Is the Information on the Higher Moments of Underlying Returns Correctly Reflected in Option Prices?

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Abstract

This study examines the information implied in options with two times to maturity. In the analysis using realized moments and the estimated risk-neutral moments following Bakshi et al (2003), we could not find any evidence about the options' mispricing. However, in the analysis using the forward moments, we find that long-term option investors, on average, seem to underestimate the third moment relative to short-term option investors, and this becomes severe when the market variance is large. In addition, we show that the third moment underestimation of long-term option investors is economically meaningful using the Corrado and Su's model and the option portfolios to trade skewness.

Keywords: implied moments, forward moments, nearby options, second nearby options, mispricing

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1. Introduction

Extensive empirical studies in finance have documented that expected returns on individual stocks are not explained by the classic capital asset pricing model that expected returns on individual stocks are determined by the covariance of their returns with the market portfolio. To explain the anomalies, new models including other risk factors have been developed, and one of them is to consider higher moments in returns such as volatility, skewness and kurtosis. The models implying that investors consider higher moments in returns, especially skewness, have a long history, but researchers have been focusing on them recently in earnest. Harvey and Siddique (2000) and Mitton and Vorkink (2007) develop an asset pricing model that incorporates the effect of co-skewness of a security with the market portfolio and idiosyncratic skewness, resepectively. The possibility that skewness is priced in asset returns as a risk factor is shown in the empirical studies of Das and Sundaram (1999), Buraschi and Jiltsov (2006), Xu (2007) and Friesen et al. (2012). Das and Sundaram (1999) show the relation between option prices and higher moments of their underlying assets. Buraschi and Jiltsov (2006) show that a model taking heterogeneity into account explains the option smile related to skewnes. Xu (2007) and Friesen et al. (2012) support the importance of skewness in asset pricing by showing that heterogeneous beliefs or variables related to belief differences are strongly related to skewness. These theoretical and empirical suggestions are also confirmed by empirical studies that document the negative relationship between skewness and future stock returns. For example, Boyer et al. (2010), Conrad et al. (2013), Bali and Murray (2013), Chang et al. (2013) show that market skewness risk or total risk skewness is a systematic risk factor that is priced in stock returns. Xing et al. (2010) show that the individual option's volatility smirk related to skewness explains future stock returns. In addition, Kozhan et al. (2013) show the existence of the negative risk premium using the realized skewness, which is calculated to be an unbiased estimator of the expected skewness proposed by Neuberger (2012).

Among the studies above, Xing et al. (2010), Friesen et al. (2012), Bali and Murray (2013), Chang et al. (2013), and Conrad et al. (2013) use the implied skewness from option prices. It might be a better choice than using historical skewness in the aspect that option prices reflect investors' expectations of future return distribution over the remaining life of the options, the same as the implied volatility has information on future realized volatility. However, different from our expectation and the research results on the information content of the implied volatility, it is not concluded yet whether the implied skewness is more informative than historical skewness. Also, it is the same for the case of the term structure of implied skewness that is related to information consistency between options with different times to maturity, even though it could affect the research results of the studies mentioned above by choosing any specific option maturity in case that there is no information consistency among options.

In this study, we examine whether option prices correctly reflect the information on the underlying asset's return distribution. In particular, we investigate the question by testing whether information from the nearby and second nearby options is consistent or not. One is to examine how consistent the implied moments are with the realized return moments of the underlying asset or the risk-neutral moments that are estimated using historical moments following Bakshi et al. (2003) and Bakshi and Madan (2006). This is similar with research on the information content of implied volatility that Canina and Figlewski (1993), Christensen and Prabhala (1998), Jiang and Tian (2005) do, but it is different from these studies in that we are interested in not only the forecasting ability itself but also the difference of the forecasting ability between the nearby and second nearby options. As a result, we find that the implied second moments have forecasting ability and are unbiased estimators of future returns' second moments, the same as the result of Jiang and Tian (2005). Moreover the third moments and fourth moments are not unbiased estimators of future moments even though the implied fourth moments have forecasting ability. However, there is no clear difference in forecasting ability of the implied moments between these two different maturities, and we could not find any evidence as to whether options of the two different maturities capture information inconsistently. In addition, this result does not change even when we examine the relation using the risk-neutral moments that are estimated using the realized moments following Bakshi et al. (2003) and Bakshi and Madan (2006) that consider the departure of the moments under two different probability measures.

Another test is to examine whether the forward information implied in the term structure of higher moments is consistent with the future spot moments. Among the studies about the information implied in option prices with different maturities, research on the term structure of the implied volatility is representative. Stein (1989) examines the term structure of implied volatilities under the assumption of the mean-reverting property of volatility and reports that the slope coefficient between implied volatilities of different times to maturity is statistically higher than the predicted value. He attributes this empirical result to the overreaction of long-term options to volatility news. Heynen et al (1994) show that the overreaction can be observed due to incorrect model specifications using tests under three different volatility models, mean-reverting, GARCH and EGARCH models. Poteshman (2001) examines the volatility relation between short-term and long-term options using a stochastic volatility model and finds mixed patterns of long horizon overreaction and short horizon underreaction. Mixon (2007) tests the expectations hypothesis of the term structure of implied volatility for stock market indexes of several countries and finds that the slope of ATM implied volatility over different times to maturity has predictive ability for future short-term implied volatility. On the other hand, Jiang and Tian (2010) use a model-free implied volatility and show that the overreaction previously documented in the literature can be the result of model misspecification or misestimation.

Extending the empirical studies about the term structure of implied volatility, we examine whether the forward implied moments are an unbiased forecast of the future spot implied moments, following Jiang and Tian (2010). We find that the forward second and fourth moments implied by the nearby and second nearby options are consistent with the future spot moments, but the forward third moment is not consistent with and lower than the future spot moment on average. Especially, in the regressions of the forecasting errors that are defined as the future spot moments minus the forward moments on the variance, skewness and kurtosis of the nearby options, the variance has predictive power for the forecasting error of the third moment and other variables have no power for all of forecasting errors. Therefore, we conclude that long-term option investors on average put too much weight on the downfall risk and this becomes severe when the market variance is large.

In addition, we examine whether the underestimated third moment of long-term option investors has economic significance using Corrado and Su's (1996) option pricing model and portfolio construction that trade skewness following Bali and Murray (2013). According to Corrado and Su's model that accommodates the skewness and kurtosis factors in the Black-Scholes framework, the mispricing or the difference of prices between the case with and without the bias caused by the underestimated third moment is large and economically significant, especially out-of-the-money options. The third moment effect results in the overpricing of long-term puts and underpricing of long-term calls. In detail, the maximum median price impacts on put options and call options are up to 1.919 and -2.546, respectively.

The empirical results using long-short portfolio that buys relatively underestimated skewness and sells relatively overestimated skewness following Bali and Murray partially confirm our expectation. The portfolio return composed of call options is positive, up to 0.56% per month, and significantly different from zero even after controlling systematic risks. This result confirms that the underestimation of the third moment of second nearby options is economically meaningful, consistent with the analysis using Corrado and Su's model. The portfolio return composed of put options is also positive, 0.43% per month, but it is not significantly different from zero after controlling the systematic risks.

The remainder of this paper is organized as follows. In Section 2, we describe the data and present the key measures from the option price data. In Section 3, we illustrate how to measure forward moments using spot moments data. In Section 4, we provide the main empirical results of the study, and in Section 5, we examine the economic significance of the empirical results. In Section 6, we present the conclusions.

