A Smiling Bear in the Equity Options Market and the Cross-section of Stock Returns

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ABSTRACT

The convexity of an option-implied volatility curve, designated *IV convexity*, is a forward-looking measure of excess tail risk contribution to the perceived variance of underlying equity returns. Using equity options data for both individual U.S.-listed stocks and Standard & Poor's 500 index for 2000-2013, we study the cross-sectional predictability of the *IV convexity* measure for future equity returns across quintile portfolios ranked by the curvature of the option-implied volatility curve. We find that the average return differential between the lowest and highest *IV convexity* quintile portfolios exceeds 1% per month, which is both economically and statistically significant on a risk-adjusted basis. The predictive power is significant for both the systematic and idiosyncratic components of *IV convexity*, and the results are robust even after controlling for the slope of option-implied volatility curve and other known predictors based on stock characteristics. Our empirical findings are consistent with earlier studies demonstrating option traders' advantage in that informed option traders anticipating heavier tail risk proactively induce leptokurtic implied distributions of underlying stock returns before equity investors express their tail-risk aversion.

Keywords: Implied volatility, Convexity, Equity options, Stock returns, Predictability

JEL classification: G12; G13; G14

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1. Introduction

The traditional mean-variance analysis (Markowitz, 1956; Sharpe, 1964; Lintner, 1965; Black, 1972) typically presumes a normally distributed asset return[1] characterized solely by its mean and variance.[2] According to Scott and Horvath (1980), however, a rational investor's utility is generally a function of higher moments as well, as they tend to have an aversion to negative skewness and high excess kurtosis in the portfolio return. The premium effect results in the market-implied asset returns with negatively-skewed and highly-leptokurtic distributions. In this context, non-normality of stock returns has been well-documented in more recent literature (e.g., Merton, 1982; Peters, 1991; Bollerslev, Chou, and Kroner, 1992) as a natural extension of the two-moment approach in portfolio optimization. Not surprisingly, considerable research has examined whether the higher moments of stock returns are indeed priced in the market; see Chi-Hsiou Hung, Shackleton, and Xinzhong (2004); Chung, Johnson, and Schill (2006); Dittmar (2002); Doan, Lin, and Zurbruegg (2010); Harvey and Siddique (2000); Kraus and Litzenberger (1976); and Smith (2007); among many others.

It is noteworthy that this higher-moment pricing effect is embedded in equity option prices in a *forward-looking* manner. As a result, there has been extensive research demonstrating that equity option markets provide informed traders with opportunities to capitalize on their information advantage. The rationale is that informed traders with private information about future stock values would have incentives to trade equity options rather than the underlying stocks to take advantage of reduced trading costs (Cox and Rubenstein, 1985), the lack of restrictions on short selling (Diamond and Verrecchia, 1987), and greater leverage effects (Black, 1975; Manaster and Rendleman, 1982). Moreover, recent researchers have shown an increased interest in inter-market inefficiency, leading to a proliferation of studies on a potential lead-lag relationship between options and stock prices. One stream of the related literature concerns the relationship between options trading volumes and the underlying stock returns, such as Anthony (1988);

[1] If the rate of return is continuously compounded, assuming a normally distributed asset return is equivalent to assuming that the asset price dynamics follow geometric Brownian motion through time.

[2] The mean-variance approach is consistent with the maximization of expected utility if either (i) the investors' utility functions are quadratic, or (ii) the assets' returns are jointly normally distributed. However, a quadratic utility function, by construction, exhibits increasing absolute risk aversion, consistent with investors who reduce the dollar amount invested in risky assets as their initial wealth increases. Accordingly, a quadratic utility formulation may be unrealistic for practical purposes. See Arrow (1971) for details.

Stephan and Whaley (1990); Easley, O'Hara, and Srinivas (1998); Chan, Chung, and Fong (2002). Others, such as Manaster, Rendleman (1982); Bhattacharya (1987); Stephen and Whaley (1990); Chan, Chung, and Johnson (1993); and Chan, Chung, and Fong (2002) have investigated the relationship between stock and options prices. Most importantly, most of the recent related literature has investigated the relationship between option-implied volatilities and future stock returns (e.g., Giot, 2005; Vijh, 1990; Chakravarty, Gulen, and Mayhew, 2004).

Interestingly, an option-implied volatility curve expresses the degree of abnormality in the market-implied distribution of the underlying stock return as a measure of the deviation between the option-implied distribution and the normal distribution with constant volatility based on the standard Black and Scholes (1973) option-pricing assumption.[3] To put it differently, the shape of an option-implied volatility curve contains the information about the higher-moment asset pricing implication beyond the standard mean-variance framework.[4] The discrepancy can be decomposed into the slope (*IV slope*) and convexity (*IV convexity*) of the option-implied volatility curve. Motivated by stochastic volatility (SV) model and stochastic-volatility jump-diffusion (SVJ) model specifications, we speculate that *IV slope* and *IV convexity* contain distinct information about future stock return and convey the information about option-implied skewness and the excess kurtosis of the underlying return distributions, respectively.

However, relevant research to date tended to focus on the asset-pricing implication of *skewness* rather than that of *excess kurtosis* of stock returns implied by option prices. For instance, Yan (2011) reports a negative predictive relationship between the slope of the implied volatility curve (as a proxy of the average size of the jump in the stock price dynamics) and the future stock return by taking the spread between the at-the-money (ATM) call and put option-implied volatilities (*IV spread*) as a measure of the slope of the implied volatility curve. In a similar vein,

[3] To better explain such a deviation originating from the positively-skewed and platokurtic preference of rational investors, prior studies have attempted to relax the unrealistic normality assumption to capture the negatively-skewed and fat-tailed distribution of stock returns implied by option prices by extending the standard Black and Scholes (1973) model to (i) stochastic volatility models: Duan (1995), Heston (1993), Hull and White (1987), Melino and Turnbull (1990, 1995), Scott (1980), Stein and Stein (1991), and Wiggins (1987); and (ii) jump-diffusion models: Bates (1996), Madan and Chang (1996), and Merton (1976).

[4] It is also claimed that the options-implied volatility curve is related to the net buying pressure of options traders; see Garleanu, Pedersen, and Poteshman (2005), Evans, Geczy, Musto, and Reed (2005), Bollen and Whaley (2004). This argument reflects the stylized market fact that the shape of option-implied volatility curve expresses the option market participants' anticipation on the future market situation, as the risk-averse intermediaries who cannot perfectly hedge their option positions in the incomplete capital market induce excess demand on options.

Cremers and Weinbaum (2010) argue that future stock returns can be predicted by the deviation from the put-call parity in the equity option market, as stocks with relatively expensive calls compared to otherwise identical puts earn approximately 50 basis points per week more in profit than the stocks with relatively expensive puts. Likewise, Xing, Zang, and Zhao (2010) propose an option-implied smirk (*IV smirk*) measure that shows its significant predictability for the cross-section of future equity returns. Recently, Chang, Christoffersen, and Jacobs (2013) estimated option-implied risk-neutral skewness and kurtosis from *index* options based on Bakshi, Kapadia, and Madan's (2003) approach to study how market-implied skewness and kurtosis affect the cross-section of stock returns. Their findings are that stocks with higher exposure to option-implied skewness have lower returns than those with lower exposure to option-implied skewness. Jin, Livnat, and Zhang (2012) find that option traders have superior abilities to process less anticipated information relative to equity traders by analyzing the slope of option-implied volatility curves. Lin and Lu (2015) argue that information asymmetry originates from analysts who offer tips to option traders about their future reports, including changes in analyst recommendations and revised forecasts. This stream of research indicates that an information discovery effect exists from the option-implied risk-neutral *skewness* in predicting future stock returns.

While considerable research has examined the predictive power of the option-implied *skewness* of stock returns (*IV slope*, *IV smirk* and *IV spread* in our study) captured by the slope of the implied volatility curve, the question whether option-implied *excess kurtosis*, proxied by *IV convexity*, predicts the cross-section of future stock returns has received less attention. It is noteworthy that Bali, Hu, and Murrray (2015) examine a set of ex-ante measures of volatility, skewness, and kurtosis derived from option-implied volatility curves in a non-parametric way. They find that options market's ex-ante view of a stock's risk profile is positively related to the stock's ex-ante expected return based on analyst price targets. However, their approach is conceptually different from ours in that they focus on the *ex-ante* expected stock returns, whereas we investigate the cross-sectional predictability of option-implied higher moment measures for future *ex-post* equity returns. Although Bali, Hu, and Murrray (2015) argue that the analyst price target is widely accepted as a proxy for ex-ante expected return, it is questionable that their target-based measures can fully capture market participants' expectation on each stock return in general. Admittedly, their finding is not directly related to information transmission from options market to stock market owing to the informational advantage of option traders, as their measure of expected returns based on analyst price target is certainly dependent on a few analysts' personal viewpoints, and subject to measurement error and potential bias.[5] In this regard, we claim that it is more appropriate to examine the relation between option-implied measures of higher moments and the ex-post realized stock returns.

Using equity options data for both individual U.S. listed stocks and Standard & Poor's 500 (S&P500) index for 2000-2013, we study the cross-sectional predictability of the *IV convexity* measure for future equity returns across quintile portfolios ranked by the curvature of the option-implied volatility curve. Specifically, the risk-neutral *excess kurtosis*, captured by *IV convexity*, can be associated with the volatility of stochastic volatility and the jump-size volatility implied by equity option prices. We find that the average return differential between the lowest and highest *IV convexity* quintile portfolios is over 1% per month, both economically and statistically significant on a risk-adjusted basis. The results are robust under the Fama-Macbeth regression specification and across different definitions of the *IV convexity* measure. All in all, the predictive power of our proposed *IV convexity* measure is significant for both the systematic and idiosyncratic components of *IV convexity*, and the results are robust even after controlling for the slope of option implied volatility curve and other known predictors based on stock characteristics. It provides strong evidence that there is a one-way information transmission from the options market to stock market. Moreover, our empirical finding that the negative relationship of *IV convexity* to future stock returns are consistent with the earlier studies demonstrating option traders' information advantage in the sense that informed option traders anticipating heavier tail risk proactively induce leptokurtic implied distributions of the underlying stock returns before equity investors express their tail-risk aversion.

From another point of view, our proposed *IV convexity* measure is related to a component of variance risk premium (VRP) in expected stock. The variance of a stock return is not a simple constant but rather a stochastic process fluctuating over time (e.g., Bollerslev, Engle, and

[5] We find that the option-implied kurtosis measure proposed by Bali, Hu, and Murray (2015) fails to show any significant prediction power in our setting; see Section 4.2 for details.

Nelson, 1994; Andersen, Bollerslev, and Diebold, 2005) and rational investors certainly demand compensation for taking the uncertainty related to the time-varying and stochastic return variance. Researchers termed this premium on the variance of stock returns as VRP, documented by Bakshi, Kapadia, and Madan (2003), Carr and Wu (2009), Bollerslev, Tauchen, and Zhou (2010), and Drechsler and Yaron (2010), among others. According to Carr and Wu (2009), the source of VRP can be decomposed into two components: (i) the correlation between the time-varying variance process and the stock return and (ii) the volatility of the variance. Nevertheless, most prior studies into VRP focus on the aggregate effect of VRP on stock returns without paying attention to the marginal contribution of each component. In this context, we conjecture that the first component is measured by *IV slope*, while the second component is captured by our proposed *IV convexity* measure. Specifically, under the stochastic volatility (SV) and stochastic-volatility jump-diffusion (SVJ) model specifications, we determine that the *IV slope* measure is associated with the option-implied skewness driven by the correlation between the stock price and its stochastic variance as well as the average size of the jump in the stock price dynamics, whereas *IV convexity* has a positive relationship to the volatility of stochastic variance and the variance of jump size. Accordingly, this paper investigates the implications of *IV slope* and *IV convexity* on VRP in the context of Carr and Wu (2009) by focusing on the impact of the second VRP component by analyzing the information delivered by our proposed *IV convexity* measure.

This paper offers several contributions to the existing literature. First, this paper examines whether *IV convexity* exhibits significant predictive power for future stock returns even after controlling for the effect of *IV slope* and other firm-specific characteristics. Although recent evidence shows that the option-implied volatility smirk (Xing, Zhang, and Zhao, 2010) and volatility spread between put and call options (Yan, 2011) predict future equity returns, our research is, to the best of our knowledge, the first study that makes a sharp distinction between the 3^{rd} and 4^{th} moments implied by option prices. It is also remarkable that our proposed measure of option-implied volatility slope and convexity measures (*IV slope* and *IV convexity)* have an advantage over Xing, Zhang, and Zhao’s (2010) proposed *IV smirk* measure, as the *IV smirk* measure, defined as defined as the implied-volatility spread between an out-of-the-money (OTM) put and ATM call, is reduced to a simple average of *IV slope* and *IV convexity*. Namely, *IV smirk* contains mixed information about higher moments and cannot distinguish between the volatility slope and convexity components addressing higher-moment implications in terms of the stock return distribution. Instead, we decompose *IV smirk* into separate *IV slope* and *IV convexity* measures. In addition, Yan's (2011) proposed *IV spread* measure simply captures the effect of the average jump size but not the effect of jump-size volatility in the SVJ model framework. We make a meaningful contribution to Yan's (2011) findings by examining how *IV convexity* explains the cross-section of future stock returns to address the jump-size volatility effect. On another note, this paper overcomes the potential caveat of ex-post information extracted from past realized returns in the previous studies on the effect of skewness (e.g., Kraus and Litzenberger, 1976; Lim, 1989; Harvey and Siddique, 2000) by estimating an ex-ante measure of skewness (*IV slope*) and excess kurtosis (*IV convexity*) from option price data in a forward-looking manner. Note that the ex-post skewness estimated from past returns is an unbiased estimator of the expected skewness only when the moments of stock returns are inter-temporally constant. Finally, this paper sheds new light on the relationship between the higher moment information extracted from individual equity option prices and the cross-section of future stock returns. Notice that Chang, Christoffersen, and Jacobs (2013) investigated how market-implied skewness and kurtosis affect the cross-section of stock returns by looking at the risk-neutral skewness and kurtosis implied by *index option prices* based on Bakshi, Kapadia, and Madan's (2003) proposed framework model. That is, their approach ignores the idiosyncratic components of option-implied higher moments in stock returns, though Yan (2011) finds that both the systematic and idiosyncratic components of *IV spread* are priced and that the latter part dominates the former in capturing the variation of cross-sectional stock returns in the future. In this context, our paper contributes Change et al.'s (2013) findings by employing firm-level equity option price data, and further decomposing *IV convexity* into systematic and idiosyncratic components to fully identify both systematic and idiosyncratic relationships between *IV convexity* and the cross-section of future stock returns.

The rest of this paper is organized as follows. Section 2 demonstrates the asset pricing implication of the proposed *IV convexity* measure through numerical analyses to develop our main research questions. Section 3 describes the data and presents the empirical results for the main hypotheses. Section 4 provides additional tests as robustness checks and Section 5 concludes the paper.

2. Asset Pricing Implication

In this section, we demonstrate the asset pricing implications of our proposed *IV convexity* and *IV slope* measures through numerical analyses. It is well documented that an option-implied risk-neutral distribution of the underlying stock return exhibits heavier tails than the normal distribution with the same mean and standard deviation, in the presence of higher moments such as skewness and excess kurtosis. Accordingly, information about these higher moments embedded in the various shapes of implied volatility curves can be examined from various perspectives.

2.1. Higher Moments and the Shape of the Implied Volatility Curve

Consider a geometric Lèvy process[6] to model the risk-neutral dynamics of the underlying stock price given by

$$\mathrm{S_t} = S_0 e^{X_t}, \tag{1}$$

where X is a Lèvy process whose increments are stationary and independent. In this context, a natural characterization of a probability distribution is specifying its cumulants.[7] To explore the effects of skewness and excess kurtosis[8] on option pricing, we can readily expand the probability distribution function $\mathrm{X_T}$, where T is the option's maturity time via the Gram-Charlier expansion, a method to express a density probability distribution in terms of another (typically Gaussian) probability distribution function using cumulant expansions [see Tanaka, Yamada, and Watanabe (2010) for details].

[Insert Figure 1 about here.]

[6] The most well-known examples of Geometric Lèvy processes are geometric brownian motion and jump diffusion models.

[7] The n^{th} cumulant is defined as the n^{th} coefficient of the Taylor expansion of the cumulant generating function, the logarithm of the moment generating function. Intuitively, the first cumulant is the expected value, and the n^{th} cumulant corresponds to the n^{th} central moment for $n=2$ or $n=3$. For $n \geq 4$, the n^{th} cumulant is the n^{th} -degree polynomial in the first n central moments.

[8] Hereafter, we use *kurtosis* and *excess kurtosis* interchangeably for simplicity, despite their conceptual differences.

Figure 1 shows the impact of skewness and excess kurtosis on the shape of its probability distribution using Gram-Charlier expansions. Skewness and excess kurtosis determine the degrees of lean and fat tails for the probability distribution function X_T, respectively. This aids in understanding how the skewness and kurtosis of X_T affect the shape of the implied volatility curves.

[Insert Figure 2 about here.]

Figure 2 illustrates the effect of different values of skewness and excess kurtosis on the shape of an implied volatility curve. We can observe that a negatively skewed distribution of X_T, *ceteris paribus*, leads to a steeper volatility smirk, whereas an increase in the excess kurtosis of X_T makes the volatility curve more convex. In this context, we define the implied volatility convexity (*IV convexity*) and the implied volatility slope (*IV slope*) as

$$IV\ Convexity = IV\left(OTM_{put}\right) + IV\left(ITM_{put}\right) - 2 \times IV(ATM), \tag{2}$$

$$IV\ Slope = IV\left(OTM_{put}\right) - IV\left(ITM_{put}\right), \tag{3}$$

where $IV(\cdot)$ denotes the implied volatility as a function of the option's moneyness.[9] Intuitively, *IV convexity* captures the degree of curvature of the implied volatility curve, whereas *IV slope* captures its slope.[10]

[Insert Figure 3 about here.]

Figure 3 confirms the option pricing implication in that the 3rd and 4th moments of X_T affect the *IV slope* and *IV convexity* of the implied volatility curve, respectively, but not vice versa.

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[9] In the absence of arbitrage opportunities, put-call parity implies that the option-implied volatilities of European call and put options should be identical when they have the same strike price and expiration date. In other words, both *IV Convexity* and *IV Slope* can be defined in terms of the implied volatilities of call options.

[10] Notice that Xing, Zhang, and Zhao (2010) propose the *IV smirk* measure given by

$$IV\ Smirk = IV\left(OTM_{put}\right) - IV(ATM),$$

which is a simple average of *IV Convexity* and *IV Slope*.

2.2. Analytical Interpretation

Although a stock return with normal distribution is extensively postulated in finance, it has long been disputed by empirical findings (e.g., Peters, 1991; Bollerslev, Chou, and Kroner, 1992) that the empirical distribution of stock returns tends to have fatter tails than those implied by the normal distribution. Earlier studies suggest *stochastic volatility* and *jump diffusion* models to capture the investors' positively-skewed and platokurtic preferences. In this context, the 3rd and 4th moments of the model-implied return distributions are worthy of investigation.

For a more in-depth exploration of the relationship between option pricing and the option-implied volatility curve, we first investigate stochastic volatility (SV) model proposed by Heston (1993). Specifically, we assume that the risk-neutral dynamics of the stock price follows a system of stochastic differential equations given by

$$dS_t = (r - q)S_t dt + \sqrt{v_t} S_t dW_t^{(1)}, \quad (4)$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t} dW_t^{(2)}, \quad (5)$$

where $\mathrm{E}[dW_t^{(1)}\, dW_t^{(2)}] = \rho \mathrm{dt}$. Here, S_t denotes the stock price at time t, r is the annualized risk-free rate under the continuous compounding rule, q is the annualized continuous dividend yield, v_t is the time-varying variance process whose evolution follows the square-root process with a long-run variance of θ, a speed of mean reversion κ, and a volatility of the variance process σ_v. In addition, $W_t^{(1)}$ and $W_t^{(2)}$ are two independent Brownian motions under the risk-neutral measure, and ρ represents the instantaneous correlation between the two Brownian motions.

