ESTIMATION OF OPTIMAL HEDGE RATIO: A WILD BOOTSTRAP APPROACH

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Abstract

We propose a new approach to estimating the minimum variance hedge ratio (MVHR) based on the wild bootstrap. We consider a range of alternative hedging strategies associated with the percentiles of the MVHR's bootstrap distribution, from conservative to aggressive ones. This can be much more informative and safer than the conventional method of hedging solely based on a single point estimate. The percentile-based hedge ratios are robust to influential outliers, non-normality, and unknown forms of heteroskedasticity. The effectiveness of the bootstrap percentile-based hedging strategies is compared with those from the naïve method and DCC-GARCH model for a range of financial assets. We find that the wild bootstrap percentiles-based hedging (particularly those associated with the 50th and 75th percentiles) outperforms its alternatives overall, in terms of hedging effectiveness, downside risk, and the return variability.

Keywords: Minimum variance hedge ratio, Wild bootstrap, DCC-GARCH, Hedging effectiveness, Heteroskedasticity

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1. Introduction

The optimal hedge ratio (or the minimum variance hedge ratio; MVHR) is widely adopted in financial risk management. Derivative instruments, such as futures contracts, are crucial to a diversified portfolio in controlling and reducing the risk associated with unfavorable price changes. Since the first establishment of the futures market in the Chicago Mercantile Exchange in 1975, there have been a large number of studies in estimating the optimal hedge ratio and evaluating its effectiveness (see, for example, Chen, Lee, and Shrestha, 2003; Chen, Ho, and Tzeng, 2014). By taking a proper position guided by the optimal hedge ratio, an investor can effectively hedge the risk associated with price change of an underlying asset. Ederington (1979) presents an earliest empirical study of the optimal hedge ratio as a means of risk minimization.

While the conventional method of estimating the optimal hedge ratio is based on the ordinary least-squares (OLS) method, a number of new alternatives have been proposed in the literature, including the vector error-correction (VEC) model (Kroner and Sultan, 1993; Li, 2010); the generalized autoregressive conditional heteroskedasticity (GARCH) model (Caporin, Jimenez-Martin, and Gonzalez-Serrano, 2014; Chang, Gonzalez-Serrano, and Jimenez-Martin, 2013; Hsu, Tseng, and Wand, 2008; Ku, Chen, and Chen, 2007; Park and Jei, 2010; Lien, Tse, and Tsui, 2002); and the Markov regime-switching method (Alizadeh and Nomikos, 2004; Chen and Tsay, 2011; Lee, 2009, 2010; Lee and Yoder, 2011; Su and Wu, 2014). These new methods are designed to overcome the well-known shortcomings of the OLS-based method, which cannot fully capture the salient features of financial data such as non-normality and heteroskedasticity. However, the superiority of these new methods over the OLS-based method has not been fully confirmed, in terms of hedging effectiveness and variance reduction. A number of studies report that the OLS-based method has been found to outperform the newly proposed methods: see, for example, Lien and Shrestha (2008) and Lien (2009).

A notable feature of the past studies is that they rely solely on the point estimators for the optimal hedge ratio. A point estimator is a single number as an estimate of the unknown population value. Although it may represent the most likely outcome from a sampling or predictive distribution, it carries no information about the degree of intrinsic uncertainty associated. As Chatfield (1993) points out in the forecasting context, an interval estimator is more informative, offering a range of possible alternatives or contingencies with a prescribed level of confidence. For this reason, one may justifiably argue that risk analysis based solely on a point estimate of the optimal hedge ratio is of limited usefulness. By presenting the percentiles within a confidence interval for the optimal hedge ratio, a researcher is able to conduct a better-informed hedging and detailed risk analysis, taking full account of the uncertainty associated with estimation. In addition, the researcher can consider a range of possible scenarios based on a group of alternative optimal hedge ratio values. For example, a number of hedging strategies within a 95% confidence interval for the optimal hedge ratio (e.g., the 25th, 50th, and 75th percentiles) can be considered.