2. Data and Risk-neutral measures

2.1 Data description

For the empirical analysis, we use the data on the S&P 500 index options traded on the CBOE from January 4, 1996 to August 30, 2013. The option data are obtained from the OptionMetrics database and include the closing bid and ask quotes for each option contract along with the corresponding strike price and maturity. We use the option data of nearby and second nearby options. The nearby call (put) option is defined as that whose remaining time to maturity is closest to 32 days among all available call (put) options on a monthly basis. In addition, we applied the following filters for the sample data. First, option quotes are included only when both bid and ask quotes are available. Option data with any missing bid or ask quotes are excluded. Second, option quotes whose bid-ask spread is larger than 0.5[3] are excluded, in order to ensure that illiquid options are excluded. Third, option quotes violating no arbitrage bounds with bid and ask quotes are excluded. Specifically, we require that both the bid and ask prices of a call (put) option with a higher (lower) exercise price should be

[3] The choice of the threshold for the bid-ask spread does not qualitatively change the results reported in this paper.

higher than those of a call (put) option with a lower (higher) exercise price, in order to ensure that any non-informative options due to minimum tick size or illiquidity are excluded. The three-month CD rate as the risk free rate and S&P 500 index are obtained from the Center for Research in Security Prices (CRSP).

2.2 Estimation of spot moments from option prices

The main variables used in this study are the implied first, second, third, and fourth moments and normalized moments for the future return distribution of the underlying asset. These measures are calculated from option prices as follows.

$$\mathrm{V(t,T)} = \int_{\mathrm{S(t)}}^{\infty} \frac{2\left(1-\ln\left[\frac{\mathrm{K}}{\mathrm{S(t)}}\right]\right)}{\mathrm{K}^2} \mathrm{C(t,T;K)dK} + \int_{0}^{\mathrm{S(t)}} \frac{2\left(1+\ln\left[\frac{\mathrm{S(t)}}{\mathrm{K}}\right]\right)}{\mathrm{K}^2} \mathrm{P(t,T;K)dK} \quad (1)$$

$$\mathrm{W(t,T)} = \int_{\mathrm{S(t)}}^{\infty} \frac{6\ln\left[\frac{\mathrm{K}}{\mathrm{S(t)}}\right] - 3\left(\ln\left[\frac{\mathrm{K}}{\mathrm{S(t)}}\right]\right)^2}{\mathrm{K}^2} \mathrm{C(t,T;K)dK} - \int_{0}^{\mathrm{S(t)}} \frac{6\ln\left[\frac{\mathrm{S(t)}}{\mathrm{K}}\right] + 3\left(\ln\left[\frac{\mathrm{S(t)}}{\mathrm{K}}\right]\right)^2}{\mathrm{K}^2} \mathrm{P(t,T;K)dK} \quad (2)$$

$$\mathrm{X(t,T)} = \int_{\mathrm{S(t)}}^{\infty} \frac{12\left(\ln\left[\frac{\mathrm{K}}{\mathrm{S(t)}}\right]\right)^2 - 4\left(\ln\left[\frac{\mathrm{K}}{\mathrm{S(t)}}\right]\right)^3}{\mathrm{K}^2} \mathrm{C(t,T;K)dK} + \int_{0}^{\mathrm{S(t)}} \frac{12\left(\ln\left[\frac{\mathrm{S(t)}}{\mathrm{K}}\right]\right)^2 + 4\left(\ln\left[\frac{\mathrm{S(t)}}{\mathrm{K}}\right]\right)^3}{\mathrm{K}^2} \mathrm{P(t,T;K)dK} \quad (3)$$

$$\mu\mathrm{(t,T)} = \mathrm{e}^{\mathrm{r(T-t)}} - 1 - \frac{\mathrm{e}^{\mathrm{r(T-t)}}}{2}\mathrm{V(t,T)} - \frac{\mathrm{e}^{\mathrm{r(T-t)}}}{6}\mathrm{W(t,T)} - \frac{\mathrm{e}^{\mathrm{r(T-t)}}}{24}\mathrm{X(t,T)} \quad (4)$$

Bakshi et al. (2003) demonstrate that the second, third, and fourth moments from t to maturity T can be calculated as the discounted values of V, W, and X, respectively, with the assumption of continuum strikes and their corresponding option prices with maturity T and the first moment is calculated using the higher moments.

Using the moments above, their normalized measures, variance, skewness, and kurtosis, can be calculated as follows.

$$VAR_t(t,T) = e^{r(T-t)}V(t,T) - \mu(t,T)^2 \quad (5)$$

$$SKEW_t(t,T) = \frac{e^{r(T-t)}W(t,T) - 3\mu(t,T)e^{r(T-t)}V(t,T) + 2\mu(t,T)^3}{VAR_t(t,T)^{\frac{3}{2}}} \quad (6)$$

$$KURT_t(t,T) = \frac{e^{r(T-t)}X(t,T) - 4\mu(t,T)e^{r(T-t)}W(t,T) + 6e^{r(T-t)}\mu(t,T)^2V(t,T) - 3\mu(t,T)^4}{VAR_t(t,T)^2} \quad (7)$$

Here, $T - t$ is the remaining time to maturity in years, and $s(t)$ is the implied stock price minus the present value of future dividends calculated from the put-call parity relation of ATM options. We calculate the moments of the two nearest maturity options, which are the nearby options and the second nearby options, using the numerical integration method described by Bakshi et al. (2003). The calculation is as follows: ATM options are chosen as the call and put options whose strike price is the closest to the S&P 500 index price. For numerical integration, the strike price interval is set as one and the domain of integration is from 50% to 150% of $s(t)$. Dennis and Mayhew (2002) show in their simulation that the skewness bias caused by the numerical integration method is less than 0.01 when the strike price interval is one and the skewness bias is pretty close to zero if the half width of the integration domain is set to be larger than 10. In order to check the robustness, we experiment with different strike price intervals and domain ranges, but different choices of the integration interval and the integration domain do not qualitatively change our results in this study. When option prices for the numerical integration are not observable, we estimate those option prices by the interpolation using the Black-Scholes implied volatilities of available options with adjacent strike prices. Specifically, when the strike price of the option to be estimated is between the maximum and minimum strike prices of available options, we choose two options with adjacent strike prices between which the strike price of the option to be estimated lies. Then, we linearly interpolate those two implied volatilities and then use the interpolated implied volatilities to obtain the option price of concern using the Black-Scholes formula. When the strike price of the option for the integration is larger (smaller) than the maximum (minimum) strike price available, the implied volatility of the option is assumed to be equal to the implied volatility of the maximum (minimum) strike price available and the option price is calculated using Black-Scholes formula. In this process, we do not assume that the Black-Scholes option pricing model holds. We use the Black-Scholes formula only for the purpose of interpolation.

Figure 1 shows the estimated variance, skewness, and kurtosis from the nearby and second nearby options. In this figure, we can see that the skewness is time-varying, as Dennis and Mayhew (2002) show, and so is the kurtosis. Also, we can see that these measures from the nearby options move together with those of the second nearby options. Table 1 shows the descriptive statistics of these measures. We can see in this table that the skewness and kurtosis are strongly negatively related with each other, while the variance and other measures are not strongly related. In addition, we can see that all of the moments are highly autocorrelated.

[Figure 1 here]

[Table 1 here]

3. Forward moments

The prices of the nearby and second nearby options contain not only the future return distribution during the options' life but also the information about the forward return distribution between two different maturities. In this study, we examine the information consistency of options with different times to maturity using the information about the forward return distribution following Jiang and Tian (2010). For this examination, we have to decide what measure we will use. First of all, we can think of forward skewness, and kurtosis because skewness and kurtosis are commonly used in the literature. However, forward skewness and kurtosis are not appropriate for this purpose because these two measures are not unbiased estimators of their future realized skewness and kurtosis. Thus, we use the forward second, third and fourth moments instead of the forward variance, skewness, and kurtosis.