[Insert Figure 4 about here.]

Based on our numerical experiments, Figure 4 demonstrates that *IV slope* reflects the leverage effect measured by the correlation coefficient (ρ), while *IV convexity* represents the degree of a large contribution of extreme events to the variance, i.e., tail risk, driven by the volatility of variance risk (σ_v). Put simply, *IV convexity* contains the information about the volatility of stochastic volatility (σ_v) and can be interpreted as a simple measure of the perceived kurtosis that addresses the option-implied tail risk in the distribution of underlying stock returns; a similar intuition is also illustrated in Figures 1-4 of Heston (1993).

On another note, *IV convexity* can be viewed as a component of VRP, as documented by Bakshi and Kapadia (2003), Carr and Wu (2009), Bollerslev, Tauchen, and Zhou (2010), and Drechsler and Yaron (2010), among others. According to Carr and Wu (2009), VRP consists of two components: (i) the correlation between the variance and the stock return and (ii) the volatility of the variance. In the SV model framework, the first component is captured by the correlation coefficient (ρ), while the second component is addressed by the volatility of stochastic volatility (σ_v). Nevertheless, recent research into VRP have focused on the aggregate effect of VRP on stock returns, but do not separately investigate how the impacts of the two VRP components differ. Thus, it is interesting to investigate the implications of *IV slope* and *IV convexity* on VRP in the context of Carr and Wu (2009). Specifically, our study aims to investigate the impact of the second component of VRP by analyzing the information delivered by the *IV convexity* measure.

We next consider the impact of jumps in the dynamics of the underlying asset price. For example, Bakshi, Cao, and Chen's (1997) study shows that jump components are necessary to explain the observed shapes of implied volatility curves in practice. In the presence of jump risk, the option-implied risk-neutral distribution of a stock price return is a function of the average jump size and jump volatility. To illustrate the implications of jump components on the shape of the option-implied volatility curve, we consider the following stochastic-volatility jump-diffusion (SVJ) model under the risk-neutral pricing measure given by

$$dS_t = (r - q - \lambda\,\mu_J)S_t dt + \sqrt{\upsilon_t}S_t dW_t^{(1)} + JS_t dN_t, \tag{6}$$

$$d\upsilon_t = \kappa(\theta - \upsilon_t)dt + \sigma_v\sqrt{\upsilon_t}dW_t^{(2)}, \tag{7}$$

where $\mathrm{E}[dW_t^{(1)}\,dW_t^{(2)}] = \rho\mathrm{dt}$, N_t is an independent Poisson process with intensity $\lambda > 0$, and J is the relative jump size, where $\log(1 + J) \sim N(\log(1 + \mu_J) - 0.5\sigma_J^2, \sigma_J^2)$. The SVJ model can be taken as an extension of the SV model with the addition of log-normal (Merton-type) jumps in the underlying asset price dynamics.[11]

[Insert Figure 5 about here.]

As we can see from Figure 5, our numerical analysis illustrates that *IV slope* is mainly driven by the average jump size (μ_J), whereas the jump size volatility (σ_J) contributes mainly to *IV convexity*. From this perspective, Yan (2011) argues that the implied-volatility spread between ATM call and put options contain information about the perceived jump risk by investigating the relationship between the implied-volatility spread and the cross-section of stock returns. Strictly speaking, in the SVJ model framework, Yan's (2011) implied-volatility spread measure simply captures the effect of μ_J but ignores the information from σ_J. In other words, the implied-volatility spread measure fails to provide any evidence in terms of whether the implied jump size volatility σ_J, can predict future stock returns. Therefore, this study contributes to Yan's (2011) finding by looking at the predictability of the *IV convexity* measure, which contains the information from σ_J, and examining how *IV convexity* affects the cross-section of future stock returns accordingly.

2.3. Hypothesis Development

We have seen that the convexity of an option-implied volatility curve is a forward-looking measure of the perceived likelihood of extreme movements in the underlying equity price originating from the perceived stochastic volatility and/or jump risk. Additionally, option prices can provide ex-ante information about the anticipated stochastic volatility and jump-diffusion due to its forward-looking nature. In this regard, the *IV slope and IV convexity* measures can be employed as proxies for the 3rd and 4th moments in the option-implied distribution of stock returns, respectively.

[11] Note that the SVJ model given by (6)-(7) can be interpreted as a version of the Bates (1996) model. Duffie, Pan and Singleton (2000) provide an illustrative example to examine the implications of SVJ model for option valuation.

Hence, the overall goal of this study is to determine if a measure of option-implied volatility convexity can show significant cross-sectional predictive power for future equity returns. This is summarized in the hypotheses as follows:

- Hypothesis 1: *If option traders have no information about the prediction for excess tail risk contributions to the perceived variance of the underlying equity returns, IV convexity cannot predict future stock returns with statistical significance.*

- Hypothesis 2-1: *If there is a one-way information transmission from the options market to the stock market, informed option traders can anticipate the excess tail risk contribution to the perceived variance of the underlying equity returns. The option investors then proactively induce leptokurtic implied distributions of stock returns before equity investors express their tail-risk aversion. Hence, IV convexity will show its predictive power for future stock price returns with a negative relationship.*

- Hypothesis 2-2: *If the information transmission occurs in both directions between option and stock markets, the efficient market hypothesis implies that options investors simultaneously induce leptokurtic implied distributions of stock returns when equity investors express their tail-risk aversion. Hence, equity investors will require compensation for taking the excess tail risk, and IV convexity will show its predictive power for future stock price returns with a positive relationship.*

If we can reject Hypothesis 1, and the relationship is negative with statistical significance, it would empirically support the existing literature demonstrating the information transmission between the options and stock markets in that informed options traders anticipating heavy tail risks proactively induce leptokurtic implied distributions before equity investors express their tail risk aversion in the stock market.

3. Empirical Analysis

This section introduces the data set and methodology to estimate option-implied convexity in a cross-sectional manner. We then test whether *IV convexity*, a proxy for the volatility of stochastic volatility (σ_v) and the jump size volatility (σ_J), has significant predictive power for future stock returns. Additionally, we compare the impact of the option implied volatility slope with that of our *IV convexity* measure on stock returns.

3.1. Data

We obtained the U.S. equity and index option data from OptionMetrics on a daily basis from January 2000 through December 2013. As this raw data includes individual equity options in the American style, Cox, Ross, and Rubinstein's (1979) binomial tree model is applied to estimate the options-implied volatility curve to account for the possibility of an early exercise with discrete dividend payments. By employing a kernel smoothing technique, OptionMetrics offers an option-implied volatility surface across different option deltas and time-to-maturities. Specifically, we obtained the fitted implied volatilities on a grid of fixed time-to-maturities, (30 days, 60 days, 90 days, 180 days, and 360 days) and option deltas (0.2, 0.25, …, 0.8 for calls and -0.8, -0.75, … , -0.2 for puts), respectively. Following Yan (2011), we then select the options with 30-day time-to-maturity on the last trading day of each month to examine the predictability of *IV convexity* for future stock returns,

[Insert Table 1 about here.]

Table 1 shows the summary statistics of the fitted implied volatility from the options with 30-day time-to-maturity chosen at the end of each month. We can clearly observe a positive convexity in the option-implied volatility curve as a function of the option's delta in that the implied volatilities from in-the-money (ITM) (calls for delta of 0.55~0.80, puts for delta of -0.80~-0.55) options and OTM (calls for delta of 0.20~0.45, puts for delta of -0.45~-0.20) options are greater on average than those near the ATM options (calls for delta of 0.50, puts for delta of -0.50).

We obtained daily and monthly individual common stock (shrcd in 10 or 11) returns from the Center for Research in Security Prices (CRSP) for stocks traded on the NYSE (exchcd=1), Amex (exchcd=2), and NASDAQ (exchcd=3). Stocks with a price less than three dollars per share are excluded to weed out very small or illiquid stocks. Accounting data is obtained from Compustat. We obtain both daily and monthly data for each factor from Kenneth R. French's Website.[12]

[12] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

3.2. Variables and Portfolio Formation

Section 2.2 demonstrated that *IV convexity* has a positive relationship with the volatility of stochastic volatility (σ_v) and jump volatility (σ_J). That is, *IV convexity* can be interpreted as a simple measure of the perceived kurtosis of the option-implied distribution of the stock returns driven by the volatility of stochastic volatility and jump size volatility. As expected, it is hard to directly calibrate the volatility of stochastic volatility (σ_v) and jump size volatility (σ_J) for each underlying stock from the cross-sectional perspective on a daily basis. We thus overcome this computational difficulty by adopting *IV convexity* as a simple proxy for the volatility of stochastic volatility (σ_v) and jump size volatility (σ_J) to investigate how the ex-ante 4$^{\text{th}}$ moment in the option-implied distribution of the stock returns affects the cross-section of future stock returns. Accordingly, we define our measure of *IV convexity* as

$$IV\ convexity = \mathrm{IV_{put}}(\Delta = -0.2) + \mathrm{IV_{put}}(\Delta = -0.8) - 2 \times \mathrm{IV_{call}}(\Delta = 0.5), \tag{8}$$

Specifically, we use the implied volatilities of OTM and ITM put and ATM call options to capture the convexity of the implied volatility curve. The rationale is that those who respond sensitively to the forthcoming tail risk would buy put options either as a protection against the potential decrease in the stock return for hedging purposes or as a leverage to grab a quick profit for speculative purposes to capitalize on private information. Therefore, those investors would have an incentive to trade OTM and/or ITM put options rather than call options. Thus, we choose OTM and ITM puts for calculating the *IV convexity* measure. As a benchmark of the option-implied volatility curve, motivated by Xing, Zhang, and Zhao (2010), we use the implied volatility of an ATM call as a representative value for the implied volatility level, as the ATM call is generally the most frequently traded option best reflecting market participants' sentiment regarding the firm's future status and condition.

As alternative measures related to the option-implied volatility curve, options implied volatility level (*IV level*), *IV slope*, *IV smirk*, and *IV spread* are defined as

$$IV\ level = 0.5[\mathrm{IV_{put}}(\Delta = -0.5) + \mathrm{IV_{call}}(\Delta = 0.5), \tag{9}$$

$$IV\ slope = \mathrm{IV_{put}}(\Delta = -0.8) - \mathrm{IV_{put}}(\Delta = -0.2), \tag{10}$$

$$IV\ smirk = \mathrm{IV}_{\mathrm{put}}(\Delta = -0.8) - \mathrm{IV}_{\mathrm{call}}(\Delta = 0.5), \tag{11}$$

$$IV\ spread = \mathrm{IV}_{\mathrm{put}}(\Delta = -0.5) - \mathrm{IV}_{\mathrm{call}}(\Delta = 0.5), \tag{12}$$

where the last two measures are motivated by Yan (2011) and Xing, Zhang, and Zhao (2010), respectively. Note that our proposed measure of *IV slope* has an advantage compared to that proposed by Xing, Zhang, and Zhao (2010), as the *IV smirk* measure is a simple average of *IV convexity* and *IV slope*. This observation implies that *IV smirk* contains mixed information about both *IV slope* and *IV convexity* and it cannot distinguish between the volatility slope and convexity components that affect the 3rd and 4th moments of the implied stock return distribution, respectively. To overcome the potential caveat of the *IV smirk* measure, we introduce our *IV slope* measure to exclude the information about *IV convexity*.

Similar to our *IV slope* measure, *IV spread* is an approximation of the slope of its tangent line near the ATM point. Namely, this *IV spread* measure cannot capture the convexity property in the options-implied curve, so by using this incomplete measure, Yan (2011) examine only the relationship between *IV slope* and stock returns, ignoring the convexity property of the option-implied volatility curve.

However, we assume in this study that *IV slope* and *IV convexity* deliver different information about the anticipated distribution of stock returns. By decomposing this into *IV slope* and *IV convexity*, we can investigate the impact of the *IV slope* and *IV convexity* on a cross-section of future stock returns and how they differ from the information extracted using Xing, Zhang, and Zhao's (2010) measure. Specifically, it is of interest to examine whether *IV convexity* makes any marginal contribution to such predictability after controlling for *IV slope* or *IV spread* between a call and a put, as Yan (2011) proposed.

At the end of each month, we compute the cross-sectional *IV level*, *IV slope*, *IV convexity*, *IV smirk*, and *IV spread* measures from 30-day time-to-maturity options. We define a firm's size (Size) as the natural logarithm of the market capitalization (prc$\times$shrout$\times$1000), which is computed at the end of each month using CRSP data. When computing book-to-market ratio (BTM), we match the yearly BE [book value of common equity (CEQ) plus deferred taxes and investment tax credit (txditc)] for all fiscal years ending at year t-1 to returns starting in July of year t, and dividing this BE by the market capitalization at month t-1. Hence, the book-to-market ratio is computed on a monthly basis. Market betas (β) are estimated by the rolling regression using the previous 36 monthly returns available up to month t-1 given by

$$(\mathrm{R_{it}} - \mathrm{R_f}) = \alpha_i + \beta_i(\mathrm{MKT_t} - \mathrm{R_{ft}}) + \varepsilon_{it}. \quad (13)$$

Following Jegadeesh and Titman (1993), momentum (MOM) is computed using cumulative returns over the past five months (t-6~t-2) skipping one month between the portfolio formation period and the computation period to exclude the reversal effect. Momentum is also rebalanced every month and assumed to be held for the next one month. Short-term reversal (REV) is estimated based on the past one-month return (t-1) as in Jegadeesh (1990) and Lehmann (1990).

Motivated by Amihud (2002), illiquidity (ILLIQ) is defined as the average of the absolute value of stock return divided by the trading volume of the stock in thousand USD using past one-month data on a daily basis.

Following Harvey and Siddique (2000), daily excess returns of individual stocks are regressed on the daily market excess return and the daily squared market excess return using the last one year data month by month, where the regression specification is given by

$$(\mathrm{R_{it}} - \mathrm{R_f})_{i,t-365\sim t} = \alpha_{\mathrm{i}} + \beta_{1,\mathrm{i}}(\mathrm{MKT_t} - \mathrm{R_{ft}})_{t-365\sim t} + \beta_{2,\mathrm{i}}(\mathrm{MKT_t} - \mathrm{R_{ft}})^2{}_{t-365\sim t} + \varepsilon_{\mathrm{i,t}}. \quad (14)$$

In this context, the coskewness (Coskew) of a stock is defined as the coefficient of the squared market excess return. To reduce the impact of infrequent trading on the coskewness estimates, a minimum of 255 trading days per month is required.

According to Ang, Hodrick, Xing, and Zhang (2006), we compute idiosyncratic volatility using daily returns. The daily excess returns of individual stocks over the last 30 days are regressed on Fama and French's (1993, 1996) three factors daily and momentum factors every month, where the regression specification is given by

$$(\mathrm{R_{it}} - \mathrm{R_f}) = \alpha_i + \beta_{1i}(\mathrm{MKT_t} - \mathrm{R_{ft}}) + \beta_{2i}\mathrm{SMB} + \beta_{3i}\mathrm{HML} + \beta_{4i}\mathrm{WML} + \varepsilon_{it}, \quad (15)$$

Idiosyncratic volatility is computed as the standard deviation of the regression residuals in every month. To reduce the impact of infrequent trading on idiosyncratic volatility estimates, a minimum of 15 trading days in a month for which CRSP reports both a daily return and non-zero trading volume is required.

Systematic volatility is estimated using the method suggested by Duan and Wei (2009): $v_{sys}^2 = \beta^2 v_m^2 / v^2$ for every month. We also computed idiosyncratic implied variance as $v_{idio}^2 = v^2 - \beta^2 v_M^2$ on a monthly basis, where v_m is the implied volatility of the S&P500 index option following Dennis, Mayhew, and Stivers (2006).

The impact of the volatility of stochastic volatility and the jump size volatility on the return dynamics of the underlying stock would be either systematic or idiosyncratic. As there are two types of options data, equity options and index options, we can disentangle *IV convexity* into systematic and the idiosyncratic components. We run the time series regression each month using the S&P 500 index options with 30-day time-to maturity as a benchmark for the market along with individual equity options with daily frequency to decompose *IV convexity* into the systematic and the idiosyncratic components as follows:

$$IV\ convexity_{i,t-30\sim t} = \alpha_i + \beta_i \times IV\ convexity_{S\&P500,t-30\sim t} + \varepsilon_{i,t}, \quad (16)$$

We define the fitted values and residual terms as *the systematic component of IV convexity* ($convexity_{sys}$) and *the idiosyncratic component of IV convexity* ($convexity_{idio}$), respectively. When constructing a single sorted *IV convexity* portfolio, we sort all stocks at the end of each month based on the *IV convexity* and matched with the subsequent monthly stock returns. The *IV convexity* portfolios (equally weighted) are rebalanced every month.

To investigate whether the anomaly of *IV convexity* persists even after controlling for other systematic risk factors, we double sort all stocks following Fama and French (1993). At the end of each month, we first sort all stocks into 5 portfolios based on the level of systematic factors (i.e., firm size, book-to-market ratio, market β, momentum, reversal etc.) and then sub-sort them into five groups based on the *IV convexity*. These constructed portfolios are matched with subsequent monthly stock returns. This process is repeated every month.

3.3. Characteristics of Portfolios Sorted by *IV convexity*

3.3.1. Predicting Cross-sectional Stock Returns

We report our empirical results in terms of the predictive power of *IV convexity* for the cross-section of future stock returns.

[Insert Table 2 about here.]

Panel A of Table 2 shows the descriptive statistics for each implied volatility measure computed at the end of each month using 30-day time-to-maturity options. As for the average values for each of variable, *IV level* has 0.4739, *IV slope* for 0.0423, *IV spread* for 0.009, *IV smirk* for 0.0687, and *IV convexity* for 0.0942, respectively. The standard deviation of *IV convexity* is 0.2624 and 0.1682 for $convexity_{sys}$, 0.2015 for $convexity_{idio}$, respectively. It seems that the $convexity_{idio}$ measure better captures the variation in *IV convexity* than the $convexity_{sys}$ measure.

Panel A of Table 2 also presents descriptive statistics for the alternative convexity measure using various OTM put deltas. The alternative *IV convexity* measures are computed by

$$\text{p}\Delta_1\text{\_c50\_p}\Delta_2 = \text{IV}_{\text{put}}(\Delta_1) + \text{IV}_{\text{put}}(\Delta_2) - 2 \times \text{IV}_{\text{call}}(0.5). \quad (17)$$

where $-0.45 \leq \Delta_1 \leq -0.25$ (for the range of OTM puts) and $-0.75 \leq \Delta_2 \leq -0.55$ (for the range of ITM puts), respectively. For example, applying Δ_1=-0.25 for OTM put and Δ_2=-0.75 for ITM put, we calculate

$$\text{p25\_c50\_p75} = \text{IV}_{\text{put}}(-0.25) + \text{IV}_{\text{put}}(-0.75) - 2 \times \text{IV}_{\text{call}}(0.5). \quad (18)$$

Similarly, p45_p50_p55 is defined as $\text{IV}_{\text{put}}(-0.45) + \text{IV}_{\text{put}}(-0.55) - 2 \times \text{IV}_{\text{call}}(0.5)$. It is natural that the *IV convexity* measure computed using deep out-of-the-money (DOTM) and deep in-the-money (DITM) options have higher options convexity values compared to measurements using OTM and ITM options. For example, *IV convexity* of p25_c50_p75 is 0.065, which is larger than the value of p45_p50_p55, 0.018.

