

# Ultimate consumption risk and investment-based stock returns

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## Abstract

We show that the ultimate consumption model proposed by Parker and Julliard (2005) well explains the cross-section of investment-based stock returns. By the generalized method of moment (GMM) estimation, we find that the ultimate consumption model with horizons from 3 years to 4 years has superior performance to the contemporaneous consumption model. The linearized model's performance is comparable to that of the Fama-French and Chen-Roll-Ross model. We argue that the better performance of the ultimate model is linked to the relationship between business-cycle frequency consumption shocks and investment-based returns.

JEL classification: G12

Keywords: investment-based portfolio; long-run risk; expected return

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# **Ultimate consumption risk and investment-based stock returns**

## **Abstract**

We show that the ultimate consumption model proposed by Parker and Julliard (2005) well explains the cross-section of investment-based stock returns. By the generalized method of moment (GMM) estimation, we find that the ultimate consumption model with horizons from 3 years to 4 years has superior performance to the contemporaneous consumption model. The linearized model's performance is comparable to that of the Fama-French and Chen-Roll-Ross model. We argue that the better performance of the ultimate model is linked to the relationship between business-cycle frequency consumption shocks and investment-based returns.

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# 1 Introduction

The asset pricing literature has its foundation in the consumption-based asset pricing model, pioneered by Lucas (1978), Breeden (1979). Intuitively, the asset prices should be related to the expectation about the future consumption growth given their payoffs at the future date. However, the consumption-based model has suffered from its poor empirical performance. Hansen and Singleton (1983) and Mehra and Prescott (1985) document that the classical consumption-based model cannot explain the empirical feature of equity and risk-free returns with reasonable parameter values in the model. After these critiques, numerous efforts are made to improve the empirical performance of the consumption-based model. Examples include Jagannathan and Wang (1996), Campbell and Cochrane (1999), Lettau and Ludvigson (2001), Bansal and Yaron (2004), Yogo (2006), and Hansen, Heaton and Li (2008). Given the intuitiveness of the model, the consumption-based model is still considered as one of the important topics in asset pricing.

After a significant theoretical improvement by Campbell and Cochrane (1999) and Bansal and Yaron (2004), many studies on the consumption-based asset pricing model incorporate the external habit or the long-run risk in the model. Calibration is generally used in proving the empirical performance of the model. Although calibration gives us good insights to see the model's performance in a large simulated sample, it might not be appropriate to examine the in-sample performance of the model.

It is well known that firms' investment is closely related to their stock returns. As empirically shown in Cooper, Gulen and Schill (2008), low investment firms earn higher returns. The relationship is quite strong, and its economic interpretation is supported by q-theory. Recently, Fama and French (2015) and Hou, Xue and Zhang (2014) propose similar asset pricing models with the investment and profitability factors. With its rich economic implications, the investment-based returns are considered as a risk factor. The relationship between the investment factor and macroeconomics is studied by Wang (2013) and Min, Kang and Lee (2015).

We take a step to estimate the relationship between the consumption risk and the investment-based portfolio returns empirically, employing the advanced model in Parker and Julliard (2005), which is called the ultimate consumption model. In this paper, we first estimate the generalized method moment (GMM) from the Euler equation to evaluate the ultimate consumption model's ability to price the investment-based returns. Second, we evaluate the ability of linearized ultimate consumption model to explain the actual spread of investment-based portfolios, and compare it to two well-known asset pricing models, which are the Fama-French three factor model and the Chen-Roll-Ross macroeconomic factor model.

We summarize the main findings of this paper as follows. First, the GMM estimates show that the ultimate consumption model with horizon  $S \geq 11$  performs better than the contemporaneous model. However, we do not observe the same pattern in Parker and Julliard (2005), which shows the hump-shaped explanatory power with its peak at  $S = 11$ . Second, the linearized model captures 48% of the actual spread of the low-minus-high investment strategy. This performance is striking compared to the other pricing models tested in this paper in that it best replicates the actual returns with comparable explanatory power as a one-factor model. Third, the performance of the ultimate consumption model can be justified by the close business-cycle relationship between the ultimate consumption growth and the investment-based returns.

The remainder of this paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the data and empirical methodology used in this paper. Section 4 reports our empirical evidence and following discussions. Section 5 concludes.

## **2 Related Literature**

Parker and Julliard (2005) argue that the consumption risk from longer-horizon consumption growth explains the cross-section of 25 size and book-to-market portfolios. There are three potential logics behind the ultimate consumption growth as a better measure: slow adjustment, measurement

error, nonseparability of marginal utility. They construct the moment conditions by cumulating the classical Euler equation and use the GMM to estimate the model. As a result, the ultimate consumption model with horizon of 3 years best prices the returns with the cross-sectional  $R^2$  of 44%. Grammig, Schrimpf and Schuppli (2009) test the ultimate consumption model in the US, UK, and German markets. They find that the model's performance gets weaker when they include industry portfolios in the test asset, although the risk aversion estimate is more reasonable in longer horizon.

A growing body of the asset pricing literature focuses on the negative relation between firms' investment and its stock returns (Cochrane (1991); Cooper, Gulen and Schill (2008); Li and Zhang (2010); Liu, Whited and Zhang (2009)). Cooper and Priestley (2011) show that the cross-section of investment-based returns are well explained by the loadings on the Chen, Roll and Ross (1986) macroeconomic factors.

Our paper makes a linkage between the consumption-based asset pricing model and the investment-based returns. Although we do not have to model the consumption side and the production side simultaneously as noted by Cochrane (2009), if the investment-based returns contain economic risk, they should be closely related to the consumption risk. Motivated by Min, Kang and Lee (2015) who show that the investment factor is closely related to macroeconomics and business cycle fluctuations, we directly investigate the relationship between the long horizon consumption growth and investment portfolio returns.

### **3 Data and Empirical Methodology**

#### **3.1 Data**

For test assets, we use quarterly returns on 10 investment-sorted portfolios and 25 size-investment sorted portfolios from Kenneth French's website.<sup>1</sup> Since we extend our ultimate consumption model

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<sup>1</sup>[http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

to the horizon  $S = 15$  (four years), we discard the last 15 quarters from the available sample. As a result, the sample period is from 1963:Q3 to 2011:Q1. Fama and French (2015) measure investment of a firm as the percentage growth of total asset (compustat item AT). This measure is also used in Cooper and Priestley (2011) which they call as asset growth (AG), and Hou, Xue and Zhang (2014) use the same definition as the measure of investment. We use returns of 3-month Treasury bonds as the risk-free rate of the quarter.