From the definition of the second, third and fourth moments, we can see that these forward moments depend on not only the spot moments from the nearby and second nearby options, but also the expected cross-product of the adjacent returns to the power of 1 to 3. The spot moments from the nearby and second nearby options are driven from options, but the forward moments are not driven from options without any assumption about the dynamics of returns. Here, we estimate the forward moments under two assumptions. The first assumption is that cross-covariance of the adjacent returns to the power of 1 to 3 is constant during the sample period. The second assumption is that the first, second and third moments from the nearby options have a linear relationship with realized moments under the risk-neutral probability measure. This assumption does not need the implied moments to be the best predictors of the realized moments, and it can be expressed as

$$\mathrm{R}(\mathrm{T}_1, \mathrm{T}_2)^{\mathrm{n}} = \alpha + E_t(\mathrm{R}(\mathrm{T}_1, \mathrm{T}_2)^{\mathrm{n}}) + \varepsilon_{\mathrm{t}} \quad for \ \ n = 1, 2, 3 \tag{8}$$

where $\mathrm{R}(\mathrm{T}_1, \mathrm{T}_2)$ is a log return between option times to maturity, T_1 and T_2 under risk-neutral probability.

Based on these assumptions, we can calculate the cross-covariance of two adjacent returns to the power 1 to 3 as the sample cross-covariance between the implied moments of each month and the implied moments of the next month from the nearby options. For example, covariance between the one month return and the squared one month forward return is estimated as the sample covariance between the first spot moment and second spot moment one month later. Cross-covariance is reported in Table 2.

[Table 2 here]

Finally, we can get recursively the first four time-t forward moments of the return from time T_1 to time T_2, $E_t[R(T_1,T_2)]$, $E_t[R(T_1,T_2)^2]$, $E_t[R(T_1,T_2)^3]$, and $E_t[R(T_1,T_2)^4]$ using the first four spot moments obtained from two options with maturities of T_1 and T_2, the constant cross-covariance reported in Table 2, and the forward moments of lower orders.

Table 3 shows the descriptive statistics of the spot moments estimated from the nearby options and second nearby options and the forward moments for the period between the maturities of the nearby options and second nearby options. In panel A and B, all the second and fourth moments estimated are statistically significantly positive on average, as they should be, and all the third moments estimated are on average negative and statistically different from zero. All the spot moments are strongly related with each other: The third moment is strongly negatively correlated with the second and fourth moments, while the second moment is strongly positively correlated with the fourth moments. All of the cross-sectional correlations reported in Table 3 are greater than 0.92 in absolute terms. The correlations among the moments reported in this table are much larger than those among the variance, skewness, and kurtosis reported in Table 1. Panel C in the Table 3 shows the descriptive statistics of the forward moments. The mean values of the forward second and fourth moments are positive and the mean values of the forward third moments are negative, the same as those of the spot moments from nearby options. However, the mean value of the forward third moments is statistically different from the mean value of the third moments from nearby options. The serial correlations of the forward moments are almost the same as those of the spot moments. Though not reported, the forward moments are also strongly correlated with the spot moments of the nearby options.

[Table 3 here]

4. Empirical results

We examine the information implied in the options with two times to maturity by two different test methods. One is to examine the relation between the implied moments and the realized moments in the physical probability measure or the risk-neutral moments that are estimated using the realized moments. Another one is to examine the relation between the forward moments and the future implied spot moments. Each examination is described in subsection 4.1 and 4.2.

4.1 The relation between the realized moments and the implied moments

The easiest way to investigate the information implied in options is to look into whether the implied moments contain information on the future return distribution because the returns of the underlying asset under the risk-neutral probability measure are not observable. The research on the information content of the implied volatility is representative. Canina and Figlewski (1993), Christensen and Prabhala (1998), and Jiang and Tian (2005) examine the information content and information efficiency of the implied volatility for realized volatility under the physical probability measure. As a result, in spite of the departure between the two different probability measures, Christensen and Prabhala (1998), and Jiang and Tian (2005) show that the implied volatility is an unbiased estimator of future realized volatility regardless of option times to maturity.

Extending the study above to higher moments, we examine the relation between the implied higher moments and realized moments of the nearby and second nearby options respectively, and look into whether there is any significant difference between the results of the nearby and second nearby options. For example, if the implied moments from short-term maturity options are unbiased estimators of realized oments while the moments from long-term options are not, it shows that long-term options could be mispriced.

The examination is as follows.

$$\mathrm{RM}^{\mathrm{n}}_{t,T} = \alpha_T + \beta_{n,T} IM^{n}_{t,T} + e_{t,T} \quad for\ T = T_1, T_2 \tag{9}$$

where $\mathrm{RM}^{n}_{t,T}$ is the realized n-th moment from t to T, $\mathrm{IM}^{n}_{t,T}$ is the n-th implied moments from t to T, and T_1 and T_2 are times to maturity of the nearby and second nearby options respectively. The implied moments are calculated using the method described in the previous section. However, in the regression above, the implied moments are the fitted implied moments by the regression of the implied moments on its lagged implied moments and lagged realized moments to correct for Error-in-Variable problem in the implied moments, following the suggestion of Christensen and Prabhala (1998). The n-th realized moments are calculated as follows:

$$Realized\ n^{th}\ moment_t = \sum_{i=1}^{s} (R_i)^n \ \ for\ n = 2, 3, 4$$

where R_i is the daily log return at time i, and s is the number of business days from t to the maturity of options.

[Table 4 here]

Table 4 shows the second stage OLS regression results of future realized moments on the fitted implied moments. First, let us look at the case of the second moment regression. The slope coefficient for the second moment regression is 0.9454, which is not significantly different from one, and the intercept coefficient (= -0.05%) is not statistically significantly different from zero at the 5% significance level, though the joint test for an intercept = 0 and a slope = 1 is rejected at the 5% significance level (p-value = 0.038). The regression result of the second nearby options is also similar to the regression result of the nearby options. The slope coefficient and the intercept coefficient for the regression are 0.9029 and -0.0013, and the coefficients are insignificantly different from one and zero at the 5% significance level, respectively. Based on these regression results that are consistent with those of Christensen and Prabhala (1998) and Jiang and Tian (2005), we could not find indirect evidence that options of two different times to maturity reflect information inconsistently, that is, any one of options are relatively mispriced in terms of the implied second moment.

In sequence, we look at the case of the third and fourth moments. First of all, the slope coefficients for the third moments of the nearby and second nearby options are 0.0028 and 0.0062 respectively, which are insignificantly different from zero at the 5% significance level. Similar to the case of the third moment, the slope coefficients for the fourth moment of the nearby and second nearby options are 0.0192 and 0.0119 respectively, and those are far from one, even though these values are significantly different from zero. The intercept coefficients for the third and fourth moments are pretty close to zero and insignificantly different from zero. And R^2s for all cases become smaller than those of the second moment regressions. These results are consistent with the result of

Conrad et al. (2013) that show a weak relationship between risk-neutral skewness and physical skewness. At the same time, the joint hypothesis of the intercept of zero and slope of one is rejected with a p-values close to zero for all cases of the third and fourth moments, regardless of time to maturity of options.

As a result, the information from options regarding the future realized third and fourth moments seem to be less accurate than for options containing information about the realized second moment, even though all of the implied moments are rejected as unbiased estimators. However, because the regression results are not changed between two times to maturity of options, there is no clear evidence that any one between the nearby and second nearby options is relatively mispriced.