Panel B of Table 2 shows the descriptive statistics of firm characteristic variables including Size, BTM, Market β, MOM, REV, ILLIQ and Coskew. While the mean and median of SIZE are 19.4607 and 19.3757, respectively, its quintile average is monotone increasing from 16.7256 to 22.4613. On the other hand, BTM has a rightly-skewed distribution, as its mean is 0.9186 and the median is 0.5472, whereas its quintile average varies from 0.1467 to 2.6192.

To examine the relationship between *IV convexity* and future stock returns, we form five portfolios according to the *IV convexity* value at the last trading day of each month. Quintile 1 is composed of stocks with the lowest *IV convexity* while Quintile 5 is composed of stocks with the highest *IV convexity*. These portfolios are equally weighted, rebalanced every month, and assuming to be held for the subsequent one-month period.

[Insert Table 3 about here.]

Table 3 reports the means and standard deviations of the five *IV convexity* quintile portfolios and average monthly portfolio returns over the entire sample period. Specifically, Panel A shows the descriptive statistics for kurtosis along with the average monthly returns of both equal-weighted (EW) and value-weighted (VW) portfolios sorted by *IV convexity*, *IV spread, and IV smirk*, where the last two measures are defined and estimated as Yan (2011) and Xing, Zhang, and Zhao (2010), respectively.

As shown, the average EW portfolio return monotonically decreases from 0.0208 for the quintile portfolio Q1 to 0.0074 for quintile portfolio Q5. The average monthly return of the arbitrage portfolio buying the lowest *IV convexity* portfolio Q1 and selling highest *IV convexity* portfolio Q5 is significantly positive (0.0134 with t-statistics of 7.87). A similar decreasing pattern is observed from the average VW portfolio returns from Q1 (0.0136) to Q5 (0.0023), and the return of zero-investment portfolio (Q1-Q5) is positive with its statistical significance (0.0113 with t-statistics of 5.08).

In addition, the EW portfolios sorted by *IV spread* show that their average returns decrease monotonically from 0.0145 for quintile portfolio Q1 to 0.0013 for quintile portfolio Q5, where the average return difference between Q1 and Q5 amounts to 0.0131 with t-statistics of 7.31, and similar patterns are observed with VW portfolios sorted by *IV spread*. These results certainly confirm Yan's (2011) empirical finding in that low *IV spread* stocks outperform high *IV spread* stocks. In a similar vein, we find that the average returns of quintile portfolios sorted by *IV smirk* are decreasing in *IV convexity*, and the returns of zero-investment portfolios (Q1-Q5) are all positive and statistically significant for both EW and VW portfolios. Note that our results are consistent with Xing, Zhang and Zhao (2010) in that there exists negative predictive relationship between *IV smirk* and future stock return.

Panel B reports descriptive statistics for average portfolio returns using the alternative *IV convexities*. We still observe decreasing patterns in portfolio returns using the alternative *IV convexity* and arbitrage portfolio returns by buying the low *IV convexity* quintile portfolio and selling the high *IV convexity* quintile portfolio are significantly positive in a consistent manner. These are consistently significantly positive for both EW and VW portfolio returns. This result confirms that the negative relationship between *IV convexity* and stock returns are robust and consistent whatever OTM put (ITM put) we use to compute convexity. These results support *Hypothesis 2-1*, indicating that information transmission between the options and stock markets in that informed options traders anticipating heavy tail risks proactively induce leptokurtic implied distributions before equity investors express their tail risk aversion in the stock market.

[Insert Figure 6 about here.]

Panel A of Figure 6 shows the monthly average *IV convexity* value for each quintile portfolio, while Panel B plots the monthly average return of the arbitrage portfolio formed by the long lowest quintile and short highest quintile portfolio (Q1-Q5). The time-varying average monthly returns of the long-short portfolio are mostly positive, confirming the results reported in Table 3.

3.3.2. Controlling Systematic Risks

Moreover, we investigate whether the positive arbitrage portfolio returns (Q1-Q5) compensate for taking systematic risk. If the positive arbitrage portfolio returns are still significant after controlling for systematic risk factors, we can argue that the decreasing pattern in the portfolio return in *IV convexity* may not be driven by systematic risks and can be recognized as an abnormal phenomenon. In this context, we test whether systematic risk factors have sufficient explanatory power for the negative relationship between *IV convexity* and stock returns. We begin by looking at two-way cuts on systematic risk and *IV convexity*, and then we conduct time-series tests by running risk factor-model [e.g., the CAPM and Fama and French (1993) factor model] regressions with the standard equity risk factors; i.e., Market β, SMB, HML, and MOM.

A. Double Sorting by Systematic risk and *IV convexity*

To examine whether the relationship between *IV convexity* and stock returns disappear after controlling for the systematic risk factors, we double-sort all stocks following Fama and French (1992). All stocks are sorted into five quintiles by ranking on systematic risk and then sorting within each quintile into five quintiles according to *IV convexity*. Fama and French (1993) suggest that firm size, book-to-market, and market β are systematic risk components of stock returns, so we adopt these three firm characteristic risks as systematic risks.

[Insert Table 4 about here.]

Table 4 reports the average monthly returns of the 25 (5×5) portfolios sorted first by firm characteristic risks (firm size, book-to-market, market β) and then by *IV convexity* and average monthly returns of the long-short arbitrage portfolios (Q1-Q5).

We can observe that the average monthly portfolio returns generally decline as the average firm-size increases. As for the results from double-sorting using firm-size and *IV convexity*, we find that the returns of the *IV convexity* quintile portfolios are still decreasing in *IV convexity* in most size quintiles, and the return of all zero-investment portfolios (Q1-Q5) in size quintiles are all positive and statistically significant. Particularly, the positive difference in the smallest quintile is largest (0.0187) compared to the other size quintile portfolios.

The two-way cuts on book-to-market and *IV convexity* show that the higher book-to-market portfolio gets more returns compared to the lower book-to-market portfolios in each *IV convexity* quintile. We can observe that decreasing patterns in *IV convexity* portfolio returns persist even after controlling the systematic compensation drawn from the book-to-market factor. Note that the overall zero-cost portfolios formed by long Q1 and short Q5 are also positive and statistically significant: 0.0097 (t-statistic = 4.82) for B1 (BTM quintile 1), 0.0121 (t-statistic = 5.76) for B2, 0.0108 (t-statistic = 5.42) for B3, 0.0134 (t-statistic = 5.67) for B4, and 0.0177 (t-statistic = 5.19) for B5.

When sorting the 25 portfolios by market β first and then by *IV convexity*, the negative relationship between *IV convexity* and stock return persists, implying that this decreasing pattern cannot be explained by market β. Note that the average monthly portfolio returns generally increase as we increase the average market β.

We also consider the other four systematic risk factors (i) the momentum effect documented by Jegadeesh and Titman (1993), (ii) the short-term reversal suggested by Jegadeesh (1990) and Lehmann (1990), (iii) the illiquidity proposed by Amihud (2002) and (iv) coskewness suggested by Harvey and Siddique (2000) to examine whether the decreasing pattern of portfolio returns in *IV convexity* disappears when controlling these systematic risk factors. Stocks are first sorted into five groups based on their momentum (or reversal, illiquidity, coskewness) measures and then sorted by *IV convexity* forming 25 ($= 5 \times 5$) portfolios.

[Insert Table 5 about here.]

Table 5 presents the returns of 25 portfolios sorted by momentum (or reversal, illiquidity, coskewness) and *IV convexity*. When we look at momentum-*IV convexity* portfolios, momentum patterns, winner portfolios achieve more abnormal returns than loser portfolios, are consistently observed, except for the lowest momentum-lowest *IV convexity* quintiles, which could be caused by a different sample of datasets compared to that in Jegadeesh and Titman (1993). While Jegadeesh and Titman (1993) use only stocks traded on the NYSE (exchcd=1) and Amex (exchcd=2), we add stocks traded on the NASDAQ (exchcd=3). For the holding period strategies, Jagadeesh and Titman (1993) adopt 3-, 6-, 9-, and 12-month holding periods, while this study assumes that portfolios are held for one month.

Even after controlling the momentum as a systematic risk, we observe the portfolio return differential between the lowest and highest *IV convexity* in each momentum quintile remains significantly positive, indicating that *IV convexity* contains economically meaningful information that cannot be explained by the momentum factor.

For the reversal-*IV convexity* double sorted portfolio case, there is a clear reversal patterns in most cases when using a reversal strategy (i.e., the past winner earns higher returns in the next month compared to past loser), though there are some distortions in lowest reversal-highest *IV convexity* quintiles. It is still observed that the Q1-Q5 strategy of buying and selling stocks based on *IV convexity* in each reversal portfolio and holding them for one month earns significantly positive returns. This implies that the same results still hold even after controlling for reversal effects.

We further incorporate the Amihud (2002) measure of illiquidity to address the role of liquidity premium in asset pricing. Amihud (2002) finds that the expected market illiquidity has positive and highly significant effect on the expected stock returns, as the investors in the equity market requires compensation for taking liquidity risk. We examine whether the market illiquidity measure (ILLIQ), as proposed by Amihud (2002), explains the higher return on the lowest *IV convexity* stock portfolio (Q1) relative to the *highest IV convexity* stock portfolio (Q5). The double-sorted quintile portfolios by ILLIQ and *IV convexity* exhibit analogous patterns in that their average returns tend to decrease in *IV convexity*. The zero-investment portfolios (Q1-Q5) based on the ILLIQ quintiles demonstrate their significantly positive average returns across different ILLIQ quintiles. This finding implies that a significantly negative *IV convexity* premium remains even after we control for the illiquidity premium effect.

Next, we consider conditional skewness, as Harvey and Siddique (2000) find that conditional skewness (*Coskew*) can explain the cross-sectional variation of expected returns even after controlling the factors based on size and book-to-market. To examine whether this *Coskew* factor captures higher returns of the lowest *IV convexity* stocks relative to the highest *IV convexity* stocks, we constructed 25 portfolios sorted by *Coskew* first and then by *IV convexity*. For the *Coskew -IV convexity* double sorted portfolios, we can clearly observe a decreasing pattern of *IV convexity* within each *Coskew* sorted quintile portfolio. The returns of zero-investment *IV convexity* portfolios (Q1-Q5) within each *Coskew* quintile portfolio are all positive and statistically significant: 0.0130 (t-statistic of 4.60) for C1 (*Coskew* quintile 1), 0.0090 (t-statistic of 3.83) for C2, 0.0040 (t-statistics 2.39) for C3, 0.0076 (t-statistics 4.18) for C4, and 0.0127 (t-statistics 5.07) for C5, respectively. The implication is that a negative IV convexity premium remains significantly even after we control for the *Coskew* premium effect.

In summary, we can conclude that the negative relationship between *IV convexity* and stock return consistently persists even after controlling for various kinds of systematic risks identified in prior research. Therefore, we argue that *IV convexity* is not caused by systematic risk components and can be considered a significantly priced risk factor.

B. Controlling for *IV slope*

It is well-known in finance that stock returns are not normally distributed but are leptokurtic (fat-tailed) and skewed to the left. To explain this property, Heston (1993) introduced the correlation between the volatility and the spot-price processes (ρ) and the volatility of stochastic volatility (σ_v) and showed that these two parameters can generate fat-tailed and left-skewed properties in stock returns. In addition, the SVJ model, an extension of SV model with the addition of log-normal (Merton-type) jumps in the underlying asset price dynamics, suggests that excess kurtosis can from both the volatility of stochastic volatility (σ_v) and jump volatility (σ_J).

As demonstrated in the theoretical development, the four parameters ($\rho,\ \sigma_v,\ \mu_J, \sigma_J$) suggested in the SV and SVJ models are deeply related to the options implied volatility curve; *IV slope* is associated with ρ and μ_J, which may generate skewness in the distribution of stock returns; whereas *IV convexity* has a positive relationship to σ_v and σ_J, which play major role in the kurtosis of stock returns.

Although we show that the impact of *IV slope* on the distribution of stock returns is different from those of *IV convexity* on the distribution of stock returns, it is still unclear whether the *IV convexity* really affects the stock return differently from *IV slope*. To answer this question, we examine whether the negative relationship between *IV convexity* and stock returns persists after controlling for the effect of the option-implied volatility slope on stock returns suggested by other researchers. For this purpose, we consider the following three measures for the slope of the option-implied volatility curve: (i) *IV slope* following our definition, (ii) *IV spread* suggested by Yan (2011), (iii) *IV smirk* proposed by Xing, Zhang, and Zhao (2010).

[Insert Table 6 about here.]

Table 6 presents the average monthly returns of 25 portfolios sorted first by *IV slope*, *IV spread*, and *IV smirk* and then sorted by *IV convexity* within each *IV slope*, *IV spread*, and *IV smirk* sorted quintile portfolio, respectively.

The first five columns show the results of our *IV slope-IV convexity* double sorted portfolio returns. We can observe the decreasing pattern with respect to *IV convexity* in each *IV slope* quintile, and the Q1-Q5 strategy based on *IV convexity* in each *IV slope* portfolio and holding them for one month returns significantly positive profits across the different specifications.

As for *IV spread* following Yan (2011), the *IV convexity* strategy that buys the lowest quintile portfolio and sells the highest quintile portfolio within each *IV spread* portfolio yield significantly positive returns in all cases, suggesting that *IV spread* does not capture the *IV convexity* effect.

It is noteworthy that *IV convexity* arbitrage portfolios (Q1-Q5) within S1 and S5 *IV smirk* portfolios produce significantly positive returns, while the portfolio returns lose their statistical significance for the S2, S3 and S4 *IV smirk* quintiles. As the *IV smirk* measure, which is a simple average of *IV slope* and *IV convexity*, contains mixed information about them, it is natural to observe that *IV smirk* explains a negative *IV convexity* premium to some extent, though *IV smirk* cannot fully capture the negative relation between *IV convexity* and future stock return in S1 and S5 portfolios.

All in all, these findings support the proposition that the negative relationship of *IV convexity* to future stock returns still holds even after considering the impact of *IV slope* and *IV spread* on stock returns. This implies that the impact of *IV convexity* on the distribution of stock returns does not come from *IV slope* and *IV spread* and that *IV convexity* is an important factor in determining the fat-tailed distribution characteristics of future stock returns.

3.3.3. Systematic and Idiosyncratic Components of *IV convexity*

The variance of stock returns are composed of two components: systematic and idiosyncratic volatility. Only systematic risk (market β) should be priced in equilibrium while idiosyncratic risk cannot capture the cross-sectional variation in stock returns. However, in the real world, investors cannot perfectly diversify away the idiosyncratic risks, so some researchers argue that idiosyncratic risk can also play important role in explaining the cross-sectional variation in stock returns. In this context, we try to decompose the volatility of stochastic volatility (σ_v) and jump size volatility (σ_J) into systematic and idiosyncratic components to further investigate the source of the relationship between *IV convexity*. Importantly, the fact that there are two types of option data, equity options and index options, allows us to decompose *IV convexity* into the market and idiosyncratic components. By analyzing the two components of *IV convexity*, we can define whether the volatility of stochastic volatility and/or the jump risk shock determines the fat-tailed property of stock returns, and if these are driven by the market and/or individual firms' properties.

[Insert Table 7 about here.]

Panel A of Table 7 shows descriptive statistics of average portfolio returns sorted by systematic components and idiosyncratic components of *IV convexity*. In Table 3, we observe that the average portfolio return monotonically decreases from Q1 to Q5 and the return differential between Q1 and Q5 is positive with statistical significance. It is worthwhile to note that the negative pattern is robust even if we decompose *IV convexity* into the systematic and idiosyncratic components. That is, $convexity_{sys}$ and $convexity_{idio}$ reveal decreasing patterns in the portfolio returns as the *IV convexity* portfolio increases.

The difference between the lowest and the highest quintile portfolios sorted by $convexity_{sys}$ and $convexity_{idio}$ are significantly positive with t-statistics of 6.56 and 5.72, respectively. This implies that both components have predictive power for future portfolio returns and are significantly priced.

Panel B of Table 7 reports the average monthly portfolio returns of the 25 quintile portfolios formed by sorting stocks based on $convexity_{\text{sys}}$ (or $convexity_{\text{idio}}$) first, and then sub-sorted by *IV convexity* in each $convexity_{\text{sys}}$ (or $convexity_{\text{idio}}$) quintile. This will allow us to figure out how the systematic or idiosyncratic components contribute to *IV convexity*. In other words, if the decreasing patterns of returns in *IV convexity* portfolio become less clearly observed under the control of $convexity_{\text{sys}}$ (or $convexity_{\text{idio}}$), this can be interpreted as a component of $convexity_{\text{sys}}$ (or $convexity_{\text{idio}}$) and can mostly explain the cross-sectional variation of return on *IV convexity* compared to the other component of $convexity_{\text{sys}}$ (or $convexity_{\text{idio}}$).

As for the results from the sample sorted by $convexity_{\text{sys}}$ first and then on *IV convexity*, the decreasing patterns persists for the $convexity_{\text{sys}}$ quintiles in general, though there are some distortions in the 1$^{\text{st}}$ and 3$^{\text{rd}}$ $convexity_{\text{sys}}$ quintiles for the highest *IV convexity* quintiles. Note that the arbitrage portfolio's returns (Q1-Q5) in each $convexity_{\text{sys}}$ quintile portfolio still remain large and statistically significant. As for the $convexity_{\text{idio}}$- *IV convexity* double sorted portfolios shown in the right-hand side of Table 8, the negative relationship between *IV convexity* and exists, though the order of portfolio returns are not perfectly preserved in the 3$^{\text{rd}}$ and 5$^{\text{th}}$ $convexity_{\text{idio}}$ case. Also, the long-short *IV convexity* portfolio returns in the $convexity_{\text{idio}}$ quintile portfolio (Q1-Q5) are significantly positive with a t-statistic higher than 2.

Thus, these results provide evidence that neither component can fully capture and explain all cross-section variations of returns on *IV convexity*, but both components ($convexity_{\text{sys}}$, $convexity_{\text{idio}}$) have decreasing patterns of portfolio returns, and are needed to capture the cross-sectional variations of returns.

3.3.4. Time-Series Analysis

In a perfectly and completely well-functioning financial market, the mean-variance efficiency of the market portfolio should hold as argued in the capital asset pricing model (CAPM), and market β is the only risk factor that captures the cross-sectional variation in expected returns. However, as many investors cannot hold perfectly diversified portfolios in practice, CAPM may not be valid in reality, the biggest drawback for this theory. Fama and French (1996) found that CAPM's measure of systematic risk is unreliable and instead, firm size and book-to-market ratio are more dependable, arguing that the three-factor model in Fama and French (1993) can capture the cross-sectional variations in returns that are not fully captured by the CAPM model. The Fama and French (1993) model has three factors: (i) $\mathrm{R_m} - \mathrm{R_f}$ (the excess return on the market), (ii) SMB (the difference in returns between small stocks and big stocks) and (iii) HML (the difference in returns between high book-to-market stocks and low book-to-market stocks).

To test whether the existing risk factor models can absorb the observed negative relationship between *IV convexity* and future stock returns, we conduct a time-series test based on CAPM and the Fama-French three factor model, respectively. Along with the Fama-French three factor model (FF3), we also use an extended four-factor model (Carhart, 1997) that includes a momentum factor (UMD) suggested by Jegadeesh and Titman (1993) (FF4).

[Insert Table 8 about here.]

