q[S_t] = \int_0^\infty \frac{xe^{h^*x}}{E^P[e^{h^*S_t}]}dF_S^P(x,t) = \frac{E^P[S_te^{h^*S_t}]}{E^P[e^{h^*S_t}]} = \frac{\partial}{\partial h^*}\{\log E^P[e^{h^*S_t}]\}. \tag{6}$$

Because $E^P[e^{h^*S_t}] = \hat{f}_S^P(-ih^*,t)$, the latter can be reduced to

$$E^Q[S_t] = \lambda\int_0^t \frac{\partial}{\partial h^*}\hat{f}_X^P(-ih^*e^{-rs})ds$$

$$= \frac{\lambda}{rh^*}\left\{E^P\left[e^{h^*X}\right] - E^P\left[e^{h^*e^{-rt}X}\right]\right\}. \tag{7}$$

The second equality can be obtained via Corollary 2 by changing the order of integration and expectation.

The arbitrage free price of stop loss contract can be determined with a retention level of *d* for the motor loss rate option pricing. Mathematically this is represented as

$$E^Q[(S_t - d)_+] = \int_d^\infty (x-d)dF_S^Q(x,t)\,. \tag{8}$$

The following equation is obtained by applying Theorem 3.4 in Dufresne et al. (2009):

$$E^Q[(S_t - d)_+] = \frac{E^Q[S_t]}{2} + \frac{1}{\pi} PV\int_0^\infty \mathrm{Re}\left[\frac{e^{-iud}(\hat{f}_S^Q(u,t)-1)}{(iu)^2}\right]du\,. \tag{9}$$

Here, PV$\int$ refers to the Cauchy principle value integral.

Substitution of (5) and (7) into (9) results in an Esscher no-arbitrage price formula for stop loss contract.[5]

If there is no arbitrage between insurance market and capital market, the discounted surplus process should be a martingale under a risk neutral measure *Q*.[6] We define the accumulated surplus process as follows,

$$U_t = u_0 e^{rt} + e^{rt} C_t - e^{rt} S_t\,, \tag{10}$$

where u_0 is the initial surplus, *r* is the risk free rate compounded continuously, and C_t is the time zero value of the risk-adjusted aggregated premiums collected on [0, t]. We define

[5] Price of a stop loss contract under the compound Poisson distribution can be calculated by using the numerical method or recursion such as the Panjer recursion formula. See Covens et al. (1979), Bühlmann (1984), Gerber (1982), and Panjer (1981) for reference. The Esscher change of measure, the general form of the Laplace transforms, and the expectations of these processes are well studied in the literature such as Jacod and Shiryaev (2003) and Dufresne et al. (2009).

[6] Same idea is used in Jang and Krvavych (2004).

$$C_t = (1+\theta)E^P[S_t] = (1+\theta)\lambda E^P[X]\bar{a}_{\bar{t}|} = C\bar{a}_{\bar{t}|},$$

with a risk adjustment parameter $\theta > 0$. We denote the continuous premium rate by $C = (1+\theta)\lambda E^P[X]$ and the present value of continuously paying annuities by $\bar{a}_{\bar{t}|} = (1-e^{-rt})/r$.

We can find an equivalent martingale measure Q that satisfies the following:[7]

$$E^Q[e^{-rt}U_t \mid F_s] \quad = \quad e^{-rs}U_s,\ Q\text{-}a.s. \tag{*}$$

for any $0 \le s \le t$. By (7), for the maturity t, we can show that h_t is the solution of the following equation. This also satisfies the existence of the Esscher transform (4).

$$E^Q[S_t] = \frac{\lambda}{rh_t}\left\{E^P\left[e^{h_t X}\right] - E^P\left[e^{h_t e^{-rt} X}\right]\right\} = C\,\bar{a}_{\bar{t}|}. \tag{11}$$

Using L'hopital's rule, it can be shown that

$$\lim_{h_t \to 0} E^Q[\mathrm{S}_\mathrm{t}] = \lambda E^P[X]\bar{a}_{\bar{t}|} \ .$$

Eq. (11) and the above identity imply that if the risk adjusted parameter θ = 0 or $C = \lambda E^P[X]$, then $h_t \equiv 0$, which means that the market takes the risk fully.

It can be shown that the constant Esscher parameter is the solution to the equation below (similar to (11)) in the presence of zero real interest rate.

$$E^P[Xe^{hX}] = \frac{C}{\lambda}.$$

From Eqs. (5) and (11), the moment generating function of the claim size distribution plays a crucial role in determining mathematical tractability of the price formula. Setting

[7] We can find a martingale measure Q or the Esscher parameter h_t directly from the market when the market becomes active and mature. This approach can be an alternative when the market is new and young.

aside empirical studies on the motor insurance claim size data, we illustrate a tractable example, [8] which is also practical.

Example 1. Generalized Erlang(n) claim size distribution

Let us assume that X follows a generalized Erlang(n) distribution with parameters $(n, \beta_1, \beta_{2,\ldots,} \beta_n)$ such that $E^P[X] = \sum_{k=1}^{n} \beta_k$. The Fourier transform is then given by $\hat{f}_X^P(u) = \prod_{k=1}^{n} (1 - i\beta_k u)^{-1}$.

To guarantee the existence of the Esscher parameter h_t, which also ensures the existence of Esscher transform, we further assume that

$$\beta_1 e^{-rt} < \beta_1 < \beta_2 e^{-rt} < \beta_2 < \cdots < \beta_n e^{-rt} < \beta_n . \tag{12}$$

This condition may hold when the interest rate r is small and the maturity t is short.

The expectation of discounted claims under Q is

$$E^Q[S_t] = \frac{\lambda}{rh^*} \left\{ \prod_{k=1}^{n} (1 - \beta_k h^*)^{-1} - \prod_{k=1}^{n} (1 - \beta_k e^{-rt} h^*)^{-1} \right\}.$$

Note that the Esscher parameter, if any, must be smaller than $1/\beta_n$ in order to guarantee the existence of the moment generating function or the Esscher transform. It can be shown that the Esscher parameter h^* is a solution of the following polynomial equation. [9]

$$p(h) = q(h)(1+\theta)\sum_{k=1}^{n} \beta_k \ , \tag{13}$$

[8] Exponential (or Gamma) distribution is often used in generalized linear models on aggregated insurance data. See Smyth and Jorgensen (2002) for examples.