We use real per capita consumption data from the National Income and Product Accounts (NIPA). We make the standard “end-of-period” assumption that consumption during period  $t$  takes place at the end of the period. Following Parker and Julliard (2005), we use nondurable consumption expenditure as our consumption measure. Our consumption data covers the sample from 1963:Q2 to 2014:Q4.

The competing asset pricing models in this paper are the Fama-French three factor model and the Chen-Roll-Ross model. Since the Fama and French (2015) and Hou, Xue and Zhang (2014) use the investment-based excess returns as one of the risk factors, we do not include those models here, because our purpose is to explain the investment-based returns themselves. We use the Fama-French factors from Kenneth French’s website. The market factor (MKT) is the value-weighted excess returns of NYSE, AMEX, and NASDAQ. The size factor (SMB) is the excess returns of the portfolios of small and big stocks, and the book-to-market factor (HML) is the excess returns of the portfolios of high book-to-market and low book-to-market stocks. Liu and Zhang (2008) argue that momentum profit can be explained by the Chen-Roll-Ross macroeconomic factors, and Cooper and Priestley (2011) show that the production-based anomalies are well explained by the loadings on the Chen-Roll-Ross factors. We follow Liu and Zhang (2008) to construct the Chen-Roll-Ross macroeconomic factors. We define marginal production (MP) as the log growth of the index of industry production from the Federal Reserve Bank of St. Louis. Unexpected inflation (UI) is defined as  $UI_t = I_t - E[I_t|t-1]$ , change of expected inflation (DEI) is  $DEI_t = E[I_{t+1}|t] - E[I_t|t-1]$ . Inflation measure is the log difference of consumer price index from the Federal Reserve Bank of St. Louis. The

expected inflation is calculated as  $E[I_t|t-1] = r_{ft} - E[RHO_t|t-1]$ , where  $r_{ft}$  is one-month Treasury bill rate and  $RHO_t = r_{ft} - I_t$  is the *ex post* real return on Treasury bills. Following Fama and Gibbons (1984),  $RHO_t$  is modeled as ARIMA(0,1,1) process as  $RHO_t - RHO_{t-1} = u_t + \theta u_{t-1}$  and the *ex ante* real rate,  $E[RHO_t|t-1]$  is calculated as  $E[RHO_t|t-1] = (r_{ft-1} - I_{t-1}) + u_t + \theta u_{t-1}$ . Term spread (UTS) is the difference between the yields of a 10-year and a 1-year government bonds. Default spread (UPR) is the difference between the yields on Moody's Baa and Aaa corporate bonds.

Table 1 describes the returns of our test assets. In Panel A, we report the mean quarterly returns of the 10 investment-sorted portfolios. The mean returns show the clear pattern that the expected returns decrease with investment. The pattern is stronger in the equally-weighted portfolios, and the difference between the lowest and the highest investment portfolios is significant. Panel B shows the mean returns of 25 size-investment double-sorted portfolios. We observe that the expected returns are quite similar in four portfolios except for the highest investment portfolios.

## 3.2 Empirical Methodology

### 3.2.1 The Ultimate Consumption Model

We introduce the ultimate consumption model in Parker and Julliard (2005) to explain the cross-section of stock returns. We start from the two-period consumption-based model, which will be documented in this paper as the “contemporaneous consumption-based model”. The model implies the Euler equation as follows:

$$E_t \left[ \delta \frac{u'(C_{t+1})}{u'(C_t)} R_{t+1}^e \right] = 0 \quad (1)$$

where  $C_t$  is the consumption expenditure at time  $t$ ,  $u(\cdot)$  denotes the utility function,  $\delta$  is the time-discount factor, and  $R_{t+1}^e$  is the excess return of any asset. The stochastic discount factor (SDF) in this model is defined as  $m_{t+1} = \delta \frac{u'(C_{t+1})}{u'(C_t)}$ , and the expected excess return is

$$E[R_{t+1}^e] = -\frac{\text{Cov}(R_{t+1}^e, m_{t+1})}{E(m_{t+1})}.$$

Parker and Julliard (2005) exploit the consumption Euler equation for the risk-free rate from time  $t + 1$  to  $t + 1 + S$  to yield the following relation:

$$u'(C_{t+1}) = \delta E_{t+1}[R_{t+1,t+1+S}^f u'(C_{t+1+S})] \quad (2)$$

When we define  $m_{t+1}^S = R_{t+1,t+1+S}^f u'(C_{t+1+S})/u'(C_t)$  and substitute equation (2) into equation (1), we end up with the following relation of expected excess return and consumption growth to the far future.

$$E[R_{t+1}^e] = -\frac{\text{Cov}(R_{t+1}^e, m_{t+1}^S)}{E(m_{t+1}^S)} \quad (3)$$

Following Parker and Julliard (2005), we call  $-\text{Cov}(R_{t+1}^e, m_{t+1}^S)$  as the *ultimate* consumption risk, and we assume the power utility function  $u(C_t) = \frac{1}{1-\gamma} C_t^{1-\gamma}$ .

Using the first-order log linearization of Lettau and Ludvigson (2001), we examine the performance of linearized consumption CAPM. With this approximation, the linearized version of stochastic discount factor has the following form:

$$m_{t+1}^S = R_{t+1,t+1+S}^f - \gamma_S R_{t+1,t+1+S}^f \Delta c_{t+1+S} \quad (4)$$

where  $\Delta c_{t+1+S} = \ln\left(\frac{C_{t+1+S}}{C_t}\right)$ . If we further assume that  $R_{t+1,t+1+S}^f$  is constant over time, the ultimate consumption model can be viewed as a one-factor model with the ultimate consumption growth as the unique factor.