In this paper, we propose a new method of hedging based on interval estimation of the optimal hedge ratio. It is possible to construct an OLS-based interval or percentile estimator for the optimal hedge ratio using a normal approximation. However, in the presence of strong non-normality and heteroskedasticity in financial data, there are shortcomings for an interval estimator based on a normal approximation. For example, it is always symmetric around the value of the optimal hedge ratio, fails to capture a high degree of volatility of financial data, and may be subject to the adverse effects of influential outliers. For this reason, we propose the wild bootstrap method (Davidson and Flachaire, 2008) to estimate a confidence interval or percentiles for the optimal hedge ratio. The wild bootstrap is a non-parametric method of approximating the sampling distribution of a statistic based on data resampling. It is well known to provide a superior alternative to the conventional normal approximation when the data shows unknown forms of (conditional) heteroskedasticity (see, for example, Kim, 2006). This paper conducts extensive empirical analyses to evaluate the hedging effectiveness based on the wild bootstrap percentiles, in comparison with the strategies based on the dynamic conditional correlation (DCC-GARCH) and the naïve method[2]. We consider two alternative wild bootstrap procedures, i.e. one based on resampling the residuals of a regression, and the other resampling the pairs of observations.

Using daily spot and futures price indices of equities, commodities (oil, gold, corn), and the U.S. dollar from January 1980 to June 2015, we find that the hedging strategies based on central percentiles (particularly the 50th and 75th) of the optimal hedge ratio distribution outperform those based on the DCC-GARCH model and the naïve hedge, in terms of hedging effectiveness, downside risk, and hedged return variability. It is also found that the optimal hedging based on pairs bootstrap is marginally better than that based on the residual bootstrap. This paper is organized as follows. Section 2 presents a brief literature review. Section 3 provides the methodological details; Section 4 the data details; and Section 5 presents the empirical results. Section 6 concludes the paper.

2. Literature Review

Chen, Lee, and Shrestha (2003) conduct a survey of different optimization functions and techniques to estimating an optimal hedge ratio. They concluded that, in general, there is

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[2] The naïve strategy is static with hedge ratio of 1, taking a hedging position equal to the exact exposure in spot market..

no particular optimal hedge ratio that is significantly superior to the alternatives. According to Chen, Ho, and Tzeng (2014), numerous studies have been conducted to provide a solution to the risk minimizing function of the MVHR. Nonetheless, the conclusion of the best hedging effectiveness from different estimation methods has still been mixed on an out-of-sample evaluation basis. It has been suggested that the major cause of these mixed results is the estimation error (Lien and Shrestha, 2008).

From the in-sample analysis, the VEC hedging model with GARCH error structure employed in Kroner and Sultan (1993) is reported to be the best currency hedging strategy based on 4.5 percent and 1.5 percent variance reduction compared to the naïve and OLS models, respectively. With regard to currency hedging, Ku, Chen, and Chen (2007) document that the estimation of hedge ratios using the dynamic conditional correlation (DCC–GARCH) model can reduce 0.14 percent of the variation in the unhedged portfolio relative to the constant OLS model, which is reported as the second most effective strategy, followed by the VEC and constant conditional correlation (CCC-GARCH) models. Park and Jei (2010), evaluating the hedging effectiveness for corn and soybeans spot and futures prices, conclude that incorporation of asymmetry and flexible distribution specification in the DCC-GARCH model cannot yield a better hedge outcome compared to the OLS hedge ratio, as the variance reduction benefits are relatively small. This finding is consistent with the study of Lien, Tse, and Tsui (2002), which compares the CCC-GARCH and constant OLS approaches to hedging a spot position with different corresponding futures indices on currency, commodity and equity securities. They also agree on indifferent hedging benefits among the models. In an effort to document the effect of the Euro sovereign debt crisis on currency hedging, Caporin, Jimenez-Martin, and Gonzalez-Serrano (2014) conclude that static OLS estimates for hedging strategy were appropriate during the calm (non-crisis) period and after the intervention of the European Central Bank. Although, the authors assert that the exponential weighted moving average filter technique outperforms several other multivariate GARCH models and the static OLS model in terms of hedging effectiveness in the aftermath of the Lehman Brothers failure.