Even though the results above show no significant difference between options with two times to maturity, it is still exposed to the issue of comparison under two different probability measures and the results may be uncomfortable to accept. Therefore, we retest the relation above using the risk-neutral moments that are estimated from the realized moments following Bakshi et al. (2003) and Bakshi and Madan (2006). In particular, Bakshi et al. (2003) and Bakshi and Madan (2006) show under the assumption of the power utility function that the implied variance, skewness and kurtosis from option prices are different from those under the physical probability measure, and the moments under two different probability measures have specific relations as follows.

$$\frac{VAR_{RN,t}-VAR_{P,t}}{VAR_{P,t}} \approx \gamma \times VAR_{p,t}^{0.5} \times SKEW_{p,t} - \frac{\gamma^2}{2}\mathrm{VAR}_{p,t} \times (KURT_{p,t} - 3) \tag{10}$$

$$\mathrm{SKEW}_{RN,t} \approx \mathrm{SKEW}_{p,t} - \gamma(\mathrm{KURT}_{p,t} - 3) \times \mathrm{VAR}_{p,t} \tag{11}$$

$$\mathrm{KURT}_{RN,t} \approx \mathrm{KURT}_{p,t} - \gamma[\big(2(\mathrm{KURT}_{p,t} + 2\big)\mathrm{SKEW}_{p,t} + \mathrm{PKEW}_{p,t}] \times \mathrm{VAR}_{p,t} \tag{12}$$

where the subscript RN and P denote the risk-neutral probability measure and physical probability measure, respectively. VAR, SKEW, KURT and PKEW mean the normalized second to fifth moments, respectively, and γ is risk aversion. First of all, we estimate the risk aversion parameter as the value to minimize the sum of the squared errors between the risk-neutral variance and the estimated risk-neutral variance in equation (10) from the nearby options, and it is 2.87, which is pretty close to the estimate in Bakshi et al. (2003). And then, using the estimated risk aversion and realized moments under the physical probability measure, we estimate the risk-neutral skewness and kurtosis. Using the estimated risk-neutral moments, we examine the relation between the estimated risk-neutral moments and the implied risk-neutral moments from option prices in equation (9).

[Table 5 here]

Table 5 presents the statistics of the estimated risk-neutral skewness and kurtosis in panel (A) and the regression results in panel (B). In panel (A), the estimated risk-neutral skewness is smaller than the implied skewness from option prices on average, and the difference is significantly different from zero at the 5% significance level for both the nearby and second nearby options. Similar to this, the estimated risk-neutral kurtosis is also smaller than the implied kurtosis on average, but the difference is not statistically significant at the 5% significance level for both the nearby and second nearby options. Panel (B) of Table 5 shows the regression results of the estimated risk-neutral moments on the implied moments. In regressions, the estimated skewness has -1.70 and 1.60 as constant terms and 0.05 and 0.02 as slope coefficients for nearby and second nearby options respectively. The estimated kurtosis has 11.22 and 9.22 as constants term and -0.32 and -0.27 as slope coefficients for nearby and second nearby options respectively. All of the constant terms in the regressions are significantly different from zero, which is inconsistent with the results in Table 4. The slope coefficients for skewness regressions are significantly different from one and insignificantly different from zero. The slope coefficients for kurtosis regressions are not only significantly different from one but also different from zero marginally. However, the regressions of both skewness and kurtosis do not show any significant difference between the two maturities, we do not find any evidence as to whether any one between the nearby and second nearby options could be mispriced, the same as in the previous examination.

4.2 The relation between the future spot implied moments and forward moments

The law of iterated expectations guarantees the following relation:

$$F^i(t;T_1,T_2) = E_t\{E_{T_1}[R(T_1,T_2)^i]\} = E_t\{S^i(T_1;T_1,T_2)\} \tag{13}$$

where $F^i(t;T_1,T_2)$ denotes the i-th forward moment at time t for the period T_1 to T_2, $S^i(T_1;T_1,T_2)$ is the i-th spot moment at time T_1 for the period T_1 to T_2. Equation (13) can be represented as

$$E_t\{\Delta M^i(T_1,T_2)\} \equiv E_t\{S^i(T_1;T_1,T_2) - F^i(t;T_1,T_2)\} = 0 \tag{14}$$

where $\Delta M^i(T_1,T_2) \equiv S^i(T_1;T_1,T_2) - F^i(t;T_1,T_2)$ is the error of using the i-th forward moment to forecast the corresponding future spot moment. These two equations imply that the forward moments derived from option prices are the best forecasts of the future spot moments given the information at time t, if the options market is efficient and all the available information is reflected in options prices.

Jiang and Tian (2010) investigate the market overreaction whether long-term implied volatility overreacts to changes in short-term implied volatility that is reported by Stein (1989) and Poteshman (2001) following equation (14) and show that there is no evidence of market misreaction. We also examine whether the forward implied moments are the best forecasts of future spot moments as follows.

$$\Delta M^i(T_1,T_2) = \alpha^i + \beta^i \times X_t(\mathrm{t},T_1) + \varepsilon(T_1) \quad for\ i = 2,3,4. \tag{15}$$

If all the available information is reflected in option prices and option prices are consistent with each other, $F^i(t;T_1,T_2)$ should be the best predictor of the future i-th moments derived from the nearby option prices at time T_1, which implies that (1) the average forecasting error should be zero and (2) α^i and β^i in equation (15) should be zero.

[Figure 2 here]

[Table 6 here]

Figure 2 shows the time-series of forecasting errors, $\Delta M^i(T_1,T_2)$. In Figure 2, there is a spike for each forecasting error around August 2008, the global financial crisis. However the spike does not affect qualitatively the result of our analysis. Table 6 shows the statistics of the forecasting errors. The average forecasting errors of the second, third, and fourth moments are -0.00035, 0.00038, and −0.000098, and their t-statistics are -1.24, 3.57, and −1.37, respectively. Thus, at the 5% significance level, we can reject the hypothesis that the average forecasting error is zero for the third moments, but we cannot reject the hypothesis for the second and fourth moments. These results show that the information reflected in the nearby options and second nearby options regarding the third moment may not contain some information available at time t and/or may not be consistent with each other. Since the average forecasting error of the third moment is significantly positive, investors in the second nearby options, on average, seem to underestimate the third moment.

In addition, we examine whether the forecasting errors of the implied moments are related to other variables in equation (15). In the regressions, we use the variance, skewness, and kurtosis calculated from the nearby options for $X_t(t, T_1)$ instead of the implied moments themselves to alleviate the multi-collinearity problem, since the variance, skewness and kurtosis are correlated less than the non-central moments. The regression results are presented in Table 7.

[Table 7 here]

Panel A of Table 7 reports the results for the forward second moment case, which is basically what Jiang and Tian (2010) examine in their study. In this case, the hypothesis that the intercept and slope coefficients should be zero cannot be rejected at any reasonable significance level. All the t-statistics are less than one in absolute values. This result is consistent with that of Jiang and Tian (2010). As a result, we do not observe the relative mispricing between the nearby and second nearby options as well as the overreaction phenomenon reported by Stein (1989) and Poteshman (2001). Panes C of Table 7 reports the results for the forward fourth moment cases. The results tell the same story as the results for the second moment. The intercept and slope coefficients of variance, skewness and kurtosis in the regressions are not statistically significant at the 5% significance level. Thus, the forward fourth moments seems to be priced consistently into option prices of the nearby and second nearby options, and also we do not observed the relative mispricing. However, the results for the third moment reported in panel B tell a different story. The nonzero average forecast error of the third moment in Table 6 is explained by variance among three explanatory variables. The slope coefficient of the variance is 0.1082, and it is significantly different from zero, and the intercept coefficient is insignificantly different from zero at the 5% significance level in the univariate regression on the variance. In other univariate regressions, skewness and kurtosis has no explanatory power. These results do not change in multivariate regressions.

In sum, the second and fourth moments embedded in the prices of long-term options seem to be estimated consistently with those embedded in the prices of short-term options. However, the third moment embedded in the price of long-term options seems to be underestimated, and the underestimation of the third moment becomes more severe as the variance of returns becomes larger. That is, long-term option investors put too much weight on downside and tail risks, and the tendency to fear the downside risk becomes larger as the volatility of returns becomes larger.

5. Economic significance of the relative bias in the long-term options

5.1 Price impact on options based on Corrado and Su's model

In the previous section, we document that the forward value of the third moment embedded in the price of the second nearby options is underestimated and it becomes serious when the market become more volatile. However, the results do not show easily the economic impact of the negative bias of the third moment on the long term options, in other words, how much the second nearby options are relatively mispriced. In this section, we evaluate the economic impact.

To evaluate the effect of the bias of the implied moment, we need to calculate the option prices without the bias to compare the option prices with the bias. However, many of the well-known models are not adequate for our task. For example, the Black-Scholes model assumes a log-normal distribution of the underlying asset price and thus, does not accommodate the third and fourth moment information that we want to reflect in option prices. On the other hand, more sophisticated models like those by Heston (1993) and Bates (1997) can accommodate the third and fourth moment information in principle but they are not easy or convenient to use for our purpose and require us to make some assumptions on the underlying return distribution. Therefore, we decide to adopt Corrado and Su's (1996) model.