Table 8 reports the coefficient estimates of CAPM, FF3, and FF4 time-series regressions for monthly excess returns on five portfolios sorted by *IV convexity* (or systematic and idiosyncratic components of *IV convexity*). The left-most six columns are the results using a portfolio sorted by *IV convexity*. When running regressions using CAPM, FF3, and FF4, we still observed the estimated intercepts in the Q1~Q3 *IV convexity* portfolio ($\hat{\alpha}_{Q1}, \hat{\alpha}_{Q2}, \hat{\alpha}_{Q3}$), which are statistically significant and have negative patterns with respect to portfolios formed by *IV convexity*. In addition, the differences in the intercept between the lowest and highest *IV convexity*, $\hat{\alpha}_{Q5} - \hat{\alpha}_{Q1}$, are 0.0116 (t-statistic = 6.82) for CAPM, 0.0118 (t-statistic = 6.9) for FF3, and 0.012 (t-statistic = 6.80) for FF4. Adopting Gibbons, Ross, and Shanken (1989), we test the null hypothesis that all estimated intercepts are jointly different from zero ($\hat{\alpha}_{Q1} = \cdots = \hat{\alpha}_{Q5} = 0$), and this is rejected with a p-value < 0.001 in the CAPM, FF3, and FF4 model specifications. These results imply that the widely-accepted existing factors ($\mathrm{R_m} - R_f$, SMB, HML, UMD) cannot fully capture and explain the negative portfolio return patterns sorted by *IV convexity*. We may then argue that cross-sectional *IV convexity* does not contain the existing systematic risk factors, thus this is one of the risk factor that can capture the cross-sectional variations in returns not explained by existing models (CAPM, FF3, and FF4).

When we conduct time-series test using portfolios sorted by decomposed components of *IV convexity* ($convexity_{\mathrm{sys}}$ $convexity_{\mathrm{idio}}$) to see which components are not explained by existing risk factors, most of the estimated intercepts are significantly positive, indicating that the CAPM, FF3, and FF4 models leaves some portion of unexplained returns for the $convexity_{\mathrm{sys}}$, $convexity_{\mathrm{idio}}$ portfolios in Q1~Q3(Q4).

The joint tests from Gibbons, Ross, and Shanken (1989) which examine whether the model explains the average portfolio returns sorted by each component of *convexity* ($convexity_{\mathrm{sys}}$, $convexity_{\mathrm{idio}}$) are strongly rejected with p-value < 0.001 for the CAPM, FF3, and FF4 models. Therefore, regardless of whether *IV convexity* is invoked by the market (systematic) or by idiosyncratic risk, both components of *IV convexity* ($convexity_{\mathrm{sys}}$, $convexity_{\mathrm{idio}}$) are not explained by existing systematic risk factors. Thus, we can infer that the negative return patterns shown in Tables 3-7 are hard to explain with existing traditional risk-based factor models. These results provide strong evidence for the information transmission hypothesis (*Hypothesis2-1*) in that informed option traders anticipating heavy tail risks proactively induce leptokurtic implied distributions before equity investors express their tail risk aversion in the stock market.

4. Robustness Checks

We address additional aspects of *IV slope* and *IV convexity* measurements for robustness. We first conduct a Fama-Macbeth regression analysis with various control variables, and then investigate a number of alternative *IV convexity* measures to check the robustness of our results.

4.1. Fama-Macbeth Regression

The time-series test results indicate that the existing factor models may not be able to perfectly capture the return predictability of *IV convexity*. As *IV convexity* can be a candidate risk factor that can explain stock returns, we conduct Fama-Macbeth (1973) cross-sectional regressions at the firm level to investigate the relationship between *IV convexity* and other measures of risks suggested in previous literature.

We consider market β [estimated following Fama and French (1992)], size (*ln_mv*), book-to-market (*btm*), momentum (*MOM*), reversal (*REV*), illiquidity (*ILLIQ*), options volatility slope (*IV spread* and *IV smirk*), idiosyncratic risk (*idio_risk*), implied volatility level (*IV level*), systematic volatility (v_{sys}^2), and idiosyncratic implied variance (v_{idio}^2)[13] as common measures of risks that explain stock returns. The market β is estimated from time-series regressions of raw stock excess returns on the R_m-R_f by month-by-month rolling over the previous three-year (36-month) returns (a minimum of 12 months). Motivated by Jegadeesh and Titman (1993), MOM is defined as the cumulative return over the past five months from t-6 to t-2 by omitting one month from the portfolio formation time point to eliminate the short-term reversal effect. MOM portfolio is rebalanced every month and held for the following one month period. REV is defined as the past one-month return as suggested by Jegadeesh (1990) and Lehmann (1990). ILLIQ is defined as the absolute monthly stock return normalized by the trading volume of the stock in thousand USD as proposed by Amihud (2002). Though our options convexity measure is computed using options implied volatility, we include the *IV level* variable to confirm that the results are not driven by implied volatility. Following Yan (2011) and Xing, Zhang, and Zhao (2010), we define the option implied volatility slope as

$$IV\ smirk = \mathrm{IV}_{\mathrm{put}}(\Delta = -0.8) - \mathrm{IV}_{\mathrm{call}}(\Delta = 0.5), \quad (20)$$

$$IV\ spread = \mathrm{IV}_{\mathrm{put}}(\Delta = -0.5) - \mathrm{IV}_{\mathrm{call}}(\Delta = 0.5), \quad (21)$$

In addition, Ang, Hodrick, Xing and Zhang (2006) argue that idiosyncratic volatility can explain the cross-sectional variation of stock returns. In this context, we estimate the idiosyncratic risk following Ang, Hodrick, Xing, and Zhang (2006) using daily returns, and include this variable in the Fama-Macbeth regression. Daily excess returns of individual stocks over the last 30 days are regressed on the three Fama-French (1993, 1996) factors daily and the momentum factors monthly as follows:

$$(\mathrm{R_{it}} - \mathrm{R_f}) = \alpha_\mathrm{i} + \beta_{1\mathrm{i}}(\mathrm{MKT_t} - \mathrm{R_{ft}}) + \beta_{2\mathrm{i}}\mathrm{SMB} + \beta_{3\mathrm{i}}\mathrm{HML} + \beta_{4\mathrm{i}}\mathrm{WML} + \varepsilon_{\mathrm{it}}, \quad (22)$$

[13] We do not include the coskewness factor when conducting Fama-Macbeth (1973) regression. Harvey and Siddique (2000) argue that coskewness is related to the momentum effect, as the low momentum portfolio returns tend to have higher skewness than high momentum portfolio returns. Thus, we exclude the coskewness in the Fama-Macbeth regression specification to avoid the multi-colinearity problem with the momentum factor.

The idiosyncratic volatility is computed as the standard deviation of the monthly regression residuals. To reduce the impact of infrequent trading on idiosyncratic volatility estimates, a minimum of 15 trading days in a month for which CRSP reports both a daily return and non-zero trading volume is required.

We also compute systematic volatility estimated by the method suggested in Duan and Wei (2009): $v_{\text{sys}}^2 = \beta^2 v_m^2 / v^2$. Idiosyncratic implied variance is $v_{idio}^2 = v^2 - \beta^2 v_M^2$, where $\mathrm{v_m}$ is the implied volatility of the S&P 500 index option, and computed following Dennis, Mayhew and Stivers (2006). We then run the monthly cross-sectional regression of individual stock returns of the subsequent month on *IV convexity* and the measures of risks presented above.

[Insert Table 9 about here.]

Panel A of Table 9 reports the averages of the monthly Fama-Macbeth (1973) cross-sectional regression coefficient estimates for individual stock returns with market β as a control variable along with the Newey-West adjusted t-statistics for the time-series average of coefficients with the lag of 3. In Model 1, the coefficient for *IV convexity* is significantly negative, in line with our previous observation in the portfolio formation approach. Other related option-implied volatility slope measures, *IV smirk* and *IV spread* in Models 2 and 3, have significantly negative coefficients, similar to Yan (2011) and Xing, Zhang, and Zhao's (2010) finding indicating a negative relationship between *IV smirk* (*IV spread*) and stock returns.

The next six columns, from Models 4 to 9, report the results with market β and other stock fundamentals including a firm's size and book-to-market ratio as control variables. Even including stock fundamentals, firm-size (*ln_mv*) and book-to-market (*btm*), we still observe significantly negative coefficients of *IV convexity*, *IV smirk,* and *IV spread*. Even if we include both *IV convexity* and *IV smirk* (or IV spread), as shown from Models 8 to 9, the coefficients of *IV convexity* are still significantly negative indicating that *IV convexity* has a strong explanatory power for stock returns which *IV smirk* and *IV spread* cannot fully capture. The significantly negative coefficients confirms the existence of size effects shown in earlier studies, whereas the coefficients of *btm* are significantly positive, supporting the existence of a value premium.

Model 10 represents the Fama-Macbeth regression result using market β, *ln_mv*, *btm*, *MOM*, *REV*, *ILLIQ* and idiosyncratic risk. Market β, *ln_mv*, *btm*, *MOM*, *REV*, and *ILLIQ* are widely accepted stock characteristics that can capture the cross-sectional variation in stock returns. However, the result is surprising, in that the coefficients of market β are insignificant, while *ln_mv* and *btm* have significantly negative and positive coefficients, respectively. The estimated coefficients of MOM take positive signs without statistical significance, whereas REV has significantly negative coefficients and ILLIQ have significantly positive coefficients.

Moreover, the estimated coefficient of idiosyncratic risk suggested by Ang, Hodrick, Xing, and Zhang (2006) is significantly negative. In an ideal asset pricing model that fully captures the cross-sectional variation in stock returns, idiosyncratic risk should not be significantly priced. The relationship between idiosyncratic risk and stock returns are inconclusive, though this is somewhat controversial among researchers. Ang, Hodrick, Xing, and Zhang (2006) show that stocks with low idiosyncratic risk earn high average returns compared to high idiosyncratic risk portfolios, and the arbitrage portfolio for long high idiosyncratic risk and short low idiosyncratic risk earns significantly negative returns. However, other researchers argue that this relationship does not persist when using different sample periods and equal-weighted returns.[14] Fu (2009) finds a significantly positive relationship between idiosyncratic risk and stock returns, and Bali and Cakici (2008) show no significant negative relationship, but insignificant positive relationships when they form equal-weighted portfolios. However, the statistical significance of the estimated coefficient on the idiosyncratic risk in Panel A of Table 10 implies that idiosyncratic risk is priced and there may exist other risk factors besides market β, *ln_mv*, *btm*, *MOM*, *REV* and *ILLIQ*. When adding *IV convexity* in Model 11, *IV convexity* has a significantly negative coefficient, with the value and significance level of the coefficient on idiosyncratic risk decreasing compared to Model 10 (from -0.121 to -0.113) and with a smaller t-value (-2.16 compared to -2.06). In this regard, we may infer that *IV convexity* can be a significant risk factor that contributes to some part of idiosyncratic risk and explains some part of the cross-sectional variation in returns that cannot be fully explained by market β, *ln_mv*, *btm*, *MOM*, *REV* or *ILLIQ*. The statistical significance of *IV convexity* still remains, even after including both *IV smirk* and *IV spread* Models 14 and 15, respectively.

[14] Note that Ang, Hodrick, Xing, and Zhang (2006) employed value-weighted returns for their research.

When all control variables are included, as in Models 16-20, the sign and significance for the *IV convexity* coefficients remain unchanged. They still have significantly negative coefficients, confirming that investors seem to require a negative premium for *IV convexity*. All in all, it can be inferred that there is no evidence that existing risk factors suggested by prior research can explain the negative return patterns in *IV convexity*, and it is possible that *IV convexity* is a priced risk factor that can capture the cross-sectional variations in returns not explained by existing models.

Finally, we conduct additional analyses with $convexity_{\mathrm{sys}}$ and $convexity_{\mathrm{idio}}$ to investigate whether the systematic and idiosyncratic components of *IV convexity* are priced. As reported in Panel B of Table 9, the univariate regressions of $convexity_{\mathrm{sys}}$ and $convexity_{\mathrm{idio}}$ in Models 1- 2 show that the estimated coefficients for $convexity_{\mathrm{sys}}$ and $convexity_{\mathrm{idio}}$ are significantly negative (-0.019 and -0.014 with t-statistics of -5.26 and -4.92, respectively), confirming our previous findings from the portfolio formation approach in Section 3.3.3.

As shown in Models 8-10, the coefficients on the $convexity_{\mathrm{sys}}$ and $convexity_{\mathrm{idio}}$ remain their statistical significance even after we control for market β, *ln_mv, btm*, *MOM*, *REV*, and idio_risk. In Model 8, $convexity_{\mathrm{sys}}$ has a significantly negative average coefficient, but the estimated coefficient on idiosyncratic risk does not change significantly from that in Model 7 with the value and significance level of the coefficient on idiosyncratic risk slightly decreasing compared to Model 7 (from -0.115 to -0.114)

Model 9 shows the results by adding the $convexity_{\mathrm{idio}}$ factor. We can observe that $convexity_{\mathrm{idio}}$ has a significantly negative coefficient, and the value of the coefficient on idiosyncratic risk decreasing compared to Model 7 (from -0.115 to -0.109) and the estimated coefficient on idiosyncratic risk is larger than that of Model 8. The significance level of the coefficient on idiosyncratic risk also becomes weaker than that in Model 7, as the t-statistics changes from -2.03 in Model 7 to -1.94 in Model 9. These results indicate that $convexity_{\mathrm{idio}}$ can be interpreted as a significantly priced risk factor, which has a meaningful explanatory power for idiosyncratic risk, albeit $convexity_{\mathrm{sys}}$ does not.

The statistical significance of both $convexity_{\text{sys}}$ and $convexity_{\text{idio}}$ is intact even if we include *IV smirk* and *IV spread* in Models 11-12, respectively. When we include all control variables in Models 13-18, we still observe the same results, confirming that the cross-sectional predictive power of *IV convexity* is statistically significant for both the systematic and idiosyncratic components.

4.2. Alternative Measures of Option-implied Volatility Convexity

In this section, we explore alternative measures of options-implied volatility convexity. We define alternative option-implied volatility convexity measures given by

$$convexity_{cp} = \frac{[\text{IV}_{\text{call}}(0.2)+\text{IV}_{\text{put}}(-0.8)]}{2} + \frac{[\text{IV}_{\text{call}}(0.8)+\text{IV}_{\text{put}}(-0.2)]}{2} - [\text{IV}_{\text{call}}(0.5) + \text{IV}_{\text{put}}(-0.5)] \quad (23)$$

$$convexity_{Bali} = \text{IV}_{\text{call}}(0.25) + \text{IV}_{\text{put}}(-0.25) - \text{IV}_{\text{call}}(0.5) - \text{IV}_{\text{put}}(-0.5) \quad (24)$$

$$convexity_{put} = \text{IV}_{\text{put}}(-0.2) + \text{IV}_{\text{call}}(0.2) - 2 \times \text{IV}_{\text{put}}(-0.5) \quad (25)$$

$$convexity_{call} = \text{IV}_{\text{call}}(0.2) + \text{IV}_{\text{call}}(0.8) - 2 \times \text{IV}_{\text{call}}(0.5). \quad (26)$$

Note that $convexity_{cp}$ incorporates comprehensive implied volatility information from call and put options, whereas our proposed *IV convexity* is constructed by deep OTM put, deep ITM put, and ATM call options. Motivated by Bali, Hu and Murray (2015), we define $convexity_{Bali}$ as the sum of OTM call and OTM put implied volatilities less the sum of the ATM call and ATM put implied volatilities. Finally, we construct a put-based *IV convexity* measure, $convexity_{put}$ and a call-based measure, $convexity_{call}$, respectively.

[Insert Table 10 about here.]

Table 10 reports the descriptive statistics of the average portfolio returns sorted by alternative measures of option-implied volatility convexity. Though there are a bit of distortions in $convexity_{cp}$ and $convexity_{put}$ quintiles, we can still observe decreasing patterns in portfolio returns with alternative measures of option-implied volatility convexity in general. Further, the returns of the arbitrage portfolio (Q1-Q5) in $convexity_{cp}$ and $convexity_{put}$ quintile portfolios remain positive with statistical significance (0.0087 for $convexity_{cp}$ with t-statistic =

5.10, and 0.0091 for $convexity_{put}$ with t-statistic = 6.09). This result confirms that the negative relationship between *IV convexity* and future stock returns are robust and consistent when we define option-implied volatility convexity in different ways.

It is remarkable that the arbitrage portfolio (Q1-Q5) return is positive but insignificant, when the portfolio is constructed based on the $convexity_{Bali}$ and $convexity_{call}$ measures. Our interpretation is consistent with the demand-based option pricing argument of Gârleanu, Pedersen and Poteshman (2009) in that the pessimistic perception of the stock's performance owing to the investors' aversion to the anticipated excess kurtosis is reflected more in the put option prices than in the call option prices. For this reason, the predictive power of the $convexity_{Bali}$ and $convexity_{call}$ measures becomes weaker and the arbitrage portfolio (Q1-Q5) returns lose their statistical significance.

4.3. Performance Evaluation based on Sharpe Ratios

Considering the risk-return trade-off, we evaluate the performance of each portfolio based on two different versions of Sharpe ratios. The standard Sharpe ratio (SR) is defined as

$$SR = \frac{\mu - r}{\sigma}, \tag{27}$$

which can be interpreted as market price of risk under the standard mean-variance framework. In the context of non-normality in the asset return distributions, however, there exists investors' preference on higher moments within expected utility function. To overcome the shortcomings of the standard Sharpe ratio, Zakamouline and Koekebakker (2009) propose a Generalized Sharpe ratio (GSR) as the ultimate generalization of the standard Sharpe ratio by accounting for all moments of distribution.[15] Assuming negative exponential utility functions with zero initial wealth, we can numerically solve an optimal capital allocation problem by maximizing the expected utility function given by

$$\mathrm{E}\left[U^{*}\left(\widetilde{W}\right)\right] = \max_{a} E[-e^{-\lambda a(x-r_f)}], \tag{28}$$

[15] The notion of the generalized Sharpe ratio is originally introduced by Hodges (1998).

and the GSR is computed in a non-parametric way using[16]

$$\text{GSR} = \sqrt{-2\log(-E[U^*(\widetilde{W})])}. \qquad (29)$$

[Insert Table 11 about here.]

Panel A of Table 11 shows the Sharpe ratios for single-sorted portfolios formed based on *IV convexity* along with alternative measures of option implied convexity. Although there exist some minor distortions in $convexity_{cp}$ quintiles, similar decreasing patterns of SR and GSR can be observed in quintile portfolios based on *IV convexity*, $convexity_{cp}$ and $convexity_{put}$. Moreover, the arbitrage portfolios (Q1-Q5) based on *IV convexity*, $convexity_{cp}$ and $convexity_{put}$ reveal positive SR and GSR (over 0.3). This result implies that one can enjoy lucrative compensation as a reward of taking excess tail risk contribution to the perceived variance from the zero-cost portfolios based on *IV convexity*. On the other hand, when we construct portfolios based on $convexity_{Bali}$ and $convexity_{call}$, the decreasing SR and GSR patterns are substantially distorted.

Panel B of Table 11 presents the SR and GSR of double-sorted quintile portfolios formed based on *IV slope* (as well as *IV spread* and *IV smirk*) first and then sub-sorted into five groups based on *IV convexity*. We can still observe that decreasing patterns in *IV convexity* portfolios' SR and GSR persist even after controlling for *IV slope (IV spread)*, and the SR and GSR of the arbitrage portfolios (Q1-Q5) are higher than 0.19. However, when we control for *IV smirk* suggested by Xing, Zhang, and Zhao (2010), *IV convexity* arbitrage portfolios' (Q1-Q5) SR and GSR within S2, S3 and S4 *IV smirk* portfolios become less than 0.09 confirming the results of Table 6.