By using the Markov Regime Switching (MRS) model, Alizadeh and Nomikos (2004) report a new technique for estimating the hedge ratio which is dependent on prevailing market conditions and also free from the non-trivial persistence effect of the distant past volatilities in the GARCH models. Although outperformance of MRS ratios over the others is found from the in-sample analysis, the results from the out-of-sample analysis are mixed. In particular, the authors report that MRS ratios provide better variance reduction for the FTSE 100 hedge, but not for the S&P 500 index in comparison with the GARCH and constant OLS hedging estimates. Following Alizadeh and Nomikos (2004), Lee and Yoder (2011) and Su and Wu (2014) combine the MRS with the GARCH models (BEKK and DCC) to allow parameters to vary over time and to be state-dependent. Then, the optimal hedge ratio is estimated from the conditional second moments of the spot and futures series that are dependent on the market states. They found better in-sample performance with the new approach, but only marginal dominance in the out-of-sample analysis as compared to the benchmark strategies based on no hedging and the constant OLS hedge ratio. The variance reductions for hedging nickel and corn in Lee and Yoder (2011), and the S&P 500 and Nikkei 225 indices in Su and Wu (2014), are reported to be within the range from 0 to 2%. In the aforementioned studies, the evidence that the alternative techniques for the MVHR estimation outperform the OLS approach has been modest in the post-sample analysis. That raises a question regarding the effective predictability of the proposed models, and the superiority of their time-varying properties to the static OLS model for estimating the optimal hedge ratio.

Based on similar concerns, Lee and Yoder (2011) implement statistical testing of forecasting superiority of the best model over a given benchmark, for instance: regime switching–GARCH compared to the traditional GARCH and the constant OLS models. They report that the tests for forecasting superiority among these methods are not statistically significant via White's method for data-snooping reality check (White, 2000).

As discussed above, the most popular methodology employed in recent studies are the GARCH-class models, which are able to capture dynamic relationships between the spots and futures, while the constant approaches, such as the static OLS model or the naïve strategy, fail in capturing such relationships. The rationale for the use of the GARCH models is the fact that high volatility in one period tends to have persistence effects in the following periods. Convergence is also a typical problem in estimating the GARCH models. In addition, Brooks, Cerny, and Miffre (2011), in an evaluation of the effectiveness of multivariate GARCH models in estimating the optimal hedge ratio, conclude that the models at best have provided very modest enhancement on an out-of-sample basis. This assertion can be re-examined by reviewing reported tables of variance reduction and hedging effectiveness employing various methods against a specific benchmark in many studies within the literature. The differences in hedging improvement among the models are minor and the estimated hedge ratios appear to be slightly different. Thus far, the main disadvantage of the OLS model is that it fails in capturing the time-varying nature of the relationship among financial time series. In addition, the OLS also assumes a constant variance of financial return and normality is often assumed for statistical inference. Despite these shortcomings, the OLS-based MVHRs are still utilized universally by financial professionals due to its computational efficiency and simplicity.

As mentioned earlier, all past studies rely exclusively on the point estimates of the optimal hedge ratio generated from alternative models. They report empirical results on the hedging effectiveness that are often mixed and inconclusive, possibly due to the point that the degree of uncertainty associated with the optimal hedging ratio estimation is not reflected in their evaluation. In this paper, we propose the percentile-based optimal hedging based on the wild bootstrap method. The method is based on the simple OLS-based regression and conducted with a time-varying framework using rolling-sub-sample windows. The wild bootstrap also provides estimation and statistical for the optimal hedge ration robust to non-normality and heteroscedasticity.

3. Methodology

In this section, we present the methodological details, including the wild bootstrap methods and DCC-GARCH model for the optimal hedge ratio. We also discuss the measures for the hedging effectiveness.