### 3.2.2 Estimation Methodology

We use a slightly modified version of moment restrictions which is used in Parker and Julliard (2005) and Yogo (2006). Taking this approach has a number of benefits. First, it allows us to compare our results directly to those of Parker and Julliard (2005). Second, it shows the model's performance in two dimensions: (1) whether it explains the overall level of equity premium and (2) whether it



explains the cross-section of the test assets. With the power utility function, the stochastic discount factor is  $m_t^S = R_{t,t+S}^f \left( \frac{C_{t+S}}{C_{t-1}} \right)^{-\gamma}$ . The moment condition for the GMM estimation is expressed as follows:

$$E[g(R_t^e, C_{t+1+S}, C_{t-1}, \mu_S, \gamma_S, \alpha_S)] = \begin{bmatrix} R_t^e - \alpha_S \mathbf{1}_N + \frac{(m_t^S - \mu_S) R_t^e}{\mu_S} \\ m_t^S - \mu_S \end{bmatrix} \quad (5)$$

where  $\alpha_S$  captures the average difference of the sample equity returns and the model-implied returns. The last moment allows us to compare the pricing models with different  $S$ s under a similar criterion.

We estimate the above model with two types of GMM. First, the GMM with a prespecified weighting matrix uses a diagonal matrix which has ones in the diagonals except for the last element and a very large weight for the last moment. Under this method, the GMM prices the exact portfolios we use in the test, not the combinations of the portfolios. To examine the model's performance, we use the distance measure in Jagannathan and Wang (1996) and Hansen and Jagannathan (1997), which is called the Hansen-Jagannathan distance. We follow the appendix of Parker and Julliard (2005) to evaluate this measure. Second, we also report the efficient GMM which we re-estimate the weighting matrix until convergence. In this case, we use Hansen (1982)'s  $J$ -test to evaluate the model's performance.

To test the linearized model's performance, we use two approaches. First, we adopt the standard cross-sectional regression of Fama and MacBeth (1973). We examine whether the cross-sectional coefficient on the ultimate consumption beta is significant when we increase the horizon  $S$  to measure the consumption growth. Second, we use the methodology used in Liu and Zhang (2008) and Cooper and Priestley (2011). More specifically, we construct the risk premium of the linear factor pricing models from a broad collection of test assets. After we build the risk premium of the factor models, we calculate the expected return of a portfolio  $P$  as follows:

$$E[R_P] = \widehat{\lambda}_F' \widehat{\beta}_F, \quad (6)$$

where  $\widehat{\beta}_F$  is the vector of factor loadings on the asset pricing factors from the time-series regression,

and  $\widehat{\lambda}_F$  is the vector of risk premiums. By doing this, we estimate the model-implied spread of low and high investment portfolios. We examine how the actual spread can be explained by the factor models.

## 4 Empirical Evidence

### 4.1 Performance of the Ultimate Consumption Model

Table 2 and Table 3 illustrate the GMM estimation results with ten equally and value weighted investment portfolios, respectively. Panel A shows the results when we use a prespecified weighting matrix, and Panel B presents the results when we use efficient GMM. We evaluate cross-sectional  $R^2$  as follows:

$$R^2 = 1 - \frac{\text{Var}(E_T[R_t^e] - \widehat{R}_t^e)}{\text{Var}(E_T[R_t^e])} \quad (7)$$

where  $E_T[R_t^e] = \frac{1}{T} \sum_{t=1}^T R_{t,t}^e$  and  $\widehat{R}_t^e = \widehat{\alpha}_S - \frac{E_T[(\widehat{m}_t^S - \widehat{\mu}_S) R_{t,t}^e]}{\widehat{\mu}_S}$ . We report estimated values of  $\alpha_S$  and  $\gamma_S$  with their GMM standard errors in brackets. In Panel A and B, we show the Hansen-Jagannathan distance and the Hansen  $J$ -statistic with their p-values in square brackets.

As documented in the previous literature, the performance of contemporaneous consumption CAPM with  $S = 0$  is disappointing for the following two reasons. First, its  $R^2$  is near zero or even negative in the efficient GMM, which means that contemporaneous consumption CAPM does not help explain the cross-section of excess returns of investment-based portfolios. Second, the term  $\alpha_0$  has the value near 2% and statistically significant. This is close to the cross-sectional average of excess returns, which implies the model's poor performance to capture the average level of equity premium.

When we increase the horizon  $S$  to measure the consumption growth, the increase in pricing performance is dramatic. First, in Table 2, the cross-sectional  $R^2$  increases to 0.671 when  $S = 15$ . In Table 3, we observe the highest  $R^2$  when  $S = 9$  with the highest value of 0.727. Although these results do not exactly replicate those of Parker and Julliard (2005) with 25 size and book-to-market

portfolios, the remarkable increase in cross-sectional explanatory power in horizons from  $S = 9$  to  $S = 15$  shows that the ultimate consumption risk with business-cycle horizon is closely connected to the cross-section of investment-sorted returns. Second, when  $S$  is greater than 5 quarters, the term to capture the equity premium,  $\alpha_S$ , has its quarterly absolute value under 1% and statistically insignificant. Compared to the contemporaneous consumption CCAPM, this shows that the ultimate consumption risk explains the overall level of equity premium.

The Hansen-Jagannathan distance in Panel A of Table 2 and 3 rejects the consumption CAPM for all values of  $S$  when we use prespecified weighting matrix for GMM estimation. In Panel B of Table 2, the consumption CAPM is not rejected in 5% significance level only when  $S = 9$ . Moreover, in Panel B of Table 3, the ultimate consumption CAPM is not rejected in 10% significance level when  $S \geq 5$ . Although the success of the model might come from the fact that ten value-weighted investment-sorted portfolios show relatively small cross-sectional variation, it can be interpreted as additional evidence of superior performance of the ultimate consumption CAPM.

We also perform the same GMM estimation with 25 equally and value weighted size-investment portfolios in Table A1 and A2 of the Appendix. The patterns of the results repeat those in Table 2 and 3. Focusing on the results from a prespecified weighting matrix, the ultimate consumption model attains the highest  $R^2$  when  $S = 11$ , repeating the results in Parker and Julliard (2005).

In sum, the ultimate consumption model performs better than the contemporaneous consumption model in explaining the cross-section of investment-based returns. Given the striking empirical performance, we further investigate the performance of the linearized model with simple regression approaches.

## 4.2 Performance of the Linearized Model

In this subsection, we examine the linearized ultimate consumption model's performance to price the investment-based portfolios and compare it to that of the other well-known asset pricing models.

