# Certification and Overinsurance in the Bond Insurance Market

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This Draft: June 19, 2015

### Abstract

Although the U.S. monoline insurers were severely downgraded during the 2008 debacle, there has been little accompanying academic interest. My paper addresses this gap by exploring a simple stylized model of bond insurance. The model reveals that some insurance contracts may not be socially desirable. This overinsurance result stems from two sources, namely (i) the insurers' equity capital cost incurred to retain the AAA rating, and (ii) a group of investors facing a rating-based certi fication constraint. Consequently, from an overall welfare perspective, some bonds may be better left uninsured in the hands of unconstrained, aggressive investors.

JEL Classification: G01; G14; G22; G28

**Keywords:** Monoline insurer; credit enhancement; financial guaranty; invest-

ment certification; capital coverage

The author is grateful to Thummim Cho, Amil Dasgupta, Young Ho Eom, Jayant Ganguli, Jaehoon Hahn, Bong-Gyu Jang, Baeho Kim, Changki Kim, Woojin Kim, Jongsub Lee, Kyoungwon Seo, Demosthenes Tambakis, and Julia Shvets for their helpful comments. The author also wishes to thank all seminar participants at the University of Cambridge, KAIST College of Business, Korea Military Academy, Korea Securities Association, Korea University Business School, Kyung Hee University, POSTECH Finance and Risk Management Research Center, Seoul National University, University of Seoul, and Yonsei School of Business, as well as all participants of Korea Securities Association's CAFM 2013.

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"It didn't need an oracle to predict Ambac's demise (The Economist, 2010)."

# 1. Introduction

One of the dramatic events witnessed during the subprime crisis was the fall of U.S. monoline insurers such as Ambac and MBIA. Since the 1970s, these firms had retained the AAA rating and specialized in credit enhancements for municipal bond issuers. By guaranteeing a bond's par and tying its credit rating to the firm's own, these insurers essentially lent their AAA ratings for business. This line of business proved popular, and they insured around a half of all U.S. municipal bonds as of 2008. Their subsequent venture into the securitized products turned out to be less successful, and following the slowdown of the U.S. housing market, unexpected losses from subprime securities led them to a painful series of rating downgrades (Drake and Neale, 2011). This resulted in "a sweeping rating downgrade across financial instruments with a face value of \$2.4 trillion (Brunnermeier, 2009, p. 87)."

This market collapse primarily emanated from a gross underestimation of the credit risks of mortgage-related products,<sup>1</sup> due to a large deterioration in the quality of late-2006 and 2007 vintage mortgage loans (Demyanyk and Van Hemert, 2011) and the breakdown of the relationship between credit observables and default likelihoods (Rajan, Seru and Vig, 2014). As a result, these insurers have been criticized in the popular press, with their decision to expand their business to mortgage-backed securities branded as a key mistake, tempted by \the housing market's siren calls (Forbes, 2010)." Yet, the accompanying academic interest in the monoline insurance debacle has been notably sparse; with the rare exception of Nanda and Singh (2004), who explore a model of bond insurance based on tax considerations,<sup>2</sup> the recent literature has witnessed very few theoretical analysis of this market.

The main aim of this paper is to fill the gap in the literature through both positive and normative analysis of the bond insurance market. More specically, I ask the following questions. First, while the popular press has focused on the market participants' mis-

<sup>&</sup>lt;sup>1</sup>As the major monoline insurers were repeatedly downgraded throughout 2008 and 2009, credit rating agencies were forced to sharply revise their loss projections. While Moody's cumulative loss rate projections for Ambac and MBIA's exposures to 2006 vintage mortgage-related products stood at around 14 to 18% in January 2008, the revised expected and stress-case projections jumped to 22% and 30% respectively by September 2008, in the space of less than eight months (Moody's, 2008).

 ${}^{2}$ In their model, the main benefit of bond insurance arises from the insurer's ability to act, in effect, as an issuer of tax-exempt security, by providing tax-exempt payments in the event of an issuer default.

handling of the credit risks of securitized products, is this really all there is to the story? Can the entirety of the problem be attributed to the participants' risk management, or are there other underlying, unaddressed issues? Second, if the participants neglect the downside risk of mortgage-related securities, as has been blamed in the popular press, what are its exact consequences for the market outcome and the participants' welfare? Both are pertinent questions, given that the insurers had \played an important role in making securities, including those based on sub-prime loans, attractive to a wide range of investors (Schich, 2008, p. 84)."

With this aim, I present a three-period, multi-asset model of bond insurance with constant absolute risk aversion (CARA) investors, two risk neutral bond insurers, multiple issuers, and a credit rating agency (CRA). Crucially, the provision of bond insurance by a AAA-rated insurer allows the issuer to gain access to a larger pool of potential investors. This is due to the investment certification role of credit ratings (DeMarzo, 2005; Boot, Milbourn and Schmeits, 2006; Benmelech and Dlugosz, 2009; Bolton, Freixas and Shapiro, 2012); many pension and money market funds face rating-based constraints on their portfolio selection, $3$  which leads to a "market segmentation among bottom tier and top tier investment grade bonds (Denison, 2003, p. 99)." Thus, my model assumes that a proportion of investors—referred to as "conservative" investors—are constrained to invest only in AAA-rated bonds.<sup>4</sup>

Furthermore, the credit ratings of insurers, as in practice, are determined by their capital adequacy against tail risk.<sup>5</sup> When this interacts with the investors' certification constraint, it yields the model's principal result, namely that the overall amount of bond insurance is likely to be excessive relative to the social optimum.

This appears counterintuitive at first. After all, the standard economic theory on insurance states risk averse participants ought to benet from transferring an asset's risk to risk neutral participants. The investors' certification constraint plays an important part in deriving this seemingly counterintuitive result. Suppose that a bond is uninsured and rated below AAA. If so, conservative investors are unable to hold the bond, leaving the remaining "aggressive" rating-unconstrained investors to clear the market. If so, each remaining aggressive investor has to hold a larger position in the asset. In order to

<sup>3</sup>For example, Cantor, Gwilym and Thomas (2007) report that around three-quarters of pension plan managers in their sample face some form of minimum rating requirements for bond purchases.

<sup>4</sup>Nearly all securities insured by a monoline insurer already held a shadow investment grade rating by at least one of three major rating agencies (Schich, 2008). Therefore, the primary question was whether they were rated AAA or not, given the stringent investment certication requirements of many investors.

 $5$ Moody's (2006), for example, explicitly required its AAA-rated bond insurers to maintain sufficient capital to cover for the 99.9 percentile portfolio loss.

induce the investors to do so, a sufficient price discount must be offered, which more than compensates for their risk aversion. Consequently, aggressive investors enjoy a utility gain from investing in an uninsured bond, as measured by the increase in their certainty equivalent wealth.

Such a utility gain no longer becomes possible when a bond is insured; the investors cannot extract any utility gain from a competitively priced riskless bond. Thus, an insured bond necessarily entails forgone utility gain for the aggressive investors, who could otherwise have benefited from purchasing an uninsured bond at a sufficient discount. Of course, the forces of standard theory on insurance is still at work; risk averse aggressive investors' forgone increase in the certainty equivalent wealth following bond insurance is always smaller than the risk neutral issuer's increase in wealth from being able to issue the bond at a higher price.

Crucially, though, the investors are not required to raise capital for their bond investment. In contrast, to insure a bond and maintain the AAA rating, an insurer needs to raise equity capital, which is bound to incur capital cost. This means that, even if the issuance price differential is larger than the investors' forgone utility gain, the magnitude of the latter can still exceed the total internalized surplus of an insurance contract when the insurer's capital cost is taken into account. However, as the negotiation process of an insurance contract only includes the insurer and the issuer, this forgone utility gain is not internalized, clearly an undesirable outcome from the overall social perspective. This result does not rely on the exclusion of a capital requirement for the investors; as long as the investors can raise capital sufficiently more cheaply than the insurer, the model's principal result of overinsurance remains intact.

While serious concerns have been voiced regarding bond insurance from a macroprudential perspective, my model's overinsurance result compounds these existing concerns regarding its social desirability from a purely microprudential standpoint. For example, in an earlier, related version of this paper (Oh, 2012), I explore how bond insurance creates systemic linkages between all insured bonds and essentially acts as a contagious transmission mechanism, transforming isolated problems within the subprime-related products into a marketwide problem. This became apparent during the 2008 monoline debacle, as the "notion that the failure of even one big bond insurer might touch off a chain reaction of losses (New York Times, 2008)" worried various market participants. My paper argues that, in addition to these systemic risk considerations, the negotiation of an insurance contract carries an inherent problem of negative externality.

Moreover, such overinsurance is fully compatible with the participants' rationality;

it stems solely from the certication needs of both the insurer, who faces a capital requirement for AAA rating, and a group of investors that face a rating-based investment constraint. This naturally leads to a comparison with Hanson and Sunderam (2013), who arrive at a similar result of "too many safe securities" in the market. While their model derives this result from endogenous information acquisition issues, whereby the issuer's excessive issuance of "safe" debt securities reduces the investors' ex ante incentive to acquire costly but valuable information, the overinsurance result in my paper does not rely on informational or agency problems; it instead highlights a different market imperfection, namely the participants' certification constraints.

However, this still leaves the question of how the market participants' risk underestimation affects the market outcome. In the second half of this paper, I address this issue by incorporating the concept of local thinking (Gennaioli and Shleifer, 2010). In a series of recent papers, Gennaioli, Shleifer and Vishny (2012, 2013) utilize this theoretical framework to highlight the problems associated with securitization. More specifically, they show that the initial neglect of a severe credit outcome gives rise to excessive issuance of securitized assets, which can result in market fragility and a systematic failure of the shadow banking system.

By employing this framework, I first show that the initial neglect of a bond's downside risk can result in an insurer downgrade. This is not surprising; if the CRA underestimates a bond's "worst case" outcome, its initial requirement for insurer capital is insufficient to cover for the tail risk. When the CRA subsequently becomes aware of this hitherto neglected outcome, its capital requirement for AAA-rated insurer becomes more stringent. This incurs additional equity capital cost, which can be substantial when the equity market conditions are unfavorable, as was the case in 2008.<sup>6</sup> If so, the insurer may choose not to defend her AAA rating even in the presence of a reputational cost of downgrade. This has a particularly harmful knock-on effect on the conservative investors, who can no longer hold a downgraded asset and have to exit the market at a "fire-sale price" (Coval and Stafford, 2007).

More importantly, though, the initial underestimation of a bond's credit risk also has an ex ante implication; the overall number of bond insurance is likely to be lower relative to the rational benchmark. Since the risk averse investors perceive the bond to be safer when the downside risk is neglected, insurance appears less beneficial. However, given the inherent negative externality associated with bond insurance, some bonds are

<sup>6</sup>For example, between Jan. 2007 and Jan. 2008, Ambac's share price fell by around 90%, with other major monoline insurers posting share price falls of comparable magnitudes.

better left uninsured in the hands of aggressive investors, and thus the local thinking participants' tendency to underinsure can actually be welfare improving. Although I do not aim to claim this as an all-encompassing, general result, it serves as a caution against a simplistic, zealous focus on the participants' risk management without addressing the underlying externality issue at the same time.

The prospect of insurance underprovision by risk-neglecting participants also has an important message regarding our current understanding of the bond insurance market. While the popularity of bond insurance in the securitized products market prior to the crisis and the market participants' risk underestimation have both been a subject of media criticism, these two regularities are susceptible to a post hoc, propter hoc fallacy. If anything, the popularity of bond insurance ought to be seen as a sign of market participants' rational decisionmaking, given the model's overinsurance result. On the other hand, if there are grounds to believe that the participants' risk assessments were inadequate, then the model argues that the overall volume of bond insurance may have been even larger had the participants been fully rational. In other words, these two observed facts ought not to be treated as a causal relationship.

In the final part of the paper, I demonstrate that the model's overinsurance result remains robust to a number of extensions. A similar phenomenon can be observed using constant relative risk aversion (CRRA) or quadratic utility specifications, and the model's key qualitative insights further remain unchanged when the number of possible credit outcomes becomes large, the bonds' credit risks become interdependent, the investors have imperfect access to the insurer's capital upon insurer default, or the three-period framework is transformed as a repeated interaction infinite horizon model.

Furthermore, the model's wider implications are also discussed. The model carries broader insights beyond the bond insurance market; my main result, namely the certication constraints as a source of market inefficiency, may be relevant for other areas of the financial market where such needs play a significant part on the participants' decisions. Moreover, given the importance of the participants' certification constraints, it places an emphasis on the role of CRAs for a more enhanced market outcome, particularly when the results of my model are taken in conjunction with macroprudential concerns as well as their observed practices prior to the crisis, such as rating shopping (Bolton, Freixas and Shapiro, 2012) and the insufficient disciplining effect of reputation (Mathis, McAndrews and Rochet, 2009; Fulghieri, Strobl and Xia, 2014).