4.4. Different Holding Period Returns of Options Implied Convexity Portfolios

We turn to examine how long the arbitrage strategy based on *IV convexity* portfolio persists to generate profits by varying investment horizons.

[Insert Table 12 about here.]

[16] It can be shown that the GSR reduces to the standard Sharpe ratio when we assume normally distributed asset returns.

Table 12 reports the average risk-adjusted monthly returns(using four-factor model) of the quintile portfolios formed on *IV convexity* for holding periods from two to six months, where 'Q1-Q5' denotes a long-short arbitrage portfolio that buys low convexity portfolio and sells high convexity portfolio. The t-statistics are computed using Newey-West procedure for adjusting serially correlated returns of overlapping samples. Though the decreasing patterns of *IV convexity* portfolio returns are slightly distorted and the decreasing patterns are less pronounced as the holding period increases, a trading strategy with a long position in low *IV convexity* stocks and a short position in high *IV convexity* stocks still yield significantly positive profits. Note that the arbitrage portfolio return decreases from 0.0012 (t=6.80) for one-month holding period to 0.0044 (t=5.97) for six-month holding period. These results imply that the opportunity of arbitrage profits using *IV convexity* information can be realized in the next first one month for the most part and gradually disappear as portfolios are held longer up to six months.

5. Conclusion

This paper finds empirical evidence that the convexity of an option-implied volatility curve has a negative predictive relationship with the cross-section of future stock returns. We demonstrate that the option-implied measure of volatility convexity, as a proxy of both the volatility of stochastic volatility and the volatility of stock jump size, reflects the degree of informed traders' anticipation of the excess tail risk contribution to the perceived variance of the underlying equity returns. Our empirical findings are consistent with earlier studies demonstrating option traders' information advantage in that informed traders anticipating heavier tail risk proactively choose the options market to capitalize on their private information. The average portfolio return sorted by *IV convexity* monotonically decreases from 0.0208 for quintile portfolio 1 (Q1) to 0.0074 for quintile portfolio 5 (Q5) on a monthly basis, implying that the average monthly return of the arbitrage portfolio buying Q1 and selling Q5 is significantly positive. This pattern remains even after decomposing *IV convexity* into systematic and idiosyncratic components, as the results still reveal decreasing patterns in the portfolio returns as the *IV convexity* specific to each portfolio increases with statistical significance. In addition, the negative relationship between *IV convexity* and future stock return consistently persists even after the various kinds of systematic risks suggested in prior research are appropriately controlled. Thus, we argue that the observed *IV* *convexity* is not absorbed by systematic risk components and should be considered as a significantly priced risk factor. Furthermore, the negative relation of *IV convexity* to future stock returns does not disappear but holds even after considering the impact of *IV slope* and other well-documented volatility skew measures. This consistency implies that *IV convexity* can be an important measure to capture the fat-tailed characteristics of stock return distributions in a forward-looking manner, as this behavior leads to the leptokurtic implied distributions of underlying stock returns before equity investors show their kurtosis risk aversion.

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Table 1. Option implied volatilities

This table reports the summary statistics (mean and standard deviation) of the fitted implied volatilities and fixed deltas of the individual equity options with one month (30 days) to expiration at the end of month obtained from OptionMetrics. DS measures the degree of accuracy in the fitting process at each point and computed by the weighted average of standard deviation. The sample periods covers Jan 2000 to Dec 2013.

	Call												
delta	20	25	30	35	40	45	50	55	60	65	70	75	80
Mean	0.493	0.480	0.471	0.466	0.463	0.463	0.464	0.468	0.472	0.479	0.486	0.497	0.511
stdev	0.279	0.278	0.277	0.276	0.274	0.273	0.273	0.274	0.275	0.277	0.279	0.281	0.285
DS	0.054	0.043	0.033	0.026	0.022	0.020	0.019	0.019	0.020	0.022	0.027	0.035	0.046

	Put												
delta	-80	-75	-70	-65	-60	-55	-50	-45	-40	-35	-30	-25	-20
Mean	0.492	0.481	0.474	0.471	0.469	0.470	0.473	0.478	0.484	0.492	0.502	0.516	0.535
stdev	0.298	0.293	0.288	0.285	0.283	0.281	0.280	0.280	0.281	0.282	0.283	0.285	0.286
DS	0.047	0.038	0.030	0.024	0.020	0.019	0.018	0.019	0.020	0.024	0.030	0.041	0.055

Table 2. Descriptive Statistics

Panel A reports the descriptive statistics of options implied volatility, skew and convexity of the equity options with one month (30 days) to expiration at the end of month. $\mathrm{IV_{put}}(\Delta_{\mathrm{put}})$ and $\mathrm{IV_{call}}(\Delta_{\mathrm{call}})$ refer to fitted implied volatilities with one month(30days) to expiration and $\Delta_{call,put}$ are options deltas. Options implied volatility is defined by $IV\ level = 0.5[\mathrm{IV_{put}}(-0.5) + \mathrm{IV_{call}}(0.5)]$ and Options volatility slopes are computed with $IV\ slope = \mathrm{IV_{put}}(-0.2) - \mathrm{IV_{put}}(-0.8)$, *IV spread*= $\mathrm{IV_{put}}(-0.5) - \mathrm{IV_{call}}(0.5)$, and *IV smirk*= $\mathrm{IV_{put}}(-0.2) - \mathrm{IV_{call}}(-0.5)$, respectively, following our definition of options volatility slope, Yan(2011) and Xing, Zhang and Zhao(2010). Option implied convexity is calculated by $IV\ convexity = \mathrm{IV_{put}}(-0.2) + \mathrm{IV_{put}}(-0.8) - 2 \times \mathrm{IV_{call}}(0.5)$. Using daily options implied convexity of equity options and S&P500 index option, we conduct time series regressions in each month to decompose options implied convexity into the systematic and idiosyncratic components given by:

$$c_{i,t-30\sim t} = \alpha_i + \beta_i c_{S\&P500,t-30\sim t} + \varepsilon_{i,t}$$

The fitted values and residual terms are the systematic components of options implied convexity ($convexity_{sys}$) and the idiosyncratic components of options implied convexity ($convexity_{idio}$), respectively. Alternative *IV convexity* are defined by $\mathrm{p}\Delta_1\mathrm{\_c50\_p}\Delta_2 = \mathrm{IV_{put}}(\Delta_1) + \mathrm{IV_{put}}(\Delta_2) - 2 \times \mathrm{IV_{call}}(0.5)$, where $-0.45 \le \Delta_1 \le -0.2$ and $-0.80 \le \Delta_2 \le -0.55$. Panel B shows the descriptive statistics of firm characteristic variables. Size (ln_mv) is computed at the end of each month and we define size as natural logarithm of the market capitalization. When computing book-to-market ratio(BTM), we match the yearly BE (book value of common equity (CEQ) plus deferred taxes and investment tax credit (txditc)) for all fiscal years ending at year t-1 to returns starting in July of year t and this BE is divided by market capitalization at month t-1. Beta (β) is estimated from time-series regressions of raw stock excess returns on the Rm-Rf by month-by-month rolling over past three year (36 months) returns (a minimum of 12 months). Momentum(MOM) is computed based on past cumulative returns over over the past 5 months (t-6 to t-2) following Jegadeesh and Titman (1993). Reversal (REV) is computed based on past one-month return (t-1) following Jegadeesh(1990) and Lehmann(1990). Illiquidity (ILLIQ) is defined as the average of the absolute value of stock return divided by the trading volume of the stock in thousand USD calculated using past one month daily data following Amihud (2002). Following Harvey and Siddique (2000), daily excess returns of individual stocks are regressed on the daily market excess return and the daily squared market excess return using the last one year data month by month given by:

$$(\mathrm{R_{it}} - \mathrm{R_f})_{i,t-365\sim t} = \alpha_i + \beta_{1,i}(\mathrm{MKT_t} - \mathrm{R_{ft}})_{t-365\sim t} + \beta_{2,i}(\mathrm{MKT_t} - \mathrm{R_{ft}})^2{}_{t-365\sim t} + \varepsilon_{i,t}$$

The coskewness of a stock is defined as the coefficient of the squared market excess return. To reduce the impact of infrequent trading on coskewness estimates, a minimum of 255 trading days in a month daily return are required.

Panel A. Option Implied Volatility, Option Implied Volatility Slope and Option Implied Volatility Convexity

	Implied Volatility	Implied Slope			Implied Convexity			Alternative Convexity Measures				
	IV level	*IV slope*	*IV spread*	*IV smirk*	IV convexity	$convexity_{sys}$	$convexity_{idio}$	p25_c50_p75	p30_c50_p70	p35_c50_p65	p40_c50_p60	p45_c50_p55
Mean	0.4739	0.0423	0.009	0.0687	0.0942	0.0942	0.0000	0.065	0.045	0.032	0.023	0.018
Stdev	0.2689	0.1614	0.1235	0.1413	0.2624	0.1682	0.2015	0.213	0.199	0.19	0.184	0.183
Median	0.4088	0.0422	0.0045	0.0516	0.0595	0.0668	-0.0049	0.039	0.027	0.019	0.014	0.01

Panel B. Firm Characteristic Variables

	Size			BTM			Beta (β)			MOM			REV			ILLIQ			Coskew		
Quintile	Mean	Median	Stdev	Mean	Median	Stdev	Mean	Median	Stdev	Mean	Median	Stdev	Mean	Median	Stdev	Mean	Median	Stdev	Mean	Median	Stdev
Q1	16.7256	16.8167	0.8923	0.1467	0.1434	0.0836	-0.0413	0.0865	0.5902	-0.2788	-0.2536	0.1693	-0.1499	-0.1231	0.1080	0.0000	0.0000	0.0000	-18.9451	-13.5341	17.1791
Q2	18.2541	18.2806	0.5998	0.3559	0.3389	0.1028	0.5764	0.5891	0.2163	-0.0699	-0.0611	0.1123	-0.0424	-0.0322	0.0534	0.0001	0.0000	0.0001	-6.3591	-5.0588	5.0064
Q3	19.3495	19.3875	0.5814	0.5771	0.5547	0.1597	0.9689	0.9783	0.2222	0.0454	0.0484	0.1086	0.0073	0.0102	0.0457	0.0003	0.0002	0.0004	-0.9856	-0.6643	2.2680
Q4	20.5129	20.5177	0.5699	0.8946	0.8553	0.2919	1.4901	1.4735	0.2625	0.1763	0.1711	0.1316	0.0603	0.0561	0.0549	0.0018	0.0009	0.0023	4.2063	3.2719	3.3457
Q5	22.4613	22.2198	1.1355	2.6192	1.6282	4.8822	2.8661	2.5030	1.2970	0.6099	0.4474	0.7128	0.2181	0.1623	0.2263	0.0359	0.0112	0.0631	15.3192	11.9180	13.5290
All	19.4607	19.3757	2.1054	0.9186	0.5472	2.3613	1.1720	0.9808	1.1860	0.0966	0.0461	0.4515	0.0187	0.0080	0.1701	0.0076	0.0002	0.0316	-1.3516	-0.5745	15.2483

Table 3. Average returns sorted by option implied volatility convexity

Panel A of this table reports descriptive statistics of the kurtosis and equal-weighted and value-weighted average portfolio monthly returns sorted by *IV convexity(IV spread and IV smirk)*. *IV convexity, IV spread* and *IV smirk* are estimated, respectively, following our definition of *IV convexity*, Yan(2011) and Xing, Zhang and Zhao(2010). On the last trading day of every each month, all firms are assigned into one of five portfolio groups based on *IV convexity*(*IV spread and IV smirk*) and we assume stocks are held for the next one-month-period. This process is repeated in every month. Panel B of this table reports descriptive statistics of equal-weighted and value-weighted average portfolio monthly returns sorted by alternative *IV convexity*. Alternative *IV convexity* are defined by $\mathrm{p}\Delta_1\_\mathrm{c50}\_\mathrm{p}\Delta_2 = \mathrm{IV_{put}}(\Delta_1) + \mathrm{IV_{put}}(\Delta_2) - 2 \times \mathrm{IV_{call}}(0.5)$, where $-0.45 \leq \Delta_1 \leq -0.2$ and $-0.80 \leq \Delta_2 \leq -0.55$. Value-weighted portfolio returns are weighted by the lag of market capitalization of the underlying stocks. Monthly stock returns are obtained from Center for Research in Security Prices (CRSP) with stocks traded on the NYSE (exchcd=1), Amex (exchcd=2) and NASDAQ (exchcd=3). We use only common shares (shrcd in 10, 11). Stocks with a price less than three dollars are excluded from the sample. 'Q1-Q5' denotes an arbitrage portfolio that buys low options implied convexity portfolio (Q1) and sells high *IV convexity* portfolio (Q5). The sample periods covers from Jan 2000 to Dec 2013. Numbers in parentheses indicates t-statistics.

Panel A. *IV convexity* (*IV spread* and *IV smirk*) and Equal-weighted (Value-weighted) portfolio return

	IV convexity						*IV spread*					*IV smirk*				
Quintile	Avg # of firms	Mean	Stdev	Avg kurtosis of return	EW Ret	VW Ret	Avg # of firms	Mean	Stdev	EW Ret	VW Ret	Avg # of firms	Mean	Stdev	EW Ret	VW Ret
Q1 (Low)	1816	-0.1364	0.2822	4.0509	0.0208	0.0136	1769.14	-0.0834	0.1491	0.0145	0.0109	1730.86	-0.0519	0.1462	0.0136	0.0112
Q2	1781	0.0156	0.0317	4.0434	0.0124	0.0061	1844.14	-0.0094	0.0131	0.0102	0.0067	1767.79	0.0292	0.0176	0.0099	0.0066
Q3	1819	0.0637	0.0394	4.1244	0.0098	0.0051	1752.86	0.0040	0.0109	0.0081	0.0044	1782.79	0.0527	0.0202	0.0073	0.0035
Q4	1869	0.1333	0.0672	4.1822	0.009	0.0025	1892.14	0.0189	0.0173	0.0066	0.0040	1824.64	0.0845	0.0286	0.0066	0.0033
Q5 (High)	1599	0.398	0.3539	4.2857	0.0074	0.0023	1700.14	0.1096	0.1844	0.0013	-0.0004	1541.21	0.2182	0.1863	0.0034	0.0019
Q1-Q5					0.0134	0.0113				0.0131	0.0113				0.0102	0.0094
t-statistic					[7.87]	[5.08]				[7.31]	[3.91]				[5.05]	[3.64]

Panel B. Alternative measure of *IV convexity* and Equal-weighted (Value-weighted) portfolio return

	p25_c50_p75				p30_c50_p70				p35_c50_p65				p40_c50_p60				p45_c50_p55			
Quintile	Avg # of firms	Mean	EW Ret	VW Ret	Avg # of firms	Mean	EW Ret	VW Ret	Avg # of firms	Mean	EW Ret	VW Ret	Avg # of firms	Mean	EW Ret	VW Ret	Avg # of firms	Mean	EW Ret	VW Ret
Q1 (Low)	1832	-0.1304	0.0202	0.0136	1831	-0.1364	0.0199	0.0106	1821	-0.1385	0.0199	0.0109	1801	-0.1388	0.0198	0.0106	1759	-0.1395	0.0204	0.0116
Q2	1790	0.001	0.012	0.0058	1804	-0.0079	0.0123	0.0064	1817	-0.013	0.0124	0.0061	1813	-0.0155	0.0124	0.0063	1814	-0.0163	0.0122	0.0069
Q3	1790	0.0421	0.0095	0.0052	1769	0.0292	0.0096	0.0049	1762	0.0206	0.0095	0.0056	1754	0.0147	0.0096	0.0057	1730	0.011	0.009	0.0052
Q4	1857	0.0977	0.0093	0.0023	1855	0.0755	0.0089	0.0031	1857	0.0604	0.0085	0.0034	1853	0.0498	0.009	0.0032	1839	0.0428	0.0088	0.0034
Q5 (High)	1585	0.312	0.008	0.0029	1572	0.2644	0.0084	0.0035	1582	0.2313	0.0088	0.0019	1583	0.2092	0.0083	0.0008	1570	0.1967	0.0086	0.0012
Q1-Q5			0.0122	0.0106			0.0115	0.0071			0.0111	0.0090			0.0115	0.0098			0.0118	0.0104
t-statistic			[7.74]	[4.85]			[7.32]	[3.36]			[7.11]	[3.89]			[7.03]	[4.07]			[6.79]	[3.87]

Table 4. Average returns of portfolio sorted by firm-size, book-to-market ratio, market beta and option implied volatility convexity

This table reports the average monthly returns of five double-sorted portfolios formed on the firm characteristic variables and *IV convexity*. For each month, stocks are sorted in five groups based on the firm characteristic variables (firm size, book-to-market ratio, market beta) and then subsorted within each quintile portfolio into one of the five portfolios according to *IV convexity*.

Using CRSP data, market capitalization (Size) is computed at the end of each month and we define size as natural logarithm of the market capitalization. When computing book-to-market ratio(BTM), we match the yearly BE (book value of common equity (CEQ) plus deferred taxes and investment tax credit (txditc)) for all fiscal years ending at year t-1 to returns starting in July of year t and this BE is divided by market capitalization(Size) at month t-1. Market betas(Beta) are estimated by the rolling regression using the previous 36 monthly returns available up to month t-1 given by

$$(\mathrm{R}_{\mathrm{i}\,\mathrm{t}-36\sim\mathrm{t}} - \mathrm{R}_{\mathrm{f}\,\mathrm{t}-36\sim\mathrm{t}}) = \alpha_i + \beta_i(\mathrm{MKT}_{\mathrm{t}-36\sim\mathrm{t}} - \mathrm{R}_{\mathrm{ft}-36\sim\mathrm{t}}) + \varepsilon_{it-36\sim\mathrm{t}}.$$

A minimum of 12 month is required when estimating market beta. Stocks are assumed to be held for one month, and portfolio returns are equally-weighted. Monthly stock returns are obtained from the Center for Research in Security Prices (CRSP) with stocks traded on the NYSE (exchcd=1), Amex (exchcd=2) and NASDAQ (exchcd=3). We use only common shares (shrcd in 10, 11). Stocks with a price less than three dollars are excluded from the sample. "Q1-Q5" denotes an arbitrage portfolio that buys a low *IV convexity* portfolio and sells a high *IV convexity* portfolio in each characteristic portfolio. The sample covers Jan 2000 to Dec 2013. Numbers in parentheses indicates t-statistics.