We first conduct the Fama-MacBeth regressions for the linearized model. First, we perform the time-series regressions for the ultimate consumption models as follows:

$$R_t^{ei} = \alpha^i + \beta_{\Delta c_{t+1+S}}^i \Delta c_{t+S} + \eta_t^i \text{ for } i = 1, 2, \dots, N \quad (8)$$

In the second-pass regressions, we use the betas from the time-series regression and perform cross-sectional regressions as follows:

$$E[R_t^{ei}] = \lambda_0 + \lambda_S \beta_{\Delta c_{t+S}}^i + \epsilon_S^i \quad (9)$$

In Table 6, we report the cross-sectional coefficients  $\lambda_S$  with their Fama-MacBeth t-statistics, the adjusted t-statistics in Shanken (1992), and the cross-sectional  $R^2$ s. Panel A, B, C, and D exhibit the results when the test assets are ten equally and value weighted investment-based portfolios and 25 equally and value weighted size-investment portfolios.

The results in Table 6 show that the ultimate consumption models perform better than the contemporaneous consumption model. Although we cannot firmly argue that the higher cross-sectional  $R^2$  in cross-sectional regression is a robust measure to examine the model's performance following Lewellen, Nagel and Shanken (2010), the cross-sectional  $R^2$  attains its maximum at  $S = 11$  in every Panel with the maximum value of 0.54 in Panel C. The size of the risk premium of consumption beta increases with the horizon  $S$ . Some of the risk premiums are significant when  $S = 11$  in Panel B, C, and D according to the Fama-MacBeth t-statistics. However, the Shanken (1992) t-statistics are not significant in any cases, which implies that the linearized model's performance is not perfectly successful in explaining the cross-section of returns. Overall, the results from the classical Fama-MacBeth regression provide evidence that the ultimate consumption model also outperforms the contemporaneous consumption model in the log-linearized version.

### 4.3 Performance Test to Explain the Investment-Based Spreads

We now turn our focus to the ability of asset pricing models to explain the spread between the low and high investment portfolios. First, we construct the risk premiums from the Fama-MacBeth regressions

of 40 test assets. The 40 test assets include ten size, ten book-to-market, ten momentum, and ten investment portfolios. The size, value, and investment portfolios are equally weighted, and the momentum portfolios are value weighted. The returns of test assets are from the Kenneth French's website. The factor pricing models include the ultimate consumption model, the Fama-French three factor model, and the Chen-Roll-Ross model. In estimating the risk premiums, we assume that the risk premium is constant over time, and perform the Fama-MacBeth full sample regressions following Griffin, Ji and Martin (2003).

We report the estimated risk premiums from the 40 test assets in Table 5. We report the cross-sectional loadings  $\widehat{\lambda}_S$  and the pricing error term  $\lambda_0$ , the Fama-MacBeth t-statistics in parentheses, and the cross-sectional  $R^2$ . In Panel A, we show the results from the ultimate consumption model. We observe that the consumption model does well in explaining the average level of stock portfolios, since the estimated  $\lambda_0$  is insignificant except for the case of  $S = 7$ . The risk premium term ranges from 0.8% to 5.7%, and is significant only when  $S = 11$ . The estimated cross-sectional  $R^2$  is from 20% to 48.5%, which are greater than those in Table 4 when we only use the investment-based portfolios. In Panel B and C, we display the results from the Fama-French three factor model and the Chen-Roll-Ross model. The Fama-French three factors exhibit significant factor loadings, where the risk premium on the market factor is negative. The risk premiums Chen-Roll-Ross factors show a similar pattern in Cooper and Priestley (2011), which is natural since the regression in Panel C is a quarterly version of the same regression. The constant terms in Panel B and C are 8.2% and 2.9% per quarter, and statistically significant. This implies that the Fama-French model cannot capture the zero-beta rate, and so the Chen-Roll-Ross model does. Finally, the cross-sectional  $R^2$ s of the Fama-French and Chen-Roll-Ross model are 65.4% and 89.8%, which are higher than the consumption-based models.

Having estimated the risk premiums of the asset pricing factors, we test whether the risk factors can well explain the lowest (highest) investment portfolio return and the low-minus-high spread. We estimate the factor loadings of the portfolios from the time-series regressions, and multiply the risk

premium estimates in Table 5 to get the expected model returns as in equation (6). The results are shown in Table 6, with the equally weighted portfolios as the test assets, respectively. The results from the value weighted portfolios are qualitatively the same. We report the factor loadings, actual returns, explained returns, and the proportion of explained and actual returns for the lowest decile, highest decile, and their differences.

In Panel A of Table 6, we report the actual and model implied returns of the lowest (highest) investment decile portfolio. For the sake of brevity, we focus on the ultimate consumption model of horizon  $S = 0, 11, 15$ , since the omitted horizons exhibit a monotonic pattern. First, although the factor loadings of the lowest and highest portfolios on the consumption growth do not differ significantly in every case, the loadings on the lowest decile is always greater than the highest decile. Second, the proportion of the explained and actual spread increases with the horizon  $S$ , which repeats our previous estimation results. The contemporaneous consumption CAPM explains only 4% of the actual spread, while the ultimate consumption CAPM captures 27% and 48% of the actual premium. The explained spread increases from 0.1% to 1.7% when  $S$  increases from 0 to 15.

In Panel B of Table 6, we display the performance of the Fama-French model. First, the negative low-high difference of the market beta and positive differences of the SMB and HML beta both help explain the positive spread, since the estimated risk premium on MKT is negative in Table 5. As a result, the proportion of the explained and the actual spread is 36%. However, the model fails to capture the individual portfolio returns. The explained returns for the lowest and highest investment portfolios are -5% and -3.8%, respectively. This comes from the big negative premium of the market beta in Table 5.

In Panel C of Table 6, the Chen-Roll-Ross model is tested. All but DEI loadings help explain the low-high spread, combined with their signs of risk premiums in Table 5. The wrong direction of DEI is negligible because its estimated risk premium in Table 5 is near zero. The ratio of the explained and actual returns is 67%, which is the highest among the three models tested.

We confirm that the ultimate consumption model performs better than the contemporaneous

consumption model in explaining the actual spread of investment-sorted portfolios. The explained part of the spread is 27% and 48% of the actual spread when  $S = 11$  and  $S = 16$ . The maximum value 48% is greater than that of the Fama-French model. As reported in Cooper and Priestley (2011), the macroeconomic factor model is successful in explaining the cross-sectional difference in investment-sorted portfolios, with 67% of explanatory power of the actual spread.