The rest of this paper is organized as follows. Section 2 provides a brief overview of the bond industry. Then, in Section 3, I present the model and outline its main assumptions.

In Section 4, I demonstrate that bond insurance entails negative externality even when the participants are fully rational. Section 5 focuses on the effect of the local thinking participants' underestimation of credit risk on the market outcome. Section 6 is devoted to various robustness checks on the model's overinsurance result and a discussion on its broader implications. Section 7 then concludes the paper.

# 2. Overview of the Bond Insurance Industry

The beginning of the bond insurance industry<sup>7</sup> in the U.S. is widely accepted to be 1971, when Ambac provided an insurance contract for Greater Juneau (Alaska) Borough Medical Art Building's obligation bond amounting to \$650,000. Other insurers quickly followed suit and established themselves, but until the 1990s, their scope was largely confined to providing nancial guarantees for municipal bond insurers. The insurers' presence in the municipal securities market was sizeable, with Ambac's 10-K lings reporting that around 54% of all municipal bonds issued in 2004 were insured by one of the monoline insurers. Nevertheless, their activities in the financial market, though significant, did not receive much public attention, with a magazine article describing their business model as "basking in profitable obscurity (The Economist,  $2008$ )."

A fierce level of competition between the insurers gradually encouraged them to look beyond the municipal securities market, $\delta$  and beginning in the early 2000s, they started providing similar guarantees to securitized and structured products. Regulatory changes also helped contributing toward this business expansion; the primary regulator in charge of the industry, namely the State of New York Insurance Department (NYID),<sup>9</sup> allowed monoline insurers to write insurance contracts for investment-grade collateralized debt obligation (CDO) tranches in 1997. Although the insurance provision had to be \indirect," with the insurer establishing a special purpose vehicle (SPV) then guaranteeing SPV's principal and interest payment guarantees to the CDO issuer, this opened the door

 $7$ Throughout this paper, I use the terms "monoline insurance," "bond insurance" and "financial" guaranty" interchangeably. For the context of this paper, the term \monoline" refers to the fact that the insurer is prevented from writing casualty or property contracts, with all businesses solely restricted to the provision of financial guaranty. This is in vivid contrast with "multi-line" insurance firms that operated within the U.S. financial markets prior to the crisis, such as AIG.

<sup>&</sup>lt;sup>8</sup>There were nine major monoline insurers operating within the U.S. at the beginning of the crisis, including Ambac, MBIA, FGIC and Assured Guaranty. Many of these firms had held and retained the AAA rating for decades at the time.

 $9<sup>9</sup>$ As most insurers—with a notable exception of Ambac—were domiciled in the State of New York, NYID was widely considered to be the primary regulator of monoline insurers. Ambac, on the other hand, was domiciled in Wisconsin.

for a whole new range of products. Insurance contracts for CDO-squared products were similarly allowed in 2004.

The insurers' presence in the structured nance market grew substantially, with their total net par outstanding in these products surpassing \$1 trillion by 2007 (Schich, 2008). As their total net par outstanding across all products were around \$2.4 trillion, structured finance accounted for a significant proportion of their business activities at the onset of the crisis. However, as the defaults in their traditional area of business, i.e., municipal securities market, were extremely rare, virtually all insurers were poorly capitalized. Schich  $(2008)$  reports that, as of end-2006, only five out of nine insurers held more than  $1\%$  of their net par outstanding as capital; even the figure for the best capitalized firm, Assured Guaranty, failed to surpass 1.5%.

Hence, these insurers were badly exposed to the writedowns in the subprime mortgage market that began in 2007. Share prices of Ambac and MBIA, the largest and most established insurers in the market, plummeted by more than 70% in one year. Credit rating agencies responded to this changing market environment; by the end of 2009, Standard and Poor's ratings for Ambac and MBIA fell to junk status, while FGIC and CIFG had their ratings completely withdrawn altogether.

As this meant automatic rating downgrades of all insured bonds worth \$2.4 trillion, the entire financial markets reacted violently. In an earlier version of this paper (Oh, 2012), I discuss how this served as a transmission mechanism through which the initial troubles in the subprime mortgage sector were transformed to a marketwide problem. In fact, the monoline insurers' troubles led to a freeze of liquidity in the asset-backed commercial paper (ABCP) market (International Monetary Fund, 2014), with devastating consequences for the money market funds (Brunnermeier, 2009). The bond insurance market itself also suffered a damaging blow to its reputation and is yet to recover fully from its devastating impact, with Assured Guaranty being the only legacy insurer still active in the business.

# 3. The model

# 3.1. Asset composition and payoff structure

I consider a three-period model with  $t = 0, 1, 2$ . At  $t = 0, I \ge 1$  bonds are issued to the public, denoted  $B_1$  to  $B_I$ . These bonds may be thought of as either municipal or securitized bonds in practice. They all have the identical maturity, which occurs at  $t = 2$ . For simplification, they are assumed zero-coupon discount bonds with the par at maturity equal to one unit of consumption numeraire. The period t market price of  $B_i$ is denoted  $p_t^i$ . All bonds' issuance volumes are normalized to 1.

A bond's credit risk is modeled as follows. At maturity, a bond may fail to repay its par value; instead, a unit of bond  $B_i$  repays  $1 - \Theta_i$ , where  $\Theta_i$  denotes its credit loss.  $\Theta_i$  can take one of three values:  $\Theta_i = \{0, \theta_i, \mu_i \theta_i\}$ , with the respective probabilities  $\Pi_i = \left\{ \pi_i^g \right\}$  $\{\xi_i^g, \pi_i^m, \pi_i^b\}$ , where  $\pi_i^b = 1 - \pi_i^g - \pi_i^m$ . Furthermore,  $\mu_i > 1$  and  $\mu_i \theta_i \leq 1$ . In other words, a bond can either experience no credit loss, small loss, or large loss, and  $\mu_i$  measures the relative severity of the tail outcome, i.e., the realization of a large loss.  $\theta_i$  reflects the overall baseline credit characteristics of a bond, as an increase in  $\theta_i$ affects both the small and large loss outcomes simultaneously. In addition, the ex ante probabilities satisfy  $\pi_i^g > \pi_i^m > \pi_i^b$ , which implies that the large loss event is least likely to occur. This payoff structure is similar to Gennaioli, Shleifer and Vishny (2012, 2013).

Although I focus on a three-point credit risk model for the ease of exposition, the model is flexible enough to yield tractable results for a larger finite number of possible loss realizations, as will be shown in Section 6.2.3. Thus, it may be considered a reduced version of the elaborate risk modeling approach employed by the market participants in practice.<sup>10</sup>

Another assumption I maintain is that a bond's credit loss realization is determined independent of other bonds. Of course, this is unlikely in practice, given the strong interdependence between bonds due to their common exposure to regional or macroeconomic factors. However, this interdependence has little impact on the qualitative results of the model, which I discuss in more detail in Section 6.2.4.

Finally, each bond is issued by a unique issuer, also indexed  $i = 1, ..., I$ . This assumption is a reasonable approximation of the actual arrangements, given that the insurance contracts are usually offered on a deal-by-deal basis. In the absence of any multi-deal bundling, even if a particular issuer issues more than one bond, her optimization problem remains identical to the case where each bond is issued by a different issuer. This, by construction, also rules out the possibility that two bonds are dierent tranches of a particular RMBS or CDO, which alleviates some of the remaining concerns regarding the credit risk independence assumption. Given that bond insurance usually involved a small subset of mezzanine, junior investment grade tranches, this is not a serious issue.

<sup>10</sup>For example, Moody's uses a correlated binomial expansion technique to calculate the idealized default probabilities for CDOs. As with any binomial expansion model, the default risk modeling is discrete in nature.

# 3.2. Market participants

#### 3.2.1. Bond issuers

As with other models of securitization, bond issuers essentially serve as a "broker." At  $t = 0$ , issuer  $i \in \{1, ..., I\}$  obtains a pool of loans by paying  $l_0^i$ . The loans last for two periods, and repay  $1 - \Theta_i$  at  $t = 2$ . There are no interest payments. This role of bond issuers as brokers is particularly true for securitized products, where the issuers hold no capacity for loss absorption due to the prevalent use of special purpose entities (SPEs) in a securitization deal.<sup>11</sup> Not surprisingly, this is the source of a bond's credit risk.

Given this set-up, a representative bond issuer's objective is simply to maximize the intermediation spread at  $t = 0$ . Prior to issuance, the issuer receives an offer to insure her bond from each insurer. If issuer  $i$  rejects all offers, then she proceeds without insurance and her intermediation spread is given by  $p_{0,U}^i - l_0^i$ , where the subscript U denotes that the bond is uninsured.<sup>12</sup> On the other hand, if she accepts insurer  $j$ 's offer, her intermediation spread is  $p_{0,I}^i - \chi_j^i - l_0^i$ , where the subscript I denotes an insured bond and  $\chi_j^i$  is the one-off insurance premium paid to insurer  $j$ <sup>13</sup> Here, I impose an implicit assumption, namely that the issuer cannot alter the quantity of bond issuance regardless of whether it is insured or not. As I will explore in Section 6.2.2, the model's main results are robust to its relaxation. Finally, An insurance can only be bought at issuance  $(t = 0)$ .

Thus, i strictly prefers bond insurance when:

$$
p_{0,I}^i - p_{0,U}^i > \chi_j^i. \tag{1}
$$

This implies that the issuance price differential  $(p_{0,I}^i - p_{0,U}^i)$  represents the issuer's maximum willingness to pay for the insurance premium. As this quantity forms a central part of the subsequent analysis, I denote  $\Delta_0^i \equiv p_{0,I}^i - p_{0,U}^i$  as a shorthand.

## 3.2.2. Bond insurers

As discussed earlier, the bond insurance industry witnessed strong competition among a number of firms. As any competition takes the form of price competition under the model

<sup>&</sup>lt;sup>11</sup>Not only do SPEs remove the loans from the originators' balance sheet, but they are also not subject to any minimum equity requirement, severely limiting loss-bearing capacities.

<sup>&</sup>lt;sup>12</sup>An implicit assumption here is that  $l_0^i$  is sufficiently low to enable bond issuance (i.e.,  $p_{0,U}^i > l_0^i$ ) even in the absence of insurance.

<sup>&</sup>lt;sup>13</sup>Since the insurance premium is one-off, it is possible to drop the time subscript t. This is a standard practice within the industry (Drake and Neale, 2011).

set-up, it suffices to restrict the attention to the case of an insurer duopoly.<sup>14</sup> These two insurers are indexed  $j = A, B$ .

At  $t = 0$ , both insurers make an offer to each issuer i. If insurer j's offer is accepted, i pays the agreed one-off premium  $(\chi^i_j)$  and j, in return, guarantees the bond's par value by covering any credit losses of bond investors at  $t = 2$ . As these insurers "typically retain most of the risk that they underwrite (Schich, 2008, p. 91)," it is reasonable to ignore reinsurance. Thus, an insurer needs to prepare for a possible claim payout at maturity by building her own capital buffer.

As in practice, capital buffer consists of two components, namely the insurance premium reserve and equity capital. Equity capital may be raised at both  $t = 0$  and  $t = 1$ , and insurer j's total stock of equity capital at t is denoted  $K_t^j$  $t<sub>t</sub><sup>j</sup>$ . More importantly, raising equity capital incurs the insurer a capital cost of  $c_t^j < 1$  per unit. This cost reflects underwriting and brokerage fees, as well as the well-documented underpricing of seasoned equity offerings (Corwin, 2003; Altinkilic and Hansen, 2003). As denoted, it may differ between the two insurers and also over time. Furthermore, given the insurers' poor capitalization prior to the crisis, I assume that the insurers enter the market at  $t = 0$  with no initial capital. In other words,  $K_{-1}^j = 0$  for  $j = A, B$ .

Reflecting the monoline insurers' long, historical AAA status, both insurers' initial credit ratings are set at AAA. In the analysis, it will become apparent that an insurer downgrade is not an issue at  $t = 0$ . However, an insurer j can be downgraded by the credit rating agency at  $t = 1$ . In this instance, a reputational cost of  $\kappa_j$  is incurred. Although the model yields meaningful results without an exogenous reputational cost, its inclusion captures the firms' reluctance to accept a rating downgrade in practice due to a loss of trust, reputation, and future business.