Avg Return															
IV convexity Quintiles	Size Quintiles					BTM Quintiles					Beta Quintiles				
	S1(Small)	S2	S3	S4	S5(Large)	B1(Low)	B2	B3	B4	B5(High)	beta1(Low)	beta2	beta3	beta4	beta5(High)
Q1 (Low *IV convexity*)	0.0255	0.0164	0.0112	0.0122	0.0093	0.0106	0.0148	0.0171	0.0225	0.0337	0.0157	0.0189	0.0189	0.0213	0.0263
Q2	0.0205	0.0118	0.0111	0.0093	0.0064	0.0035	0.0089	0.0121	0.0153	0.0274	0.0100	0.0118	0.0125	0.0132	0.0148
Q3	0.016	0.0111	0.008	0.0109	0.0071	0.0063	0.008	0.0106	0.0134	0.0195	0.0096	0.0121	0.0099	0.0109	0.0102
Q4	0.0139	0.0097	0.0081	0.0081	0.0057	0.0033	0.006	0.0082	0.0126	0.0189	0.0084	0.0109	0.0109	0.0098	0.0084
Q5 (High *IV convexity*)	0.0068	0.0034	0.0064	0.0065	0.0034	0.0009	0.0028	0.0063	0.0091	0.016	0.0076	0.0083	0.0072	0.0089	0.0050
Q1-Q5	0.0187	0.013	0.0048	0.0057	0.0059	0.0097	0.0121	0.0108	0.0134	0.0177	0.0081	0.0106	0.0118	0.0124	0.0213
t-statistic	[6.49]	[5.86]	[2.26]	[3.23]	[3.97]	[4.82]	[5.76]	[5.42]	[5.67]	[5.19]	[4.73]	[6.15]	[6.13]	[5.13]	[6.32]

Avg # of firms															
IV convexity Quintiles	Size Quintiles					BTM Quintiles					Beta Quintiles				
	S1(Small)	S2	S3	S4	S5(Large)	B1(Low)	B2	B3	B4	B5(High)	beta1(Low)	beta2	beta3	beta4	beta5(High)
Q1 (Low *IV convexity*)	416	461	448	416	379	408	467	486	470	377	425	488	493	457	378
Q2	441	465	447	416	371	392	439	467	464	403	398	468	478	462	405
Q3	446	468	461	429	381	400	457	479	478	401	419	480	486	468	405
Q4	444	479	467	433	389	426	482	498	489	406	429	493	508	488	416
Q5 (High *IV convexity*)	401	431	422	373	348	369	436	457	436	362	366	438	454	435	376

Table 5. Average returns of portfolio sorted by momentum, reversal, illiquidity, coskewness and option implied volatility convexity

This table reports the average monthly returns of a double-sorted quintile portfolio formed based on momentum (reversal, illiquidity, coskewness) and *IV convexity*. Momentum(MOM) is computed based on past cumulative returns over the past 5 months (t-6 to t-2) following Jegadeesh and Titman (1993). Reversal(REV) is computed based on previous one-month return (t-1) following Jegadeesh(1990) and Lehmann(1990). Illiquidity (ILLIQ) is defined as the average of the absolute value of stock return divided by the trading volume of the stock in thousand USD calculated using past one month daily data following Amihud (2002). Following Harvey and Siddique (2000), daily excess returns of individual stocks are regressed on the daily market excess return and the daily squared market excess return month by month using the last one year data as below:

$$(R_{it} - R_f)_{i,t-365\sim t} = \alpha_i + \beta_{1,i}(MKT_t - R_{ft})_{t-365\sim t} + \beta_{2,i}(MKT_t - R_{ft})^2_{t-365\sim t} + \varepsilon_{i,t}$$

The coskewness of a stock is defined as the coefficient of the squared market excess return. Daily stock returns are obtained from the Center for Research in Security Prices (CRSP). To reduce the impact of infrequent trading on coskewness estimates, a minimum of 255 trading days in a month for which CRSP reports daily return are required. Monthly stock returns are obtained from the Center for Research in Security Prices (CRSP). The sample covers Jan 2000 to Dec 2013 with stocks traded on the NYSE (exchcd=1), Amex (exchcd=2) and NASDAQ (exchcd=3). For each month, stocks are sorted into five groups based on momentum(reversal, liquidity, coskewness) and then subsorted within each quintile portfolio into one of the five portfolios according to *IV convexity*. Stocks are assumed to be held for one month, and portfolio returns are equally-weighted. We use only common shares (shrcd in 10, 11). Stocks with a price less than three dollars are excluded from the sample. "Q1-Q5" denotes an arbitrage portfolio that buys a low *IV convexity* portfolio and sells a high *IV convexity* portfolio in each momentum (reversal, illiquidity, coskew) portfolio. Numbers in parentheses indicate t-statistics.

	Avg Return																			
IV convexity Quintiles	MOM(t-6~t-2) Quintiles					REV Quintiles					ILLIQ Quintiles					Coskew Quintiles				
	M1(Loser)	M2	M3	M4	M5(Winner)	R1(Loser)	R2	R3	R4	R5(Winner)	I1(Low)	I2	I3	I4	I5(High)	C1(Low)	C2	C3	C4	C5(High)
Q1 (Low *IV convexity*)	0.0225	0.0157	0.013	0.0147	0.0273	0.0237	0.017	0.016	0.0136	0.0134	0.01	0.0116	0.0145	0.0181	0.0288	0.0128	0.0169	0.0131	0.0132	0.0140
Q2	0.0107	0.0094	0.0103	0.01	0.0185	0.016	0.0132	0.0096	0.0092	0.0083	0.0071	0.009	0.0119	0.0134	0.0242	0.0115	0.0111	0.0116	0.0085	0.0082
Q3	0.0025	0.006	0.0078	0.0087	0.0183	0.0123	0.0119	0.0098	0.0075	0.0065	0.0063	0.0102	0.0101	0.0116	0.0173	0.0098	0.0099	0.0087	0.0098	0.0057
Q4	-0.0012	0.0071	0.007	0.0086	0.0177	0.0084	0.0092	0.0087	0.0066	0.0068	0.0055	0.0076	0.0089	0.0108	0.0157	0.0056	0.0086	0.0092	0.0074	0.0037
Q5 (High *IV convexity*)	-0.0029	0.0036	0.0072	0.0082	0.0166	0.0061	0.0095	0.007	0.0055	0.0013	0.0029	0.006	0.0076	0.0023	0.0084	-0.0003	0.0079	0.0091	0.0056	0.0013
Q1-Q5	0.0254	0.0121	0.0058	0.0066	0.0108	0.0176	0.0076	0.0089	0.0081	0.0122	0.0071	0.0056	0.0069	0.0158	0.0204	0.0130	0.0090	0.0040	0.0076	0.0127
t-statistic	[8.36]	[5.92]	[3.76]	[4.51]	[5.39]	[6.99]	[3.64]	[4.69]	[4.49]	[5.73]	[4.21]	[2.89]	[3.15]	[6.01]	[6.85]	[4.6]	[3.83]	[2.39]	[4.18]	[5.07]

	Avg # of firms																			
IV convexity Quintiles	MOM Quintiles					REV Quintiles					ILLIQ Quintiles					Coskew Quintiles				
	M1(Loser)	M2	M3	M4	M5(Winner)	R1(Loser)	R2	R3	R4	R5(Winner)	I1(Low)	I2	I3	I4	I5(High)	C1(Low)	C2	C3	C4	C5(High)
Q1 (Low *IV convexity*)	519	716	761	740	596	679	732	740	745	716	421	477	526	527	428	501	608	620	608	501
Q2	574	690	711	694	602	745	730	700	718	742	401	470	513	521	462	524	574	585	575	511
Q3	574	703	733	698	604	750	748	720	731	745	413	477	525	527	453	529	594	603	585	516
Q4	571	717	741	715	602	745	753	736	753	758	429	486	526	533	449	537	609	621	610	529
Q5 (High *IV convexity*)	497	646	650	628	522	667	697	684	700	695	389	420	460	463	408	473	552	570	558	480

Table 6. Average returns of portfolio sorted by option implied volatility slope (spread, smirk) and option implied volatility convexity

This table reports the average monthly returns of a double-sorted quintile portfolio formed based on *IV slope (IV spread, IV smirk)* and *IV convexity*. Portfolios are sorted in five groups at the end of each month based on *IV slope (IV spread, IV smirk)* first and then sub-sorted into five groups based on *IV convexity*. Options volatility slopes are computed by $IV\ slope = \mathrm{IV_{put}}(-0.2) - \mathrm{IV_{put}}(-0.8)$, *IV spread*= $\mathrm{IV_{put}}(-0.5) - \mathrm{IV_{call}}(0.5)$, and *IV smirk*= $\mathrm{IV_{put}}(-0.2) - \mathrm{IV_{call}}(-0.5)$, respectively, following our definition of options volatility slope, Yan(2011) and Xing, Zhang and Zhao(2010).

Stocks are held for one month, and portfolio returns are equal-weighted. Monthly stock returns are obtained from the Center for Research in Security Prices (CRSP) with stocks traded on the NYSE (exchcd=1), Amex (exchcd=2) and NASDAQ (exc hcd=3). We use only common shares (shrcd in 10, 11). Stocks with a price less than three dollars are excluded from the sample. "Q1-Q5" denotes an arbitrage portfolio that buys a low *IV convexity* portfolio and sells a high *IV convexity* portfolio in each *IV slope (IV spread, IV smirk)* portfolio. The sample periods cover Jan 2000 to Dec 2013. Numbers in parentheses indicates t-statistics.

IV convexity Quintiles	Avg Return														
	IV slope Quintiles					*IV spread* Quintiles					*IV smirk* Quintiles				
	S1(Low)	S2	S3	S4	S5(High)	S1(Low)	S2	S3	S4	S5(High)	S1(Low)	S2	S3	S4	S5(High)
Q1 (Low *IV convexity*)	0.0267	0.027	0.0178	0.016	0.0151	0.0320	0.0163	0.0123	0.0138	0.0130	0.0314	0.0143	0.0104	0.0098	0.0125
Q2	0.0165	0.0164	0.0115	0.0111	0.0136	0.0228	0.0132	0.0092	0.0085	0.0122	0.0224	0.0126	0.0084	0.0085	0.0093
Q3	0.0126	0.0101	0.0079	0.0089	0.0123	0.0189	0.0102	0.0073	0.0067	0.0091	0.0198	0.0118	0.0084	0.0081	0.0087
Q4	0.0079	0.007	0.0081	0.007	0.0089	0.0150	0.0111	0.0091	0.0074	0.0059	0.0191	0.0101	0.0078	0.0089	0.0059
Q5 (High *IV convexity*)	0.0039	0.0085	0.0066	0.0113	0.0087	0.0167	0.0104	0.0078	0.0076	0.0028	0.0200	0.0132	0.0077	0.0078	0.0029
Q1-Q5	0.0227	0.0185	0.0112	0.0047	0.0064	0.0153	0.0059	0.0045	0.0063	0.0102	0.0114	0.0011	0.0026	0.0020	0.0096
t-statistic	[6.84]	[6.30]	[4.59]	[2.14]	[2.52]	[5.21]	[3.22]	[2.61]	[3.17]	[3.77]	[3.9]	[0.62]	[1.24]	[0.93]	[3.75]
p-value	<.0001	<.0001	<.0001	0.0337	0.0128	<.0001	0.0016	0.0098	0.0018	0.0002	0.0001	0.5339	0.2163	0.3523	0.0002

Curvature Quintiles	Avg # of firms														
	s Quintiles					*IV spread* Quintiles					*IV smirk* Quintiles				
	S1(Low)	S2	S3	S4	S5(High)	S1(Low)	S2	S3	S4	S5(High)	S1(Low)	S2	S3	S4	S5(High)
Q1(Low)	611	637	754	778	693	557	771	740	739	655	557	779	749	725	654
Q2	688	669	681	740	735	705	718	666	698	657	711	701	714	731	676
Q3	693	698	682	720	689	749	737	696	713	653	738	682	719	741	657
Q4	669	726	719	727	636	770	781	751	729	619	691	691	728	722	610
Q5(High)	548	689	729	704	527	669	736	724	683	511	618	689	704	675	504

Table 7. Average portfolio returns sorted by systematic components of *IV convexity* (idiosyncratic components of *IV convexity*) and *IV convexity*

Panel A reports descriptive statistics of the average portfolio returns sorted by systematic components of *IV convexity* ($convexity_{sys}$) and idiosyncratic components of *IV convexity* ($convexity_{idio}$). Using daily *IV convexity* of equity options and S&P500 index option, we conduct time series regressions in each month to decompose *IV convexity* into the systematic and idiosyncratic components given by:

$$IV\ convexity_{i,t-30\sim t} = \alpha_i + \beta_i IV\ convexity_{S\&P500,t-30\sim t} + \varepsilon_{i,t}$$

The fitted values and residual terms are the systematic components of *IV convexity* ($convexity_{sys}$) and the idiosyncratic components of *IV convexity* ($convexity_{idio}$), respectively. On the last trading day of every each month, all firms are assigned into one of five portfolio groups based on *IV convexity*, $convexity_{sys}$($conve\square\square ty_{idio}$) and we assume stocks are held for the next one-month-period. This process is repeated in every month. Panel B reports the average monthly returns of a double-sorted quintile portfolio using systematic components of *IV convexity* ($convexity_{sys}$) and idiosyncratic components of *IV convexity* ($convexity_{idio}$). Portfolios are sorted into five groups at the end of each month based on $convexity_{sys}$ (or $convexity_{idio}$) first, and then sub-sorted into five groups based on *IV convexity*. Stocks are held for one month, and portfolio returns are equal-weighted. Stocks are assumed to be held for one month, and portfolio returns are equally-weighted. Monthly stock returns are obtained from the Center for Research in Security Prices (CRSP) with stocks traded on the NYSE (exchcd=1), Amex (exchcd=2) and NASDAQ (exchcd=3). We use only common shares (shrcd in 10, 11). Stocks with a price less than three dollars are excluded from the sample. "Q1-Q5" denotes an arbitrage portfolio that buys a low *IV convexity* portfolio and sells a high *IV convexity* portfolio in each $convexity_{sys}$ (or $convexity_{idio}$) portfolio. The sample periods covers Jan 2000 to Dec 2013. Numbers in parentheses indicate t-statistics.

Panel A. Option implied convexity and averaged portfolio return

	$convexity_{sys}$				$convexity_{idio}$			
Quintile	Avg # of firms	Mean	Stdev	Avg Ret	Avg # of firms	Mean	Stdev	Avg Ret
Q1 (Low *IV convexity*)	1650	-0.0434	0.1517	0.0199	1776	-0.2113	0.2402	0.0174
Q2	1674	0.0373	0.0199	0.0111	1895	-0.0518	0.0383	0.0117
Q3	1777	0.0682	0.026	0.01	1794	-0.0039	0.033	0.0116
Q4	1733	0.1145	0.0425	0.0094	1891	0.047	0.0483	0.0101
Q5 (High *IV convexity*)	1355	0.2773	0.2113	0.0089	1811	0.2379	0.2781	0.0087
Q1-Q5				0.011				0.0086
t-statistic				[6.56]				[5.72]

Panel B. Double sorted quintile portfolio using $convexity_{sys}$ and $convexity_{idio}$

	$convexity_{sys}$										$convexity_{idio}$									
IV convexity Quintiles	Avg Return					Avg # of firms					Avg Return					Avg # of firms				
	$c_{sys}1$	$c_{sys}2$	$c_{sys}3$	$c_{sys}4$	$c_{sys}5$	$c_{sys}1$	$c_{sys}2$	$c_{sys}3$	$c_{sys}4$	$c_{sys}5$	$c_{idio}1$	$c_{idio}2$	$c_{idio}3$	$c_{idio}4$	$c_{idio}5$	$c_{idio}1$	$c_{idio}2$	$c_{idio}3$	$c_{idio}4$	$c_{idio}5$
Q1 (Low *IV convexity*)	0.0319	0.0158	0.0156	0.0143	0.0155	553	775	776	762	665	0.0152	0.0145	0.0130	0.0111	0.0049	631	767	713	741	737
Q2	0.0224	0.0122	0.0091	0.0091	0.0095	691	676	736	741	644	0.0129	0.0107	0.0105	0.0081	0.0044	783	753	659	732	790
Q3	0.0188	0.0090	0.0079	0.0101	0.0094	677	646	713	713	604	0.0115	0.0093	0.0079	0.0075	0.0059	801	753	696	755	754
Q4	0.0131	0.0102	0.0088	0.0077	0.0070	702	696	726	709	561	0.0101	0.0064	0.0095	0.0068	0.0007	755	754	730	745	709
Q5 (High *IV convexity*)	0.0140	0.0086	0.0090	0.0059	0.0029	686	766	767	721	478	0.0049	0.0065	0.0079	0.0057	-0.0033	605	672	686	663	590
Q1-Q5	0.0179	0.0072	0.0066	0.0084	0.0126						0.0102	0.0080	0.0051	0.0053	0.0082					
t-statistic	[5.93]	[3.49]	[3.24]	[4.07]	[5.45]						[3.16]	[3.66]	[2.63]	[2.71]	[3.3]					

Table 8. Time series tests of 3- and 4- factor models using options implied volatility convexity quintiles

This table presents the coefficient estimates of CAPM, Fama-French three (four)-factor models for monthly excess returns on *IV convexity* quintiles portfolios. Fama-French factors [$R_M - R_f$], small market capitalization minus big (SMB), and high book-to-market ratio minus low(HML), and momentum factor(UMD)] are obtained from Kenneth French's website. *IV convexity* quintiles are formed as in Table4. The sample period covers Jan 2000 to Dec 2013 with stocks traded on the NYSE (exchcd=1), Amex (exchcd=2) and NASDAQ (exchcd=3). Stocks with a price less than three dollars are excluded from the sample, and Newey-west (1987) adjusted t-statistics are reported in square brackets. The last row in each model labeled "Joint test p-value" reports a Gibbons, Ross and Shanken(1989) results that tests the null hypothesis whether all intercept are jointly different from zero ($\hat{\alpha}_{Q1} = \cdots = \hat{\alpha}_{Q5} = 0$) or not.