We argue that the performance of the ultimate model is remarkable by the following reasons. First, the explained portfolio returns of the ultimate models are close to the actual ones. The actual returns are 5.9% and 2.3%, and the model implied returns of the ultimate model with  $S = 15$  are 4.3% and 2.6%, respectively. The Fama-French model desperately fails in that its explained returns are -5.1% and -3.8%. Even though the Chen-Roll-Ross model generates the highest spread, the explained returns are 2.5% and 0.1%, which are far from the actual returns. Second, the performance of the ultimate model can be highlighted since it is one-factor model. Although its performance is not better than the Chen-Roll-Ross model in explaining the actual spread, the ultimate consumption model gives a one-factor explanation of portfolio returns, with its comparable explanatory power.

#### **4.4 Business-Cycle Frequency Relation between Consumption and Investment-based Returns**

In this subsection, we see what is behind the success of the ultimate consumption model. Parker and Julliard (2005) explain their success of the ultimate consumption model in pricing the 25 size and book-to-market portfolios with the predictability of future consumption by SMB and HML factors. If the investment-based factor predicts the future consumption, the covariance of the investment-based portfolios and the ultimate consumption would increase with the horizon. This results in the better performance of the ultimate consumption model. We follow Parker and Julliard (2005) to test the predictability. We construct the investment factor, INV, as the difference of the lowest and highest equally weighted investment decile portfolio returns. Then we perform the predictive regressions of

the long-horizon consumption growths on the investment factor.

In Table 7, we show the results of predictive regressions as the same way in Parker and Julliard (2005). Panel A displays our predictive regression results. When  $S = 0$ , the coefficient is 0.7%, with the regression  $R^2$  of 0.2%. The regression coefficient is significantly different from zero with 10% confidence when  $S \geq 11$ , and is significant with 5% confidence when  $S \geq 15$ . The regression  $R^2$  is almost monotonically increasing with  $S$ , with its maximum of 2.8% when  $S = 15$ . The increase in  $R^2$  is remarkable after we consider the increase in the variance of consumption growth in Panel C. In Panel B, we report the results of reverse regression to examine whether the covariance future consumption growth, and the results confirm that we cannot find correlation between the investment factor and contemporaneous consumption growth where there is significantly positive relationship when  $S \geq 11$ .

When we accept the predictability of future consumption growth by the investment factor, as shown here and in Min, Kang and Lee (2015), the superior performance of the ultimate consumption model can be justified. Moreover, the  $R^2$  in Table 7 does not exhibit the hump-shaped pattern in Parker and Julliard (2005). Rather, from the horizons we examined, the longest horizon,  $S = 15$ , has the highest  $R^2$ . This can be connected to our previous results, which show the best performance when  $S = 15$  in most cases. Therefore,  $S = 11$  may not be the “best” horizon in explaining the investment-based returns.

Why the best horizon differs with the test assets? A potential explanation could be made when we focus on the business-cycle consumption risk by Bandi and Tamoni (2015). They decompose the consumption growth into its subcomponents based on their persistence. In their model, the business-cycle components, with periodicity between 2 years and 8 years, play an important role in explaining the cross-section of returns. The emphasis on the business-cycle component may correspond to the small persistent component in Bansal and Yaron (2004). Taking the long-horizon consumption growth, which is called *aggregation*, can be a good way to eliminate the short-horizon components (Bandi, Perron, Tamoni and Tebaldi (2014)). Therefore, we do not restrict ourselves to define the “best”



horizon, which may differ from asset to asset, and focus on the “business-cycle” frequency consumption shocks as the potential driver of the success of the ultimate consumption model.

## **5 Conclusion**

Recently, the investment-based returns are spotlighted by researchers, and the investment factor is considered as an important risk measure. If the investment-based returns carry economic risk, it is natural to investigate if it is related to the consumption risk. Given its close linkage to the business cycle variables, we examine whether the ultimate consumption growth, which has the simplest form as a unique asset pricing factor, is related to the investment-based returns.

Employing the GMM approach by Parker and Julliard (2005), we find that the ultimate consumption model performs significantly better than the contemporaneous consumption model. We find that the ultimate consumption model’s performance is comparable to the Fama-French three factor model and the Chen-Roll-Ross model, which implies that the ultimate consumption risk does a significant role in describing the investment-based returns. However, we do not conclude that the “best” horizon for the ultimate consumption growth is three years when size and book-to-market portfolios are used in Parker and Julliard (2005). We interpret our empirical evidence as a result of the close relationship between investment-based returns, with some slack in determining the best horizon.

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**Table 1. Summary Statistics**

Panel A presents the mean quarterly returns of 10 investment-sorted portfolios. Panel B shows the mean quarterly returns of 25 size-investment double-sorted portfolios.  $p(dif)$  shows the p-value of the test whether the mean lowest-minus-highest investment portfolio return is different from zero. The sample period is from 1963:Q3 to 2011:Q1 (187 quarters).

Panel A: 10 INV-sorted portfolios											
INV	Low	2	3	4	5	6	7	8	9	High	p(dif)
EW	0.059	0.050	0.045	0.043	0.042	0.040	0.041	0.039	0.036	0.023	0.000
VW	0.036	0.035	0.031	0.030	0.029	0.028	0.031	0.027	0.029	0.022	0.003
Panel B: 25 SIZE/INV double-sorted portfolios											
	EW	INV	1	2	3	4	5	(1-5)	p(dif)		
SIZE			1	0.062	0.051	0.047	0.045	0.030	0.031	0.000	
			2	0.041	0.042	0.042	0.042	0.026	0.015	0.001	
			3	0.043	0.042	0.037	0.040	0.026	0.017	0.000	
			4	0.037	0.036	0.036	0.035	0.029	0.008	0.080	
			5	0.037	0.031	0.031	0.029	0.023	0.013	0.006	
	VW	INV	1	2	3	4	5	(1-5)	p(dif)		
SIZE			1	0.045	0.044	0.043	0.041	0.026	0.019	0.000	
			2	0.041	0.040	0.041	0.041	0.028	0.013	0.002	
			3	0.040	0.041	0.036	0.038	0.028	0.012	0.008	
			4	0.036	0.034	0.035	0.035	0.029	0.007	0.163	
			5	0.033	0.028	0.027	0.027	0.025	0.008	0.088	

**Table 2. GMM Estimation Results**

This table displays the GMM estimation results of the ultimate consumption models from equation (4) and (5) with 10 equally weighted investment-sorted portfolios as test assets. Panel A shows the results from the GMM with prespecified weighting matrix, and Panel B depicts the results from the efficient GMM. The numbers in the brackets are the standard errors, and the numbers in the squared brackets are p-values of test statistics. The column HJ shows the Hansen-Jagannathan distance and its p-value is calculated from 10,000 simulations. The column J shows the Hansen's J-test statistic. The sample period is from 1963:Q3 to 2011:Q1.