Corrado and Su (1996) suggest an option pricing model to accommodate the skewness and kurtosis of the underlying asset return, extending the Black-Scholes option pricing model to capture the volatility smile without any other assumptions. They approximate a non-normal probability density function by a Gram-Charlier series expansion[4] and provide the following approximate formula for option pricing:

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[4] A Gram-Charlier series expansion of the density function f(x) is defined as

$$\text{Call price} = Call\ price_{BS} + skewness * Q_3 + (kurtosis - 3) * Q_4 \qquad (16)$$

where

$$Call\ price_{BS} = S_0 N(d) - Ke^{-rT} N\left(d - \sigma\sqrt{T}\right)$$

$$Q_3 = \frac{1}{3!} S_0 \sigma\sqrt{t}\left(\left(2\sigma\sqrt{T} - d\right)n(d) - \sigma^2 T N(d)\right)$$

$$Q_4 = \frac{1}{4!} S_0 \sigma\sqrt{t}\left(\left(d^2 - 1 - 3\sigma\sqrt{T}(d - \sigma\sqrt{T})\right)n(d) + \sigma^3 T^{\frac{3}{2}} N(d)\right)$$

$$d = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

Using the equation above, we can evaluate how much 'mispricing' may occur by the bias of the third moment, where mispricing means the price difference between the cases with and without the bias. The option prices without the bias are defined as option prices that are estimated using adjusted skewness and kurtosis using the adjusted forward third moment. The forward third moment without the bias is the adjusted forward third moment as much as the average forecasting error of the third moment, 0.0003808, that is reported in Table 6 or as much as -0.00004+0.10820 x variance of the nearby options that is shown in Table 7. The adjusted skewness for the second nearby options with time-varying bias of the forward third moment is bigger than the implied skewness of the second nearby options, and the average difference is 0.507.

The price impact on options is reported in Table 8.

[Table 8 here]

In general, the underestimated third moment implies that put options are overestimated while call options are

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$$f(x) = \sum_{n=0}^{\infty} c_n H_n(x)\varphi(x)$$

where $\varphi(x)$ is a normal density function, $H_n(x)$ are Hermite polynomials derived from successively higher derivatives of $\varphi(x)$, and the coefficients c_n are determined by moments of the distribution function F(x). If we sum the infinite series with infinite moment values, we can generate any distribution. However, we have only fourth moments values and the density function f(x) are truncated to exclude terms beyond the fourth moment.

underestimated when other things are equal because investors put too much weight on the downfall risk. The estimated price impact on put options (panel A) and call options (panel B) in the Table 8 is consistent with our expectation. In detail, put options with the bias are overpriced as much as from 0.731 to 1.919 across the moneyness, and it is up to 3% to 21.5% of the median value of the options prices that we calculate using the interpolated Black-Scholes volatilities that are observed in the market. Contrary to this, call options with the bias are underpriced as much as from 0.198 to 2.546 across the moneyness, and it is up to 0.6% to 42.5% in our sample. Not surprisingly, the price impact on the put and call options becomes stronger as options become more out-of-the-money. As a result, based on Corrado and Su's model, the underestimated third moment of the second nearby options seems to have economically meaningful price impact on options, especially out-of-the-money.

5.2 Abnormal Returns from Trading Skewness

Since Corrado and Su's model used in the previous section is an approximation model using the Gram-Charlier series, the estimated effect could be far from the real impact of the bias of the third moment due to the model specification issue or approximation error. This concern is underpinned considering the fact that the option price estimated based on Corrado and Su's model tends to be negative depending on the values of skewness and kurtosis, especially for deep out-of-the money options, even though the option price is restricted to be non-negative.

In this section, we reexamine the economic significance of the bias of the third moment by analyzing the returns of portfolios suggested by Bali and Murray (2013). The portfolios suggested by Bali and Murray are exposed to skewness, then, the portfolios have a positive (negative) return if the skewness of the portfolio increases (decreases). The portfolios called skewness assets are composed of options to avoid an exposure to vega risk, and its underlying asset to hedge delta risk. For the investigation of the economic significance, we use two skewness assets[5] that are composed as follows.

[5] Bali and Murray also suggest the CallPut skewness asset that is composed of call and put options to trade skewness. In this paper, we do not use the CallPut skewness asset as the portfolio has negative investment money, which makes difficult to interpret the value change in terms of return.

i) The call skewness assets is a portfolio that is composed of call options and the underlying stock: buying an OTM call option and selling ATM call options as much as $-vega_{(call\ OTM)}/vega_{(call\ ATM)}$ for vega hedge, and buying the underlying stock as much as $-(delta_{(call\ OTM)}$ + call ATM options position $\times$ $delta_{(call\ ATM)})$ for delta hedge.

ii) The put skewness asset is a portfolio that is composed of put options and the underlying stock: selling an OTM put option and buying ATM put options as much as $vega_{(put\ OTM)}/vega_{(put\ ATM)}$ to hedge vega risk, and buying the underlying stock as much as $-(delta_{(put\ OTM)}$+put ATM option position $\times$ $delta_{(put\ ATM)})$ to hedge delta risk.

Some more details to construct these skewness assets are as follow: We choose the ATM option as an option whose delta is the closest to 0.5 in absolute terms, and the OTM option is chosen as an option having delta closest to 0.1 (or 0.2) in absolute terms. The skewness assets are created on the following business day of the monthly option expirations, and liquidated at the expiration date of the next month. The delta, vega for hedging are calculated using the Black-Scholes option pricing model. The dividends from the underlying stock of the assets are assumed to be invested in a bank account with the risk-free rate from the dividend payment date to the option expiration date.

We define the excess return on a skewness asset as

$$\text{Excess Return on a skewness asset} = \frac{price_{t+1} - price_t}{price_t} - (e^{rt} - 1)$$

where the price is the sum of the position times the market price of each security comprising the skewnes asset at the time. Especially, the price at t+1 for the skewness asset that is comprised of nearby options is the sum of the position times the payoff of each security including the future value of the received dividends at the expiration.

[Table 9 here]

Panel A in Table 9 shows the average returns on the two skewness assets using conditions, |OTM delta| is 0.1

or 0.2 and |ATM delta| is 0.5. The average returns are from -0.4% to 0% and all of the values are statistically insignificantly different from zero. However, that the average return that is insignificantly different from zero does not mean that the skewness assets are priced fairly, since the systematic risks of the skewness assets are not taken into account. Panel B of the table shows the returns after the systematic risks are controlled for the case that the delta of OTM options is 0.2[6] in absolute. For the risk factors capturing the systematic risks, we use the Fama-French three factors[7] (Market excess return, SMB, HML), delta-hedged excess return as a proxy of the variance risk premium, and the skewness risk premium. The delta-hedged excess return of the S&P 500 index is the one month holding period excess return on the portfolio composed of an ATM call option and ATM put options (delta hedged portfolio) using the nearby options as Goyal and Saretto (2009) do. The skewness risk premium[8] is the return difference between 10 and 1 decile portfolios in Bali and Murray (2013).

In panel B, the columns (A) and (B) show the returns of skewness assets after controlling the systematic risks. Call skewness asset returns of the nearby options and second nearby options are explained by the market excess return and delta-hedged excess return, and the constant terms for the two portfolios after controlling the systematic risks are not different from zero. Put skewness asset returns of the nearby and second nearby options also have constant terms that are insignificant from zero. But, contrary to the case of call skewness assets, all risk factors that are commonly used in the market have no explanation power for the returns of the put skewness assets and the R^2s are close to zero, lower than the case of the call skewness assets.