Finally, as in standard models of insurance, both insurers are risk neutral. At  $t = 0$ , 1, insurer  $j \in \{A, B\}$  maximizes her expected terminal wealth at  $t = 2$ , denoted  $V_2^j$  $\frac{z^{\jmath}}{2},$ defined as:

$$
V_2^j \equiv \sum_{i=1}^I I_j^i \left( \chi_j^i - \Theta_i \right) - c_0^j K_0^j - c_1^j \left( K_1^j - K_0^j \right), \tag{2}
$$

if insurer j retains AAA rating at  $t = 1$ , and:

$$
V_2^j \equiv \sum_{i=1}^I I_j^i \left( \chi_j^i - \Theta_i \right) - c_0^j K_0^j - c_1^j \left( K_1^j - K_0^j \right) - \kappa_j, \tag{3}
$$

<sup>14</sup>The case of  $N > 2$  insurers carries the identical economic intuition.

if insurer j is downgraded at  $t = 1$ . In both (2) and (3),  $I_j^i$  is an indicator function that takes the value of 1 if and only if bond  $i$  is insured by insurer  $j$ . Both equations also implicitly assume no discounting, since an inclusion of discount rate has no impact on the qualitative results of the model.

 $(2)$  and  $(3)$  still require another implicit assumption. When an insurer's capital buffer falls short of the claim demand at maturity, limited liability becomes an issue. In other words, an insurer's ex ante decision may incorporate the possibility that she cannot be held responsible beyond her capital buffer in the event of bankruptcy. However, this complicates the optimization problem and renders a solution intractable. To overcome this issue, the insurer is assumed to receive a negative utility equal to the size of her capital shortfall when she holds insufficient capital to meet all claims. A similar assumption is imposed in other theoretical studies such as Brunnermeier and Pedersen (2009), and given the costly legal processes following an insurer bankruptcy, the prospect of negative utility is not particularly controversial.

### 3.2.3. Credit rating agency

In the model, there is a single credit rating agency (CRA) in charge of rating all bonds and bond insurers. The possibility of "rating shopping" (Bolton, Freixas and Shapiro, 2012; Bongaerts, Cremers and Goetzmann, 2012) is deliberately ruled out as it is not the main interest of the paper. CRA's rating disclosure is not contingent upon payment. Then, due to the lump sum nature of the payment, it is possible to simply assume the ratings are determined free of charge. Thus, the provision of credit ratings in this model may be seen as a public good, as posited in Duan and Van Laere (2012).

Given the earlier discussion, I consider a simplied rating structure whereby a bond or an insurer is either rated AAA or below-AAA. All ratings are updated every period. Then, the essence of various rating criteria in practice is distilled in the following set of conditions. First, a bond is rated AAA at t if and only if  $E_t(\Theta_i) = 0$ . Therefore, unless  $\pi_i^g = 1$ , a bond  $B_i$  cannot be rated AAA on its own merit.

Second, a bond insurer's credit rating is solely determined by her capital adequacy. In the U.S. bond market, whether or not an insurer has built up sufficient capital to cover for the downside risk of her insurance portfolio remains the most important rating factor. To reflect this, a bond insurer has to hold sufficient capital to cover for the "worst" case" portfolio loss to be rated AAA. Due to the independence of credit risk, a worst case portfolio loss arises when the worst case loss is realized for each insured bond. While such an event may seem unlikely to occur for a large portfolio due to the law of large numbers, the bonds' close interdependence in practice implies that it is not as unrealistic as what the simple asymptotics would suggest.

#### 3.2.4. Bond investors

The single most important distinction in this model is made with regards to the investor type. There are two different types of potential investors. First, a proportion of "aggressive" investors are free to invest regardless of a bond's credit rating. The remaining "conservative" investors, however, can only invest in AAA-rated bonds. Thus, if a bond loses its AAA rating, then they are immediately required to sell off any existing holdings of the downgraded bond.

In order to make bond prices directly comparable, each bond  $B_i$  at  $t = 0$  attracts a continuum of potential investors of measure one, a proportion  $\lambda_i \in [0, 1]$  of whom are aggressive and the remaining  $1 - \lambda_i$  are conservative. Each bond's investor pool is distinct, implying that an investor only considers her potential investment decision over one particular bond. This assumption is particularly appropriate for the municipal bond market, which has an unusual characteristic of small buy-and-hold retail traders exerting a dominance presence in the market, $^{15}$  often considering municipal bonds as low risk,  $\text{tax-exempt}^{16}$  investment opportunities for their pensions.

In short, both types of investors can hold a AAA-rated asset; the ex post proportions of aggressive and conservative investors are  $\lambda_i$  and  $1 - \lambda_i$  respectively. In contrast, for a bond rated below AAA, the actual investor pool consists entirely of aggressive investors of measure  $\lambda_i$ .

All investors follow standard CARA utility, take the market price as given at each period, and consume only at  $t = 2$ . This means that the investors maximize the expected utility associated with their terminal wealth. More formally, if a representative investor j's cumulative holding of bond  $B_i$  at t is denoted  $x_{i,t}^k(j)$ , with  $k \in \{agg, con\}$ distinguishing the investor type, then her objective function is given by:

$$
\max_{x_{i,t}^k(j)} -E_t \exp\left\{-\gamma W_2^k(j)\right\},\tag{4}
$$

<sup>&</sup>lt;sup>15</sup>For example, Securities and Exchange Commission (2012) reports that 50.2% all outstanding municipal bonds are held directly by individuals, with additional 25% held indirectly through mutual or money market funds.

<sup>&</sup>lt;sup>16</sup>Securities and Exchange Commission (2012) reports that the vast majority of municipal bonds have enjoyed tax-exempt status prior to the crisis, with taxable bonds only accounting for 11% of total principal amount of issuance in 2008.

where  $W_t^k(j)$  denotes the wealth of investor j of type k at t, and  $\gamma$  is the risk aversion parameter. While  $\gamma$  is assumed to be homogeneous across all investor pools, allowing for heterogeneity has no qualitative effect on the main results. The evolution of wealth is mark-to-market, given as follows:

$$
W_{t}^{k}(j) = W_{t-1}^{k}(j) + (p_{t}^{i} - p_{t-1}^{i}) x_{i,t-1}^{k}(j); \quad k \in \{agg, \; con\}.
$$
 (5)

Due to the constant risk aversion, the investors' initial wealth does not affect their optimization problem, and I therefore assume  $W_0^{agg}$  $C_0^{agg}(j) = W_0^{con}(j) = 0.$ 

Another implicit assumption is that the investors completely disregard an insurer's payout capacity at  $t = 0$  if she is rated below AAA at issuance. This shortcut prevents bond insurance by an insurer without AAA rating. For a small number of bonds in the asset universe, and with reasonable parameter values, it is possible to show numerically that the intermediate option of bond insurance without AAA rating is dominated by either bond insurance with AAA rating or leaving the bond uninsured.

Finally, for each bond  $B_i \in \{1, ..., I\}$ , the market clearing condition is given by:

$$
\int_0^{\lambda_i} x_{i,t}^{agg}(j) \, dj + \int_{\lambda_i}^1 x_{i,t}^{con}(j) \, dj = 1,\tag{6}
$$

since the volume of each bond issuance is normalized to 1.

# 4. Overinsurance under the rational benchmark

In this section, I derive the model's principal result of overinsurance under the fully rational benchmark. In other words, all participants' perceived probabilities correspond to the objective probabilities specied in Section 3.1.

# 4.1. Insurance decision at issuance  $(t = 0)$

Since the bond insurance market is a duopoly, I consider without loss of generality whether issuer i has an incentive to accept insurer  $A$ 's offer or not.  $A$ 's offer is accepted for sure if the following conditions are met. First,  $A$ 's insurance premium offer must be less than the issuance price differential  $(\chi^i_A < \Delta_0^i)$ . Second, it must also be less than the competitor's offer  $(\chi_A^i < \chi_B^i)$ . If  $\chi_A^i = \chi_B^i < \Delta_0^i$ , then the standard assumption applies and both insurers' offers are equally likely to be accepted.

However, from the insurer's perspective, the insurance premium must be actuarially fair. It is worth noting that, in the absence of a further shock at  $t = 1$ , an insurer automatically satisfies the AAA rating criteria at  $t = 1$  if she has raised sufficient capital at  $t = 0$ . In other words, a rating downgrade is not an issue at  $t = 1$ . If so, using (2), and given the CRA's rating criteria, the insurer's reservation premium for bond insurance is given by the sum of a bond's expected credit loss and the additional cost of equity capital incurred by the insurance of  $B_i$ . Formally, an insurance offer is made whenever:

$$
\chi_j^i \ge E_0(\Theta_i) + c_0^j (\mu_i \theta_i - E_0(\Theta_i)), \quad j \in \{A, B\}.
$$
 (7)

I denote the right hand side, namely insurer j's minimum required insurance premium of  $B_i$ , as  $\Gamma^i_{j,0}$ . Since both insurers' risk assessments of  $B_i$  are identical, any difference in  $\Gamma^i_{j,0}$  arises only from a difference in their respective costs of equity capital at  $t = 0$ . More specifically,  $\Gamma_{A,0}^i < \Gamma_{B,0}^i$  for all  $B_i \in \{B_1, ..., B_I\}$  if and only if  $c_0^A < c_0^B$  and vice versa.

In short,  $B_i$  will be insured for sure if:

$$
\Delta_0^i > \min\left(\Gamma_{A,0}^i, \ \Gamma_{B,0}^i\right). \tag{8}
$$

Then, the investors' optimization conditions, when combined with (8), yield Proposition 1, which reveals that bond insurance occurs only when at least one of the insurers' costs of equity capital is sufficiently low:

**Proposition 1 (bond insurance at**  $t = 0$ ). Let  $\varsigma_i \equiv \mu_i \pi_i^g + (\mu_i - 1) \pi_i^m$ . Then,  $B_i \in$  ${B_1, ..., B_I}$  is insured with certainty at  $t = 0$  if and only if:

$$
\min\left(c_0^A, c_0^B\right) < \frac{\varsigma_i \pi_i^b \exp\left(\frac{\gamma \mu_i \theta_i}{\lambda_i}\right) + \left(\varsigma_i - \mu_i + 1\right) \pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) + \left(\varsigma_i - \mu_i\right) \pi_i^g}{\varsigma_i \pi_i^b \exp\left(\frac{\gamma \mu_i \theta_i}{\lambda_i}\right) + \varsigma_i \pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) + \varsigma_i \pi_i^g} < 1,\tag{9}
$$

where  $\pi_i^b = 1 - \pi_i^g - \pi_i^m$  as before.

### Proof. See Appendix. ■

In other words, bond insurance only becomes viable when an insurer can raise equity capital cheaply enough so that she expects non-negative profits and fulfills the CRA's rating criteria at the same time. The right hand side of (9) then yields the maximum cost of equity capital at which bond insurance would occur for bond  $B_i$ , referred to as the "insurance threshold" throughout the paper.

For a better understanding of this proposition, consider a bond  $B_i$  with the following set of parameters. Suppose  $\pi_i^g = 99\%$ ,  $\pi_i^m = 0.9\%$ , and  $\pi_i^b = 0.1\%$ . The baseline loss,  $\theta_i$ , is set at 25%, implying a recovery rate of 75%. In addition,  $\mu_i = 2$  such that, under the large loss scenario, the bond's recovery rate drops to 50%. In practice, such a bond holds a shadow rating of either AA or A. If the risk aversion parameter  $(\gamma)$  is set at 2 and the proportion of aggressive investors in the potential investor pool is  $25\%$ , then  $B_i$  will be insured as long as one of the insurers' costs of equity capital is lower than 7:4%.

Holding the equity market conditions of bond insurers constant, a bond is more likely to be insured when its insurance threshold is high. Then, a bond may be casually referred to as "more likely to be insured" when its insurance threshold increases. As this threshold depends on a number of model parameters, Proposition 2 claries their respective relationships:

### Proposition 2 (model parameters and the likelihood of bond insurance).

- (i) An increase in  $\gamma$  or  $\theta_i$  makes bond insurance more likely.
- (ii) An increase in  $\lambda_i$  makes bond insurance less likely.
- (iii) An increase in  $\mu_i$  has an ambiguous effect on the likelihood of bond insurance.
- (iv) For sufficiently large  $\mu_i$ , a further increase in  $\mu_i$  makes bond insurance more likely.