Panel A: Prespecified weighting matrix					Panel B: Efficient GMM			
S	$R^2$	$\alpha$	$\gamma$	HJ	$R^2$	$\alpha$	$\gamma$	J
0	0.000	0.027 (0.008)	5.002 (82.881)	0.383 [0.001]	-0.033	0.019 (0.007)	57.764 (51.205)	40.910 [0]
1	0.003	0.028 (0.01)	0.792 (35.722)	0.384 [0]	-0.071	0.014 (0.009)	33.898 (19.683)	42.066 [0]
3	0.059	0.016 (0.015)	19.038 (27.776)	0.342 [0]	-0.012	0.012 (0.011)	30.733 (15.665)	30.982 [0]
5	0.164	-0.006 (0.015)	30.282 (13.337)	0.279 [0]	0.145	0.021 (0.01)	32.012 (9.556)	22.021 [0.005]
7	0.125	0.003 (0.013)	20.432 (11.971)	0.323 [0]	0.053	0.022 (0.01)	24.146 (8.728)	24.875 [0.002]
9	0.478	-0.009 (0.013)	40.591 (12.871)	0.198 [0]	0.448	0.027 (0.008)	33.248 (8.432)	15.380 [0.052]
11	0.498	-0.005 (0.012)	42.078 (16.84)	0.214 [0]	0.438	0.022 (0.008)	34.600 (9.037)	15.902 [0.044]
13	0.535	0.003 (0.009)	42.476 (18.151)	0.204 [0]	0.496	0.021 (0.007)	42.425 (10.166)	18.904 [0.015]
15	0.671	0.006 (0.011)	48.951 (22.59)	0.197 [0]	0.650	0.025 (0.007)	41.244 (10.403)	21.156 [0.007]

**Table 3. GMM Estimation Results**

This table displays the GMM estimation results of the ultimate consumption models from equation (4) and (5) with 10 value weighted investment-sorted portfolios as test assets. Panel A shows the results from the GMM with prespecified weighting matrix, and Panel B depicts the results from the efficient GMM. The numbers in the brackets are the standard errors, and the numbers in the squared brackets are p-values of test statistics. The column HJ shows the Hansen-Jagannathan distance and its p-value is calculated from 10,000 simulations. The column J shows the Hansen's J-test statistic. The sample period is from 1963:Q3 to 2011:Q1.

Panel A: Prespecified weighting matrix					Panel B: Efficient GMM			
S	$R^2$	$\alpha$	$\gamma$	HJ	$R^2$	$\alpha$	$\gamma$	J
0	0.052	0.022 (0.006)	-49.824 (53.022)	0.146 [0.008]	-0.117	0.016 (0.006)	42.098 (32.823)	18.919 [0.015]
1	0.106	0.024 (0.007)	-28.772 (32.389)	0.139 [0]	-0.342	0.013 (0.007)	23.558 (19.549)	17.849 [0.022]
3	0.002	0.018 (0.008)	-3.901 (25.636)	0.129 [0]	-0.150	0.009 (0.009)	24.655 (14.017)	13.073 [0.109]
5	0.023	0.022 (0.007)	-12.174 (19.667)	0.149 [0]	-0.137	0.008 (0.007)	20.308 (8.654)	12.334 [0.137]
7	0.287	-0.002 (0.009)	22.849 (11.448)	0.065 [0]	0.285	0.007 (0.007)	21.724 (7.602)	9.322 [0.316]
9	0.727	0.003 (0.009)	30.941 (10.425)	0.051 [0]	0.662	0.010 (0.006)	24.129 (7.782)	6.617 [0.578]
11	0.615	0.004 (0.009)	30.983 (13.166)	0.073 [0]	0.510	0.009 (0.006)	20.343 (8.344)	9.311 [0.317]
13	0.437	0.005 (0.009)	31.283 (19.237)	0.068 [0]	0.404	0.006 (0.006)	24.614 (11.982)	7.437 [0.49]
15	0.282	0.007 (0.009)	29.304 (17.455)	0.083 [0]	0.245	0.007 (0.006)	21.953 (11.803)	10.027 [0.263]

**Table 4. Cross-sectional regressions of Linearized Ultimate Consumption Models**

This table displays the results of the following cross-sectional regressions.

$$E[R_t^{ei}] = \lambda_0 + \lambda_S \beta_{\Delta c_{t+S}}^i + \epsilon_S^i,$$

where  $\beta_{\Delta c_{t+S}}^i$  is the loading from the time-series regressions  $R_t^{ei} = \alpha^i + \beta_{\Delta c_{t+1+S}}^i \Delta c_{t+S} + \eta_t^i$ .

Panel A and B show the results with ten equally and value weighted investment portfolios, respectively. Panel C and D show the results with 25 equally and value weighted size-investment portfolios, respectively. The sample period is from 1963:Q3 to 2011:Q1.