3.1. Background

The hedge ratio is defined as the number of contracts to be taken out by an investor in the futures market in order to hedge her position in the spot market. The value of the hedge ratio is a dollar amount in futures contracts entered into by an investor or a hedger to protect against the risk of any loss from holding every one-dollar in the spot market. The minimum variance hedge ratio (MVHR) can be estimated by regressing a spot return against the return of the corresponding futures, using the OLS method. The MVHR minimizes the variance of the hedged return, which can be expressed as

$$R_h = R_S + \beta \ R_F \tag{1}$$

where R_h is a vector of the hedged portfolio's returns, R_S is a vector of spot returns of a risky asset, R_F is a vector of futures return of the risky asset and β is the hedge ratio reflecting the dollar amount of futures contracts that needs to be entered into for the sake of hedging the risk of $1 in the spot market. Note that each vector of the above returns has a length of *n*, which also denotes the sample size.

The purpose of taking a position in the futures market is to protect the value of a position in the spot market. In other words, a hedge through the use of futures contracts with a size determined by the hedge ratio is aimed to protect the initial value of an investment or commodity position from any changes in price: $\Delta R_h = \Delta R_S + \beta\ \Delta R_F = 0$. The conventional approach to this task is to use the estimate of β based on the regression of the form:

$$R_S = c + \beta\ R_F + \varepsilon \tag{2}$$

The OLS estimator for β in (2) is expressed as:

$$\hat{\beta} = (R_F^T\ R_F)^{-1} R_F^T R_S, \tag{3}$$

where the superscript T refers to the transpose of the corresponding vector.

3.2. Hedging with wild bootstrap percentiles

As mentioned earlier, we propose a new approach to estimation of the MVHR with the wild bootstrap, accounting for the heteroskedasticity and non-normality issues of the error term in equation (2). We consider the percentiles within a confidence interval for the MVHR, which covers the true value of the MVHR with a prescribed level of confidence. By constructing a confidence interval and its percentiles for the MVHR, the degree of estimation uncertainty is explicitly presented. We first consider the wild bootstrap based on residual resampling, which is employed in conjunction with the heteroskedasticity consistent covariance matrix estimator (HCCME) as proposed in Flachaire (2005), Davidson and

Flachaire (2008), and Cribari-neto and Lima (2009). In addition, we also consider the wild bootstrap based on resampling the pairs of observations. The former bootstrap method assumes that R_F in regression (2) is exogenous and uncorrelated with the error term; while the latter assumes that it is random. The pairs bootstrap addresses the potential endogeneity problem, when both R_S and R_F may be driven by the same market shocks. The two wild bootstrap methods are proposed as alternatives for the percentile-based hedging strategy.

The wild bootstrap based on residual resampling can be described as follows:

Step 1: Estimate the optimal hedge ratio $\hat{\beta}$ given in (3) for the regression (2).

Step 2: Draw a bootstrap sample (R_{Si}^{*}, R_{Fi}) based on $\hat{\beta}$ for each i^th observation from 1 to *n*:

$$R_{Si}^{*} = \hat{\beta}\ R_{Fi} + t_i^{*}\ \hat{\varepsilon}_i/(1 - h_i)$$

where t_i^{*} is a independent random variable with zero mean and unit variance and $\hat{\varepsilon}_i/(1 - h_i)$ is the transformed residual from the regression (2), robust to heteroskedasticity[3].

Step 3: Compute the new estimate of the hedge ratio $\hat{\beta}^{*}$ with the bootstrap sample (R_{Si}^{*}, R_{Fi}) (for i = 1, …, *n*) following the regression (2).

Step 4: Repeat Step 2 and 3 many times, say *B*, to form the bootstrap distribution of $\left\{\hat{\beta}^{*}(i)\right\}_{i=1}^{B}$ for $\hat{\beta}$.

Step 5: The (1- α)100% wild bootstrapping confidence interval is constructed with the lower limit and upper limits representing the 0.5α percentile and (1 – 0.5α) percentile, respectively, of the bootstrap distribution $\left\{\hat{\beta}^{*}(i)\right\}_{i=1}^{B}$. The percentiles within the confidence interval can be estimated in a similar way. The number of bootstrap iterations *B* is set at 1000. The wild bootstrap based on resampling the pairs is identical to the above-mentioned procedure, except

[3] h_i is the i^th diagonal element of the orthogonal projection matrix $H = R_F(R_F^T R_F)^{-1} R_F^T$ (see Long and Ervin, 2000).

in Steps 2 and 3, resampling and estimations are conducted as $(R_{Si}^*, R_{Fi}^*) = (t_i^* R_{Si}, t_i^* R_{Fi})$; and $\hat{\beta}^*$ is the OLS estimator from (R_{Si}^*, R_{Fi}^*) (for i = 1, …, *n*).

