Effect of Liquidity on the Implied Volatility Surface in Interest Options Markets

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ABSTRACT

The volatility implied in the option price exhibits the systematic bias with respect to different levels of exercise prices for different maturities, and this anomaly has been arousing the attentions of many financial economists. This paper investigates the bias of volatility surface implied in options markets, and relate it to various measures of liquidities in Eurodollar futures and futures options. We find the effect of liquidity and the level of previous period implied volatility on the shape and change of volatility is significant in the interest rate options market. The implied volatility bias is larger for deep in-the-money and out-of-the-money options and for short maturity options than for at-the-money and for long maturity options.

I. Introduction

The volatility as a measure of risk in the financial market has motivated many financial researchers and industry professionals, and induced the innovation in the financial market. Since the introduction of the Black-Scholes option pricing model, the derivative security markets have been expanded quite rapidly for more than past 30 years. Derivative securities can be used for hedging and mitigating risk, as well as for speculation and arbitrage which sometimes cause turbulences in the financial markets.

The only unknown parameter in the Black-Scholes model is the volatility of the underlying asset. If the option market is efficient, all relevant information should be contained in the option price, and the implied volatility should represent a rational forecast of future volatility when the appropriate option pricing model is employed. However, the volatility implied in the option price exhibits the systematic bias with respect to different levels of exercise prices for different maturities, and this anomaly has been arousing the attentions of many financial economists as well as industry practitioners in the derivative security markets.

Different patterns of volatility functions have been observed for different underlying assets in different markets. For the equity markets, the implied volatility as a function of exercise price shows monotonically downward sloping volatility “skew,” especially for the equity index options since the market crash of October 1987. This suggests that the in-the-money calls and out-of-the-money puts are in greater demand compared to out-of-the-money calls and in-the-money puts. For the currency markets, the implied volatility function is often reported to exhibit more of the symmetric valley-shaped curve where the implied volatilities for the in-the-money and out-of-the-money options are higher than those for the at-the-money options. This pattern of implied volatility bias with respect to exercise price is known as a volatility “smile.” For the commodity markets, the implied volatility typically shows upwards sloping forward skew, which exhibits reverse pattern to equities. When supply is tight in the commodity markets, businesses would rather pay more to secure supply than to risk supply disruptions.

The implied volatility also shows characteristic differences for options of different maturities. This relationship between the implied volatility and the time to maturity is known as the term structure of volatility. The volatility smile or skew tends to be less pronounced as the option maturity increases. Also, volatility tends to be increasing function of maturity when short-dated volatility are historically low because there is an expectation that volatilities will increase, and vice versa. The combination of the term structure of volatility with the volatility smile is called the implied volatility surface. It is the 3-D plot of implied volatilities with respect to the different levels of exercise prices and maturities.

Most of previous researches on volatility bias concentrate on the validity of the stochastic movements of asset prices with no completely satisfactory explanation. Evolving from the option valuation model, the relationship between the volatility implied in the option's price and the various liquidity measures has been empirically investigated, mostly for the equity markets. Many attempts have been made to explain the anomaly in the shape of the volatility implied in option's price. Most of them try to relax the Black-Scholes assumption of constant volatility by allowing the deterministic or stochastic local volatility rate of underlying security returns. Deterministic volatility structure models include Emanuel and MacBeth [1982], Dupire [1994], and Rubinstein [1994]. Dumas, Fleming, and Whaley [1998] claim that models based on a simple deterministic volatility structure generate highly unstable parameters through time, and suggest that deterministic volatility models cannot explain the time-series variation in option prices.

Option valuation models based on stochastic volatility or jumps in the underlying price process include Bakshi, Cao, and Chen [1997], Jorion [1989], Bates [2000], and Anderson, Benzoni, and Lund [2002]. A stochastic volatility model can generate the observed downward sloping implied volatility function if innovations to volatility are negatively correlated with underlying asset returns. While a stochastic volatility model appears to perform better than the Black–Scholes or deterministic volatility structure model, some of the implied parameter estimates differ from the ones estimated directly from actual returns, and it provides only a partial explanation of the shape of the implied volatility functions.