# Proof. See Appendix. ■

First, it is worth noting that, as investors become more risk averse, their demand for bond insurance also increases. This is the likely factor that explains the traditional popularity of bond insurance in the municipal bond market; while the magnitudes of losses were generally small, the dominant presence of buy-and-hold retail investors with pension investment considerations may have implied that the investors' risk aversion was higher than in other bond markets. On the other hand, an increase in  $\lambda_i$  raises the number of aggressive investors who compete with each other in the absence of bond insurance, raising the price of an uninsured bond. This makes bond insurance a less attractive option for the issuer.

The effect of  $\mu_i$  is not monotonic due to the nature of the rating criteria. More specifically, the insurers are required to raise equity capital to cover for the worst case loss  $(\mu_i \theta_i)$  even if it is extremely unlikely to occur. If so, the extra burden of equity capital cost may be disproportionately high compared to the increase in the overall expected loss, making bond insurance an unattractive option even for the risk averse investors. However, as  $\mu_i$  increases, the investors eventually become strongly concerned about the sheer magnitude of the tail outcome. Their desire to insure against such a catastrophic outcome would then be strengthened, making bond insurance more likely.

### FIGURE 1 HERE

This is graphically illustrated in Figure 1. Using the same set of parameter values as in the earlier numerical example, Figure 1 plots how the insurance threshold changes as the four model parameters  $(\mu_i, \gamma, \theta_i)$  and  $\lambda_i$  are varied in turn. Each parameter behaves as discussed in Proposition 2.

Having derived the conditions under which a bond is insured, I briefly explain the outcome of competition in this market. Given the earlier argument, it is apparent that the insurer with a lower cost of equity capital emerges as the winner. For example, when  $c_0^A < c_0^B$ , insurer A takes over the entire market and offer min  $(\Delta_0^i, \Gamma_{B,0}^i)$  to every bond  $B_i$  that satisfies (9). When  $c_0^A = c_0^B = c_0$ , both insurers offer their reservation prices, i.e.,  $\Gamma_{A,0}^i = \Gamma_{B,0}^i = \Gamma_0^i$ , to issuer i and divide the insurance market in half. In this instance, all surplus from bond insurance is transferred to the issuers as a result of price competition.

# 4.2. Subsequent decisions at  $t = 1$  and  $t = 2$

In the absence of an external shock, no further trading occurs at  $t = 1$ , and the insurers' credit ratings remain at AAA. Both types of investors continue to hold insured bonds, while all uninsured bonds remain in the hands of aggressive investors. At maturity  $(t = 2)$ , credit losses are realized. The par of an insured bond is guaranteed by the insurer under all circumstances, and the aggressive investors bear the brunt of any credit loss realization for uninsured bonds.

# 4.3. Negative externality and the prospect of overinsurance

How does the provision of bond insurance affect the participants' overall welfare? In order to address this issue, an appropriate measure of "market welfare" must first be constructed, which I denote  $\Omega_t$ . In order to minimize any normative judgement over the definition of market welfare, it follows a standard definition, namely a simple sum of each agent's expected surplus evaluated prior to maturity at a given point in time, i.e., either  $t = 0$  or  $t = 1$ . This includes all issuers, insurers, and investors participating in the market. Cruciall, as it requires a welfare comparison of participants with different risk attitudes, I use the certainty equivalent wealth measure for the continuum of risk averse investors, with each type weighted by its respective proportion  $(\lambda_i)$ .

Since the agents' choices and information sets are unchanged between  $t = 0$  and  $t = 1$ under the rational benchmark, it must be that  $\Omega_0 = \Omega_1$ . Then, a simple way to calculate  $\Omega_t$  is to add up the agents' welfare derived from each  $B_i \in \{B_1, ..., B_I\}$ , which I denote  $\Omega_t^i$ . In other words,  $\Omega_0$  satisfies:

$$
\Omega_0 = \sum_{i=1}^I \Omega_0^i.
$$
\n(10)

This bond-by-bond analysis of market welfare yields Proposition 3, which states that bond insurance may not always be beneficial from a social perspective:

**Proposition 3** (negative externality of bond insurance). For all  $B_i$  that satisfies

$$
\min\left(\Gamma_{A,0}^i,\ \Gamma_{B,0}^i\right)\in\left(\Upsilon_0^i,\ \Delta_0^i\right),\tag{11}
$$

where  $\Upsilon_0^i \equiv \frac{\lambda_i}{\gamma}$  $\frac{\lambda_i}{\gamma}\log\left(\pi_i^g+\pi_i^m\exp\left(\frac{\gamma\theta_i}{\lambda_i}\right)\right)$  $\lambda_i$  $\left(1-\pi_i^g-\pi_i^m\right)\exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right)$  $\left(\frac{\mu_i\theta_i}{\lambda_i}\right)\big)>0,$  bond insurance occurs but is not socially desirable.

### **Proof.** See Appendix. ■

In other words, under the rational benchmark, the amount of bond insurance may be excessive relative to the social optimum. This is the paper's principal result that may initially appear counterintuitive. After all, the standard economic understanding of insurance is that risk averse investors ought to benet from transferring the bond's credit risk to the risk neutral insurer via the issuer's acceptance of an insurance contract.

This is not the case in my model for the following reason. When a bond is issued without insurance, conservative investors cannot hold the asset, leaving the remaining aggressive investors to clear the market, each of whom now has to hold a larger position in the asset. Given that their risk averse demand for the asset is a finite and negative function of the market price, the price of an uninsured bond has to be sufficiently discounted. This price discount acts as a source of welfare gain, as measured by an increase in their certainty equivalent wealth; the bond's price falls to the extent that they are more than compensated for their risk aversion.

On the other hand, by investing in a competitively priced insured bond, both conservative and aggressive investors merely receive their reservation utility. In other words, insurance entails some forgone utility gain from the aggressive investors' perspective. This forgone utility gain, from an overall social perspective, is equal to  $\Delta_0^i - \Upsilon_0^i$ , smaller than the issuance price differential  $(\Delta_0^i)$  enjoyed by the issuer, as predicted by the standard economic theory on insurance.

However, crucially, the insurer also faces a certification constraint. More specifically, an insurer has to raise equity capital for each insured bond to cover for its tail risk and abide by the credit rating agency's requirement for AAA rating. This capital cost reduces the overall internalized surplus of an insurance contract that can be shared between the issuer and the insurer, i.e.,  $\Delta_0^i$  – min  $(\Gamma_{A,0}^i, \Gamma_{B,0}^i)$ . From a social planner's perspective, this quantity has to be compared to the utility gain that the aggressive investors can generate in the absence of an insurance contract, i.e.,  $\Delta_0^i - \Upsilon_0^i$ . However, since the negotiation for an insurance contract is a private process solely between the issuer and the insurer, any bond that satisfies  $\Delta_0^i$  > min  $(\Gamma_{A,0}^i, \Gamma_{B,0}^i)$  will be insured, regardless of whether the aggressive investors can generate a greater surplus in its absence or not. Hence, insurance provision can be excessive relative to the social optimum.

It is worth noting that this result does not depend on the assumption that the investors do not face a capital requirement for their bond investment. Even for a special class of investors who face such a requirement, the model's result still stands as long as they can raise capital at sufficiently more favorable terms than the insurer. More specifically, as long as their cost of raising equity capital following the investment is less than  $\Upsilon_0^i$  –  $\min(\Gamma^i_{A,0}, \Gamma^i_{B,0}),$  the model continues to yield the overinsurance result.

This overinsurance result raises an interesting possibility, namely that the aggressive investors, while risk averse, may be willing to pay the issuer not to accept the insurance contract and increase a bond's credit risk. This occurs when the insurer's capital cost falls within an intermediate range where it is low enough for a bond insurance contract to be accepted privately but high enough that the aggressive investors can generate a greater surplus in the absence of bond insurance. It highlights the powerful role of certification constraints—either on the part of insurers in the form of a capital requirement or the investment constraint on the part of conservative investors—in the dynamics of the bond insurance market; in its absence, it is difficult to envisage a scenario where a risk averse participant is willing to deliberately make the asset riskier.

While the possible overprovision of "safe" securities has also been explored in Hanson and Sunderam (2013), the models' starting points are markedly different. In their model, making securities "safe" leads to an underproduction of valuable information; the investors' ex ante incentive to become informed is reduced when the asset is already safe and informationally insensitive. This is socially undesirable as the presence of informed investors can facilitate efficient market functioning when a bad state arises. The contribution of my model is that a similar "rational overinsurance" may be generated without resorting to informational or agency-related issues. It arises specifically from a different type of market friction, namely the participants' certication constraints.

I numerically elaborate this prospect of overinsurance by revisiting the earlier example in Section 4.1. Here, when  $\min\left(c_0^A, c_0^B\right) = 7\%$ , the cost of equity capital lies below the insurance threshold of 7.4% and  $B_i$  is thus insured. However, the internalized surplus from bond insurance is very small at around 0:002, but the negative externality of bond insurance, i.e., the  $\lambda$ -weighted increase in the aggressive investors' certainty equivalent wealth in the absence of bond insurance, is 0.026. This is a clear example of how bond insurance may not always be beneficial even with realistic parameter values.

### FIGURE 2 HERE

Furthermore, a lengthy algebraic inspection of  $U_{i,U}^{agg}$  derived in the Appendix yields that the negative externality of bond insurance increases in  $\mu_i$  and  $\theta_i$  but is non-monotonic in  $\gamma$  and  $\lambda_i$ <sup>17</sup>. In particular, the non-monotonicity of  $\lambda_i$  arises from two conflicting forces. When  $\lambda_i$  is low, a small number of aggressive investors have to clear the market. This leads to a larger price discount and stronger increase in the certainty equivalent wealth. However, this gain in certainty equivalent wealth is assigned a lower weight precisely because they are few in number. Figure 2 illustrates this relationship graphically using the identical set of parameter values as in Figure 1. At low values of  $\lambda_i$ , the magnitude of negative externality  $(\Delta_0^i - \Upsilon_0^i)$  increases rapidly, reaching its maximum at around  $\lambda_i = 0.103$ . Then, a further increase in  $\lambda_i$  leads to a rapid decay of its magnitude, approaching the lower bound of 0 as  $\lambda_i$  tends to 1.

#### FIGURE 3 HERE

Using (11), it is also possible to derive a socially optimal insurance threshold that takes into account of bond insurance's negative externality on aggressive investors, in the identical manner to Proposition  $1<sup>18</sup>$  Figure 3 graphically illustrates this discrepancy between the actual insurance threshold and the socially optimal insurance threshold.

<sup>&</sup>lt;sup>17</sup>The proof is omitted for the brevity of exposition.

<sup>&</sup>lt;sup>18</sup>As the derivation is very straightforward but the closed form solution of the socially optimal insurance threshold is signicantly lengthier, I omit the solution for expositional clarity.

It is worth noting that the magnitude of such discrepancy can be substantial in many instances. The region between the two thresholds represents where insurance contracts are socially undesirable, i.e., where overinsurance occurs.

# 5. Effect of neglected risk on market dynamics

Section 4 has explored the model's key result, namely that the participants' certification constraints could result in overinsurance in the bond insurance market. However, this still leaves the question of how the participants' underestimation of downside risk affects market dynamics. After all, it is not immediate that the key characteristic of the bond insurance market, namely the popularity of bond insurance business prior to the crisis, may be seen as a result of the participants' neglect of downside risk.

In order to address this question, I now change the participants' risk perception to be "local thinking." More specifically, following Gennaioli, Shleifer and Vishny (2012), I assume that the participants assess the credit risk of a bond at  $t = 0$  using only the two most likely states, ignoring the least likely scenario. Given the earlier assumption, this implies that the large loss scenario is initially neglected. For simplicity, the relative probabilities of the remaining likely states are assumed to remain the same, but an alteration to this probability assignment rule has no qualitative effect on the model's results. Formally, the participants' perceived probabilities of no loss and small loss for  $B_i$ are  $\pi_i^g$  $\pi_i^g / (\pi_i^g + \pi_i^m)$  and  $\pi_i^m / (\pi_i^g + \pi_i^m)$  respectively, even though the objective probabilities are governed as before.

Using this framework, I highlight two key results regarding neglected risk. In the first subsection, I highlight the  $ex$  ante implications of local thinking and their respective welfare consequences. In the second subsection, I show that the neglected risk could result in a potentially damaging insurer downgrade, and through this framework, I rationalize the 2008 monoline insurer debacle within the model framework.