The bootstrap (Efron, 1979) is a method of approximating the sampling distribution of a statistic in a non-parametric way by repeated resampling of the observed data. The wild bootstrap (Liu, 1988; Mammen, 1993) is a bootstrap method designed for the data that shows an unknown form of heteroskedasticity, which has been shown to be asymptotically valid (Cribari-neto and Lima, 2009; Cribari-Neto, Souza, and Vasconcellos, 2007; Davidson and Flachaire, 2008; Flachaire, 2005). Considering $X_i^* = t_i^* X_i$ and $Y_i^* = t_i^* Y_i$ where *X* and *Y* are random variables, the variance and covariance of resampled data X^* and Y^*, conditional on *X* and *Y* respectively, can effectively replicate those of *X* and *Y*. That is,

$$Var(X_i^* \mid X_i) = X_i^2; Var(Y_i^* \mid Y_i) = Y_i^2; Cov(X_i^*, Y_i^* \mid X_i, Y_i) = X_i Y_i ,$$

A choice of an distribution should be made for t_i^*. In this paper, we use Mammen's (1993) two-point distribution:

$$t_i^* = \begin{cases} -\dfrac{\sqrt{5}-1}{2} \; with\; probability\; p = \dfrac{\sqrt{5}+1}{2\sqrt{5}} \\ \dfrac{\sqrt{5}+1}{2} \; with\; probability\; (1-p) \end{cases}$$

which is well-known to give higher-order refinements.

In this study, we construct the 95 percent confidence interval, paying attention to the 10th, 25th, 50th, 75th, and 90th percentiles. Hedging strategies are then produced based on the percentile hedge ratios. A hedged position based on an upper percentile may be regarded as an aggressive strategy, whereas that based on a lower percentile may be considered as a conservative one. The 50th percentile (median) hedging position may be regarded as a neutral one. We argue that these percentile-based hedging strategies effectively provide different scenarios under different market conditions, since the interval will be tighter in normal times, but wider under turbulent market conditions. As a result, these hedging strategies are much more informative than the one based on a single point estimate of the hedge ratio. In addition, it can be more beneficial in terms of hedging effectiveness, downside risk, and the hedged return fluctuation that are discussed hereafter.

3.3. Hedging based on the DCC-GARCH

As an alternative to the wild bootstrap, we use the bivariate DCC-GARCH(1,1) model developed by Engle (2002), which is widely used in the prior literature The model has been popular due to its superiority to the OLS approach in considering time-varying conditional variance and covariance of the spot and futures returns. This model is defined as follows:

$$R_{S,t} = \mu_S + u_{S,t}$$

$$R_{F,t} = \mu_F + u_{F,t}$$

$$\begin{bmatrix} u_{S,t} \\ u_{F,t} \end{bmatrix} / \Omega_{t-1} \sim N(0, H_t)$$

$$h_{S,t} = c_S + a_S u_{S,t}^2 + b_S h_{S,t-1}$$

$$h_{F,t} = c_F + a_F u_{F,t}^2 + b_F h_{F,t-1}$$

$$h_{SF,t} = \rho_{SF,t} \sqrt{h_{S,t}} \sqrt{h_{F,t}}$$

$$\rho_{SF,t} = \frac{q_{SF,t}}{\sqrt{q_{SS,t} q_{FF,t}}}$$

$$q_{SF,t} = \bar{\rho}_{SF} + \gamma\left(z_{S,t-1} z_{F,t-1} - \bar{\rho}_{SF}\right) + \delta(\, q_{SF,t-1} - \bar{\rho}_{SF}) \qquad (5)$$

where $\rho_{SF,t}$ is the dynamic conditional correlation, $q_{SF,t}$ is the conditional correlation, $\bar{\rho}_{SF}$ is the constant unconditional correlation, $z_{S,t-1} and\ z_{F,t-1}$ are the standardized residuals of the spot and futures returns, respectively. Hence, the time-varying hedge ratio $\widehat{\beta_t}$ can be calculated based on the conditional covariance matrix from the DCC-GARCH(1,1) model as follows:

$$\widehat{\beta_t} = \frac{h_{SF,t}}{h_{F,t}} \tag{6}$$

3.4. Computational details and evaluation of hedging strategies

To conduct the hedging with time-varying estimators, we adopt the rolling sub-sample window of 250 trading days[4] for both the wild bootstrap and the DCC-GARCH(1,1) model. From each sub-sample window, one-step ahead prediction from the DCC-GARCH(1,1) model is generated, along with the wild bootstrap percentiles. To compare the predictive ability of the alternatives, hedged portfolios are constructed by combining the long position in the spot market and the short position in the futures market from all windows. These hedging strategies are evaluated in terms of variance reduction, downside risk and stability of the hedged portfolio returns.

The hedging effectiveness of the strategies is measured by the percentage decrease in volatility of the hedged portfolio return relative to the unhedged portfolio return. An optimal hedging strategy is expected to have a relatively stable hedge ratio and to provide the largest variance reduction from the unhedged position, which is the purely long position in the spot market in this study. The hedging effectiveness (*HE*) is given by

$$HE = \frac{Var(U) - Var(H)}{Var(U)}, \tag{7}$$

where *Var*(*U*) and *Var*(*H*) denotes the variances of the unhedged return and hedged return respectively. We also use the semi-variance (*SV*), which is the average squared deviation of the observations below the mean of the hedged returns, which can be written as

[4] The length of the time window is constructed for rebalancing needs of a portfolio for every 12 months when the portfolio manager can revise their hedging position based on the time-varying spot-futures relationship. The choice of the time window length is also to facilitate the convergence issue of the rolling DCC-GARCH model.

$$SV = \frac{\sum_{i=1}^{m}(X_i - \overline{X})^2}{m-1}, \tag{8}$$

where m is the number of hedged return observations X_i below the average. By comparing the *SV* values across the proposed hedging strategies, we can identify which strategy may potentially expose an investor to higher downside risks that are associated with losses. It provides information as to which of the available strategies is safer in situations of adverse market conditions or situations of high volatility in market movements. In addition to these measures, we compare the inter-quartile range (difference between the 3rd and 1st quartiles) and 95% range (difference between the 97.5th and 2.5th percentiles) of the return distributions.

Finally, a variance equality test between hedged returns using different models is performed with an F-test. This is to examine whether the strategy using time-varying DCC-GARCH hedge ratios is equal or larger than the one from the bootstrap distribution of the optimal hedge ratio. The test statistic is calculated as the ratio of the sample variance of the DCC-GARCH hedged returns to that of an alternative strategy's hedged returns[5], following an F-distribution with appropriate degrees of freedom. The alternative hypothesis is that the variance ratio is greater than 1, which means the DCC-GARCH hedge is riskier than its alternative. Rejection of the null hypothesis that the ratio of these variances is equal to one indicates that daily returns from the DDC-GARCH model are more volatile than those of its alternative. The F-statistic for this test is written as

$$F = \sigma_{DCC}^2 / \sigma_i^2 , \tag{9}$$

where σ_{DCC}^2 denotes the sample variance of the hedged returns on the DCC-GARCH hedge; and σ_i^2 the sample variance of the hedged returns on an alternative hedge.

[5] Alternative hedging strategies to the DCC-GARCH approach are, within this study, the naïve strategy and percentiles from the bootstrap distribution of the MVHR (at 10th, 25th, 50th, 75th, 90th percentiles)