In spite of the complex development of option pricing and hedging models, the behavior of the observed volatility bias has not been successfully explained. Another approach to understand the structure of implied volatility is to relate the implied volatility to the market frictions for different option series in different option markets. Bakshi, Kapadia, and Madan [2003] study the risk-neutral skewness implicit in the prices of index options and individual stock options. Negatively sloped implied volatility function tends to correspond to negative implicit risk-neutral skewness. They show how risk-neutral skewness is related to the coefficient of relative risk aversion, and that the risk neutral skewness implicit in individual stock options will be less negative than the risk-neutral skewness of the index. Cetin, Jarrow, Protter and Warachka [2006] model the liquidity risk as a stochastic supply curve in Black-Scholes economy. Their empirical results show that the liquidity costs are a significant component of the option's price and increase quadratically in the number of options being hedged, and non-optimal Black-Scholes hedges cause the impact of illiquidity to depend on the option's moneyness.

Bollen and Whaley [2004] examines the relationship between the shape of the implied volatility function and net buying pressure in the individual and index equity option markets. They support the "limits to arbitrage hypothesis" over the "learning hypothesis," where the implied volatility changes are reversed in short period of time and option's own net buying pressure is a significant factor in explaining changes in implied volatility. They claim that although the empirical asset return distributions for the index and individual equity options are very similar, the shapes of implied volatilities are dramatically different. Their results suggest that net buying pressure plays an important role in determining the shape of implied volatility functions, especially for equity index options. Garleanu, Pedersen and Poteshman [2009] recognize that the prices of index and individual equity option display quite different properties even though the dynamics of underlying assets are similar. They also find the importance of buying pressure in the options market and that end users tend to have net long positions in equity index options, particularly with regard to out-of-the-money puts, and net short positions in individual stock options. They conclude that the net demand of non-market makers for equity options, across different levels of moneyness, is directly related to their expensiveness and skew patterns where the expensiveness of an option is defined as the difference between the Black-Scholes implied volatility and a proxy for the expected volatility over the life of option.

In contrast to the extensively documented cross-sectional features in the equity option markets, only a few researches have studied the effect of volatility bias in the interest rate option markets.

Jarrow, Li and Zhao [2007] examine the volatility smile in interest rate caps and floors which are long-term interest rate instruments. They find that even a multifactor term structure models augmented with stochastic volatility and jumps do not fully capture the volatility smile, and claim that the volatility smile contains information that is not available using only at-the-money options. Deuskar, Gupta and Subrahmanyam [2008] investigate the economic determinants of interest rate volatility bias for the over-the-counter interest rate caps and floors, and find the strong volatility smile patterns in the interest rate caps and floors markets for different maturities. In addition, they explore the determinants of volatility smile, and find that the shape of the smile is positively related to the short-term interest rate and the liquidity costs, and negatively related to the slope of the term structure of interest rates especially for longer maturity options.

In this paper, we investigate the relationship between the movement and shape of the implied volatility surface and the various liquidity measures in Eurodollar futures and futures option markets. Eurodollar futures and futures options in Chicago Mercantile Exchange are the most active short-term interest rate instruments, and growing rapidly in the international financial markets. Futures and options are the instruments that allow investors to capitalize the available information in the market while limiting risk to a predetermined level, and they offer effective means of managing the interest rate risk of fixed income portfolios. Various proxies are used for the liquidity measures to capture the liquidity in the Eurodollar futures and options markets. A better understanding of the liquidity structure and its impact on the volatility and pricing of interest rate options is critical to improving the efficiency and stability of financial markets and the overall health of the economy, as evidenced by the past financial crises.

II. Eurodollar Futures and Futures Options

A. Markets for Eurodollar Futures and Options

Eurodollar futures and futures option markets started trading in early 1980's in the Chicago Mercantile Exchange (CME), and they are the most active interest rate markets on short-term instruments. The Eurodollar is a U.S. dollar deposit in a foreign bank or subsidiary of a U.S. bank outside the jurisdiction of U.S. government. Since the Eurodollar is not subject to Federal Reserve requirements and is subject to default risk, its yield is expected to be higher than that of domestic deposits in the United States. Yet, it still represents a low risk investment tool and is often used as a proxy for risk-free interest rate.