# 5.1. Ex ante insurance decisions and welfare consequences

A change in the participants' risk perception alters both the uninsured price of a bond and the insurers' minimum premia. Using the identical procedure as in Section 4.1, the key condition for the acceptance of an insurance contract at  $t = 0$  may be derived as follows:<sup>19</sup>

$$
\min\left(c_0^A, \ c_0^B\right) < \frac{\pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) - \pi_i^m}{\pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) + \pi_i^g} < 1. \tag{12}
$$

Let the insurance threshold associated with (12) be denoted  $\tilde{c}_0^i$ . This value needs to be compared against the insurance threshold under the rational benchmark  $(\bar{c}_0^i)$ , which determines whether the participants' initial neglect of the large loss outcome makes bond insurance more or less likely relative to the rational benchmark. While neither threshold is always unambiguously higher, an important result is obtained:

**Proposition 4 (neglected risk and underinsurance).** For sufficiently large values of  $\mu_i$ , the initial neglect of a large loss outcome makes bond insurance less likely relative to the rational benchmark.

**Proof.** From part (ii) of Proposition 2,  $\vec{c}_0^i$  increases in  $\mu_i$  whenever  $\mu_i$  is sufficiently large. Since  $\tilde{c}_0^i$  is unaffected by a change in  $\mu_i$ , eventually it must be that  $\bar{c}_0^i > \tilde{c}_0^i$  for some large enough  $\mu_i$ .

This is another key result of the paper. When  $\mu_i$  is high, there is a possibility that the bond would suffer a severe credit loss. As the magnitude of this loss increases, the risk averse investors' desire for insurance also increases. Conversely, when this risk is neglected, the bond is perceived to be safer and their desire for insurance subsides.

While the monoline insurers were criticized for neglecting the possibility of a catastrophic loss, Proposition 4 reveals that this alone cannot account for the popularity of bond insurance prior to the crisis. If anything, the fact that monoline insurers rapidly expanded in the bond insurance market, when taken alone, favors the hypothesis that the participants were rational in their risk management. On the other hand, if there are good grounds to believe that the participants neglected the downside risk, then Proposition 4 suggests there would have been even greater demand for bond insurance had the participants been fully rational. Thus, the monoline insurers' active presence in the market and their neglect of the tail outcome are susceptible to a post hoc, propter hoc fallacy; the fact that these two characteristics coexisted should not be interpreted as a causal relationship.

### FIGURE 4 HERE

 $19\text{As the procedure remains identical to the proof of Proposition 1, the derivation is omitted for brevity.}$ 

As for other model parameters, i.e.,  $\gamma$ ,  $\theta_i$ , and  $\lambda_i$ , the results are less clear cut. A change in one of these parameters affects both  $\tilde{c}_0^i$  and  $\bar{c}_0^i$  in the same direction, and its effect on the *relative* magnitude of these two thresholds is generally ambiguous. However, simulation results indicate that, for reasonable parameter values,  $\tilde{c}_0^i$  is generally less sensitive to a parameter change than  $\bar{c}_0^i$ . <sup>20</sup> Figure 4 illustrates this graphically using the identical set of parameter values as before. In particular, there is a strong divergence between  $\tilde{c}_0^i$  and  $\bar{c}_0^i$  as  $\mu_i$  increases;  $\bar{c}_0^i$  rapidly increases but  $\tilde{c}_0^i$  remains unaffected, with  $\bar{c}_0^i$ lying below  $\tilde{c}_0^i$  only for a restricted parameter range of  $\mu_i \in [1.2, 1.6]$ .

How does this tendency to underinsure affect the overall market welfare? I now repeat the welfare analysis of Section 4.3. Before I proceed, however, the definition of "welfare" has to be clarified. I maintain that the participants' welfare is computed from an omniscient perspective, using the objective probabilities instead of their perceived probabilities. In other words, all objectively possible scenarios are included regardless of whether the agents themselves take into account of a particular outcome or not. Then, another important result is obtained in Proposition 5:

Proposition 5 (possible welfare improvement under local thinking). Suppose  $\mu_i$ is sufficiently high that  $\tilde{c}_0^i < \bar{c}_0^i$  for bond  $B_i$ . If so, the agents' welfare derived from  $B_i$  could be higher relative to the rational benchmark when the participants engage in local thinking.

### **Proof.** See the numerical example below.

How does this surprising welfare improvement occur? Proposition 3 has already shown that some bond insurance contracts are socially undesirable under the rational benchmark. Proposition 4, however, has also shown that the participants' desire for bond insurance subsides when they neglect the downside risk. If this tendency to underinsure reduces the extent of overinsurance—allowing the aggressive investors to take advantage of an uninsured bond's price discount—then the overall market welfare could be increased accordingly.

I demonstrate this using the earlier numerical example, where  $\bar{c}_0^i = 7.4\%$ . A simple computation also yields  $\tilde{c}_0^i = 5.4\%$ . Thus, if min  $(c_0^A, c_0^B) = 7\%$  as before, a bond would be insured under the rational benchmark but not when the participants engage in local thinking. Now, let  $\tilde{\Omega}_{0,U}^{i}$  denote the overall social welfare of local thinking participants derived from an uninsured  $B_i$ , with their welfare calculated using objective, not perceived,

<sup>20</sup>Simulation codes are available from the author.

probabilities. First, in this instance,  $\tilde{p}_{0,U}^i = 0.984$ , which is larger than  $p_{0,U}^i = 0.961$ . In other words, from an omniscient perspective, the bond is mispriced upward as a result of the participants' local thinking. Yet, it still turns out that  $\Omega_{0,I}^{i} - \tilde{\Omega}_{0,U}^{i} = -0.001$ , implying that the welfare is lower under the rational benchmark. By ameliorating the problem of overinsurance, the local thinking participants' tendency to underinsure could lead to a welfare improvement even when the bond's initial price is set "incorrectly."

# 5.2. Subsequent prospect of insurer downgrade

Based on the analysis of the previous subsection alone, one may be left with the impression that the underestimation of credit risk is desirable from a welfare perspective. However, it would be dangerous to present an unconditionally positive picture of neglected risk. The main reason for this is that an insurer downgrade can occur as a result of local thinking.

In contrast to the rational benchmark, a local thinking insurer does not raise sufficient capital to cover for the true worst case loss. This leads to two possible scenarios. On one hand, the participants' perceived probabilities may not change at  $t = 1$ . If so, no trading would occur at  $t = 1$ . At maturity, when a hitherto neglected large loss outcome is realized, an insurer's capital buffer may fall short of the claim payout demand. Formally, the amount of insurer j's capital shortfall at maturity is given by  $\max\left(0, \sum_{i=1}^{I} I_j^i (\Theta_i - \theta_i)\right)$ . If such a shortfall occurs, general arrangements in practice demand that all investors requiring insurance payout take a "haircut." This haircut is defined as a proportion of their demanded payout, given by  $\sum_{i=1}^{I} I_j^i (\Theta_i - \theta_i) / \sum_{i=1}^{I} I_j^i \Theta_i$ .

On the other hand, consider a more interesting case where an exogenous signal arrives at  $t = 1$ . I assume that, following this signal, the posterior possibilities of no loss for all bonds decrease to the extent that they are completely discounted by local thinking participants. For convenience, further assume that the relative posterior probabilities of small loss and large loss remain identical, although this assignment rule, once again, has no bearing on the qualitative results of the paper. In other words, the perceived probabilities of local thinking agents for bond  $B_i$  at  $t = 1$  changes to  $\pi_i^m/(1 - \pi_i^g)$  $\binom{g}{i}$  for small loss and  $\left(1 - \pi_i^g - \pi_i^m\right) / \left(1 - \pi_i^g\right)$  $\binom{g}{i}$  for large loss. Let these perceived probabilities be denoted  $\tilde{\pi}_i^m$  and  $\tilde{\pi}_i^b$  respectively.

Following the arrival of a signal, the CRA now demands that an insurer's capital buffer increases to  $\mu_i \theta_i$  for each insured  $B_i$  at  $t = 1$ . If the required capital is raised by an insurer, then her AAA rating is retained. However, she may choose not to raise more capital at  $t = 1$  and accept a rating downgrade. More formally, this occurs when the equity capital cost of abiding by the CRA's new capital requirement is larger than the reputational cost of downgrade, which occurs whenever:

$$
c_1^j \sum_{i=1}^I I_j^i (\mu_i - 1) \theta_i > \kappa_j.
$$
 (13)

This highlights the important role of the insurers' equity market conditions in determining their ability to defend the AAA rating. Indeed, the first round of rating downgrade for Ambac in January 2008 occurred immediately after its plan to raise \$1 billion of fresh equity capital was canceled due to depressed share price.

When the insurers raise additional capital, the investors' decisions remain unchanged at  $t = 1$ , and the market outcome is comparable to Section 4.2. The problem arises following an insurer downgrade. Then, all conservative investors holding an asset insured by the downgraded insurer have to liquidate their holdings. This exit price is determined in equilibrium through an interaction of aggressive and conservative investors.

In order to describe how this exit price is calculated in more detail, suppose that there are two bonds in the downgraded insurer's portfolio, namely  $B_1$  and  $B_2$ . Since the downgraded insurer's capital amounting to  $\theta_1 + \theta_2$ , local thinking aggressive investors are faced with the following scenarios. With probability  $\tilde{\pi}_1^m \tilde{\pi}_2^m$ , both bonds incur small losses and the capital buffer is sufficient. With probability  $\tilde{\pi}_1^m \tilde{\pi}_2^b$ ,  $B_1$  incurs a small loss but  $B_2$ 's credit loss is large, so the haircut amounts to  $(\mu_2 - 1) \theta_2 / (\theta_1 + \mu_2 \theta_2)$ . With probability  $\tilde{\pi}_1^b \tilde{\pi}_2^m$ , the haircut is given by  $(\mu_1 - 1) \theta_1 / (\mu_1 \theta_1 + \theta_2)$ , and lastly, when both bonds incur large losses, the haircut is  $\left(\sum_{i=1}^2 (\mu_i - 1) \theta_i\right) / \left(\sum_{i=1}^2 \mu_i \theta_i\right)$ . Substituting these into the aggressive investors' optimization conditions yield the bond's downgraded price, which remains in fully tractable, closed form. However, while theoretically possible, it quickly becomes computationally cumbersome as the number of bonds in the insurer's portfolio increases.

I now present this graphically. Suppose the two bonds are identical to each other in all respects, with the parameter values corresponding to the earlier numerical example. Let  $c_0^A = 5\% < c_0^B$  so that these bonds are insured by insurer A at  $t = 0$ , and further suppose  $c_1^A$  increases to 10%. Finally, let  $\kappa_A = 0.03$ , which is smaller than the additional cost of equity capital.

## FIGURE 5 HERE

In this case, both bonds experience a price discount of around 6:2% upon insurer

downgrade. Figure 5 also plots the period 1 downgraded price of  $B_1$  as a function of various model parameters. While a change in  $\gamma$  or  $\lambda_1$  does not affect the insurer's downgrade decision, it does affect bond's price upon insurer downgrade. As expected, an increase in risk aversion  $(\gamma)$  reduces the bond's downgraded price while an increase in the proportion of aggressive investors  $(\lambda_1)$  has the opposite effect. A change in  $\mu_1$  is somewhat different; the price of  $B_1$  initially remains at 1 but it jumps downward to 0.982 at  $\mu_1 = 1.2$ , where the insurer begins to accept a rating downgrade. Beyond this point, a further increase in  $\mu_1$  reduces its price.

How does this affect the market welfare at  $t = 1$ ? Without loss of generality, suppose insurer A is facing this threat of a rating downgrade. Furthermore, let  $\Omega_{1,def}^A$  denote the total welfare generated by insurer A if she defends her credit rating, and  $\Omega_{1,down}^A$  denote the corresponding measure when the insurer accepts a rating downgrade. Then, using a similar line of reasoning as in the proof of Proposition 3, it is possible to show that:

$$
\Omega_{1,down}^{A} - \Omega_{1,def}^{A} = c_1^{A} \sum_{i=1}^{I} I_A^i \left(\mu_i - 1\right) \theta_i - \kappa_A \qquad (14)
$$
\n
$$
+ \sum_{i=1}^{I} I_A^i \left\{ -\frac{\lambda_i}{\gamma} \log \left( -\tilde{U}_{i,U}^{agg} \left(p_{1,D}^i\right) \right) - \frac{\left(1 - \lambda_i\right)}{\gamma} \log \left( -\tilde{U}_{i,U}^{con} \left(p_{1,D}^i\right) \right) \right\},
$$

where  $p_{1,D}^i$  is the price of downgraded  $B_i$ , while  $\tilde{U}_{i,U}^{agg}(p_{1,D}^i)$  and  $\tilde{U}_{i,U}^{con}(p_{1,D}^i)$  denote the expected utilities of aggressive and conservative investors trading at this downgraded price following an insurer downgrade.