4. Data Details

Hedging effectiveness is evaluated for a range of assets, including equites (S&P500, FTSE100), commodities (crude oil mid-price, S&P gold spot price index, and corn number 2 yellow), and the currency (US dollar index[6]). The futures contracts employed for hedging instruments are continuous settlement price indices available from DataStream[7]. The data ranges from January 1980 to June 2015, although some assets have different starting values due to data availability, covering the periods of a number of economic and financial crises. Descriptive statistics for the spot and futures returns of each asset are presented in Table 1. The mean and variance properties of the returns are typical of financial returns. The Lagrange multiplier test for the ARCH effect indicates the presence of conditional heteroskedasticity, which justifies the use of a dynamic strategy to reduce the price risk exposure in the spot market, such as the rolling OLS and DCC-GARCH models. The Jarque-Bera test for non-normality indicates the non-normal returns. The strong evidence of heteroskedasticity and non-normality justifies the use of the wild bootstrap. For all assets, the Pearson correlations among the spot and futures returns indicate strong linear association. Although the details are not reported, we find that the spot and futures prices of all employed assets are co-integrated. Figures 1 and 2 present the time plots of the price and return of the different financial assets and their futures. The futures price indices appear to be more sensitive and volatile relative to changes in the spot price indices. The price reaction in the futures market is stronger than in

[6] The US dollar index, which has been existed since 1973, is a geometrically weighted average of a basket of six currencies against the US dollar, i.e. British pound, Canadian dollar, the Euro, Japanese yen, Swedish krona and Swiss franc. Since the US dollar is freely floated against all other foreign currencies, the Federal Reserve Bank initiated the measure of the US dollar index to provide an external bilateral trade-weighted average of the US dollar.

[7] The continuous futures indices are perpetual series of futures prices, volumes and open interest derived from individual futures contracts. They starts at the nearest contract month, which forms the first price values for the continuous series until either the contract reaches its expiry date or until the first business day of the notional contract month, whichever is sooner. At this point prices from the next trading contract month are taken. No adjustment for price differentials is made. Thomson Reuters DataStream provides the methodology.

the spot market, as shown by the sharper spikes in the return plots. This suggests that an incoherent strategy involving futures transaction may result in higher risk than anticipated.

5. Empirical result

In this section, we present the optimal hedge ratio estimates based on two alternative bootstrap methods and DCC-GARCH model. In addition, we evaluate and compare hedging effectiveness of the strategies across the alternative methods.

5.1. Optimal hedge ratio estimates

The time-varying optimal hedge ratios are shown from Figures 3 to 8 for all six assets, generated using 250-trading-day moving sub-sample window. In each figure, the bootstrap percentile-based hedge ratios, using the residual resampling procedure, are reported in the top panel; and those based on the pairs bootstrap appear in the middle panel. These plots present 95% confidence band with the percentiles indicated within, which are used for percentile-based hedging. The naïve strategy is also indicated by the horizontal line at the hedge ratio of 1, in each panel for comparison. The two alternative bootstrap confidence intervals appear to show similar pattern over time, but the one based on pairs bootstrap is slightly more stable overall. By looking at the width of the bootstrap confidence band, we can assess the degree of uncertainty associated over time. The interval between appropriate percentiles represents the wild bootstrap confidence interval for the optimal hedge ratio: for example, the 2.5^{h} and 97.5^{th} percentiles are the lower bound and upper bound of the 95 percent confidence interval. A wider band indicates a higher degree of uncertainty in estimation. It is likely that the width of the band changes depending on the prevailing market conditions that drive the degree of risk. We observe tighter confidence bands for the equities than the commodities and the US dollar. This may be explained by more complicated fundamentals affecting the commodities spot and futures returns, especially the global supply and demand (Ai, Chatrath, and Song, 2006; Groen and Pesenti, 2010; Gorton, Hayashi, and Rouwenhorst, 2013), effects of US dollar strength (Akram, 2009), and the speculative activities and active positions of hedge funds in both commodity and equity futures markets (Büyükşahin and Robe, 2014; Gorton, Hayashi, and Rouwenhorst, 2013).

Another advantage of using the confidence band is that the hedger can conduct a test for statistical significance. For example, if the confidence band does not cover the value of 1, we cannot accept the null hypothesis that the optimal hedge ratio is equal to one, at the prescribed level of significance, which means that the optimal hedge ratio is statistically different from the naïve hedge ratio. It appears that, for most of the assets, the optimal hedge ratio is statistically different from 1 frequently over time, at the 5% level of significance. This tendency has been particularly strong before 2000. For the U.S. dollar, the naïve strategy has never been optimal over the entire period at the 5% level of significance, since the 95% confidence band does not cover 1 for nearly the entire period. For gold, the wild bootstrap confidence band has become tighter and increasingly convergent to 1.