Eurodollar futures contract is an agreement to purchase or sell three month Eurodollar time deposit with a principal value of 1 million Eurodollars at some specified price at the maturity of the contract. On the delivery day of the Eurodollar futures contract, it calls for cash settlement instead of actual delivery of the underlying instrument. The futures price is quoted based on an IMM (International Monetary Market) index, which is the difference between 100 and the Eurodollar yield. At the maturity of Eurodollar futures contract, the value of the three month Eurodollar time deposit must converge to a principal value of $1 million. Hence, the volatility of the futures price is expected to decline as the contract matures. Investors buy the Eurodollar futures contract to protect against falling interest rates and sell to hedge against rising interest rates. An option on Eurodollar futures in CME is a right to purchase or sell the underlying Eurodollar futures contract at the specified exercise price for a given period of time.

Daily settlement prices for Eurodollar futures and futures options are available from CME for the 25-year period from March 1985 to November 2009. Eurodollar futures are issued every quarter with maturities ranging from 3 months to 10 years. The American style quarterly and serial options are offered in CME with maturities up to 2 years across different moneyness levels. Hence, there are total of eight quarterly options along with two front month serial options. The standard quarterly options maturing in March, June, September and December and options with up to one year maturities are more liquid in the market.

B. Derivation of Implied Volatility Surface

We derive the implied volatilities from the modified version of the Black [1976] option pricing model for the pricing of Eurodollar futures options. Although there may be more complex alternative interest rate models that explain at least part of the smile and term structure of volatility, they would not be able to fully explain these volatility biases without considering the effect of market frictions. Also, it is needed to understand the empirical regularities of volatility structure using more standardized model.

Based on the assumption of the lognormal distribution of futures prices, Black derived the option pricing functions for the futures option. In Black's pricing formula, the option on a futures contract can be treated in the same way as the option on a security paying a continuous dividend at risk-free rate. In other words, the value of a call or put option on a futures contract can be determined by replacing the underlying spot price with the discounted futures price, $F \cdot e^{-r\tau}$, where F is the futures price, r is the risk-free rate and τ is the time to maturity of futures option. However, a Eurodollar time deposit with a principal value of $1 million matures three months after the futures and futures option expiration, and an infinitely large futures price three months before the maturity is not plausible. The assumption of lognormal price is inappropriate for short term interest rate instruments since the lognormal distribution allow for the possibility of infinitely large prices. Therefore, we modify the option pricing model of Black to apply to the Eurodollar futures call and put options, assuming the Eurodollar yield, rather than the Eurodollar futures price, has a lognormal distribution at the expiration of the underlying contract.

The implied volatility can be calculated by inverting the option pricing function given the other parameters. Since the option pricing function is not easily invertible, we can numerically approximate the volatility implied in the option price by equating the model price with the market price of the call or put option. The quasi-Newton method and a finite difference gradient can be employed to the option pricing model for the futures options. In this paper, we construct the time series of futures contracts with less than three, six, nine, and twelve months to maturity. That is, we construct the daily time series of futures and futures option contracts with the first-nearby (three-month), second-nearby (six-month), third-nearby (nine-month), and fourth-nearby (twelve-month) maturity. The time series of the first-nearby maturity contract has maturity up to three months; the second-nearby maturity contract has maturity from three months to six months; the third-nearby maturity contract has maturity from six months to nine months; and the fourth-nearby maturity contract has maturity from nine months to twelve months. The behaviors of implied volatility expected in the market tend to exhibit different patterns for different maturities.

III. Empirical Results

A. Volatility Bias and Liquidity

Liquidity, in general, can be defined as the degree to which an asset, in any quantity or amount, can be bought or sold in the market within a short period of time and without causing significant movement in its price. Empirical studies have found that the liquidity effect is an important economic factor and significant in many asset prices. Although the liquidity effects have extensively been studied for the equity markets, relatively little is investigated about the liquidity effect for the interest rate markets in the finance literature, especially for short-term futures and options. In this section, we examine the relation between the volatility bias implied in the valuation model and various liquidity measures in Eurodollar futures options markets. We use the option trading volume and option open interest as liquidity measures. The strike price bias of the implied volatility, popularly known as the *smile effect*, is expected to be negatively related to these liquidity measures.