The columns (B)-(A) in panel B show the returns of long-short portfolios buying the skewness asset of the second nearby options and selling the skewness asset of the nearby options. The portfolios are constructed to test that the relatively underestimated implied skewness of the second nearby is economically significant. The empirical results partially confirm our expectation. The long-short portfolio constructed by call options has a significant positive return, 0.56% per month, after controlling systematic risks, which is consistent with our previous analysis. Market excess return and delta-hedged excess return still have explanatory power, the same as

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6 The qualitative result does not change when the OTM options are chosen differently.

7 We obtain the data from Ken French's web site.

8 The skewness risk premium is obtained from the author of Bali and Murray (2013). The data includes returns from Jan 1996 to Sep 2010.

the result for each call skewness asset. The long-short portfolio composed of put options also has a positive return, 0.43% per month, after controlling the systematic risks, but it is not significantly different from zero which is the same as the constant terms for each put skewness asset. The only difference from the result for each put skewness asset is that market excess return and delta-hedged excess return explain the return on the put skewness asset. Even though we construct portfolios to trade skewness, the portfolio returns that are composed of call and put options after controlling systematic risks are different. The difference between the two seems to be consistent with the different price impact on call and put options reported in Table 8. However, further research on this seems to be needed.

6. Conclusion

If options with every possible strike price exist in the market, we can complete the market or generate any possible payoff returns (Ross (1976)), which means that if we have a sufficient number of options with different strike prices, then we can extract the full information on the distribution of underlying asset returns or moments of asset returns. The existing literature mainly focuses on the information regarding the second moment or the variance of the underlying asset returns and examines the relation between the implied volatility (or model-free volatility) and the realized volatility. Our study extends the literature by investigating the third and fourth moments as well as the second moment.

Our study suggests a way to construct forward moments iteratively using the model-free second, third, and fourth moments. Using these moments, we document the following:

(1) The third forward measure of the option with two different maturities is not an unbiased estimator of the future realized third moments, though the second and fourth forward measure can be regarded as unbiased estimators of the future second and fourth moment.

(2) Long-term option investors, on average, seem to underestimate the third moment relative to short-term option investors. The underestimation of long-term option investors becomes severe when the variance is large.

(3) The estimation results show that the mispricing or the difference of prices between the cases with and

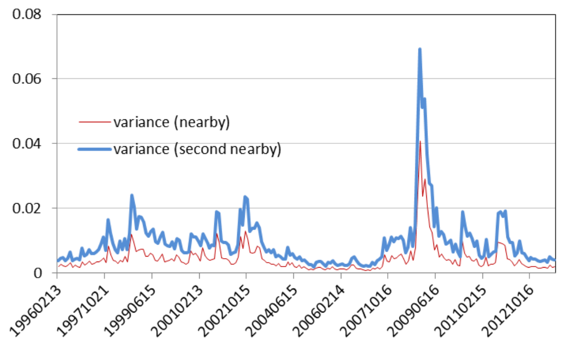
without the biases of the third moment is large and economically significant under Corrado and Su’s model. The third moment effect results in the overpricing of long-term puts and underpricing of long-term calls.

(4) The long-short portfolio consisting of buying long-term skewness and selling short-term skewness using the skewness assets constructed from call (put) options has a positive return that is significantly (insignificantly) different from zero after controlling for the systematic risks. This partially confirms that the underestimation of the implied third moment is economically meaningful.

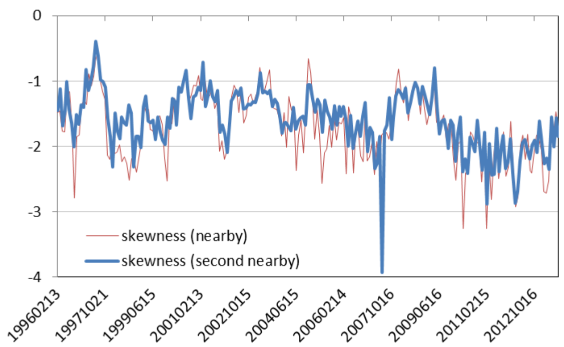
Figure 1. The implied variance, skewness, and kurtosis

Figures show the implied variance, skewness and kurtosis from the nearby and second nearby S&P 500 index options from January 1996 to August 2013. We calculate the risk-neutral variance, skewness, and kurtosis by the methodology of Bakshi et al. (2003) using option price data. The data are from the OptionMetrics database and include the closing bid and ask quotes for each option contract along with the corresponding strike prices and maturity information. We applied the following filters. First, option quotes are included only when both bid and ask quotes are available. Option data with any missing bid or ask quotes are excluded. Second, option quotes whose bid-ask spread is larger than 0.5 are excluded, in order to ensure that illiquid options are excluded. Third, option quotes violating no arbitrage bound with bid and ask quotes are excluded. Specifically, we require that both bid and ask prices of a call (put) option with a higher (lower) exercise price should be higher than those of a call (put) option with a lower (higher) exercise price, in order to ensure that any non-informative options due to minimum tick size or illiquidity are excluded. We use the three-month CD rate as the risk free rate. The S&P 500 index is also used. These data are obtained from the Center for Research in Security Prices.

Panel A. Implied variance



Panel B. Implied skewness



Panel C. Implied kurtosis

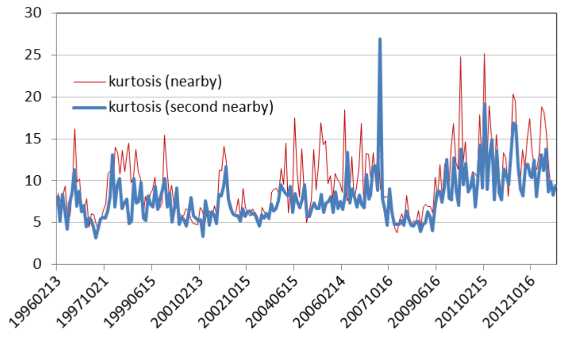
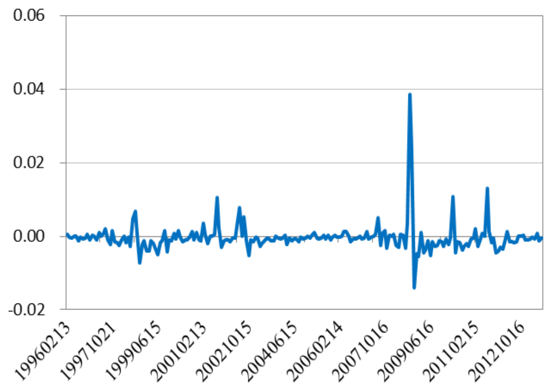


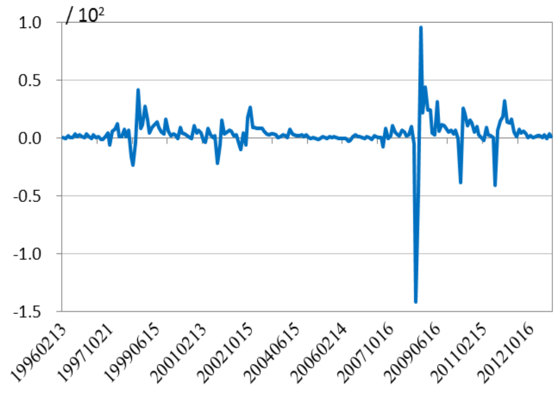
Figure 2. The forecasting errors of the forward moments

Figures show the forecasting errors of the forward moments. The forecasting error is defined as the future implied moment minus the forward moments. The future implied moments are the implied moments from the second nearby options at the expiration date of the nearby options.

Panel A. Forecasting error of the second moment



Panel B. Forecasting error of the third moment



Panel C. Forecasting error of the fourth moment

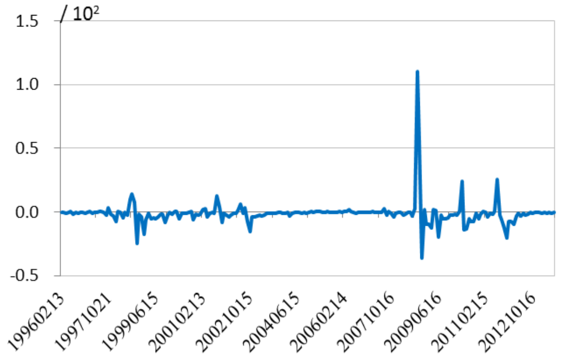


Table 1. Descriptive statistics of the variance, skewness, and kurtosis

This table shows the descriptive statistics of the variance, skewness, and kurtosis from the nearby and second nearby option prices.