[9] We assume that the Esscher parameter is constant once the maturity t is fixed. Thus, we drop t in the expression.

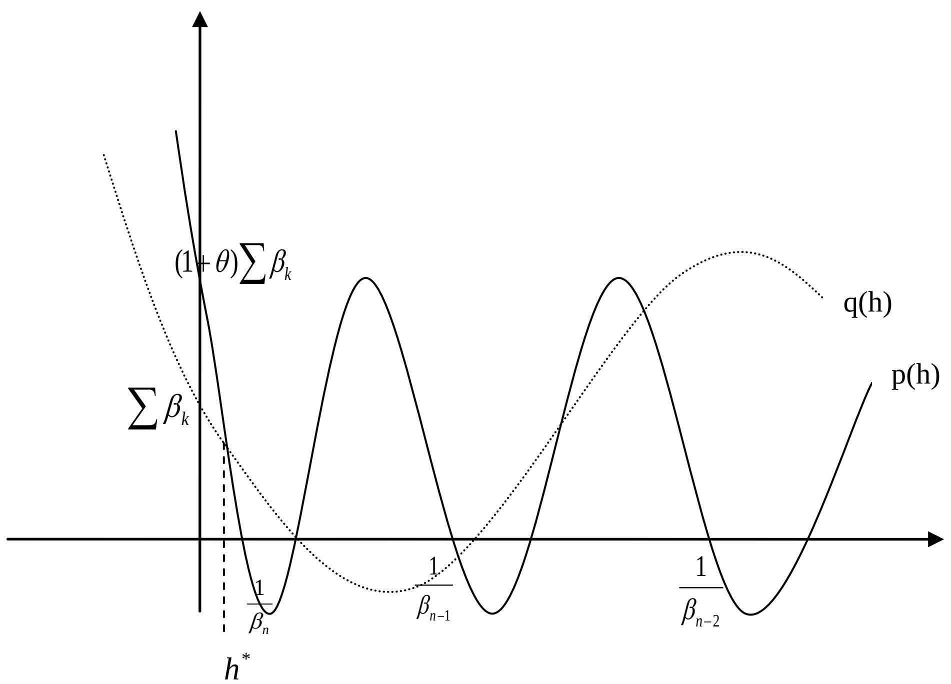
where

$$p(h)=\sum_{k=1}^{n}\beta_k-h(1+e^{-rt})\sum_{k\neq l}\beta_k\beta_l+h^2(1+e^{-rt}+e^{-2rt})\sum_{k\neq l\neq m}\beta_k\beta_l\beta_m+\cdots+(-1)^{n-1}h^{n-1}\left(\sum_{q=0}^{n-1}e^{-qt}\right)\prod_{k=1}^{n}\beta_k$$

$$q(h)=\prod_{k=1}^{n}(1-\beta_k h)(1-\beta_k e^{-rt}h)\ .$$

Clearly, *q(h)* has *2n* zeros at $\left\{\frac{1}{\beta_n},\frac{1}{\beta_n e^{-rt}},\cdots,\frac{1}{\beta_1},\frac{1}{\beta_1 e^{-rt}}\right\}$. From (12) and the fact that $p(1/\beta_j)=\frac{1}{\beta_j^{n-2}}\prod_{k\neq j}(\beta_j-\beta_k e^{-rt})$, we can show that *p(h)* has *n*-1 zeros and the *j*th smallest zero lies in the interval $(1/\beta_{n-j+1},\ 1/\beta_{n-j})$. Therefore, there exists only one root *h*, which is smaller than $1/\beta_n$ as illustrated in Figure 1.

Figure 1 Finding h* using graphs



Note that the exponential distribution is a special case when n =1. The Esscher parameter in this case is

$$h^* = \frac{1}{2\beta}\left\{1+e^{rt}-\sqrt{(1+e^{rt})^2-4e^{rt}\theta/(1+\theta)}\right\}. \tag{14}$$

It follows that

$$k(u,\ \eta,\ h^*,\ l,\ t) := \lambda\int_{(l,t]}\hat{f}_X^P((u(1-\eta)-ih^*)e^{-rs})ds$$

$$=\frac{\lambda}{2r}\sum_{k=1}^{n}\left[\frac{2\beta_k^{n-1}\left\{\arctan\left(\frac{u(1-\eta)\beta_k}{\beta_k h^*-e^{rt}}\right)-\arctan\left(\frac{u(1-\eta)\beta_k}{\beta_k h^*-e^{rl}}\right)\right\}i+\beta_k\ \log\left\{\frac{u^2(1-\eta)^2\beta_k^2+(e^{rt}-\beta_k h^*)^2}{u^2(1-\eta)^2\beta_k^2+(e^{rl}-\beta_k h^*)^2}\right\}}{\prod_{j\neq k}(\beta_k-\beta_j)}\right]. \tag{15}$$

For notational simplicity, we denote

$$A(u,\eta,l,t)=\prod_{k=1}^{n}\left\{\frac{u^2(1-\eta)^2\beta_k^2+(e^{rt}-\beta_k h^*)^2}{u^2(1-\eta)^2\beta_k^2+(e^{rl}-\beta_k h^*)^2}\right\}^{\frac{\lambda\beta_k}{2r\prod_{j\neq k}(\beta_k-\beta_j)}},$$

and

$$B(u,\eta,l,t)=\frac{\lambda}{r}\sum_{k=1}^{n}\left[\frac{\beta_k^{n-1}\left\{\arctan\left(\frac{u(1-\eta)\beta_k}{\beta_k h^*-e^{rt}}\right)-\arctan\left(\frac{u(1-\eta)\beta_k}{\beta_k h^*-e^{rl}}\right)\right\}}{\prod_{j\neq k}(\beta_k-\beta_j)}\right].$$

Then, the Fourier transform of the distribution of S_t can be written as

$$\hat{f}_S^Q(u,t)=\exp\left\{k(u,\ 0,\ h^*,\ 0,\ t)-k(0,\ 0,\ h^*,\ 0,\ t)\right\}=\frac{A(u,0,0,t)}{A(0,0,0,t)}e^{iB(u,0,0,t)}. \tag{16}$$

Substituting the above into (9) gives the market price of the stop loss contract.