The strike price bias of the implied volatility is related to option trading volume and option open interest in Figure 1 and Figure 2, respectively, for Eurodollar futures call and put options. In the upper panels of Figure 1 and Figure 2, the implied volatility calculated from the option pricing formula for each maturity option is plotted against the ratio of the futures price to the exercise price, *F/X*. That is, the implied volatilities in all of the options in our dataset are calculated and averaged for each interval of the *F/X* ratio. The strike price bias of the implied volatility is more severe for deep in-the-money call and put options and for deep out-of-the-money put options, and less severe for at-the-money call and put options. In addition, short maturity options have larger biases than the long maturity call and put options. In other words, the *volatility smile effect* is larger for short maturity options and for deep in-the-money and out-of-the-money options than for long maturity and at-the-money options.

Figure 1. Implied Volatility, Trading Volume and Open Interest in Eurodollar Futures Call Options

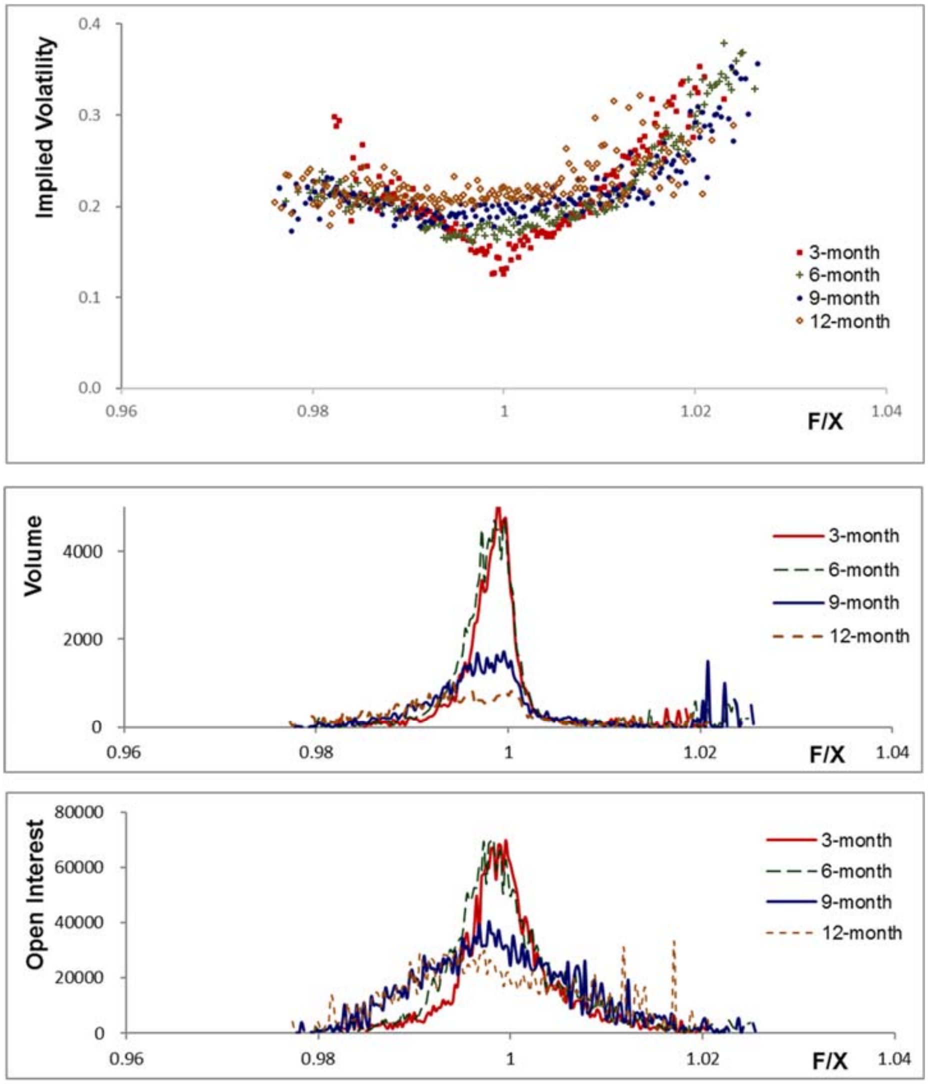
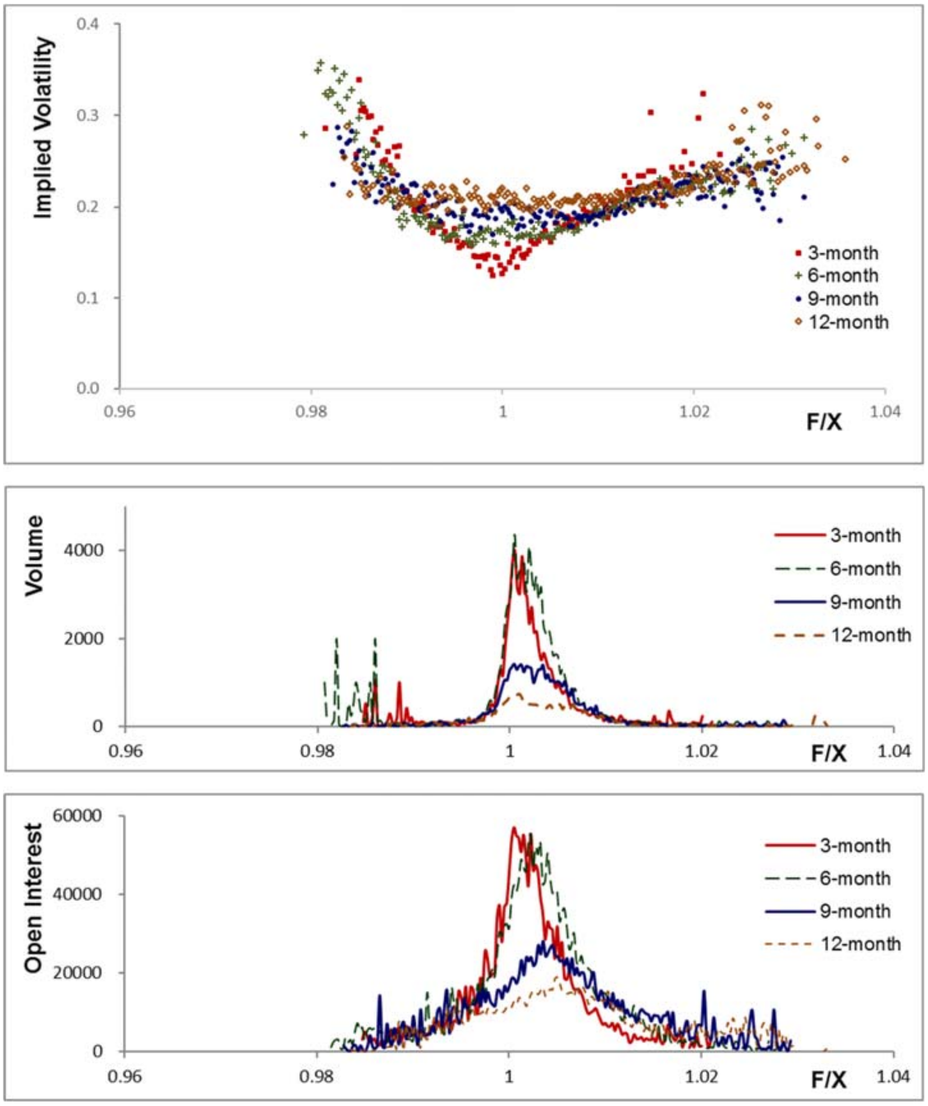


Figure 2. Implied Volatility, Trading Volume and Open Interest in Eurodollar Futures Put Options



The middle and lower panels of Figure 1 and Figure 2, respectively, exhibit the option trading volume and open interest for each maturity option relative to the ratio of the futures price to the exercise price, *F/X*. Trading volume represents the number of contracts traded over a given time interval, and open interest is the cumulative number of unliquidated contracts outstanding at any point in time. Whereas the volume shows the level of market activity, the open interest indicates the size of a market. Both trading volume and open interest increase as the underlying option approaches its maturity, and open interest drops to zero when it matures.