Model	Factor sensitivities		*IV convexity*						$convexity_{sys}$						$convexity_{idio}$					
		Statistics	Q1	Q2	Q3	Q4	Q5	Q1-Q5	Q1	Q2	Q3	Q4	Q5	Q1-Q5	Q1	Q2	Q3	Q4	Q5	Q1-Q5
CAPM	Alpha	Coefficient	0.0146	0.0069	0.0042	0.0032	0.0014	0.0116	0.0138	0.0056	0.0043	0.0035	0.0031	0.0091	0.0113	0.0061	0.0061	0.0044	0.0026	0.0070
		t-stat	[5.60]	[4.75]	[2.88]	[2.04]	[0.66]	[6.82]	[5.51]	[3.96]	[2.83]	[2.05]	[1.53]	[5.39]	[4.70]	[3.92]	[4.41]	[2.88]	[1.20]	[4.63]
	MKTRF	Coefficient	1.4472	1.2075	1.2505	1.3057	1.3830	0.0694	1.4207	1.2283	1.2811	1.3393	1.3338	0.0923	1.4086	1.2623	1.2307	1.2831	1.4110	0.0029
		t-stat	[23.11]	[31.82]	[36.13]	[34.53]	[27.20]	[1.48]	[25.71]	[33.44]	[35.29]	[30.46]	[27.65]	[2.12]	[25.73]	[30.01]	[33.77]	[35.37]	[29.21]	[0.07]
	Adj.R		0.792	0.8968	0.9065	0.9007	0.85	0.0162	0.8006	0.9057	0.9003	0.8858	0.8494	0.0337	0.8101	0.8932	0.9128	0.9017	0.8442	0
	Joint test: p-value				[0.00]						[0.00]						[0.00]			
FF3	Alpha	Coefficient	0.0124	0.0048	0.0023	0.0011	-0.0010	0.0118	0.0118	0.0039	0.0024	0.0011	0.0004	0.0098	0.0087	0.0038	0.0042	0.0025	0.0005	0.0066
		t-stat	[5.87]	[4.52]	[2.34]	[1.03]	[-0.64]	[6.90]	[6.00]	[3.69]	[2.47]	[0.90]	[0.26]	[6.46]	[4.60]	[3.62]	[4.12]	[2.49]	[0.31]	[4.38]
	MKTRF	Coefficient	1.3027	1.1149	1.1512	1.2026	1.2650	0.0426	1.2767	1.1361	1.1724	1.2278	1.2313	0.0503	1.2713	1.1624	1.1475	1.1776	1.2796	-0.0035
		t-stat	[23.58]	[37.48]	[44.49]	[40.90]	[28.50]	[0.96]	[28.30]	[38.33]	[46.30]	[34.29]	[29.01]	[1.49]	[25.92]	[36.13]	[38.38]	[43.02]	[29.43]	[-0.08]
	SMB	Coefficient	0.6328	0.4493	0.4557	0.4824	0.5520	0.0795	0.6199	0.4238	0.4955	0.5324	0.5111	0.1075	0.6300	0.4786	0.4016	0.4772	0.5825	0.0463
		t-stat	[5.33]	[8.39]	[9.74]	[8.46]	[5.76]	[1.43]	[5.55]	[8.44]	[11.93]	[7.58]	[5.01]	[2.39]	[5.85]	[8.64]	[7.18]	[9.60]	[5.91]	[0.90]
	HML	Coefficient	0.0211	0.1346	0.0717	0.1006	0.1138	-0.1013	-0.0094	0.0685	0.0690	0.1377	0.1863	-0.2042	0.0995	0.1277	0.1146	0.0573	0.0386	0.0523
		t-stat	[0.23]	[2.71]	[1.70]	[2.44]	[1.67]	[-1.42]	[-0.12]	[1.42]	[1.73]	[2.51]	[2.70]	[-3.64]	[1.19]	[2.39]	[2.65]	[1.31]	[0.61]	[0.74]
	Adj.R		0.8685	0.9541	0.9642	0.9585	0.9133	0.0579	0.8803	0.9572	0.9654	0.9508	0.9063	0.1901	0.887	0.9527	0.9576	0.9625	0.9159	0.01
	Joint test: p-value				[0.00]						[0.00]						[0.00]			
FF4	Alpha	Coefficient	0.0132	0.0050	0.0024	0.0014	-0.0003	0.0120	0.0126	0.0040	0.0026	0.0015	0.0011	0.0099	0.0094	0.0042	0.0044	0.0028	0.0011	0.0067
		t-stat	[7.33]	[5.10]	[2.48]	[1.54]	[-0.29]	[6.80]	[7.55]	[3.94]	[2.90]	[1.48]	[0.90]	[6.34]	[5.85]	[4.63]	[4.58]	[2.93]	[0.88]	[4.37]
	MKTRF	Coefficient	1.1125	1.0580	1.1264	1.1304	1.1112	0.0059	1.1050	1.1005	1.1200	1.1355	1.0804	0.0291	1.1044	1.0793	1.0974	1.1272	1.1380	-0.0290
		t-stat	[18.13]	[34.90]	[37.00]	[38.48]	[27.91]	[0.11]	[19.37]	[34.59]	[42.41]	[34.70]	[29.53]	[0.64]	[20.55]	[40.32]	[35.53]	[36.75]	[25.76]	[-0.61]
	SMB	Coefficient	0.7588	0.4870	0.4722	0.5302	0.6539	0.1038	0.7336	0.4474	0.5302	0.5935	0.6110	0.1215	0.7406	0.5336	0.4348	0.5106	0.6763	0.0632
		t-stat	[7.28]	[11.09]	[9.97]	[11.01]	[8.59]	[1.54]	[7.50]	[9.27]	[13.78]	[10.08]	[7.54]	[2.13]	[7.64]	[12.92]	[8.60]	[11.41]	[8.01]	[1.05]
	HML	Coefficient	-0.0143	0.1240	0.0671	0.0871	0.0852	-0.1081	-0.0413	0.0619	0.0592	0.1206	0.1582	-0.2081	0.0684	0.1122	0.1053	0.0480	0.0123	0.0475
		t-stat	[-0.22]	[3.02]	[1.64]	[2.52]	[1.86]	[-1.52]	[-0.68]	[1.35]	[1.78]	[3.06]	[3.89]	[-3.47]	[1.20]	[3.10]	[2.89]	[1.22]	[0.23]	[0.70]
	UMD	Coefficient	-0.3321	-0.0994	-0.0433	-0.1261	-0.2686	-0.0640	-0.2999	-0.0622	-0.0915	-0.1612	-0.2635	-0.0369	-0.2916	-0.1452	-0.0875	-0.0880	-0.2474	-0.0446
		t-stat	[-4.35]	[-4.37]	[-1.77]	[-7.22]	[-11.29]	[-1.01]	[-3.96]	[-2.40]	[-4.13]	[-7.41]	[-9.08]	[-0.64]	[-4.59]	[-5.23]	[-3.86]	[-4.63]	[-6.43]	[-0.88]
	Adj.R		0.9232	0.9619	0.9654	0.9694	0.9553	0.0771	0.9269	0.96	0.9713	0.9675	0.9497	0.1935	0.9324	0.9681	0.9635	0.9679	0.9499	0.0013
	Joint test: p-value				[0.00]						[0.00]						[0.00]			

Table 9. Fama-MacBeth regressions

Panel A of this table reports the averages of month-by–month Fama and Macbeth (1973) cross-sectional regression coefficient estimates for individual stock returns on *IV convexity* and control Panel A of this table reports the averages of month-by–month Fama and Macbeth (1973) cross-sectional regression coefficient estimates for individual stock returns on *IV convexity* and control variables. Panel B shows the averages of month-by–month Fama and Macbeth (1973) cross-sectional regression coefficient estimates for individual stock returns on $convexity_{\mathrm{sys}}$ and $convexity_{\mathrm{idio}}$ and control variables. The cross-section of expected stock returns is regressed on control variables. Control variables include market β estimated following Fama and French (1992), size (*ln_mv*), book-to-market (*btm*), momentum(MOM), reversal(REV), illiquidity(ILLIQ), *IV slope (IV spread, IV spread)*, idiosyncratic risk (*idio_risk*), implied volatility level (*IV level*), systematic volatility (v_{sys}^2), idiosyncratic implied variance (v_{isio}^2). Market β is estimated from time-series regressions of raw stock excess returns on the Rm-Rf by month-by-month rolling over the past three year (36 months) returns (a minimum of 12 months). Following Ang, Hodrick, Xing, and Zhang (2006), daily excess returns of individual stocks are regressed on the four Fama-French (1993,1996) factors daily in every month as:

$$(\mathrm{R_{pt}} - \mathrm{R_f}) = \alpha_\mathrm{p} + \beta_{1\mathrm{p}}(\mathrm{MKT_t} - \mathrm{R_{ft}}) + \beta_{2\mathrm{p}}\mathrm{SMB} + \beta_{3\mathrm{p}}\mathrm{HML} + \beta_{4\mathrm{p}}\mathrm{WML} + \varepsilon_{\mathrm{pt}}$$

The idiosyncratic volatility of a stock is computed as the standard deviation of the regression residuals. Daily stock returns are obtained from the Center for Research in Security Prices (CRSP). Momentum(MOM) is computed based on past cumulative returns over over the past 5 months (t-6 to t-2) following Jegadeesh and Titman (1993). Reversal(REV) is computed based on past one-month return (t-1) following Jegadeesh(1990) and Lehmann(1990). Illiquidity (ILLIQ) is defined as the absolute monthly stock return devided by the dollar trading volume in the stock (in $thousands) following Amihud (2002). Systematic volatility is estimated by the method suggested by Duan and Wei(2009) as $v_{sys}^2 = \beta^2 v_m^2 / v^2$. Idiosyncratic implied variance as $v_{idio}^2 = v^2 - \beta^2 v_M^2$, where $\mathrm{v_m}$ is the implied volatility of S&P500 index option, is also computed following Dennis, Mayhey and Stivers (2006). The daily factor data are downloaded from Kenneth R. French's web site. To reduce the impact of infrequent trading on idiosyncratic volatility estimates, a minimum of 15 trading days in a month for which CRSP reports both a daily return and non-zero trading volume are required. The sample period covers Jan 2000 to Dec 2013 with stocks traded on the NYSE (exchcd=1), Amex (exchcd=2) and NASDAQ (exchcd=3) and stocks with a price less than three dollars are excluded from the sample. Newey-west adjusted t-statistics for the time-series average of coefficients using lag3 are reported. Numbers in parentheses indicate the t -statistic.

Panel A. *IV convexity*

Variable	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5	MODEL6	MODEL7	MODEL8	MODEL9	MODEL10	MODEL11	MODEL12	MODEL13	MODEL14	MODEL15	MODEL16	MODEL17	MODEL18	MODEL19	MODEL20
IV convexity	**-0.015*** **	**-0.009*** **	**-0.011*** **		**-0.015*** **			**-0.011*** **	**-0.013*** **		**-0.015*** **			**-0.011*** **	**-0.012*** **	**-0.016*** **			**-0.011*** **	**-0.013*** **
	(-6.76)	**(-3.12)**	**(-2.81)**		**(-6.60)**			**(-3.86)**	**(-4.12)**		**(-6.76)**			**(-4.14)**	**(-3.89)**	**(-7.08)**			**(-4.25)**	**(-4.12)**
IV smirk		-0.025***				-0.038***		-0.018**				-0.038***		-0.017**			-0.041***		-0.019**	
		(-2.79)				(-5.70)		(-2.22)				(-5.71)		(-2.19)			(-5.90)		(-2.20)	
IV spread			-0.012				-0.028***		-0.007				-0.028***		-0.009			-0.030***		-0.010
			(-1.21)				(-5.29)		(-0.98)				(-5.54)		(-1.21)			(-5.39)		(-1.31)
beta	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002	0.002	0.002	0.002
	(0.67)	(0.69)	(0.68)	(0.32)	(0.33)	(0.34)	(0.32)	(0.34)	(0.34)	(0.22)	(0.20)	(0.22)	(0.20)	(0.22)	(0.22)	(0.88)	(0.90)	(0.92)	(0.86)	(0.92)
ln_mv				-0.001**	-0.002**	-0.002**	-0.002**	-0.002**	-0.002**	-0.002**	-0.002***	-0.002**	-0.002***	-0.002**	-0.002**	-0.002***	-0.002***	-0.002***	-0.002***	-0.002***
				(-2.27)	(-2.39)	(-2.34)	(-2.34)	(-2.37)	(-2.30)	(-2.52)	(-2.64)	(-2.54)	(-2.62)	(-2.60)	(-2.52)	(-3.10)	(-2.79)	(-3.02)	(-3.00)	(-2.95)
btm				0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.003**	0.003**	0.003**	0.003**	0.003**
				(2.08)	(2.14)	(2.06)	(2.15)	(2.13)	(2.15)	(2.15)	(2.19)	(2.12)	(2.23)	(2.18)	(2.22)	(2.06)	(1.98)	(2.09)	(2.04)	(2.09)
MOM										0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002
										(0.23)	(0.19)	(0.24)	(0.28)	(0.19)	(0.22)	(0.44)	(0.51)	(0.54)	(0.45)	(0.46)
REV										-0.017***	-0.017***	-0.017***	-0.017***	-0.017***	-0.017***	-0.017***	-0.017***	-0.017***	-0.017***	-0.017***
										(-3.46)	(-3.43)	(-3.41)	(-3.36)	(-3.43)	(-3.42)	(-3.56)	(-3.53)	(-3.49)	(-3.57)	(-3.55)
ILLIQ										0.037*	0.035*	0.038*	0.034*	0.036*	0.036*	0.047**	0.048**	0.045**	0.047**	0.046**
										(1.83)	(1.70)	(1.83)	(1.67)	(1.72)	(1.71)	(2.18)	(2.24)	(2.14)	(2.20)	(2.16)
idio_risk										-0.121**	-0.113**	-0.112**	-0.117**	-0.111**	-0.107*					
										(-2.16)	(-2.06)	(-2.01)	(-2.12)	(-2.02)	(-1.96)					
IV level																-0.012	-0.010	-0.012	-0.011	-0.012
																(-1.06)	(-0.87)	(-1.07)	(-0.94)	(-1.04)
v_{sys}^2																-0.002**	-0.002**	-0.002**	-0.002**	-0.002**
																(-2.01)	(-2.12)	(-2.03)	(-2.04)	(-2.05)
v_{isio}^2																-0.000	-0.000	0.000	-0.001	-0.000
																(-0.04)	(-0.03)	(0.05)	(-0.12)	(-0.03)
$\overline{Adj}\ R^2$	0.035	0.036	0.036	0.049	0.050	0.050	0.050	0.051	0.051	0.063	0.064	0.064	0.064	0.065	0.065	0.071	0.071	0.071	0.071	0.071

Panel B. $convexity_{\mathrm{sys}}$ and $convexity_{\mathrm{idio}}$

Variable	MODEL1	MODEL2	MODEL3	MODEL4	MODEL5	MODEL6	MODEL7	MODEL8	MODEL9	MODEL10	MODEL11	MODEL12	MODEL13	MODEL14	MODEL15	MODEL16	MODEL17	MODEL18
$\boldsymbol{convexity_{\mathrm{sys}}}$	-0.019***	-0.020***		-0.022***		-0.022***		-0.021***		-0.022***	-0.018***	-0.019***	-0.022***	-0.018***	-0.020***	-0.022***	-0.018***	-0.019***
	(-5.26)	(-6.13)		(-7.20)		(-7.23)		(-7.66)		(-7.70)	(-5.27)	(-5.51)	(-7.41)	(-5.14)	(-5.69)	(-7.92)	(-5.28)	(-5.67)
$\boldsymbol{convexity_{\mathrm{idio}}}$	-0.014***	-0.014***			-0.011***	-0.012***			-0.011***	-0.012***	-0.007**	-0.010***	-0.012***	-0.007**	-0.011***	-0.013***	-0.007**	-0.011***
	(-4.92)	(-4.90)			(-4.29)	(-4.68)			(-4.28)	(-4.68)	(-2.23)	(-2.97)	(-4.67)	(-2.33)	(-3.24)	(-4.89)	(-2.47)	(-3.18)
beta		0.002	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002	0.002
		(0.68)	(0.32)	(0.33)	(0.34)	(0.33)	(0.22)	(0.22)	(0.22)	(0.22)	(0.24)	(0.24)	(0.06)	(0.08)	(0.08)	(0.90)	(0.87)	(0.93)
ln_mv			-0.001**	-0.002**	-0.002**	-0.002**	-0.002***	-0.002***	-0.002***	-0.002***	-0.002***	-0.002***	-0.001**	-0.001**	-0.001**	-0.002***	-0.002***	-0.002***
			(-2.27)	(-2.47)	(-2.30)	(-2.47)	(-2.84)	(-3.05)	(-2.85)	(-3.04)	(-3.01)	(-2.91)	(-2.25)	(-2.22)	(-2.14)	(-3.26)	(-3.15)	(-3.10)
BTM			0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.004**	0.003**	0.003**	0.003**
			(2.08)	(2.15)	(2.06)	(2.14)	(2.23)	(2.29)	(2.20)	(2.27)	(2.26)	(2.30)	(2.32)	(2.31)	(2.35)	(2.06)	(2.04)	(2.08)
MOM							0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000	0.002	0.002	0.002
							(0.19)	(0.14)	(0.20)	(0.15)	(0.16)	(0.18)	(0.09)	(0.10)	(0.11)	(0.44)	(0.45)	(0.46)
REV							-0.017***	-0.017***	-0.018***	-0.017***	-0.017***	-0.017***	-0.017***	-0.017***	-0.017***	-0.017***	-0.017***	-0.017***
							(-3.45)	(-3.38)	(-3.48)	(-3.43)	(-3.42)	(-3.41)	(-3.36)	(-3.35)	(-3.36)	(-3.56)	(-3.56)	(-3.55)
idio_risk							-0.115**	-0.114**	-0.109*	-0.110**	-0.108*	-0.104*						
							(-2.03)	(-2.04)	(-1.94)	(-1.98)	(-1.94)	(-1.88)						
IV spread											-0.020**			-0.021***			-0.022**	
											(-2.58)			(-2.70)			(-2.56)	
IV smirk												-0.008			-0.005			-0.009
												(-1.07)			(-0.77)			(-1.15)
ILLIQ													0.026	0.028	0.027	0.043**	0.044**	0.043**
													(1.25)	(1.31)	(1.29)	(2.04)	(2.06)	(2.02)
IV level																-0.012	-0.011	-0.012
																(-1.09)	(-0.96)	(-1.07)
v_{sys}^2																-0.002**	-0.002**	-0.002**
																(-2.03)	(-2.05)	(-2.07)
v_{isio}^2																-0.000	-0.001	-0.000
																(-0.02)	(-0.09)	(-0.01)
$\overline{Adj}\ R^2$	0.003	0.036	0.049	0.050	0.050	0.051	0.061	0.063	0.063	0.063	0.064	0.064	0.062	0.062	0.062	0.071	0.072	0.072

Table 10. Alternative Measure of Options implied Convexity

This table reports the descriptive statistics of the average portfolio returns sorted by alternative measures of option implied convexity. $convexity_{cp}$ is an unbiased pure kurtosis measure without loss of information and is calculated by;

$$IV\ convexity_{cp} = \frac{[\mathrm{IV_{call}}(0.8) + \mathrm{IV_{put}}(-0.2)]}{2} + \frac{[\mathrm{IV_{call}}(0.2) + \mathrm{IV_{put}}(-0.8)]}{2} - [\mathrm{IV_{call}}(0.5) + \mathrm{IV_{put}}(-0.5)]$$

Alternative option implied convexities are calculated as below;

$$IV\ convexity_{Bali} = \mathrm{IV_{call}}(0.25) + \mathrm{IV_{put}}(-0.25) - \mathrm{IV_{call}}(0.5) - \mathrm{IV_{put}}(-0.5)$$

$$IV\ convexity_{put} = \mathrm{IV_{put}}(-0.2) + \mathrm{IV_{call}}(0.2) - 2 \times \mathrm{IV_{put}}(-0.5)$$

$$IV\ convexity_{call} = \mathrm{IV_{call}}(0.2) + \mathrm{IV_{call}}(0.8) - 2 \times \mathrm{IV_{call}}(0.5)$$

On the last trading day of each month, all firms are assigned to one of five portfolio groups based on alternative options implied convexity assuming that stocks are held for the next one-month-period. This process is repeated in every month. Monthly stock returns are obtained from the Center for Research in Security Prices (CRSP) with stocks traded on the NYSE (exchcd=1), Amex (exchcd=2), and NASDAQ (exchcd=3). We use only common shares (shrcd in 10, 11). Stocks with a price less than three dollars are excluded from the sample. "Q1-Q5" denotes an arbitrage portfolio that buys a low options implied convexity portfolio (Q1) and sells a high options implied convexity portfolio (Q5). The sample covers Jan 2000 to Dec 2013. Numbers in parentheses indicate t-statistics.