■

4. Risk Neutral Distribution of Increment of Loss Processes and a Ratchet Option Price

We now derive the Fourier transform of the risk neutral distribution of the increment of the loss process. The price formula of a ratchet option is shown.

For a fixed $\eta \geq 0$ and $0 \leq l \leq t \leq T$, let us consider a general increment of the loss process,

$$Z_{l,t}(\eta) := S_t - \eta S_l = \int_{(0,t]} \int_{R^+} e^{-ru} x N(du, dx) - \eta \int_{(0,l]} \int_{R^+} e^{-ru} x N(du, dx)$$

$$= \int_{(0,l]} \int_{R^+} (1-\eta) e^{-ru} x N(du, dx) + \int_{(l,t]} \int_{R^+} e^{-ru} x N(du, dx) . \qquad (17)$$

As noted in the Remark 1, under *Q*, the jump process follows an inhomogeneous Poisson process with the intensity function $\lambda_s^Q = \lambda \hat{f}_X^P(-ih^* e^{-rs})$, which depends on time. The claim size distribution $dF_X^Q(x,s) = \dfrac{\exp\{h^* e^{-rs} x\}}{\hat{f}_X^P(-ih^* e^{-rs})} dF_X^P(x)$ is also a function of time. Fortunately, the distribution still has an independent increments property. The first and the second terms in (17) are non-overlapping and thus independent under both measures *P* and *Q*. Based on Theorem 1, the property provides the following expression for the Fourier transform of $Z_{l,t}(\eta)$ under *Q*.

Corollary 3. *For a fixed $\eta \geq 0$ and $0 \leq l \leq t \leq T$, the Fourier transform of risk neutral distribution of the general increment process $Z_{l,t}(\eta)$ is expressed as*

$$\hat{f}_{Z_{l,t}}^Q(u,\eta) = E^Q\left[e^{iu(S_t - \eta S_l)}\right]$$

$$= E^Q\left[\exp\left\{iu\left(\int_{(0,l]}\int_{R^+}(1-\eta)e^{-rs}xN(ds,dx)+\int_{(l,t]}\int_{R^+}e^{-rs}xN(ds,dx)\right)\right\}\right]$$

$$= E^Q\left[\exp\left\{\int_{(0,l]}\int_{R^+}iu(1-\eta)e^{-rs}xN(ds,dx)\right\}\right]E^Q\left[\exp\left\{\int_{(l,t]}\int_{R^+}iue^{-rs}xN(ds,dx)\right\}\right]$$

$$= \exp\left\{\lambda\left(\int_{(0,l]}\hat{f}_X^P((u(1-\eta)-ih^*)e^{-rs})ds+\int_{(l,t]}\hat{f}_X^P((u-ih^*)e^{-rs})ds-\int_{(0,t]}\hat{f}_X^P(-ih^*e^{-rs})ds\right)\right\}. \quad (18)$$

The expectation of $Z_{l,t}(\eta)$ is

$$E^Q[Z_{l,t}(\eta)] = \frac{\lambda}{rh^*}\left\{(1-\eta)E^P[e^{h^*X}]-E^P[e^{h^*e^{-rt}X}]+\eta E^P[e^{h^*e^{-rl}X}]\right\}. \quad (19)$$

By substituting the Fourier transform (18) and the expectation (19) into the stop-loss price formula (9), we can obtain

$$E^Q[(S_t-\eta S_l)_+] = E^Q[(Z_{l,t}(\eta))_+] = \frac{E^Q[Z_{l,t}(\eta)]}{2}+\frac{1}{\pi}PV\int_0^\infty \mathrm{Re}\left[\frac{(\hat{f}^Q_{Z_{l,t}}(u,\eta)-1)}{(iu)^2}\right]du. \quad (20)$$

Example continued: Generalized Erlang(n) claim size distribution

The expectation of $Z_{l,t}(\eta)$ is

$$E^Q[Z_{l,t}(\eta)] = \frac{\lambda}{rh^*}\left\{(1-\eta)\prod_{k=1}^n(1-\beta_k h^*)^{-1}-\prod_{k=1}^n(1-\beta_k e^{-rt}h^*)^{-1}+\eta\prod_{k=1}^n(1-\beta_k e^{-rl}h^*)^{-1}\right\}.$$

Recalling Eq. (15), the Fourier transform (18) can be written as

$$\hat{f}^Q_{Z_{l,t}}(u,\eta) = \exp\{k(u,\,\eta,\,h^*,\;0,\,t)+k(u,\,0,\,h^*,\;l,\,t)-k(0,\,\eta,\,h^*,\;0,\,t)\}$$

$$= \frac{A(u,\eta,0,l)\cdot A(u,0,l,t)}{A(0,\eta,0,t)}\exp[\{B(u,\eta,0,l)+B(u,0,l,t)\}i].$$

By substituting these formulae into the price formula (20), we can obtain an integral expression of stop loss contract where the threshold depends on the loss history. ■

Ratchet option on motor loss rate

The actual loss ratio is defined as the actual aggregate loss divided by the total gross premium over a period of time [0, t]. In practice, the fixed loss rate is essentially determined by the historical claims data and is usually accomplished by simulating the future loss that can be assumed to be retained by the insurance companies.

We denote gross aggregate premium collected on [0, t] by

$$G_t = (1+\theta^*)E^P[S_t] = (1+\alpha)C_t = (1+\alpha)(1+\theta)\lambda E^P[X]\bar{a}_{\overline{t|}} = G\bar{a}_{\overline{t|}}, \tag{21}$$

where θ is a risk adjustment parameter, and α is a security loading factor (expense rate).