As anticipated, the strike price bias of the implied volatility taken from deep in-the-money and out-of-the-money options is due mainly to the lack of liquidity in the market. For both call and put options, the trading volume rapidly declines for options that are either deep in-the-money or deep out-of-the-money. Furthermore, the illiquidity of in-the-money and out-of-the-money options is exaggerated for shorter maturities.

B. GMM Regression Tests of Volatility Smile

In this section, the relationship between the liquidity and volatility of the Eurodollar market is investigated using the generalized method of moments (GMM) technique and adjusting for the autocorrelation and heteroscedasticity of the residual errors. We construct the daily time series of futures and futures option contracts for different maturity categories, and measure the moneyness of options based on the relative difference between the futures price and the exercise price of option. Then, we assess the time series relationship between the implied volatility and various measures of liquidity in the Eurodollar futures and options markets. For each maturity time-series, the GMM regression specification for the option implied volatility is set as follows:

$$\sigma_t - \sigma_{ATM,t} = a_1 + b_1 \cdot \mathrm{Vol}_t + b_2 \cdot \mathrm{OI}_t + b_3 \cdot \sigma_{t-1} + \varepsilon_{1,t},$$

where $\sigma_t - \sigma_{ATM,t}$ is the deviation of the implied volatility from the at-the-money implied volatility on each trading day for each maturity category. Vol_t is the trading volume and OI_t is the open interest of the options. In order to maintain the stationarity of data, the log linear forms of option trading volume and open interest are employed for different maturity series.

OLS estimation of the linear statistical model assumes that errors are specified as homoscedastic and the sampling process for residual error and regressor is uncorrelated. However, the above regression involves the overlapping error structure defined by the maturity cycle of the underlying security, and yesterday's forecast error tends to be transmitted to today's volatility forecast. In addition, since the volatility time series are calculated from a different number of price observations over different lengths of the option's life at each time, the forecasting errors for different time periods are expected to have different precisions. In other words, the forecasting errors are heteroscedastic, and this should be reflected in the estimation process with different weights. While OLS estimation would generate unbiased and consistent parameter estimates as long as error terms are uncorrelated over time, the OLS covariance matrix of parameters would be inconsistent because of autocorrelation and heteroscedasticity. The GMM estimator, initially developed by Hansen (1982), is known to be consistent, asymptotically normal, and efficient in large samples. In addition, Newey and West (1987) propose a consistent and positive semi-definite covariance estimator, where the resulting standard error and *t*-statistics correct for the autocorrelation and heteroscedasticity are consistent.[1]

To test the relationship between the liquidity and volatility in the Eurodollar market, the GMM regression equation is fitted separately for samples of different maturity options. The equation is estimated using the ordinary least squares method, and the covariance matrix is adjusted for heteroscedasticity and serial dependence in the time series of forecast errors. In the above regression specification, we include the lagged implied volatility as an independent variable that can test the learning hypothesis and the limits to arbitrage hypothesis. Under the learning hypothesis, there should be no serial correlation in the changes of implied volatility, and under the limits to arbitrage hypothesis, the change in implied volatility will reverse in short period of time, implying significantly negative coefficient.

The test results from the GMM regression of the implied volatility bias on the liquidity variables and lagged variable of implied volatility are reported in Table 1. The negative relationship between the implied volatility and option trading volume is statistically significant at the one percent level for the shorter maturities of Eurodollar futures call and put options, but relatively less significant

[1] Application of the GMM technique may not result in asymptotically efficient estimators compared with the generalized least squares procedures. However, the GLS procedure can result in inconsistent parameter estimates and requires the complete specification of the nature of the serial correlation and heteroscedasticity, while the GMM technique implicitly permits the disturbance terms to be both serially correlated and heteroscedastic in the construction of the orthogonality conditions.

for longer maturity options where the implied volatility exhibits flatter smile. Open interest which is an increasing function of maturity is less significant for shorter maturity options, but show stronger significance as maturity of the option series increases. The level of previous period implied volatility has significantly negative impact on the deviation of volatility from its at-the-money implied volatility.