Crucially, the last term in  $(14)$  represents the knock-on effect of an insurer's downgrade decision on the investors' welfare, not internalized in her decisionmaking. In particular, an insurer's rating downgrade triggers a forced sale of the conservative investors' holdings at a "fire-sale price" (Coval and Stafford, 2007), harming their utility. The aggressive investors also suffer mark-to-market losses on their existing holdings, but they absorb the conservative investors' positions at a discount, rendering the overall effect on their welfare to be ambiguous.

For the earlier numerical example, the externality term in  $(14)$  amounts to  $-0.043$ . Even when the internalized benefit of the insurer is accounted for, the overall welfare impact of an insurer downgrade, i.e.,  $\Omega_{1,down}^A - \Omega_{1,def}^A$ , remains negative at -0.023. In other words, although it is in the insurer's interest not to defend her credit rating, its adverse spillover effect on the investors dominates her private benefit.

## FIGURE 6 HERE

Figure 6 further demonstrates that the social undesirability of insurer downgrade holds more generally. Regardless of how the parameter values are varied, the welfare differential  $(\Omega_{1,down}^A - \Omega_{1,def}^A)$  is always negative, implying that it is always better to encourage the bond insurer to retain her AAA rating. In particular, there is a large downward jump in the welfare differential at  $\mu_1 = 1.2$  where the insurer switches from defending the AAA rating to accepting a downgrade. However, beyond this point, a further increase in  $\mu_1$ leads to an improvement in the welfare differential. In other words, while the conservative investors suffer from a forced exit at a depressed price following an insurer downgrade, the insurer's capital cost needed to defend her rating also becomes an important welfare consideration for large values of  $\mu_1$ .

# 6. Discussion

# 6.1. Main predictions of the model

The model has derived three main predictions regarding the bond insurance market. Before I proceed with the discussion, I reiterate them in an explicit form for the clarity of exposition:

- (P1) Participants' certication constraints lead to overinsurance in the bond insurance market.
- (P2) Participants' underestimation of downside risk results in a tendency to underinsure relative to the rational benchmark, which can potentially ameliorate the aforementioned problem of overinsurance.
- (P3) However, this neglected downside risk also opens up the possibility of an insurer downgrade. Though privately optimal from an insurer's perspective, it leads to a fire sale of the conservative investors' existing holdings in the market and harms their welfare.

# 6.2. Extensions and robustness checks

Since the model relies on a number of important assumptions, I consider various extensions in order to check whether or not the model's main results hold when some of these assumptions are relaxed or altered. As it will become clear, the model's key qualitative results remain intact for a variety of scenarios.

#### 6.2.1. Alternative investor utility specifications

In the baseline model, the investors' preference was characterized by the exponential CARA utility. The reason for this was that the CARA utility yields a neat closed form solution for the price of an uninsured bond, which cannot be guaranteed by other types of utility specications. However, it is worth checking whether or not the model's overinsurance result is driven by the choice of a particular utility specification. Thus, I consider three other popular risk averse utility specications, namely: (i) CRRA utility with  $U=\frac{W^{1-\alpha}}{1-\alpha}$  $\frac{W^{1-\alpha}}{1-\alpha}$ , (ii) log utility with  $U = \log W$ , and (iii) quadratic utility with  $U = W - \frac{\beta}{2}W^2$ .

For the ease of exposition, I revisit the numerical example considered throughout Sections 3 and 4. I first consider the CRRA utility with  $\alpha = 2$ . As the investors' initial wealth now matters, both the conservative and aggressive investors' initial wealth levels are set homogeneously at 1. Furthermore, let  $\min (c_0^A, c_0^B) = 3.5\%$ . While a closed form solution cannot be attained in this instance, numerical method yields  $p_{0,U}^i \approx 0.945$ . Since the internalized surplus of the issuer and the insurer is 0:035, an insurance contract is accepted. However, as the forgone utility gain of aggressive investors is 0:176, the overinsurance result remains unchanged. As local thinking continues to result in a tendency to underinsure, the potentially welfare improving nature of insurance underprovision can also be shown with a suitable set of parameters.

When the log utility specification is employed with the investors' initial wealth levels unchanged at 1 and min  $(c_0^A, c_0^B) = 3.5\%$ , then  $p_{0,U}^i \approx 0.977$ . Once again, while the internalized surplus is 0:003, the aggressive investors' forgone utility gain from insurance is 0:017. A similar result may be obtained with quadratic utility. If the investors' initial wealth is set at 0,  $\beta$  at 3, and  $\min(c_0^A, c_0^B) = 3.5\%$ , the internalized surplus of an insurance contract is 0:055 but its negative externality on aggressive investors stands at 0:211. Furthermore, in both instances, the initial neglect of downside risk continues to result in a tendency to underinsure, with similar welfare consequences.

Thus, as long as the investors are risk averse—which leads them to value a fall in the shadow price of an uninsured bond in a different manner to the "brokering" bond issuer—the exact choice of their utility specification is not a crucial factor behind the model's main results.

### 6.2.2. Issuer's ex ante decision on issuance volume

The prospect of aggressive investors' welfare gain from a discounted uninsured bond may be sensitive to the assumption that the issuer has to issue 1 unit regardless of whether the bond is insured or not. This assumption may be justied from the fact that the decision to insure a bond is generally made late in the issuance stage, by which point it is not straightforward to alter the issuance volume. However, it is still worth analyzing a more flexible set-up, because an issuer with sufficient knowledge of investor composition and risk characteristics can anticipate how the aggressive investors would react to a change in issuance volume in the absence of insurance, which may in turn affect her initial loan acquisition decision.

Thus, I consider a following variant of the model. For simplicity, suppose issuer  $i$  is allowed to choose her issuance volume after the insurance contract negotiation. More specifically, she may purchase  $q_i \leq 1$  unit of loans at a constant price of  $l_0^i < 1$  following the negotiation but prior to issuance. Further suppose issuer  $i$  has full knowledge of the composition of the investor pool and the investors' risk attitude, as well as the insurers' respective capital costs.

Then, I obtain the following results. First, since  $p_{0,I}^i$  is independent of issuance volume, an insured bond will always be issued with issuance volume of 1. However, given the aggressive investors' optimal demand,  $p_{0,U}^i$  increases as the issuance volume falls. Thus, when computing the profit from an uninsured bond, i.e.,  $(p_{0,U}^i - l_0^i) q_i$ , the issuer has to balance both price and quantity effects, and the optimal, profit-maximizing  $q_i$  may not turn out to be 1. In other words, the issuer may reduce the issuance volume in order to increase the price of an uninsured bond and consequently her prot.

For example, I consider the case of earlier numerical example and set min  $(c_0^A, c_0^B)$  =  $6\%$  and  $l_0^i = 0.9$ . In other words, loans may be purchased up to 1 unit at a constant discount of 10%. In the absence of bond insurance, a prot-maximizing issuer would only issue 0:88 units of bond, raising the uninsured bond's price to 0:972 as opposed to the baseline price of 0:960. The ensuing prot of bond issuer from an uninsured bond is 0:064, which yields her profit differential from insurance to be  $0.1 - 0.064 = 0.036$ . Moreover, as a cost-effective insurer's minimum offer is 0.032, insurance occurs under this scenario, with the issuer and the insurer's internalized surplus standing at 0.004. However, this still entails forgone welfare gain for aggressive investors amounting to 0.016, substantially larger than the internalized surplus computed above.

In other words, the model's main result of rational overinsurance is robust to relaxing the assumption that bond issuer has to issue 1 unit of bond under all circumstances. The main reason for this is that the issuer's prot maximizing strategy generally retains some degree of price discount. If so, there will always be some extent of forgone welfare gain as a result of insurance. Then, it is always possible to find a suitable level of the insurers'

equity capital cost where the internalized surplus of the issuer and the insurer is positive but arbitrarily small, making the internalized surplus from an insurance contract smaller than its negative externality on aggressive investors.

### 6.2.3. Large number of possible credit outcomes

A salient aspect of this model is its tractability in a more general setting. Suppose that each  $B_i$  has more than three possible credit outcomes at maturity. Formally,  $\Theta_i =$  $\{\mu_1\theta_i, \mu_2\theta_i, ..., \mu_M\theta_i\}$ , where  $M > 3$ ,  $\mu_1 = 0$ ,  $\mu_2 = 1$ ,  $\mu_M\theta_i \le 1$ , and  $\mu_j > \mu_k$  for all  $j > k$ . M may be arbitrarily large as long as the nature of credit risk remains discrete. The corresponding probability set is given by  $\Pi_i = {\pi_1, \pi_2, ..., \pi_M}$ . Then, under the rational benchmark, the bond's issuance price differential is given by:

$$
\Delta_0^i = \frac{\theta_i \sum_{j=1}^M \pi_j \mu_j \exp\left(\frac{\gamma \mu_j \theta_i}{\lambda_i}\right)}{\sum_{j=1}^M \pi_j \exp\left(\frac{\gamma \mu_j \theta_i}{\lambda_i}\right)},
$$
\n(15)

while each insurer j's reservation price for insuring  $B_i$  is given by:

$$
\Gamma_{j,0}^{i} = \theta_{i} \left[ \sum_{j=1}^{M} \pi_{j} \mu_{j} + c_{0}^{j} \left( \mu_{M} - \sum_{j=1}^{M} \pi_{j} \mu_{j} \right) \right].
$$
 (16)

A simple inspection of (15) and (16) reveals that, even when there are more than three possible credit outcomes at maturity, the insurance decision will continue to be characterized by a single insurance threshold at  $t = 0$ , denoted  $\hat{c}_0^i$ :

$$
\hat{c}_0^i = \frac{\sum_{j=1}^M \pi_j \mu_j \exp\left(\frac{\gamma \mu_j \theta_i}{\lambda_i}\right) - \left(\sum_{j=1}^M \pi_j \mu_j\right) \left(\sum_{j=1}^M \pi_j \exp\left(\frac{\gamma \mu_j \theta_i}{\lambda_i}\right)\right)}{\left(\mu_M - \sum_{j=1}^M \pi_j \mu_j\right) \left(\sum_{j=1}^M \pi_j \exp\left(\frac{\gamma \mu_j \theta_i}{\lambda_i}\right)\right)}.
$$
(17)

The same technique can be applied to the case of local thinking when the agents consider  $L < M$  most likely scenarios among the possible credit outcomes. Using this framework, it is straightforward to show that the model's main results—namely overinsurance under the rational benchmark, the tendency to underinsure under local thinking, and the damaging prospect of an insurer downgrade—all continue to hold. Thus, the results of my model are not by-products of a three-point discrete credit risk model.

### 6.2.4. Credit risk interdependence

Due to the nature of the model set-up, any interdependence of the bonds' credit risks has little bearing on the agents' decisionmaking process. First, since the issuer only earns the intermediation spread at issuance, she has little concern over how her bond's credit risk relates to other bonds. Second, the CRA has no incentive to revise its rating criteria if it believes that the bonds' credit risks are positively correlated; if anything, it strengthens the rationale behind its emphasis on the capital coverage for the \worst case" portfolio loss. Third, an increase in the correlation of the bonds' credit risks does not affect the insurers' decisions because of their risk neutrality and the rating criteria.

Thus, any effect of the bonds' interdependence is restricted to the investors' optimization problem. However, since the investors in this model only consider investing in one particular bond, its interdependence with other bonds does not affect their decisions. Thus, all of the model's results remain intact regardless of whether the bonds' credit risks are interrelated or not. Nevertheless, this critically depends on the assumption that investors have access to one particular bond. While it is a suitable assumption for the municipal bond market, with the dominant presence of buy-and-hold retail investors, it may be a less suitable assumption in securitized and structured product markets.

However, a combination of credit risk independence and the investors' limited access to one particular bond yields very similar results to the case where the investors are allowed to invest freely across the asset universe but the bonds are highly correlated, in close accordance with the investors' optimization problem in practice. After all, risk averse investors' desire to hold multiple assets stem from the benefits of diversification, which become eroded when the bonds' credit risks are highly correlated to each other. Thus, while the two assumptions are less realistic in nature when considered in isolation, their interaction can nevertheless yield results that are close to the actual market.

#### 6.2.5. Insurer default and imperfect access to insurer capital

Throughout the paper, I maintain an implicit assumption, namely that the guarantee by a AAA-rated insurer makes the asset completely riskless. In other words,  $p_{0,I}^i = 1$ is always guaranteed for all  $B_i \in \{B_1, ..., B_I\}$ . In the baseline model setting, insurer default is not an issue; as the insurer builds sufficient capital to cover for the worst case portfolio loss, the claimholders are always protected as long as they have full access to the insurer's capital without hindrance.