Let us denote the actual cumulative loss rate by q_t,

$$q_t = \frac{S_t}{G\bar{a}_{\overline{t|}}} = \frac{L_t}{G\bar{s}_{\overline{t|}}}, \tag{22}$$

where $L_t = S_t e^{rt}$ is the cumulative loss until the time *t*; and $\bar{a}_{\overline{t|}}$ and $\bar{s}_{\overline{t|}} = e^{rt}\bar{a}_{\overline{t|}}$ are the present value and the accumulated value, respectively, of continuously paying annuities with unit of annual payment.

For each settlement point, the threshold of the ratchet option is defined as

$$\hat{q}_{t_i}(\pi) = \begin{cases} (1+\pi)E^Q[q_{t_1}] = (1+\pi)\dfrac{E^Q[S_{t_1}]}{G\bar{a}_{\overline{t_1|}}}, & i = 1, \\ (1+\pi)q_{t_{i-1}} = (1+\pi)\dfrac{S_{t_{i-1}}}{G\bar{a}_{\overline{t_{i-1}|}}}, & i > 1. \end{cases} \tag{23}$$

Note that the threshold evolves over time and depends on the actual loss of the previous settlement point. The parameter $\pi\ (> -1)$ determines the level of the subsequent strike thresholds and must be specified upfront. If an insurer would like to hedge the extreme losses above the previous period's realized loss, a large π is preferred.

At each settlement date, the protection seller (issuer) will pay the premium collected multiplied by the excess loss rate beyond the prefixed threshold. Thus, a no-arbitrage price of the n-year ratchet option is given as

$$V(\pi;0) = E^Q\left[\sum_{i=1}^{n} G\bar{a}_{\overline{t_i}|}\left(q_{t_i} - \hat{q}_{t_i}(\pi)\right)_+\right]$$

$$= E^Q\left[G\bar{a}_{\overline{t_1}|}\left(q_{t_1} - (1+\pi)E^Q[q_{t_1}]\right)_+\right] + E^Q\left[\sum_{i=2}^{n} G\bar{a}_{\overline{t_i}|}\left(q_{t_i} - (1+\pi)q_{t_{i-1}}\right)_+\right]$$

$$= E^Q\left[\left(S_{t_1} - (1+\pi)E^Q[S_{t_1}]\right)_+\right] + \sum_{i=2}^{n} E^Q\left[\left(S_{t_i} - (1+\pi)\delta(t_{i-1},t_i)S_{t_{i-1}}\right)_+\right], \qquad (24)$$

where $\delta(t_{i-1},t_i) = \dfrac{1-e^{-rt_i}}{1-e^{-rt_{i-1}}}$.

If we assume $\eta = (1+\pi)\delta(t_{i-1},t_i)$, then from (20) we obtain an integral expression of each summand in the ratchet option price formula (24).

5. Pricing Motor Loss Rate Swaps

Plain Vanilla Motor Loss Rate Swaps

First, we consider a fixed-for-floating plain vanilla motor loss rate swap settled in arrears. Even though we have a continuous time loss rate model, we consider only the finite collection of discrete future dates $\{T_j, j = 0, 1, \ldots, n\}$ with $T_0 = 0$. The dates $T_0,\ldots,T_{n-1}$ are known as reset dates, and the dates $T_1,\ldots,T_n$ are known as settlement dates. The payments are made on the settlement dates and the number of payments n is called the length of a swap. The first date T_0 is referred to as the start date of a swap, and we assume it is today for the sake of simplicity. The period $[T_{j-1}, T_j]$ is called the j-th accrual period. We assume that

Party A agrees to pay Party B a fixed amount of losses derived from a pre-agreed fixed loss ratio denoted by $\hat{q}_{T_j}$ at each settlement date T_j, $j = 1, \ldots, n$. In return, Party B agrees to pay Party A a floating amount of losses realized until each settlement date T_j, $j = 1, \ldots, n$. The two parties usually need to pay the net amount, that is, the difference between the two mutual obligations. Therefore, Party B should pay when the actual loss ratio exceeds the predetermined fixed loss ratio $q_{T_j} > (1+s)\ \hat{q}_{T_j}$, where s is a real number to be determined. That is, $q_{T_j} > (1+s)\hat{q}_{T_j}$ implies that $L_{T_j} > (1+s)\hat{q}_{T_j}\ G\ \bar{s}_{\overline{T_j}|}$, and Party B should pay $L_{T_j} -$ $(1+s)\hat{q}_{T_j}\ G\ \bar{s}_{\overline{T_j}|}$. If $q_{T_j} \leq (1+s)\hat{q}_{T_j}$, then Party A should pay $(1+s)\hat{q}_{T_j}\ G\ \bar{s}_{\overline{T_j}|} - L_{T_j}$ at time T_j.

We consider the value of a motor loss rate swap as a function of a real number s at time $0 = T_0$,

$$\mathrm{MS}(s) = E^Q\left[\sum_{j=1}^{n} e^{-rT_j}(L_{T_j} - (1+s)\hat{q}_{T_j} G\bar{s}_{\overline{T_j}|})\right]$$

$$= E^Q\left[\sum_{j=1}^{n}(S_{T_j} - (1+s)\hat{q}_{T_j} G\bar{a}_{\overline{T_j}|})\right]. \quad (25)$$

We know that a swap value is zero at initiation; therefore, we naturally define the spread of a motor loss rate swap.