Panel A. nearby options

	Variance	Skewness	Kurtosis
mean	0.005	-1.702	9.683
standard deviation	0.005	0.513	3.899
correlation	1.000	0.160	-0.275
	0.160	1.000	-0.932
	-0.275	-0.932	1.000
Autocorrelation	0.808	0.658	0.525
	0.652	0.549	0.493
	0.520	0.478	0.428
	0.370	0.413	0.362
	0.266	0.327	0.317

Panel B. Second nearby options

	Variance	Skewness	Kurtosis
mean	0.009	-1.601	7.920
standard deviation	0.008	0.452	2.966
correlation	1.000	0.141	-0.246
	0.141	1.000	-0.943
	-0.246	-0.943	1.000
Autocorrelation	0.844	0.612	0.436
	0.692	0.571	0.448
	0.554	0.546	0.447
	0.407	0.446	0.303
	0.316	0.375	0.304

Table 2. Cross-covariance between the adjacent returns to the power of 1 to 3

Cross-covariance between the adjacent returns to the power of 1 to 3 is estimated as the sample cross-covariance between the implied moments of each month and the implied moments of the next month from the nearby options from January 4, 1996 to August 30, 2013.

		Each month		
		First moment	Second moment	Third moment
Next month	First moment	0.0000081	-0.0000084	0.0000022
	Second moment	-0.0000092	0.0000172	-0.0000044
	Third moment	0.0000025	-0.0000045	0.0000012

Table 3. Descriptive statistics of the implied second, third, and fourth moments and their forward moments

This table shows the descriptive statistics of the implied second, third, and fourth moments and their forward moments. Panels A and B show the statistics of the nearby options and second nearby options, respectively. Panel C shows the statistics of the forward moments calculated as follows, using constant cross-covariance between the two adjacent returns to the power of 1 to 3 in Table 2.

$$E_t[R(T_1,T_2)^2] = E_t[R(t,T_2)^2] - E_t[R(t,T_1)^2] - 2(\mathrm{Cov}(R(t,T_1),R(T_1,T_2)) + E_t[R(t,T_1)]E_t[R(T_1,T_2)])$$

$$E_t[R(T_1,T_2)^3] = E_t[R(t,T_2)^3] - E_t[R(t,T_1)^3] - 3(\mathrm{Cov}(R(t,T_1)^2,R(T_1,T_2)) + E_t[R(t,T_1)^2]E_t[R(T_1,T_2)]) - 3(\mathrm{Cov}(R(t,T_1),R(T_1,T_2)^2) + E_t[R(t,T_1)]E_t[R(T_1,T_2)^2])$$

$$E_t[R(T_1,T_2)^4] = E_t[R(t,T_2)^4] - E_t[R(t,T_1)^4] - 4(\mathrm{Cov}(R(t,T_1)^3,R(T_1,T_2)) + E_t[R(t,T_1)^3]E_t[R(T_1,T_2)]) - 6(\mathrm{Cov}(R(t,T_1)^2,R(T_1,T_2)^2) + E_t[R(t,T_1)^2]E_t[R(T_1,T_2)^2]) - 4(\mathrm{Cov}(R(t,T_1),R(T_1,T_2)^3) + E_t[R(t,T_1)]E_t[R(T_1,T_2)^3])$$

where $R(t,T_1)$ means a log return between t to T_1.

Panel A. nearby options

	Second moment	Third moment	Fourth moment
mean	0.0046	-0.0006	0.0003
standard deviation	0.0046	0.0013	0.0009
correlation	1.0000		
	-0.9638	1.0000	
	0.9406	-0.9918	1.0000

Panel B. Second nearby options

	Second moment	Third moment	Fourth moment
mean	0.0095	-0.0018	0.0010
standard deviation	0.0084	0.0030	0.0024
correlation	1.0000		
	-0.9652	1.0000	
	0.9456	-0.9926	1.0000

Panel C. forward moments

	Second moment	Third moment	Fourth moment
mean	0.0049	-0.0011	0.0005
standard deviation	0.0039	0.0016	0.0008
correlation	1.0000		
	-0.9610	1.0000	
	0.9309	-0.9875	1.0000

Table 4. The relation between the realized moments and the implied moments from options

This table presents the second stage results of IV regressions that test whether the implied moments from options are unbiased estimators of realized moments from t to the maturity of options. In these regressions, the realized n-th moments are calculated as

$$Realized\ n^{th}\ moment_t = \sum_{i=1}^{s} (R_i)^n \quad for\ n = 2, 3, 4$$

where R_i is the daily log return at time i, and s is the number of business days from t to the maturity of options. The standard errors are calculated following the Newey-West method with four lags.

		Constant	Implied Second moment	R^2	constant x 100	Implied Third moment	R^2	constant x 100	Implied Fourth moment	R^2
Nearby options	coefficient	-0.0005	0.9454	0.40	-0.0008	0.0028	0.00	0.0000	0.0192	0.19
	standard error	0.0006	0.1882		0.0006	0.0099		0.0002	0.0064	
Second nearby options	coefficient	-0.0013	0.9029	0.41	-0.0006	0.0062	0.01	0.0000	0.0119	0.19
	standard error	0.0014	0.2160		0.0011	0.0085		0.0003	0.0056	

Table 5. The relation between the estimated risk-neutral moments and the implied moments from options

This table shows the relation between the implied moments from options and the estimated risk-neutral moments following Bakshi et al. (2003) and Bakshi and Madan (2006) as

$$\frac{VAR_{RN,t}-VAR_{P,t}}{VAR_{P,t}} \approx \gamma \times VAR_{p,t}^{0.5} \times SKEW_{p,t} - \frac{\gamma^2}{2}\mathrm{VAR}_{p,t} \times (KURT_{p,t} - 3)$$

$$\mathrm{SKEW}_{RN,t} \approx \mathrm{SKEW}_{p,t} - \gamma(\mathrm{KURT}_{p,t} - 3) \times \mathrm{VAR}_{p,t}$$

$$\mathrm{KURT}_{RN,t} \approx \mathrm{KURT}_{p,t} - \gamma[\big(2(\mathrm{KURT}_{p,t} + 2\big)\mathrm{SKEW}_{p,t} + \mathrm{PKEW}_{p,t}] \times \mathrm{VAR}_{p,t}$$

where VAR, SKEW, KURT, PKEW means normalized second to fifth moments, P, RN subscripts means physical and risk-neutral probability measures respectively, and γ is risk aversion. In this analysis, we estimate γ as the value to minimize the error between the implied variance and the realized variance using realized second to fourth normalized moments. The risk-neutral skewness and kurtosis are estimated using realized moments and the estimated risk aversion.

Panel A presents the statistics of the estimated risk-neutral moments against the implied moments from options prices. Panel B shows the regression results.

Panel A. The estimated risk-neutral moment versus the implied moment

	Average implied skewness (1)	Average estimated skewness (2)	Difference (1-2)	Standard deviation of difference	t statistic
Nearby options	-1.70	-0.05	-1.65	0.69	-2.39
Second nearby options	-1.60	-0.05	-1.55	0.66	-2.35

	Average implied kurtosis (1)	Average estimated kurtosis (2)	Difference (1-2)	Standard deviation of difference	t statistic
Nearby options	9.68	3.18	6.51	4.39	1.48
Second nearby options	7.92	3.18	4.74	3.75	1.27

Panel B. Regression of the estimated risk-neutral moment on its implied moment

		Constant	t statistic	Implied moment	t statistic
skewness	Nearby options	-1.70	-47.57	0.05	0.75
	Second nearby options	-1.60	-50.74	0.02	0.34
kurtosis	Nearby options	11.22	12.77	-0.32	-1.83
	Second nearby options	9.22	13.83	-0.27	-2.05

Table 6. The forecasting errors of the forward moments

This table shows the summary statistics of the forecasting errors of the forward moments that are defined as the future implied moments of the nearby options minus the forward moments.