	$IV\ convexity_{cp}$				$IV\ convexity_{Bali}$				IV $convexity_{put}$				IV $convexity_{call}$			
Quintile	Avg # of firms	Mean	Stdev	Avg Return	Avg # of firms	Mean	Stdev	Avg Ret	Avg # of firms	Mean	Stdev	Avg Return	Avg # of firms	Mean	Stdev	Avg Return
Q1 (Low curvature)	1721	-0.0407	0.1191	0.019	1795	-0.0517	0.1324	0.0138	1778.79	-0.0571	0.1627	0.0177	1794.43	-0.0562	0.1588	0.0154
Q2	1718	0.0227	0.0134	0.011	1722	0.0056	0.0070	0.0155	1782.93	0.0166	0.013	0.014	1766.71	0.0139	0.0138	0.0125
Q3	1807	0.0501	0.02	0.0098	1715	0.0245	0.0096	0.0082	1794.71	0.047	0.0213	0.0095	1800	0.0434	0.0229	0.0093
Q4	1831	0.0923	0.035	0.0096	1791	0.0584	0.0229	0.0095	1854.86	0.0955	0.0413	0.0099	1863.79	0.0904	0.0416	0.0097
Q5 (High curvature)	1556	0.235	0.1566	0.0103	1456	0.2238	0.2183	0.0127	1624.5	0.2747	0.2034	0.0086	1590.93	0.2533	0.1893	0.0128
Q1-Q5				0.0087				0.0011				0.0091				0.0026
t-statistic				[5.10]				[0.86]				[6.09]				[1.76]

Table 11. Alternative Measure of Portfolio Performance: Sharpe Ratio (SR) and Generalized Sharpe Ratio (GSR)

Panel A reports the Sharpe ratio for single-sorted portfolios formed based on *IV convexity* or alternative measures of option implied convexity. *IV convexity* is estimated following our definition of *IV convexity* and alternative measures of option implied convexities are computed as in Table 10. Panel B presents the Sharp ratio of double-sorted quintile portfolios formed based on *IV slope (IV spread, IV smirk)* first and then sub-sorted into five groups based on *IV convexity*. Options volatility slopes are computed by $IV\ slope = \mathrm{IV_{put}}(-0.2) - \mathrm{IV_{put}}(-0.8)$, *IV spread*= $\mathrm{IV_{put}}(-0.5) - \mathrm{IV_{call}}(0.5)$, and *IV smirk*= $\mathrm{IV_{put}}(-0.2) - \mathrm{IV_{call}}(-0.5)$, respectively, following our definition of options volatility slope, Yan (2011) and Xing, Zhang and Zhao (2010). Sharpe ratios are estimated by standard Sharpe ratio(SR) and Generalized Sharpe ratio(GSR) suggested by Zakamouline and Koekebakker (2009). SR is defined as $\frac{\mu-r}{\sigma}$ and GSR is computed as $\sqrt{-2\log(-E[U^*(\widetilde{W})])}$, where $\mathrm{E}[U^*(\widetilde{W})] = \max_a E[-e^{-\lambda a(x-r_f}]$. Monthly stock returns are obtained from the Center for Research in Security Prices (CRSP) with stocks traded on the NYSE (exchcd=1), Amex (exchcd=2) and NASDAQ (exchcd=3). We use only common shares (shrcd in 10, 11). Stocks with a price less than three dollars are excluded from the sample. "Q1-Q5" denotes an arbitrage portfolio that buys a low *IV convexity* portfolio and sells a high *IV convexity* portfolio. The sample covers Jan 2000 to Dec 2013.

Panel A. Sharpe ratio for single-sorted portfolios: *IV convexity* (alternative convexity)

	Sharpe Ratio									
	SR					GSR				
Quintile	*IV convexity*	$convexity_{cp}$	$convexity_{Bali}$	$convexity_{put}$	$convexity_{call}$	*IV_convexity*	$convexity_{cp}$	$convexity_{Bali}$	$convexity_{put}$	$convexity_{call}$
Q1 (Low)	0.2526	0.2294	0.1715	0.2160	0.1866	0.2575	0.2325	0.1717	0.2170	0.1868
Q2	0.1815	0.1545	0.2118	0.1976	0.1723	0.1799	0.1533	0.2113	0.1973	0.1717
Q3	0.1334	0.1334	0.1088	0.1314	0.1248	0.1322	0.1321	0.1080	0.1303	0.1237
Q4	0.115	0.1220	0.1200	0.1277	0.1239	0.1139	0.1210	0.1192	0.1266	0.1228
Q5 (High)	0.0826	0.1298	0.1672	0.1025	0.1716	0.0823	0.1289	0.1659	0.1019	0.1706
Q1-Q5	0.6087	0.3950	0.0667	0.4711	0.1362	0.7598	0.4018	0.0669	0.4829	0.1362

Panel B. Sharp ratio of double-sorted quintile portfolios: *IV slope (IV spread, IV smirk)* first and then on *IV convexity*

	SR														
IV convexity Quintiles	*IV slope* Quintiles					*IV spread* Quintiles					*IV smirk* Quintiles				
	S1(Low)	S2	S3	S4	S5(High)	S1(Low)	S2	S3	S4	S5(High)	S1(Low)	S2	S3	S4	S5(High)
Q1 (Low *IV convexity*)	0.2695	0.3063	0.2363	0.2016	0.1684	0.3326	0.2129	0.1672	0.1762	0.1476	0.3252	0.1872	0.1278	0.1145	0.1453
Q2	0.2068	0.2391	0.1872	0.1616	0.1607	0.255	0.203	0.1335	0.112	0.1412	0.2502	0.1966	0.1132	0.1018	0.1097
Q3	0.1452	0.1476	0.1163	0.1161	0.1479	0.2276	0.149	0.0973	0.0815	0.1015	0.2391	0.1965	0.1156	0.1004	0.1034
Q4	0.0854	0.0903	0.1126	0.0838	0.1008	0.1935	0.1508	0.1244	0.0901	0.0569	0.2597	0.1467	0.1088	0.1084	0.062
Q5 (High *IV convexity*)	0.0291	0.0991	0.0755	0.1397	0.1108	0.2124	0.1412	0.0988	0.0906	0.0143	0.2445	0.1738	0.0916	0.0869	0.0164
Q1-Q5	0.5292	0.4877	0.3549	0.1657	0.1946	0.403	0.249	0.2022	0.2452	0.2916	0.302	0.0482	0.096	0.0722	0.2904

	GSR														
IV convexity Quintiles	*IV slope* Quintiles					*IV spread* Quintiles					*IV smirk* Quintiles				
	S1(Low)	S2	S3	S4	S5(High)	S1(Low)	S2	S3	S4	S5(High)	S1(Low)	S2	S3	S4	S5(High)
Q1 (Low *IV convexity*)	0.2804	0.3274	0.2379	0.2024	0.1699	0.3478	0.2148	0.1656	0.1752	0.1476	0.3405	0.1866	0.1277	0.1141	0.145
Q2	0.2071	0.2391	0.1834	0.1608	0.161	0.2657	0.2021	0.1319	0.111	0.1406	0.2595	0.1949	0.1123	0.1011	0.1096
Q3	0.1457	0.146	0.1153	0.1151	0.1471	0.2296	0.1475	0.0966	0.0808	0.1008	0.2428	0.193	0.1146	0.0995	0.1027
Q4	0.0849	0.0895	0.1113	0.0833	0.1008	0.1942	0.1494	0.1234	0.0894	0.0566	0.2612	0.145	0.1078	0.1072	0.0617
Q5 (High *IV convexity*)	0.0291	0.0987	0.0749	0.1388	0.1103	0.2122	0.1404	0.0979	0.0901	0.0143	0.2472	0.1735	0.0909	0.0864	0.0164
Q1-Q5	0.6034	0.627	0.4378	0.1741	0.2004	0.4331	0.2575	0.1938	0.251	0.2942	0.3079	0.0485	0.0958	0.0722	0.2869

Table 12. Different holding period returns

This table reports the average risk-adjusted monthly returns (using the four-factor model) of the quintile portfolios formed on c for holding period of two months to six months. 'Q1-Q5' denotes a long-short arbitrage portfolio that buys low convexity portfolio and sells high convexity portfolio. The t-statistics are computed using Newey-West procedure for adjusting serially correlated returns of overlapping samples.

	Q1 (Low)	Q2	Q3	Q4	Q5 (High)	Q1-Q5
One month	0.0132	0.005	0.0024	0.0014	-0.0003	0.012
						[6.80]
Two months	0.0110	0.0047	0.0024	0.0028	0.0015	0.0079
						[6.78]
Three months	0.0106	0.0048	0.0026	0.0028	0.0025	0.0064
						[6.37]
Four months	0.0101	0.0046	0.0027	0.0032	0.0033	0.0053
						[5.92]
Five months	0.0099	0.0043	0.0029	0.0035	0.0037	0.0046
						[5.87]
Six months	0.0099	0.0042	0.0030	0.0038	0.0039	0.0044
						[5.97]

Figure 1. Density of a Lèvy Process X_T under the Gram-Charlier Expansion

This figure shows the impact of skewness and excess kurtosis on the shape of its probability distribution using Gram-Charlier expansions. The base parameter set is taken as $(\mu, \sigma, skewness, excess\ kurtosis) = (0.05, 0.3, -0.5, 1.0)$ where μ is mean and σ is standard deviation.

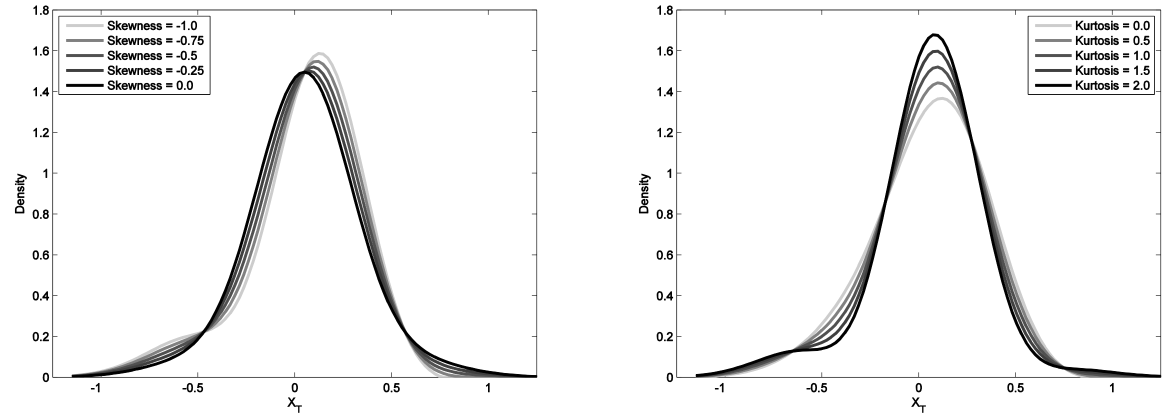


Figure 2. The Shape of Implied Volatility Curves

This figure illustrates the effect of different values of skewness and excess kurtosis on the shape of implied volatility curve. The base parameters are consistent with those of Figure 1 and $(S_0, T, r) = (100, 0.5, 0.05)$.

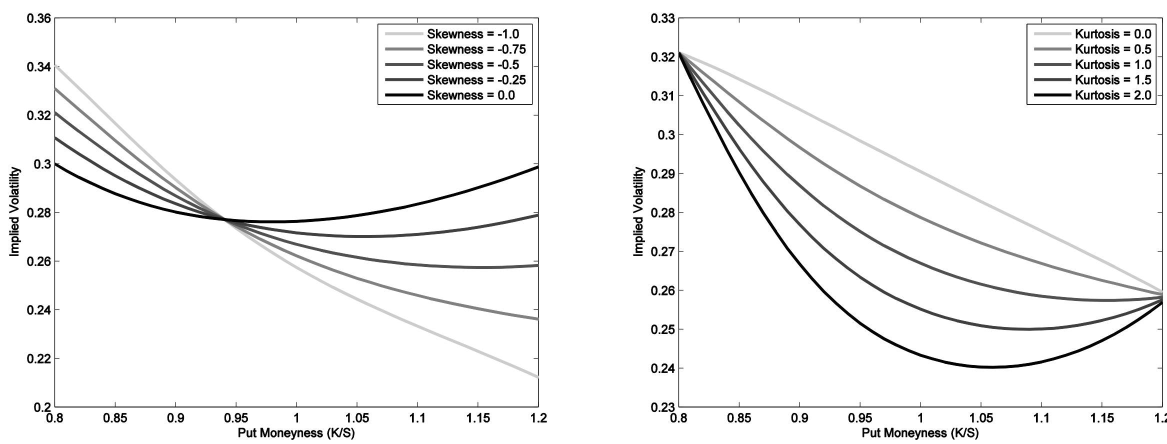


Figure 3. Higher Moments of Underlying Asset Return, *IV slope* and *IV convexity*

This figure shows the effect of skewness and excess kurtosis of underlying asset returns on *IV slope* and *IV convexity*. The base parameters are consistent with those of Figure 1 and 2. We take 0.8, 1.0 and 1.2 as the moneyness (K/S) points for *IV slope* and *IV convexity*, respectively.

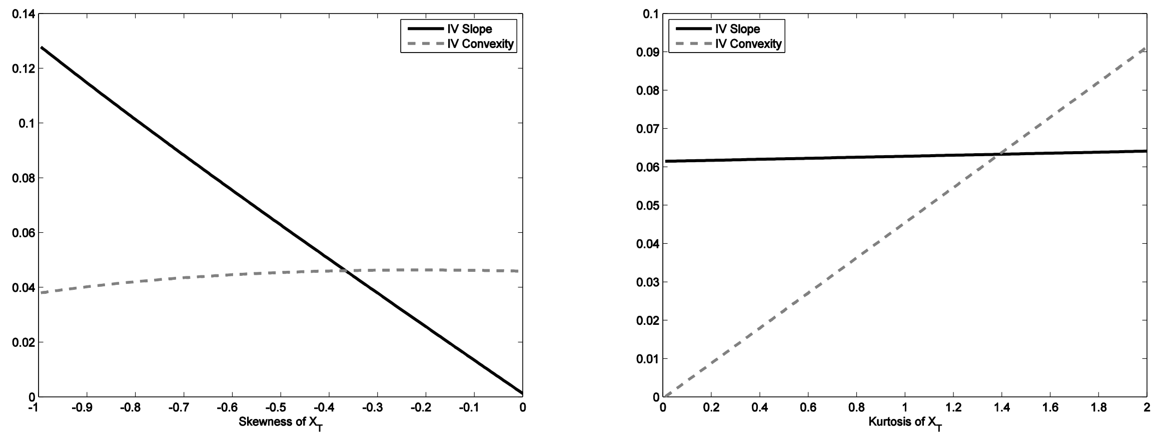


Figure 4. The Impacts of ρ and σ_v on *IV slope* and *IV convexity* under the SV Model

This figure shows the effect of ρ and σ_v on *IV slope* and *IV convexity* under the SV model given by (4)-(5). The base parameter set $(S, v_0, \kappa, \theta, \sigma_v, \rho, T, r, q) = (100, 0.01, 2.0, 0.01, 0.1, 0.0, 0.5, 0.0, 0.0)$ is taken from Heston (1993).

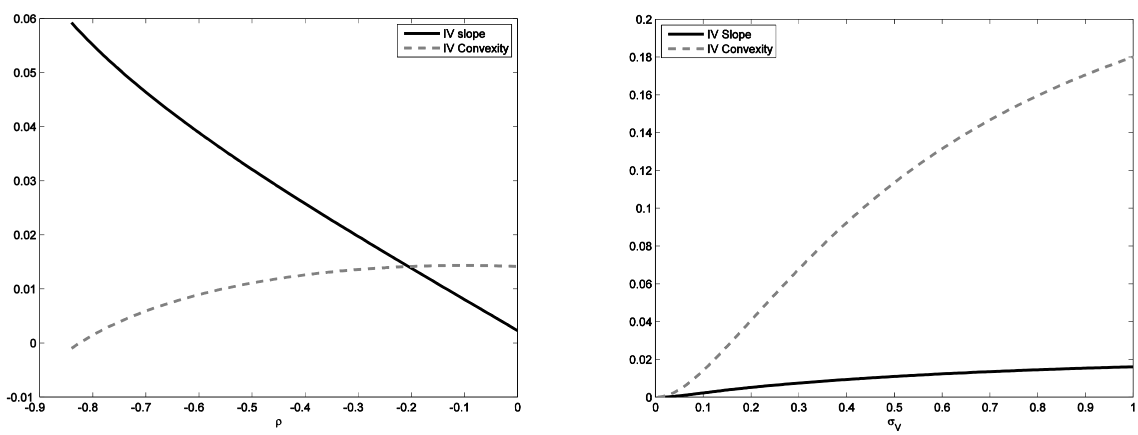


Figure 5. The Impacts of μ_J and σ_J on *IV slope* and *IV convexity* under the SVJ Model

This figure shows the effect of μ_J and σ_J on *IV slope* and *IV convexity* under SVJ model given by (6)-(7). The base parameter set $(S, v_0, \kappa, \theta, \sigma_v, \rho, \lambda, \mu_J, \sigma_J, T, r, q) = (100, 0.094^2, 3.99, 0.014, 0.27, -0.79, 0.11, -0.12, 0.15, 0.5, 0.0319, 0.0)$ is taken from Duffie, Pan and Singleton (2000).

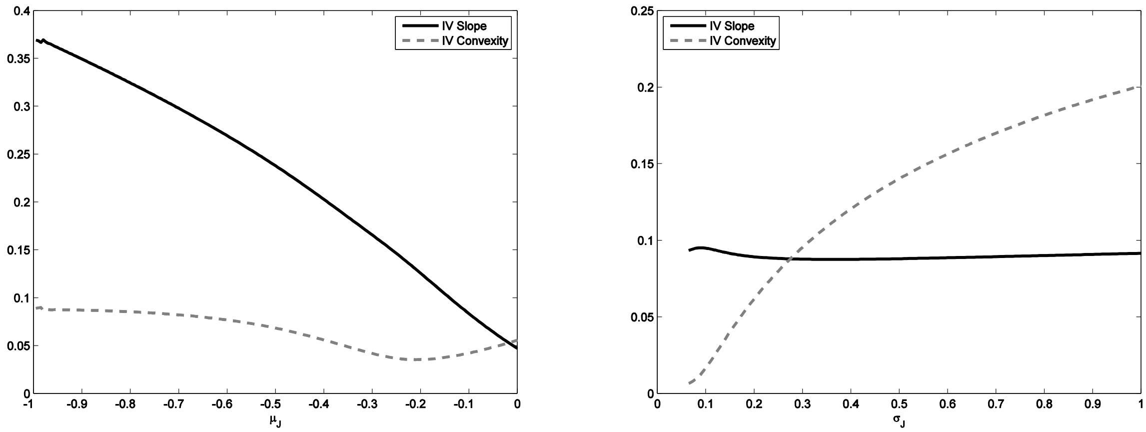
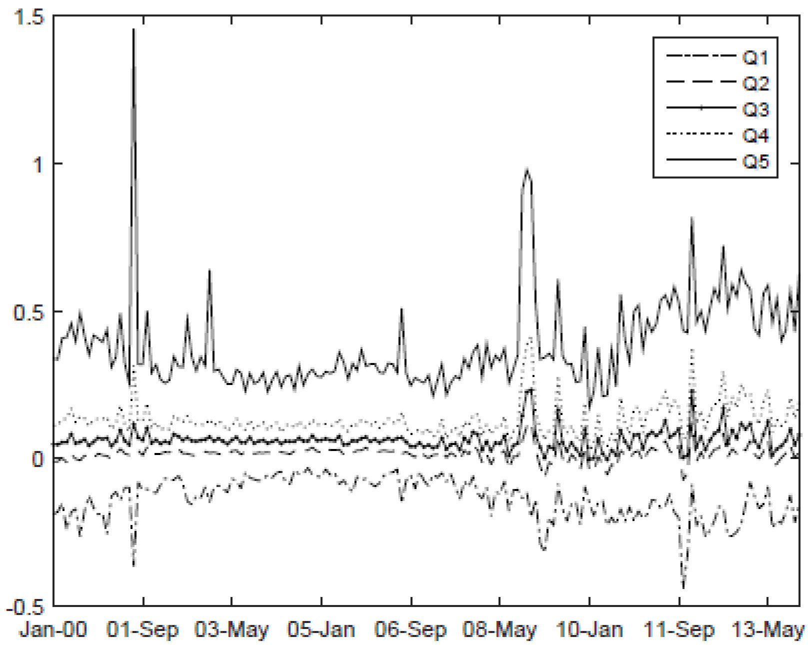


Figure 6. Average IV convexity and returns of quintile portfolios

This figure shows the time-series behavior of average IV convexity and returns of quintile porfolios from January 20000 to December 2013. Panel A plots the monthly average *IV convexity* of the quintile portfolios. Panel B shows the monthly average returns of the long-short portfolio Q1-Q5.

Panel A. Average *IV convexity* of quintile portfolios



Panel B. Returns of Q1-Q5