Definition 3. *The spread of a motor loss rate swap is the value of s that makes the value of a motor loss rate swap zero, i.e., the value of s for which MS(s) = 0.*

Using definition 3, we obtain an explicit formula for a motor loss rate swap spread,

$$s = \frac{\sum_{j=1}^{n} E^Q[S_{T_j}]}{\sum_{j=1}^{n} \hat{q}_{T_j} G\bar{a}_{\overline{T_j}|}} - 1 \quad = \quad \frac{1}{1+\alpha} \; \frac{\sum_{j=1}^{n} \bar{a}_{\overline{T_j}|}}{\sum_{j=1}^{n} \hat{q}_{T_j} \bar{a}_{\overline{T_j}|}} - 1. \tag{26}$$

When we consider only a one-period swap, that is, $n = 1$, the price is

$$s = \frac{1}{1+\alpha} \; \frac{1}{\hat{q}_{T_1}} - 1.$$

If the expected loss rate $\hat{q}_{T_1}$ is produced well enough to predict the real loss rate for the next period, then the swap prices should be zero and the expense rate should be

$$\alpha = \frac{1}{\hat{q}_{T_1}} - 1.$$

Given the value of s determined above, we can calculate the market price of the swap at any time $t \geq T_0$. The following Corollary 4 of Theorem 1 can be shown based on the fact that the loss distribution S_t still has the property of independent increments after the change in measure. This can also be seen in remark 1 in the previous section.

Corollary 4. Under the Esscher transformed measure Q, the Fourier transform of the conditional distribution S_t given by $S_l = y$ has the following expression:

$$\hat{f}_S^Q(u,t;y,l) = E^Q\left[e^{iuS_t} \mid S_l = y\right] = E^Q\left[\exp\left\{iuy + \int_{(l,t]}\int_{R^+} iuxe^{-rs} N(ds,dx)\right\} \Big| S_l = y\right]$$

$$= \exp\left\{iuy + \lambda \int_l^t \left\{\hat{f}_X^P((u - ih_t)e^{-rs}) - \hat{f}_X^P(-ih_t e^{-rs})\right\} ds\right\}. \tag{27}$$

Then, the conditional expectation can be written as

$$E^Q[S_t \mid S_l = y] = \frac{1}{i}\frac{d}{du} \hat{f}_S^Q(u,t;y,l) \Big|_{u=0}$$

$$= y + \frac{\lambda}{rh_t}\left\{E^P[e^{h_t e^{-rt} X}] - E^P[e^{h_t e^{-rt} X}]\right\}. \tag{28}$$

By using the above conditional expectation under the measure *Q*, the market price of the swap at a time in the *j*-th accrual period, $t \in (T_{j-1}, T_j)$, can be expressed as

$$V(t) = E^Q\left[e^{rt}\sum_{i=j}^{n}(S_{T_i} - (1+s)\hat{q}_{T_i} G\bar{a}_{\overline{T_i}|})\middle|S_t\right] = e^{rt}\left(\sum_{i=j}^{n} E^Q[S_{T_i}|S_t] - \frac{\sum_{i=j}^{n}\hat{q}_{T_i}\bar{a}_{\overline{T_i}|}}{\sum_{i=1}^{n}\hat{q}_{T_i}\bar{a}_{\overline{T_i}|}}\sum_{i=1}^{n}E^Q\left[S_{T_i}\right]\right)$$

$$= e^{rt}\left((n-j+1)S_t + \sum_{i=j}^{n}\frac{\lambda}{rh^*}\left\{E^P[e^{h^* e^{-rt} X}] - E^P[e^{h^* e^{-rT_i} X}]\right\} - C\sum_{i=1}^{n}\bar{a}_{\overline{T_i}|}\frac{\sum_{i=j}^{n}\hat{q}_{T_i}\bar{a}_{\overline{T_i}|}}{\sum_{i=1}^{n}\hat{q}_{T_i}\bar{a}_{\overline{T_i}|}}\right). \tag{29}$$

6. Numerical Examples and Problems

Here, we consider some numerical examples under certain specific assumptions on the distribution of the discounted losses and the parameters.[10] We also discuss a few issues on the development of motor insurance-linked securities (MILS). For example, we assume that the losses follow a discounted compound Poisson process with generalized Erlang(2) claim size distribution. We assume that the Poisson parameter λ *=12,* $\beta_1 = 5$, $\beta_2 = 15$, [11] and maturity *T = 5.*

[10] Numerical integrations are implemented using the R-function *integrate*. A hundred thousand simulations of S_t are conducted under Q based on Remark 1.For each simulation, risk neutral arrival times of the inhomogeneous Poisson process with rate function λ_t are simulated by a thinning algorithm. For each arrival time, a risk neutral claim amount is generated by the density $f_X^Q(x,t)$ using rejection algorithm. See Ross (2002) for details on these algorithms.

[11] The choice of loss frequency and claim size parameters is more or less arbitrary. We roughly use the 2007 US private passenger insurance losses data given by Insurance Information Institute, in Auto Insurance (2009),

Table 1 summarizes the risk adjusted premium rate $C = (1+\theta)\lambda E^{P}[X]$ and the Esscher parameter $h^{*} = h_{5}$ for several different choices of the risk adjustment parameter θ and the interest rate r.

Table 1 Premium rate and Esscher parameters

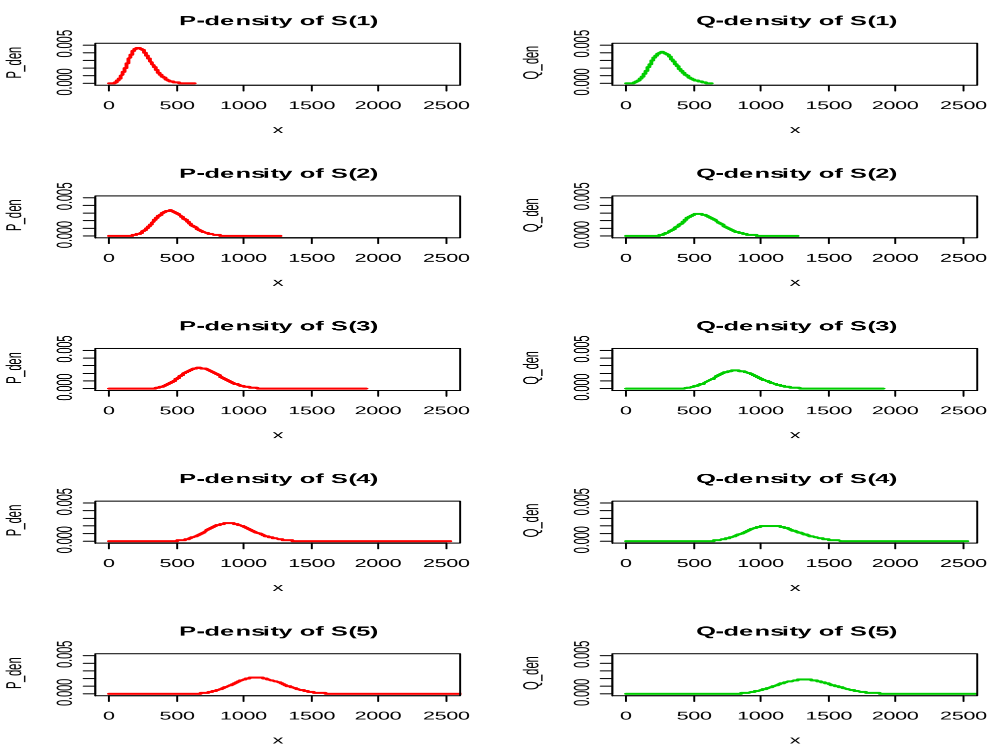
θ	r	C	h^{*}
0.1	0.01	264	0.002967
	0.03		0.003108
	0.05		0.003248
	0.07		0.003386
0.2	0.01	288	0.005534
	0.03		0.005802
	0.05		0.006066
	0.07		0.006326
0.3	0.01	312	0.007828
	0.03		0.008207
	0.05		0.008581
	0.07		0.008946