	Second moment	Third moment	Fourth moment
average	-0.0003486	0.0003808	-0.0000978
standard deviation	0.0002813	0.0001067	0.0000715
t-statistic	-1.24	3.57	-1.37

Table 7. Regression results of the forecasting errors

This table presents the OLS regression results of the forecasting errors on the normalized moments as

$$\Delta \mathrm{M}^{\mathrm{i}}(T_1, T_2) = \alpha^{\mathrm{i}} + \beta^{\mathrm{i}} * X_t(\mathrm{t}, T_1) + \varepsilon_t$$

$$\text{where } \Delta \mathrm{M}^{\mathrm{i}}(T_1, T_2) = \mathrm{M}^{\mathrm{i}}(T_1, T_1\!: T_2) - \mathrm{M}^{\mathrm{i}}(\mathrm{t}, \mathrm{T1\!: T2})$$

$\mathrm{M}^{\mathrm{i}}(\mathrm{t}, \mathrm{T1\!: T2})$ is the i-th forward moment from T_1 to T_2 calculated at time t, and $\mathrm{M}^{\mathrm{i}}(T_1, T_1\!: T_2)$ is i-th future implied moment calculated at time T_1, the monthly option expiration date. $\Delta \mathrm{M}^{\mathrm{i}}(T_1, T_2)$ is the forecasting error for the i-th moment. The explanatory variables in the regression are the variance, skewness, and kurtosis from the nearby options.

Panels A, B, and C show the OLS regression results for the second, third, and fourth moments, respectively. The Newey-West t-statistics in parentheses are calculated with four lags.

Panel A: The forecasting error of the second moment

Constant	Variance	Skewness	Kurtosis	R^2
-0.00033	-0.00240			0.000
(-1.12)	(-0.03)			
0.00010		0.00026		0.002
(0.11)		(0.62)		
-0.00026			-0.00001	0.000
(-0.34)			(-0.13)	
0.00015	-0.00716	0.00027		0.002
(0.24)	(-0.08)	(0.76)		
-0.00023	-0.00463		-0.00001	0.000
(-0.42)	(-0.05)		(-0.19)	

Panel B: The forecasting error of the third moment

Constant	Variance	Skewness	Kurtosis	R^2
-0.00004	0.10820			0.202
(-0.43)	(3.39)			
0.00038		-0.00004		0.000
(1.39)		(-0.28)		
0.00065			-0.00002	0.005
(2.56)			(-0.98)	
-0.00040	0.11175	-0.00020		0.211
(-1.95)	(3.54)	(-1.62)		
-0.00021	0.11193		0.00002	0.205
(-1.21)	(3.56)		(0.94)	

Panel C: The forecasting error of the fourth moment

Constant	Variance	Skewness	Kurtosis	R^2
-0.00009	-0.01284			0.008
(-1.16)	(-0.47)			
0.00003		0.00011		0.007
(0.15)		(1.17)		
-0.00011			0.00000	0.001
(-0.71)			(-0.32)	
0.00013	-0.01512	0.00013		0.017
(1.01)	(-0.56)	(1.64)		
0.00000	-0.01495		-0.00001	0.010
(0.01)	(-0.55)		(-0.92)	

Table 8. The price impact of the relatively underestimated third moment in the second nearby options

This table presents the median price impact of the underestimated third moment described in Table 7. Based on Corrado and Su's model, the price impact is estimated as the model option price with the volatility, skewness, and kurtosis from the second nearby options minus the model option price with the adjusted third moment when other things are equal. The adjusted third moment is the value that is adjusted from the forward third moments as much as the average forecasting error of the third moment, 0.0003808, in Table 6 or −0.00004 + 0.10820 × variance of the nearby options based on the regression result in Table 7. In ratio presents the median price impact reported in the column "In value" divided by the median value of option prices that we calculate using the interpolated Black-Scholes volatilities observed in the market.

Panel A. The median mispricing on OTM put options

	(K/S)x100=90		(K/S)x100=92.5		(K/S)x100=95		(K/S)x100=97.5	
	In value	In ratio	In value	In ratio	In value	In ratio	In value	In ratio
Constant bias	1.727	0.215	1.919	0.167	1.669	0.098	0.865	0.036
Time varying bias	1.460	0.181	1.513	0.132	1.309	0.077	0.731	0.030

Panel B. The median mispricing on ATM and OTM call options

	(K/S)x100=100		(K/S)x100=102.5		(K/S)x100=105		(K/S)x100=107.5	
	In value	In ratio	In value	In ratio	In value	In ratio	In value	In ratio
Constant bias	-0.198	-0.006	-1.186	-0.050	-2.069	-0.163	-2.546	-0.425
Time varying bias	-0.253	-0.007	-1.131	-0.048	-1.827	-0.144	-2.154	-0.360

Table 9. Returns on skewness assets

Panel A shows the statistics of the returns on the call and put skewness portfolios suggested in Bali and Murray (2013). Panel B shows the portfolio returns after controlling systematic risks. Systematic risk factors are Fama-French three factors, delta-hedged excess return as a proxy of variance risk premium and skewness risk premium from January 1996 to September 2010. In detail, delta-hedged excess return of the S&P 500 index is the one month holding period excess return on the delta-hedged portfolio composed of ATM call and put options of the nearby options. Skewness risk premium is the return difference between 10 and 1 decile portfolios in Bali and Murray (2013). The skewness asset are constructed using ATM and OTM options whose deltas are closest to 0.5 and 0.1 or 0.2 in absolute terms, respectively. Panel B shows the return analysis of the portfolios constructed by OTM options whose deltas are closest to 0.2 in absolute terms. The values in brackets are Newey-West t-statistics with lag 4.

Panel A. Descriptive statistics of the returns on skewness assets

		|OTM delta|=0.1		|OTM delta|=0.2	
		nearby options	second nearby options	nearby options	second nearby options
Call skewness asset	mean	-0.004	0.001	-0.004	0.001
	standard error	0.004	0.002	0.004	0.002
Put skewness asset	mean	0.000	-0.001	-0.003	0.000
	standard error	0.004	0.002	0.003	0.001

Panel B. Returns on skewness assets after controlling risk factors

	Call skewness asset			Put skewness asset		
	nearby options (A)	second nearby options (B)	(B-A)	nearby options (A)	second nearby options (B)	(B-A)
constant	-0.0057	-0.0001	0.0056	-0.0036	-0.0007	0.0043
	(-1.46)	(-0.05)	(2.31)	(-0.7)	(-0.32)	(1.29)
Maket excess return	-0.0025	-0.0007	0.0018	0.0000	-0.0003	0.0022
	(-2.09)	(-1.11)	(2.51)	(0.01)	(-0.51)	(2.16)
SMB	-0.0007	0.0001	0.0007	0.0000	0.0001	0.0009
	(-0.75)	(0.12)	(1.06)	(-0.04)	(0.33)	(1.02)
HML	0.0000	-0.0001	-0.0001	0.0003	0.0003	0.0005
	(0.01)	(-0.07)	(-0.08)	(0.21)	(0.47)	(0.45)
Delta-hedged excess return	-0.0413	-0.0156	0.0257	0.0044	-0.0002	0.0410
	(-5.54)	(-3.81)	(6.18)	(0.51)	(-0.06)	(6.78)
Skewness risk premium	0.0471	0.0096	-0.0375	0.0376	-0.0233	-0.0978
	(1.04)	(0.43)	(-1.11)	(0.26)	(-0.51)	(-1.56)
R^2	29.0%	20.3%	24.4%	0.6%	1.0%	32.7%

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