Table 1. GMM Regression of the Strike Price Bias of Implied Volatility on Liquidity Measures

$$\sigma_t - \sigma_{ATM,t} = a_1 + b_1 \cdot \mathrm{Vol}_t + b_2 \cdot \mathrm{OI}_t + b_3 \cdot \sigma_{t-1} + \varepsilon_{1,t},$$

	a_1	b_1*1000	b_2*1000	b_3	# obs
Panel A: Eurodollar Futures Call Option					
3-month	0.044 **	-2.880 **	0.317	-0.014 **	23248
	(-26.16)	(-20.1)	(-1.61)	(-6.27)	
6-month	0.026 **	-0.687 **	-0.311 *	-0.019 **	33668
	(-24.34)	(-6.64)	(-2.29)	(-13.05)	
9-month	0.013 **	0.617 **	-0.133	-0.024 **	28203
	(-11.33)	(-5.43)	(-0.89)	(-15.17)	
12-month	0.000	0.538 **	1.070 **	-0.019 **	22034
	(-0.07)	(-4.55)	(-6.81)	(-12.04)	
Panel B: Eurodollar Futures Put Option					
3-month	0.053 **	-4.530 **	-0.038	-0.024 **	21649
	(-32.85)	(-36.1)	(-0.22)	(-11.45)	
6-month	0.048 **	-1.580 **	-2.470 **	-0.016 **	33759
	(-45.75)	(-17.16)	(-20.17)	(-11.31)	
9-month	0.038 **	-0.680 **	-2.820 **	-0.026 **	30300
	(-43.78)	(-8.95)	(-26.54)	(-22.2)	
12-month	0.037 **	-0.032	-3.440 **	-0.037 **	24481
	(-33.69)	(-0.34)	(-26.24)	(-26.17)	

Notes: The deviation of implied volatility from its at-the-money volatility is regressed against the logarithms of the option trading volume, open interest and the lagged value of implied volatility in the market for Eurodollar futures call and put options. 3-month, 6-month, 9-month, and 12-month stand for the underlying futures and futures option contracts with 0-3 months, 3-6 months, 6-9 months, and 9-12 months to maturity at each time, respectively. *T*-statistics are reported in parentheses. * denotes significance at the 5% level and ** denotes significance at the 1% level.

The smile effect of implied volatility can result from the fatter tails and higher peak in the middle than those of the lognormal distribution of Eurodollar yield. This fat tail characteristic reflects a belief that there is a greater chance of a large move in the price of the underlying asset, and a high peak of the distribution reflects a belief that the probability of very small changes in the price is also greater than is predicted by the lognormal distribution. Another possible suggestion by Rubinstein [1985] is that the exercise price bias of the implied volatility may be correlated with the macroeconomic variables such as the level of stock market price, the level of the stock market volatility, and the level of interest rates.

IV. Concluding Remarks

The relationship between asset return and risk and, hence, the volatility of asset prices has been of interest to financial economists. When market is efficient, the volatility implied in the option price reflects investors' assessments of future market volatility. This research investigates the liquidity effect on the shape and change of the volatility surface in the Eurodollar futures and options markets which is the most active short-term interest rate markets in the world. This liquidity effect on implied volatility surface is examined for each moneyness and maturity series of options. We compare various liquidity measures with the implied volatility surface which is a combination of the smile and term structure of volatilities for different moneyness and maturity categories.

We report the empirical results relating to the liquidity effects on the implied volatility surface using the comprehensive data set covering longer time span since Eurodollar futures and options have been traded for over 25 years including the periods of several financial crises. We show that the volatility smile effect for different maturity is strongly present in the Eurodollar futures option markets, and establish that the exercise price bias of the implied volatility, or the smile effect, is negatively related to various measures of liquidity. It tends to be more severe for deep in-the-money and deep out-of-the-money options, and less severe for at-the-money call and put options. In addition, short maturity options have larger biases than the long maturity options. In other words, the volatility smile effect is larger for short maturity options and for deep in-the-money and out-of-the-money options than for long maturity and at-the-money options, respectively. The results of this research will have important implications for the modeling and risk management of interest rate instruments, especially for Eurodollar futures and options markets.

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