available at http://www.iii.org/media/facts/statsbyissue/auto/. Statistics indicate that the frequency of liability claims is about 5 per 100 vehicles, and the average claim severity is about $15,000 when we add bodily injury and property damage. For collision and comprehensive coverage, the frequency is about 7 per 100 vehicles and the average per claim severity is roughly $5,000. We use the Poisson parameter λ = 12 (= 5+7), β_1 = 5 and β_2 = 15 for illustrative purposes. The results may vary when the assumptions are changed.

It is evident from Table 1 that the Esscher parameter increases in both the loading factor and the interest rate. The risk adjustment parameter and the interest rate are the two main input components that determine the riskiness of the underlying loss process. The higher value of the Esscher parameter results in a shift of loss distribution to the right.

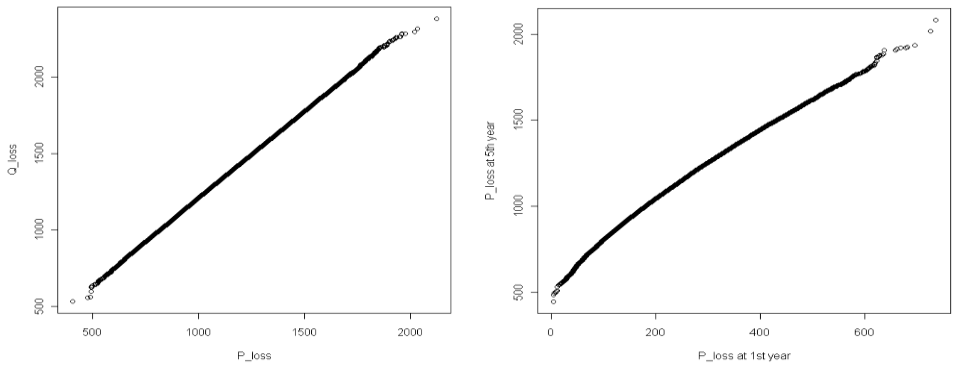
Figure 2 shows the evolution of the discounted loss distribution S_t over time and compares the two densities $f_S^P(x,t)$ and $f_S^Q(x,t)$ when θ =0.2 and r = 0.03.

Figure 2 Evolution of loss process



Figures 2 and 3 show that the distribution of the discounted losses under the Esscher transform are translated to the right and have a slightly heavier left tail and a lighter right tail. This implies that the Esscher transform puts more weight on smaller extreme values.

Figure 3 qq-plots



The second plot of Figure 3 compares the distributions of *S(1)* and *S(5)* under *P*. We see that the left tail of *S(5)* is heavier, and the right tail becomes thinner than that of *S(1)*. This is because of the effect of discounting. In other words, the discounted values of claims that happen in the fifth year contribute less to the total aggregated loss than claims that arrive in the first year.

Table 2 gives the time zero prices of five-year ratchet options for different choices of θ corresponding to different coverage levels π. We assume that the interest rate r is 0.03.

Table 2 Time zero price of five-year ratchet options

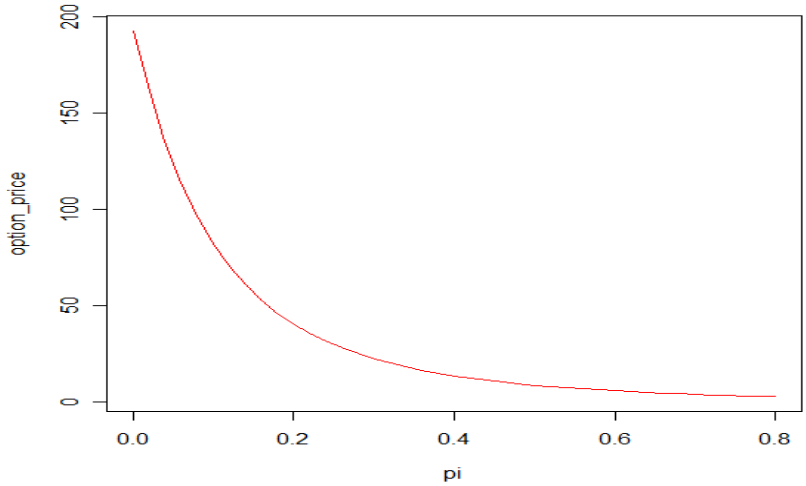
Θ	π	$V(\pi;0)$	Θ	π	$V(\pi;0)$
	0.0	183.65		0.0	201.59
	0.1	80.27		0.1	83.81
0.1	0.2	40.29	0.3	0.2	40.62
	0.3	22.84		0.3	22.36
	0.4	14.02		0.4	13.34

	0.5	9.11		0.5	8.42
	0.6	6.18		0.6	5.56
	0.7	4.35		0.7	3.81
	0.8	3.15		0.8	2.70
0.2	0.0	192.85	0.4	0.0	209.84
	0.1	82.14		0.1	85.27
	0.2	40.49		0.2	40.69
	0.3	22.61		0.3	22.09
	0.4	13.68		0.4	13.00
	0.5	8.76		0.5	8.10
	0.6	5.86		0.6	5.28
	0.7	4.07		0.7	3.57
	0.8	2.91		0.8	2.49

High-risk adjustment parameters make the option price large for moderate coverage levels (π = 0, 0.1, or 0.2). The prices become smaller for extreme coverage cases. This is because the left tail of the Esscher transformed distribution becomes heavier as the risk adjustment parameter gets larger.