However, in practice, market participants do worry about insurer defaults. While

the CDS premia of Ambac and MBIA both remained below 20 basis points in early 2007, they both reached 800 basis points by early 2008. Thus, to make the scenario more interesting, suppose insurer  $j$  defaults for some exogenous reason at maturity with probability  $\varepsilon_j$ . Insurer default occurs independent of the bonds' credit loss realizations, although relaxing this assumption has no qualitative effect on the model's results. More importantly, upon default, only a fraction  $\psi_j < 1$  of her capital is accessible to the claimholders. Finally, for expositional convenience, let  $\psi_i \mu_i \geq 1$ .

In this instance, the insurer continues to be rated at AAA, as she holds sufficient capital base to cover for the worst case loss. Moreover, her capital cost remains unchanged. However, from the investors' perspective, the insured bond now carries some credit risk as a credit loss of  $(1 - \psi_j) \mu_i \theta_i$  occurs with probability  $\varepsilon_j \pi_i^b$ . If so, its price,  $p_{0,I}^i$ , will be strictly less than 1. For the numerical example considered throughout the paper, if  $\varepsilon_j = 0.5$  and  $\psi_j = 0.5$  then  $p_{0,I}^i$  falls to 0.999. However, the overinsurance result remains intact; the internalized surplus of the issuer and the insurer is 0:006 but its weighted negative utility on conservative and aggressive investors amounts to 0:024. Thus, the baseline model's overinsurance result can still be obtained in a more realistic setting with "risky" insured bonds.

### 6.2.6. Infinite time horizon with repeated bond issuance

Throughout the paper, I considered a one-off issuance of bonds. However, it is possible to extend the model toward a more realistic setting where new bonds are issued every period and the time horizon is innite. The agents' choices can be modeled in a number of ways, but consider the following scenario. Suppose the insurers maximize the sum of their discounted expected utility while a new pool of potential investors arrive for each bond every period. Further suppose that an exogenous signal arrives at each t for local thinking participants, which tilts their perceived posterior probabilities. Lastly, an insurer's AAA rating is deemed credible regardless of its past downgrade history as long as she is perceived by the CRA to hold a sufficient capital buffer for all her outstanding insurance claims.

Then, a simple inspection of this problem reveals that the reputational cost of a downgrade at a certain  $t < \infty$ , an exogenous parameter in the baseline model, corresponds to the loss of the insurer's expected profit from new insurance business at  $t$ . This simple thought experiment demonstrates that the inclusion of an exogenous reputation cost in the model is not arbitrary; it reflects the essence of repeated interactions in practice.

# 6.3. Discussions and policy implications

The model raises a number of important regulatory implications. Regulatory concerns regarding the bond insurance industry have mainly centered on its role as contagious transmission mechanism from a macroprudential perspective. By holding a large number of bonds in the insurance portfolio, these insurers create systemic linkages across various financial instruments; credit ratings of municipal, corporate, and securitized bonds all become tied to the capital prudence of a small number of bond insurers. Thus, unexpected losses in the structured bond market can affect investors in the municipal bond market and vice versa. Moody's (2015) highlights this problem in their updated rating methodology, where it calls for the monoline insurers to be aware of these systemic considerations as well as the increased tail dependence of the constituent assets in their portfolio.

Thus, it is no surprise that the regulatory response to the 2008 monoline insurer debacle has largely been confined to damage limitation for more traditional areas of the bond market. NYID brokered a deal between MBIA and FGIC in August 2008, through which MBIA acquired most of the FGIC's relatively safe municipal bond portfolio. Following the deal, FGIC was left with a badly exposed portfolio and the CRAs eventually withdrew their credit ratings, while MBIA was essentially given a lifeline. As for Ambac, the regulators in Wisconsin ordered a segregation of Ambac's liabilities, creating separate accounts for municipal bonds and "toxic" products. The common aim of both actions, which essentially created seniority among the claimholders in favor of municipal bond investors, was to prevent a contagion of mortgage-related problems out of these systemic, macroprudential concerns.

My model adds to these regulatory concerns from a microprudential perspective. Not only does the provision of bond insurance creates dangerous systemic links, but it also has unfavorable welfare implications for the investors without a rating-based certication constraint. The model's result thus ought to be viewed in a complementary manner to the existing macroprudential concerns. In other words, some insurance contracts ought to be discouraged not only because it creates systemic issues when the insurer's financial health is in doubt, but also because there exist a group of investors who can extract more surplus out of the bond in its absence. In this regard, it provides a stronger theoretical basis for the regulatory limits on bond insurance practices following the crisis, such as Circular Letter No. 19 issued by the NYID in September 2008.<sup>21</sup>

 $^{21}$ In the letter, the regulators set strict limits on offering insurance policies for CDOs and noninvestment-grade bonds, called for a larger capital buffer against mezzanine junior investment-grade bonds, and encouraged the insurers to re-evaluate their current risk management practices.

Moreover, given the critical role of the participants' certification constraints in my model, it places an important burden on the CRAs in improving the market functioning. While a bond's inherent risk characteristics or the investors' certification constraints cannot be controlled, the CRA can still have a direct impact on the market outcome through its capital requirement for AAA-rated insurers. Suppose that, due to favorable equity market conditions, bond insurance activities are vibrant. Furthermore, suppose there are reasons to suspect that a substantial proportion of these contracts are not socially desirable. Then, the CRA can always revise its rating criteria and make its capital requirement more stringent. For example, instead of covering for the worst case loss, the insurer may be asked to cover for a multiple  $(1.2x \text{ or } 1.4x, \text{ for example})$  of the worst case loss. By doing so, the CRA directly affects the overall welfare of the participants in the bond insurance market.

In this respect, the model supports Duan and Van Laere's (2012) notion that \credit ratings bear the characteristics of a public good (p. 3240)." Not only do these ratings provide clear, accessible information on the bond issuers' creditworthiness, my model highlights that they can effectively determine which investors can hold a given bond. In other words, in the presence of certication constraints, rating models act as a powerful tool of asset allocation. Whether or not the CRA fully realizes this vital role of their rating model development is open to question. Given the recent literature on rating shopping as a source of rating inflation (Bolton, Freixas and Shapiro, 2012), for example, the answer does not appear wholly favorable.

This is an important area of future regulatory discussion, because the model's main theme carries beyond the bond insurance market. For example, due to the nature of the Basel II capital regulations, a bank's purchase of a credit default swap (CDS) contract from an institution with high credit rating also had broadly similar effects prior to the crisis; it allowed a bank to free up a sizeable proportion of its regulatory capital (Basel Committee on Banking Supervision, 2004). If so, my model suggests that their consequent increased presence in other market segments, such as the market for asset-backed commercial papers (ABCPs), could have had adverse repercussions on the overall market welfare. Thus, while discussed within the narrow framework of bond insurance in this paper, the model's main message may prove relevant for other areas of the financial market.

# 7. Conclusion

This paper has engaged in a comprehensive theoretical analysis of the market for bond insurance. The model's principal result was that the participants' certification constraints opened up a possibility of overinsurance, where some accepted insurance contracts were not desirable from an overall welfare perspective. By turning junior investment-grade bonds into AAA-rated securities, the provision of bond insurance eliminated the price discount of an uninsured bond, which could have otherwise yielded welfare gain for the rating-unconstrained aggressive investors. As the issuer and the insurer had no incentive to fully internalize this forgone welfare gain, an insurance contract resulted in negative externality. Crucially, its extent could potentially dominate the privately internalized surplus for an intermediate range of the insurer's cost of equity capital.

Furthermore, by considering the case of local thinking participants who initially neglect a bond's downside risk, the paper has also shown that the criticism of monoline insurers in the popular press since the crisis might be susceptible to a post hoc, propter hoc fallacy. Although the market participants' underestimation of mortgage-related credit risk and the popularity of bond insurance in the securitized products market were both observed prior to the crisis, my model argued that these two regularities ought not to be treated as a causal link. If anything, the active presence of bond insurers in the securitized market, when taken alone, favored the hypothesis of participants dealing with credit risk in a rational manner. The model also cautioned against a simplistic focus on instituting proper risk management practices without first acknowledging the subtle, underlying welfare effects of the participants' certification constraints.

While these certification constraints were discussed within the framework of bond insurance in this paper, their welfare implications might prove relevant for other areas of the nancial market where such needs play a signicant part in the security design. Interestingly, the obvious candidates, such as the CDS or ABCP markets, also happen to be the areas where the regulators consistently voice macroprudential systemic concerns. My model's adds to their concerns from a microprudential standpoint. Creating \certied" securities through these means of credit enhancement ought to be discouraged under certain circumstances not just out of fear for a systemic failure, but also because there exist a group of investors who can extract more surplus from the security when it fails to satisfy the certication constraint. Thus, identifying such groups of investors and analyzing their scope for welfare improvement would prove valuable for enhanced regulatory efforts in these markets.

# Appendix: Proofs

**Proof of Proposition 1.** I prove the proposition in a number of steps. First of all, I derive issuer is maximum willingness to pay for bond insurance, given by the issuance price differential with vs. without bond insurance  $(\Delta_0^i)$ . Since the insurer holds sufficient capital to cover for the worst case credit loss, the bond is perceived to be completely riskless if it is insured. If so,  $p_{0,I}^i = 1$  follows trivially. Then:

**Lemma A.1.** The issuance price differential of  $B_i \in \{B_1, ..., B_I\}$  is given by:

$$
\Delta_0^i = \frac{\theta_i \pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) + \mu_i \theta_i \left(1 - \pi_i^g - \pi_i^m\right) \exp\left(\frac{\gamma \mu_i \theta_i}{\lambda_i}\right)}{\pi_i^g + \pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) + \left(1 - \pi_i^g - \pi_i^m\right) \exp\left(\frac{\gamma \mu_i \theta_i}{\lambda_i}\right)}.
$$
\n(A.1)

**Proof.** To compute  $\Delta_0^i$ , it is necessary to derive the bond's price without insurance, i.e.,  $p_{0,U}^{i}$ . Since a conservative investor j cannot hold an uninsured bond,  $x_{i,t}^{con}(j)$  = 0. To derive  $x_{i,t}^{agg}(j)$ , rearranging (4) and (5) along with  $W_0^{agg}$  $\int_0^{agg}(j) = 0$  gives a representative aggressive investor  $j$ 's objective function as:

$$
\max_{x_{i,t}^{agg}(j)} \left[ \begin{array}{c} -\pi_i^g \exp \left\{ -\gamma \left( 1 - p_{0,U}^i \right) x_{i,t}^{agg}(j) \right\} - \pi_i^m \exp \left\{ -\gamma \left( 1 - \theta_i - p_{0,U}^i \right) x_{i,t}^{agg}(j) \right\} \\ -\left( 1 - \pi_i^m - \pi_i^g \right) \exp \left\{ -\gamma \left( 1 - \mu_i \theta_i - p_{0,U}^i \right) x_{i,t}^{agg}(j) \right\} \end{array} \right].
$$
\n(A.2)

This may be rearranged as:

$$
\max_{x_{i,t}^{agg}(j)} - \exp\left(-\gamma \left(1 - p_{0,U}^i\right) x_{i,t}^{agg}\left(j\right)\right) \left\{\begin{array}{c} \pi_i^g + \pi_i^m \exp\left(\gamma \theta_i x_{i,t}^{agg}\left(j\right)\right) \\ + \left(1 - \pi_i^m - \pi_i^g\right) \exp\left(\gamma \mu_i \theta_i x_{i,t}^{agg}\left(j\right)\right) \end{array}\right\}.
$$
\n(A.3)

The first order condition yields:

$$
\begin{split} & \left(1 - p_{0,U}^i\right) \left\{\pi_i^g + \pi_i^m \exp\left(\gamma \theta_i x_{i,t}^{agg}\left(j\right)\right) + \left(1 - \pi_i^m - \pi_i^g\right) \exp\left(\gamma \mu_i \theta_i x_{i,t}^{agg}\left(j\right)\right)\right\} \\ &= \theta_i \pi_i^m \exp\left(\gamma \theta_i x_{i,t}^{agg}\left(j\right)\right) + \mu_i \theta_i \left(1 - \pi_i^m - \pi_i^g\right) \exp\left(\gamma \mu_i \theta_i x_{i,t}^{agg}\left(j\right)\right). \end{split} \tag{A.4}
$$

Since every investor of the same type are homogeneous in all other respects, I look for a symmetric equilibrium. Then, the market clearing condition requires  $x_{i,t}^{agg}(j) = \frac{1}{\lambda_i}$ . Using this, a simple rearrangement of (A.4) yields:

$$
p_{0,U}^{i} = \frac{\pi_i^g + (1 - \theta_i) \pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) + (1 - \mu_i \theta_i) (1 - \pi_i^g - \pi_i^m) \exp\left(\frac{\gamma \mu_i \theta_i}{\lambda_i}\right)}{\pi_i^g + \pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) + (1 - \pi_i^g - \pi_i^m) \exp\left(\frac{\gamma \mu_i \theta_i}{\lambda_i}\right)}.
$$
 (A.5)

Notice that  $p_{0,U}^i \in (0,1)$ , with  $p_{0,U}^i = 1$  if and only if  $\pi_i^g = 1$  or  $\theta_i = 0$ . Using  $p_{0,I}^i = 1$  and  $(A.5)$ ,  $(A.1)$  can be derived immediately.