Horizon	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Panel A: INV EW																
coeff	0.003	0.002	0.006	0.008	0.006	0.010	0.013	0.015	0.030	0.032	0.041	0.055	0.054	0.058	0.070	0.073
FM t	1.206	0.451	0.958	1.081	0.726	0.926	1.090	0.987	1.587	1.532	1.680	2.086	2.024	2.359	3.703	4.549
SH t	1.017	0.439	0.843	0.955	0.685	0.829	0.928	0.838	1.037	1.004	0.978	1.021	1.036	1.186	1.664	2.036
R <sup>2</sup>	0.080	0.011	0.049	0.063	0.028	0.045	0.061	0.049	0.118	0.108	0.120	0.150	0.122	0.096	0.138	0.151
Panel B: INV VW																
coeff	0.000	-0.002	-0.003	-0.002	-0.003	-0.004	0.000	0.006	0.010	0.014	0.017	0.019	0.018	0.019	0.019	0.015
FM t	0.089	-0.747	-0.798	-0.410	-0.498	-0.487	0.041	0.546	0.998	1.289	1.476	1.672	1.496	1.628	1.595	1.093
SH t	0.089	-0.722	-0.762	-0.404	-0.488	-0.476	0.041	0.531	0.924	1.149	1.274	1.432	1.312	1.415	1.397	1.013
R <sup>2</sup>	0.001	0.078	0.081	0.021	0.031	0.027	0.000	0.026	0.057	0.083	0.092	0.090	0.069	0.069	0.062	0.033
Panel C: SIZEINV EW																
coeff	0.005	0.006	0.010	0.013	0.015	0.020	0.025	0.030	0.041	0.045	0.053	0.063	0.071	0.086	0.092	0.092
FM t	2.176	1.712	1.638	1.903	1.801	1.992	2.179	2.243	2.547	2.571	2.668	2.832	2.861	3.092	3.419	3.476
SH t	1.507	1.338	1.250	1.387	1.359	1.383	1.428	1.389	1.361	1.346	1.293	1.246	1.188	1.122	1.219	1.278
R <sup>2</sup>	0.341	0.211	0.147	0.257	0.250	0.307	0.384	0.415	0.489	0.506	0.536	0.542	0.513	0.470	0.441	0.421
Panel D: SIZEINV VW																
coeff	0.002	0.003	0.003	0.005	0.006	0.009	0.015	0.020	0.030	0.033	0.039	0.047	0.053	0.066	0.068	0.067
FM t	1.149	0.800	0.509	0.765	0.797	1.004	1.328	1.548	1.966	2.012	2.151	2.359	2.412	2.737	3.048	3.042
SH t	1.036	0.761	0.495	0.723	0.752	0.901	1.104	1.186	1.285	1.288	1.282	1.289	1.254	1.244	1.391	1.455
R <sup>2</sup>	0.129	0.063	0.020	0.053	0.063	0.096	0.172	0.235	0.319	0.343	0.387	0.397	0.375	0.363	0.347	0.326

**Table 5. Estimates of Risk Premiums**

We estimate the risk premiums of the risk factors in three asset pricing models: the consumption CAPM, the Fama-French three factor model, the Chen-Roll-Ross model. We estimate the factor loadings of the asset pricing factors in time-series regressions of 40 test assets in the first stage. In the second stage, we use those loadings to estimate the risk premiums. For the test assets, we use ten equally weighted size portfolios, ten equally weighted book-to-market portfolios, ten value weighted momentum portfolios, and ten equally weighted investment portfolios. We report the estimated risk premium coefficients from the second stage.

Panel A: CCAPM							
	$\lambda_0$	$\Delta c$					$R^2$
0	0.017 (1.84)	0.008 (1.623)					0.199
3	0.017 (1.955)	0.014 (1.674)					0.178
7	0.019 (2.267)	0.018 (1.581)					0.155
11	0.016 (1.594)	0.037 (2.063)					0.358
15	0.011 (0.905)	0.057 (1.961)					0.485
Panel B: Fama-French three factor model							
	$\lambda_0$	MKT	SMB	HML			$R^2$
	0.082 (5.13)	-0.056 (-3.225)	0.011 (2.101)	0.013 (2.31)			0.654
Panel C: Chen-Roll-Ross model							
	$\lambda_0$	MP	UI	DEI	UTS	UPR	$R^2$
	0.029 (2.718)	0.037 (4.207)	0.003 (0.697)	0.000 (0.186)	0.041 (2.869)	-0.017 (-3.425)	0.898



**Table 6. Average Returns and Expected Return Spreads**

This table displays the loadings on the mimicking portfolios of the asset pricing factors for the lowest and highest equally weighted investment decile.

$$E[R_t^{ei}] = \lambda_F' \beta$$

where  $\beta$  is the vector of factor loadings from the time-series regression, and  $\lambda_F$  is the vector of risk premiums estimated in Table 5. “Actual” column is the sample mean of the portfolio. “Explained” column displays the model-implied returns. “Proportion” column shows the ratio of the model-implied spread and actual spread. Panel A, B, and C show the results from the ultimate consumption CAPM, Fama-French three factor model, and Chen-Roll-Ross model, respectively.

Panel A: ultimate CCAPM						Panel B: Fama-French three factor model								
S		$\Delta c$	actual	explained	proportion		MKT	SMB	HML		actual	explained	proportion	
0	Low	3.479	0.059	0.028		Low	1.102	1.694	0.363		0.059	-0.051		
		(2.023)						(11.184)	(11.571)	(2.31)				
	High	3.300	0.023	0.027		High	1.147	1.363	-0.144		0.023	-0.038		
		(2.272)						(20.283)	(15.649)	(-1.672)				
	dif	0.179	0.036	0.001	0.040	dif	-0.046	0.331	0.507		0.036	0.013	0.356	
p(dif)	(0.468)				p(dif)	(0.657)	(0.027)	(0.003)						
11	Low	0.884	0.059	0.033		Panel C: Chen-Roll-Ross model								
		(2.621)					MP	UI	DEI	UTS	UPR	actual	explained	proportion
	High	0.623	0.023	0.023		Low	0.727	0.046	4.529	0.541	1.463	0.059	0.025	
		(1.97)						(13.297)	(0.381)	(5.698)	(13.144)	(20.125)		
	dif	0.260	0.036	0.010	0.267	High	0.344	-1.206	5.578	0.464	1.658	0.023	0.001	
p(dif)	(0.287)						(6.721)	(-10.164)	(14.252)	(21.059)	(24.093)			
15	Low	0.762	0.059	0.043		dif	0.383	1.252	-1.049	0.077	-0.195	0.036	0.024	0.673
		(2.533)				p(dif)	(0)	(0)	(0.881)	(0.05)	(0.974)			
	High	0.457	0.023	0.026										
		(1.688)												
	dif	0.305	0.036	0.017	0.481									
p(dif)	(0.226)													

**Table 7. Forecasting the Future Consumption by Investment-based Factors**

We construct the INV factor as the difference of the lowest and highest equally weighted investment decile portfolios. In Panel A, we show the time-series regression coefficients and  $R^2$ s when we forecast the ultimate consumption growth with the INV factor. In Panel B, we report the results of reverse-regressions, which forecasts future INV factor with the ultimate consumption growth. Panel C displays the variances of ultimate consumption growth.