Figure 4 shows that the option prices decrease rapidly as the coverage rate π increases when $\theta = 0.2$ and $r = 0.03$.

Figure 4 Market prices of five-year ratchet options according to π



Swap spread depends on the trigger $\hat{q}_t$, which determines the level of insurance coverage of the swap contract. In this example, we use percentile values of the loss ratio process q_t under the physical measure P.

Table 3 Evolution of percentiles of loss ratio under P **(**$\theta = 0.2,\ r = 0.03,\ \alpha = 0.1$**)**

Percentiles	yr1	yr2	yr3	yr4	yr5
10%	0.42	0.51	0.56	0.58	0.60
20%	0.52	0.59	0.62	0.64	0.65
30%	0.59	0.65	0.67	0.68	0.69
40%	0.67	0.70	0.71	0.72	0.72
50%	0.73	0.75	0.75	0.75	0.75
60%	0.81	0.80	0.79	0.79	0.78
70%	0.88	0.85	0.84	0.83	0.82
80%	0.98	0.92	0.89	0.87	0.86

Table 3 provides the percentiles of the loss ratio process at the end of each year. We can see that the lower percentiles increase and upper percentiles decrease with time. This is also the effect of discounting.

Table 4 Swap spreads for $\theta = 0.2,\ r = 0.03,\ \alpha = 0.1$

$\hat{q}$	10%tile	20%tile	30%tile	40%tile	50%tile	60%tile	70%tile	80%tile
s	0.616	0.458	0.358	0.280	0.213	0.152	0.090	0.025

For given parameters, Table 4 gives the five-year swap spread for each percentile-based threshold $\hat{q}_t$. Let us choose the 80th percentile for the trigger $\hat{q}_1 = 0.98$. Then, the protection seller should pay 1.025*0.98*$G\ \bar{s}_{\overline{1}|}$ to the protection buyer if the actual loss ratio is greater than 1.025*0.98. The protection buyer has to pay the same amount when the actual loss ratio is less than 1.025*0.98.

As one can see from the spread formula (26), a fair spread has an inverse relationship with the expense rate α and the trigger $\hat{q}_t$. In general, counterparties can agree to use arbitrary thresholds. For some cases, the spread can even be negative when the expense rate is chosen to be much higher than the ideal level. The spread can be equal to zero when the expense rate $\alpha = \sum_{j=1}^{n} \bar{a}_{\overline{T_j}|} \Big/ \sum_{j=1}^{n} \hat{q}_{T_j} \bar{a}_{\overline{T_j}|} - 1$. When the P-percentile-based thresholds are used (as in this example), the effect of the increase in expense rate is cancelled out due to the decrease in the threshold at the same rate. Thus, the swap spread remains unchanged regardless of the choice of expense rates. This suggests that *P*-percentile is a sensible choice for the trigger $\hat{q}_t$.

Now, a few issues on the MILS[12] are presented. Bae et al. (2009) derived the theoretical framework and methodology and showed the usefulness of motor loss rate securitization. There are certain benefits of using options and swaps over the existing market instruments for motor insurance securitization. The main advantages of the hybrid derivatives over tranche notes are their easiness and flexibility. They can design the derivatives according to their needs. The swaps and options are over-the-counter contracts; they would require smaller transaction costs. However, over-the-counter contracts may have counterparty risks, and this can be a disadvantage.

For tranche notes with fixed loss rate trigger, there would be possibility of moral hazard by insurers. As maturity approaches, insures may mark up their insurance losses ratio to obtain reimbursement from the protection seller (investors). Also, it is possible that motor insurers may put less effort in underwriting and loss assessment to minimize insurance losses. These moral hazards can be reduced by establishing independent special purpose vehicles (SPVs) responsible for auditing the determination of loss ratio as well as issuing and pricing the securities. The motor insurance loss rate ratchet option proposed in this paper dissuades insurers from inflating their insurance losses in the middle of the security tenure. The multi-term ratchet option resets the loss trigger periodically according to the realized loss ratios of the previous years. The insurer may obtain benefits by manipulating the loss ratio of one period. However, the benefit will be cancelled in the following period due to a high loss

[12] We briefly discuss a few issues in this paper. More detailed discussions regarding MILS can be found in Bae and Kim (2010).

rate trigger. Also, there can be legal issues in almost all aspects of the MILS operation. They can resolve the legal issues using regulations or laws. In particular, regulators should focus on the financial soundness of issuers and market participants.

Motor insurance-linked securities can be an effective alternative to the reinsurance markets, which have a strong development potential for high-risk groups that may not be (re)insured under the current insurance/reinsurance practices. Capital market investors are likely to welcome the risks expecting high returns as compensation for taking risks. Insurance companies benefit from the securitization because they can expand the risk pool by accepting high-risk groups that are otherwise uninsurable. In addition, the ability to insure the high-risk groups makes the insurance industry competitive and gives it a level playing field with other financial institutions such as banks.

7. Conclusions

When an insurer decides to sell motor insurance contracts covering the losses from motor accidents, it should consider appropriate models to estimate the expected amount of annual claims. However, the insurer can be exposed to the risk of the actual loss rate being higher than expected. As a hedging method, we suggest the use of hybrid derivatives for motor insurance loss rate risk transfer. We consider a few motor insurance-linked derivatives such as motor insurance loss rate options and swaps, which can be traded over the counter in a capital market. They are designed not only to provide the insurer with innovative hedging methods for its loss rate risks but also to give more investment choices to the potential investors in the financial market.