This quantity ought to be compared to the insurers' respective reservation prices, i.e., min  $(\Gamma_{A,0}^i, \Gamma_{B,0}^i)$ . Using the definition of  $\Gamma_{A,0}^i$  and  $\Gamma_{B,0}^i$ , it must be that:

$$
\Gamma_{A,0}^{i} = \pi_{i}^{m} \theta_{i} + (1 - \pi_{i}^{g} - \pi_{i}^{m}) \mu_{i} \theta_{i} + c_{0}^{A} (\mu_{i} \theta_{i} - \pi_{i}^{m} \theta_{i} - (1 - \pi_{i}^{g} - \pi_{i}^{m}) \mu_{i} \theta_{i}),
$$
 (A.6)  
\n
$$
\Gamma_{B,0}^{i} = \pi_{i}^{m} \theta_{i} + (1 - \pi_{i}^{g} - \pi_{i}^{m}) \mu_{i} \theta_{i} + c_{0}^{B} (\mu_{i} \theta_{i} - \pi_{i}^{m} \theta_{i} - (1 - \pi_{i}^{g} - \pi_{i}^{m}) \mu_{i} \theta_{i}).
$$
 (A.7)

This in turn implies that:

$$
\min\left(\Gamma_{A,0}^{i}, \ \Gamma_{B,0}^{i}\right) = \pi_{i}^{m} \theta_{i} + \left(1 - \pi_{i}^{g} - \pi_{i}^{m}\right) \mu_{i} \theta_{i} + \min\left(c_{0}^{A}, \ c_{0}^{B}\right) \left(\mu_{i} \theta_{i} - \pi_{i}^{m} \theta_{i} - \left(1 - \pi_{i}^{g} - \pi_{i}^{m}\right) \mu_{i} \theta_{i}\right), \quad (A.8)
$$

Now, let  $\varsigma_i \equiv \mu_i \pi_i^g + (\mu_i - 1) \pi_i^m$ . Notice that  $\varsigma_i > 0$  since  $\mu_i > 1$ . Using (8), (A.1), and (A.8), the following quantity must be positive, i.e.,

$$
-\pi_i^g \left(\pi_i^m + \left(1 - \pi_i^g - \pi_i^m\right) \mu_i + \varsigma_i \min\left(c_0^A, c_0^B\right)\right)
$$

$$
+\left[\pi_i^g \mu_i + \left(1 - \pi_i^m\right) \left(1 - \mu_i\right) - \varsigma_i \min\left(c_0^A, c_0^B\right)\right] \pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right)
$$

$$
+\varsigma_i \left\{1 - \min\left(c_0^A, c_0^B\right)\right\} \left(1 - \pi_i^g - \pi_i^m\right) \exp\left(\frac{\gamma \mu_i \theta_i}{\lambda_i}\right) > 0. \tag{A.9}
$$

Rearranging the inequality regarding (A.9) in terms of min  $(c_0^A, c_0^B)$  yields (9) in the proposition.

**Proof of Proposition 2.** I prove the last two parts of the proposition first, as this requires more careful consideration. Let  $\bar{c}_0^i$  denote the insurance threshold in (9), i.e.,

$$
\bar{c}_0^i \equiv \frac{\varsigma_i \pi_i^b \exp\left(\frac{\gamma \mu_i \theta_i}{\lambda_i}\right) + \left(\varsigma_i - \mu_i + 1\right) \pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) + \left(\varsigma_i - \mu_i\right) \pi_i^g}{\varsigma_i \pi_i^b \exp\left(\frac{\gamma \mu_i \theta_i}{\lambda_i}\right) + \varsigma_i \pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) + \varsigma_i \pi_i^g}.\tag{A.10}
$$

Then, using the chain and quotient rules, a tedious algebraic manipulation yields that the sign of  $\frac{\partial \vec{c}_0^i}{\partial \mu_i}$  is determined by the sign of:

$$
(1 - \pi_i^g - \pi_i^m) \left\{ \lambda_i \pi_i^g + \gamma \theta_i \left( \mu_i - 1 \right) \varsigma_i \right\} \exp \left( \frac{\gamma \left( \mu_i + 1 \right) \theta_i}{\lambda_i} \right)
$$
  
+ 
$$
\pi_i^g \left( 1 - \pi_i^g - \pi_i^m \right) \left( \lambda_i + \mu_i \varsigma_i \right) \exp \left( \frac{\gamma \mu_i \theta_i}{\lambda_i} \right) + \lambda_i \left( \pi_i^g \right)^2
$$
  
+ 
$$
\lambda_i \pi_i^g \left( \pi_i^m - \pi_i^g \right) \exp \left( \frac{\gamma \theta_i}{\lambda_i} \right).
$$
 (A.11)

The first three terms of  $(A.11)$  are always positive but the sign of the last term is ambiguous, depending on whether  $\pi_i^m > \pi_i^g$  or  $\pi_i^m < \pi_i^g$ . In particular, for small values of  $1-\pi_i^g-\pi_i^m$  and  $\mu_i$  close to 1, it is conceivable that the last term dominates. Thus, although an increase in  $\mu_i$  unambiguously raises  $\bar{c}_0^i$  when  $\pi_i^m > \pi_i^g$ , its effect is otherwise ambiguous. This proves part (iii) of the proposition. However, from (A.11), it is immediately possible to deduce that  $\frac{\partial (\bar{c}_0^i)^2}{\partial^2 u}$  $\frac{C_{0j}}{\partial^{2} \mu_{i}} > 0$ , as the first two terms are always increasing in  $\mu_{i}$  but the last two terms are unaffected. This proves part (iv) of the proposition.

As for the remaining parts of the proposition, notice that  $\gamma$ ,  $\theta_i$ , and  $\lambda_i$  do not affect  $\bar{c}_0^i$  individually but instead in a combination of  $\frac{\gamma \theta_i}{\lambda_i}$ . Now, let  $\varphi_i \equiv \frac{\gamma \theta_i}{\lambda_i}$  $\frac{\gamma \theta_i}{\lambda_i}$  then:

$$
\bar{c}_0^i \equiv \frac{\varsigma_i \pi_i^b \exp\left(\mu_i \varphi_i\right) + \left(\varsigma_i - \mu_i + 1\right) \pi_i^m \exp\left(\varphi_i\right) + \left(\varsigma_i - \mu_i\right) \pi_i^g}{\varsigma_i \pi_i^b \exp\left(\mu_i \varphi_i\right) + \varsigma_i \pi_i^m \exp\left(\varphi_i\right) + \varsigma_i \pi_i^g}.\tag{A.12}
$$

Then, after some algebraic manipulation, the sign of  $\frac{\partial \vec{c}_0^i}{\partial \varphi_i}$  is determined by the sign of

$$
\pi_i^m (1 - \pi_i^g - \pi_i^m) (\mu_i - 1)^2 \exp \{ (\mu_i + 1) \varphi_i \} + \pi_i^g (1 - \pi_i^g - \pi_i^m) (\mu_i)^2 \exp (\mu_i \varphi_i) + \pi_i^g \pi_i^m \exp (\varphi_i),
$$
\n(A.13)

which is always positive. This implies that  $\frac{\partial \bar{c}_0^i}{\partial \varphi_i} > 0$ , which in turn implies that  $\frac{\partial \bar{c}_0^i}{\partial \gamma} > 0$ ,  $\frac{\partial \bar{c}_0^i}{\partial \theta_i} > 0$ , and  $\frac{\partial \bar{c}_0^i}{\partial \lambda_i} < 0$ , as stated in the proposition.

**Proof of Proposition 3.** Consider a bond  $B_i \in \{B_1, ..., B_I\}$  without loss of generality. The proof is obtained from a comparison of the agents' welfare when  $B_i$  is insured against when it is issued without bond insurance. I denote the former welfare measure as  $\Omega_{0,l}^i$ and the latter as  $\Omega_{0,U}^i$ .

Suppose first that  $B_i$  is insured. Then, the surplus from bond insurance to be divided between the issuer and the insurer is  $p_{0,I}^i - l_0^i - \min(\Gamma_{A,0}^i, \Gamma_{B,0}^i)$ . Since  $p_{0,I}^i = 1$  and the insured bond's par is always guaranteed, it is trivial to show using (4) and (5) that both types of investors in bond  $B_i$  receive ex ante utility of  $-1$ . This is equivalent to receiving a certainty equivalent wealth of 0. Then, it must be that:

$$
\Omega_{0,I}^{i} = p_{0,I}^{i} - l_0^{i} - \min\left(\Gamma_{A,0}^{i}, \ \Gamma_{B,0}^{i}\right). \tag{A.14}
$$

On the other hand, suppose now that  $B_i$  is not insured. If so, its issuance price is given by  $p_{0,U}^i$  and only the aggressive investors hold the asset. Conservative investors remain outside the market, with their certainty equivalent wealth standing at 0. The issuer earns  $p_{0,U}^i - l_0^i$  while the insurer has zero payoff. Finally, the aggressive investors' ex ante expected utility  $(U_{i,U}^{agg})$  can be computed using  $(A.5)$ :

$$
U_{i,U}^{agg} = -\left(\pi_i^g + \pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) + (1 - \pi_i^g - \pi_i^m) \exp\left(\frac{\gamma \mu_i \theta_i}{\lambda_i}\right)\right) \exp\left(-\frac{\gamma}{\lambda_i} \Delta_0^i\right). \tag{A.15}
$$

The certainty equivalent wealth associated with this utility level is  $-\frac{1}{2}$  $\frac{1}{\gamma} \log \left(-U_{i,U}^{agg}\right).$ However, using (A.15), it may be rearranged as:

$$
-\frac{1}{\gamma}\log\left(-U_{i,U}^{agg}\right) = \frac{\Delta_0^i}{\lambda_i} - \frac{1}{\gamma}\log\left(\pi_i^g + \pi_i^m \exp\left(\frac{\gamma\theta_i}{\lambda_i}\right) + (1 - \pi_i^g - \pi_i^m)\exp\left(\frac{\gamma\mu_i\theta_i}{\lambda_i}\right)\right).
$$
(A.16)

This needs to be weighted by the aggressive investors' proportion, i.e.,  $\lambda_i$ . Then,  $\Omega^i_{0,U}$ is equal to:

$$
\Omega_{0,U}^{i} = p_{0,U}^{i} - l_0^{i} + \Delta_0^{i} - \frac{\lambda_i}{\gamma} \log \left( \pi_i^g + \pi_i^m \exp\left(\frac{\gamma \theta_i}{\lambda_i}\right) + (1 - \pi_i^g - \pi_i^m) \exp\left(\frac{\gamma \mu_i \theta_i}{\lambda_i}\right) \right).
$$
\n(A.17)

Obviously, bond insurance is strictly preferred from an overall social perspective only when  $\Omega_{0,I}^i > \Omega_{0,U}^i$ . Using (A.14) and (A.17), and also knowing that  $\Delta_0^i \equiv p_{0,I}^i - p_{0,U}^i$ , this condition reduces to:

$$
\min\left(\Gamma_{A,0}^{i},\ \Gamma_{B,0}^{i}\right) > \frac{\lambda_{i}}{\gamma} \log\left(\pi_{i}^{g} + \pi_{i}^{m} \exp\left(\frac{\gamma \theta_{i}}{\lambda_{i}}\right) + (1 - \pi_{i}^{g} - \pi_{i}^{m}) \exp\left(\frac{\gamma \mu_{i} \theta_{i}}{\lambda_{i}}\right)\right). \tag{A.18}
$$

Combining  $(A.18)$  with  $(7)$  completes the proof.  $\blacksquare$ 

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Figure 1: Insurance threshold as a function of model parameters (rational benchmark)

Figure 2: Negative externality of bond insurance as a function of model parameters (rational benchmark)







Figure 4: Comparison of insurance thresholds under full rationality and local thinking





Figure 5: Period 1 price of  $B_1$  and the model parameters (following insurer downgrade)

Figure 6: Overall social welfare and the model parameters (following insurer downgrade)