Panel A: cons. Growth on INV			Panel B: INV on future cons. Growth		Panel C: Variance of cons. Growth
S	coeff	$R^2$	coeff	$R^2$	
0	0.002 0.227	0.000	0.375 0.576	0.002	0.662
1	0.007 0.516	0.002	0.466 1.169	0.006	1.427
2	0.016 0.940	0.005	0.417 1.291	0.008	2.350
3	0.023 1.154	0.007	0.211 0.821	0.003	3.410
4	0.018 0.718	0.003	0.237 1.103	0.006	4.435
5	0.025 0.927	0.005	0.231 1.209	0.007	5.419
6	0.030 0.974	0.006	0.186 1.056	0.005	6.375
7	0.030 0.875	0.005	0.210 1.457	0.008	7.339
8	0.038 1.115	0.008	0.218 1.718	0.009	8.015
9	0.044 1.240	0.009	0.211 1.759	0.009	8.726
10	0.046 1.286	0.009	0.277 2.561	0.017	9.366
11	0.064 1.787	0.017	0.274 2.629	0.018	10.149
12	0.067 1.882	0.017	0.253 2.489	0.016	10.994
13	0.066 1.804	0.016	0.301 2.867	0.025	11.829
14	0.083 2.225	0.024	0.318 3.026	0.029	12.656
15	0.092 2.435	0.028	0.304 2.875	0.027	13.367

## Appendix: The GMM estimation results for the size-investment double-sorted portfolios

**Table A1. GMM Estimation Results**

This table displays the GMM estimation results of the ultimate consumption models from equation (4) and (5) with 25 equally weighted size-investment double-sorted portfolios as test assets. Panel A shows the results from the GMM with prespecified weighting matrix, and Panel B depicts the results from the efficient GMM. The numbers in the brackets are the standard errors, and the numbers in the squared brackets are p-values of test statistics. The column HJ shows the Hansen-Jagannathan distance and its p-value is calculated from 10,000 simulations. The column J shows the Hansen's J-test statistic. The sample period is from 1963:Q3 to 2011:Q1.

Panel A: Prespecified weighting matrix					Panel B: Efficient GMM			
S	rsq	alpha	gamma	HJ	rsq	alpha	gamma	jtest
0	0.141	0.009 (0.016)	100.426 (78.127)	0.838 [0]	0.085	0.026 (0.006)	45.388 (23.535)	68.613 [0]
1	0.045	0.016 (0.011)	18.540 (32.624)	0.762 [0]	0.046	0.020 (0.005)	20.447 (13.899)	69.441 [0]
3	0.114	0.013 (0.012)	18.448 (23.262)	0.800 [0]	-0.074	0.025 (0.005)	-6.985 (11.872)	70.005 [0]
5	0.344	-0.003 (0.014)	27.277 (12.944)	0.960 [0]	0.210	0.025 (0.005)	4.215 (6.722)	98.898 [0]
7	0.368	-0.001 (0.012)	22.276 (10.297)	0.919 [0]	0.007	0.024 (0.005)	-3.300 (6.736)	77.099 [0]
9	0.574	0.005 (0.01)	25.115 (8.493)	0.894 [0]	-0.028	0.025 (0.005)	-1.951 (6.402)	71.659 [0]
11	0.630	0.002 (0.01)	30.221 (11.879)	0.831 [0]	0.552	0.021 (0.005)	19.802 (5.419)	66.100 [0]
13	0.643	0.004 (0.009)	37.245 (18.381)	0.886 [0]	0.602	0.016 (0.005)	29.662 (7.061)	59.514 [0]
15	0.727	0.007 (0.01)	46.580 (28.243)	0.836 [0]	0.656	0.018 (0.005)	31.869 (7.71)	56.332 [0]

**Table A2. GMM Estimation Results**

This table displays the GMM estimation results of the ultimate consumption models from equation (4) and (5) with 25 value weighted size-investment double-sorted portfolios as test assets. Panel A shows the results from the GMM with prespecified weighting matrix, and Panel B depicts the results from the efficient GMM. The numbers in the brackets are the standard errors, and the numbers in the squared brackets are p-values of test statistics. The column HJ shows the Hansen-Jagannathan distance and its p-value is calculated from 10,000 simulations. The column J shows the Hansen's J-test statistic. The sample period is from 1963:Q3 to 2011:Q1.

Panel A: Prespecified weighting matrix					Panel B: Efficient GMM			
S	rsq	alpha	gamma	HJ	rsq	alpha	gamma	jtest
0	0.063	0.015 (0.008)	47.314 (63.013)	0.554 [0.001]	-0.033	0.022 (0.005)	-10.754 (25.985)	56.072 [0]
1	0.022	0.017 (0.009)	10.979 (33.469)	0.525 [0]	0.015	0.021 (0.005)	4.964 (15.51)	55.289 [0]
3	0.058	0.015 (0.011)	13.091 (26.086)	0.553 [0]	-0.205	0.024 (0.005)	-20.344 (10.837)	53.327 [0]
5	0.267	0.002 (0.011)	22.062 (14.649)	0.628 [0]	-0.217	0.024 (0.005)	-12.128 (8.732)	53.957 [0]
7	0.348	0.004 (0.008)	17.726 (10.11)	0.605 [0]	-0.054	0.022 (0.005)	-2.300 (7.032)	55.836 [0]
9	0.528	0.008 (0.008)	19.916 (8.657)	0.618 [0]	0.087	0.021 (0.005)	2.222 (6.066)	56.430 [0]
11	0.587	0.007 (0.007)	22.946 (10.837)	0.644 [0]	0.117	0.021 (0.005)	2.981 (5.852)	57.030 [0]
13	0.552	0.007 (0.008)	28.250 (17.08)	0.702 [0]	0.091	0.021 (0.005)	2.999 (6.687)	57.455 [0]
15	0.555	0.009 (0.009)	34.331 (22.718)	0.732 [0]	0.275	0.019 (0.005)	9.957 (6.021)	58.332 [0]