The insurer may want to exchange random cash flows in the future based on the outcome of motor loss rates with prefixed values. In this case, the insurance company would use motor loss rate swaps. When an insurer is particularly interested in managing extreme loss rate risks, a swap contract would be inappropriate. Alternative methods considered is a motor loss rate ratchet option that can be used to protect the motor insurance providers against the risks of higher motor loss rates without giving up the possible benefits of lower motor loss rates. The pricing formulas of a ratchet option and swaps on motor insurance loss rates are given under a few assumptions made on the aggregate loss process. We choose the Esscher transform to link between the insurance market and capital market. The risk neutral pricing formulas for the ratchet option and swaps are obtained by using Fourier inversion and are expressed in integral forms.

We show numerical examples using numerical integration and simulation methods to illustrate the derivative prices and their characteristics when the claim amount distribution follows a generalized Erlang distribution.

References

Auto Insurance, 2009, Retrieved Dec. 10, 2009, from http://www.iii.org/media/facts/statsbyissue/auto/.

AXA Financial Protection, 2005, AXA launches the first securitization of a motor insurance portfolio. Press Release, November 3, 2005. Retrieved Nov. 10, 2009, from www.axa.com/hidden/lib/en/uploads/pr/group/2005/AXA_PR_20051103.pdf.

Bae, T., C. Kim, and R.J. Kulperger, 2009, Securitization of motor insurance loss rate risks. *Insurance: Mathematics and Economics*, 44: 48–58.

Bae, T., and C. Kim, 2010, Motor Insurance Linked Securities: An Area of Financial Innovation, *UNSW working paper*.

Barrieu, P., and H. Loubergé, 2009, Hybrid Cat Bonds, *Journal of Risk and Insurance*, 76 (3): 547-578.

Bühlmann, H. ,1984, Numerical evaluation of the compound Poisson distribution: Recursion or Fast-Fourier transform, *Scandinavian Actuarial Journal*, 116–126.

Chang, C.-C., S.-K. Lin, and M.-T. Yu, 2011, Valuation of Catastrophe Equity Puts with Markov-Modulated Poisson Processes, *Journal of Risk and Insurance*, 78 (2): 447-473.

Covens, F., V. Wouwe, and M. Goovaerts, 1979, On the numerical evaluation of stop-loss premium, *Astin Bulletin*, 10: 318–324.

Cummins, J. D., and M. A. Weiss, 2009, Convergence of insurance and financial markets: Hybrid and securitized risk transfer solutions, *Journal of Risk and Insurance*, 76 (3): 493-545.

Delbaen, F., and J. Haezendonck, 1987, Classical risk theory in an economic environment, *Insurance: Mathematics and Economics*, 6: 85–116.

De Mey, J. ,2007, Insurance and the capital markets, *The Geneva Papers*, 32: 35–41.

Dufresne, D., J. Garrido, and M. Morales, 2009, Fourier inversion formulas in option pricing and insurance, *Methodology and Computing in Applied Probability*, 11(3): 359-383.

Deringer, F. B., 2006, Insurance and reinsurance news, April 2006. Retrieved Dec. 10, 2009, from http://www.freshfields.com/publications/pdfs/2006/14736.pdf.

Gerber, H., 1982, On the numerical evaluation of the distribution of aggregate claims and its stop-loss premiums, *Insurance: Mathematics and Economics*, 1: 13–18.

Gerber, H., and E. S. Shiu, 1994, Option pricing by Esscher transforms, *Transactions of Society of Actuaries*, 46: 99–140.

Härdle, W. K. and B. L. Cabrera, 2010, Calibrating CAT Bonds for Mexican Earthquakes, *Journal of Risk and Insurance*, 77 (3): 625–650.

Jacod, J. and A. Shiryaev, 2003, *Limit Theorems for Stochastic Processes*, Springer-Verlag.

Jang, J., and Y. Krvavych, 2004, Arbitrage-free premium calculation for extreme losses using the shot noise process and the Esscher transform, *Insurance: Mathematics and Economics*, 35: 97–111.

Klein, R. W., and S. Wang, 2009, Catastrophe Risk Financing in the United States and the European Union: A Comparative Analysis of Alternative Regulatory Approaches, *Journal of Risk and Insurance*, 76 (3): 607-637.

Lakdawalla, D. and G. Zanjani, 2011, Catastrophe Bonds, Reinsurance, and the Optimal Collateralization of Risk Transfer, *Journal of Risk and Insurance*, In press.

Lee, J. P. and M.-T. Yu, 2002, Pricing Default-Risky CAT Bonds With Moral Hazard and Basis Risk, *Journal of Risk and Insurance*, 69 (1): 25–44.

Miyahara, Y., and T. Fujiwara, 2003, The minimal entropy martingale measures for geometric Lévy processes, *Finance and Stochastics*, 7: 509–531.

Nilsen, T., and J. Paulsen, 1996, On the distribution of a randomly discounted compound Poisson process, *Stochastic Processes and their Applications*, 61: 305–310.

Panjer, H.H., 1981, Recursive evaluation of a family of compound distributions, *ASTIN Bulletin*, 12: 175–76.

Paulsen, J., 1993, Risk theory in a stochastic economic environment, *Stochastic Processes and their Applications*, 46: 327–361.

Resnik, S. I., 1986, Point processes, regular variation and weak convergence, *Advances in Applied Probability*, 18 (1): 66–138.

Ross, S., 2002, *Simulation*, 3rd edition, Academic Press, San Diego.

Smyth, G. K., and B. Jorgensen, 2002, Fitting Tweedie's compound Poisson model to insurance claims data: Dispersion modeling, *Astin Bulletin*, 32 (1): 143–157.

Towers Perrin Tillinghast, 2006, AXA's motor insurance book securitization. Towers Perrin, January 2006. Retrieved Dec. 10, 2009, from http://www.towersperrin.com/tp/getwebcachedoc?webc=TILL/USA/2006/200601/UpdateAXA.pdf.