A high degree of estimation uncertainty of the optimal hedge ratio for most of the assets obviously appears in some common periods: 1987-1988 (effect of Black Monday), 1992-1993 (European Currency crisis), 1997-1998 (Asian Financial crisis), 2001-2003 (Dot-com bubble that caused a deep decrease in the US dollar index) and 2007-2010 (the US Subprime Housing crisis and the Global Financial crisis). These turbulent episodes have influenced not only the US market, but have also impacted on the global financial markets. However, the gold appears as an exception among the analyzed asset classes, and especially during and after the crisis in 2007. This may represent the popularity of gold as a safe haven asset, as discussed and identified by Nguyen and Liu (2017).

It is evident that the DCC-GARCH hedge ratios are considerably more volatile than the wild bootstrap hedge ratios, especially during the periods of turbulent financial markets. Visually, substantial swings are observed in the time-varying DCC-GARCH ratios across the assets. The high volatility of daily MVHRs using the DCC-GARCH is also documented in previous studies (Park and Jei, 2010; Chang, Gonzalez-Serrano, and Jimenez-Martin, 2013; Caporin, Jimenez-Martin, and Gonzalez-Serrano, 2014). From Figures 3 to 8, it is noteworthy that the movements in the DCC-GARCH hedge ratios are well outside the bootstrap confidence bands. The wild bootstrap hedge ratios are stable, mostly well above 0 and less than 1. In contrast, the DCC-GARCH hedge ratios are often negative and can be greater than 2. The negative hedge ratio reflects a negative relationship between the spot and futures indices, indicating an inverse co-movement of the futures price to the current spot price, a signal of speculation opportunity. The remarkable observed volatility of the DCC-GARCH hedge ratios is probably due to the highly parametric nature of the DCC-GARCH model.

5.2. Comparing the hedging effectiveness:

A hedged portfolio is constructed taking a combined position in spot and futures returns of an asset so as to minimize the exposed risk in the physical market, using the optimal hedge ratios obtained from alternative methods. The hedged return in percentage is calculated daily. Table 2 presents the statistics for the hedging effectiveness and Table 3 reports the F-test statistics for variance equality discussed in Section 3. Overall, the wild bootstrap percentile hedging strategies are found to be more effective than those based on the

DCC-GARCH and the naïve methods, in terms of the variance reduction, downside risk, and stability in the hedged returns.

We first compare the hedging effectiveness between the bootstrap percentiles-based methods with the naïve one. For S&P500, the variance of hedged returns are consistently lower with the bootstrap methods. The naïve hedge shows the variance of 0.11 while the bootstrap methods show the variance in the neighborhood of 0.08. The *SV* value of the naïve hedge is around 0.33, much higher than the bootstrap values which are around 0.29. The *HE* values also indicate a higher reduction in variance for the bootstrap methods. The *IQ* and 95% ranges also indicate a lower variability in the hedged returns based on the wild bootstrap. Similar results are evident for the FTSE100 and USD where the bootstrap-based optimal hedging shows a better performance than the naïve method. For oil and corn, the evidence is not as strong as the other assets, but there is a tendency that the bootstrap-based methods are slightly better in terms of overall variance reduction. For the gold, all measures indicate that the naïve method beats the wild bootstrap alternatives.

Now we compare the hedging performance between the wild bootstrap and the DCC-GARCH model. For all assets considered, all measures of hedging performance indicate that the wild bootstrap outperforms the DCC-GARCH. The latter is found to beat the naïve method for the equities and USD, but not in the commodities. As we find above, the bootstrap method beats the native method in hedging performance for all assets, except for the gold. Overall, the bootstrap percentile-based hedge ratios, especially at the 50^{th} and 75^{th} percentiles of the bootstrap distribution, which may be regard as the neutral strategy and aggressive strategy respectively, provide more effective hedging. The variance reduction in the hedged portfolio returns using the pairs bootstrap is marginally higher than the residual bootstrap, especially for the S&P500 index and corn.

To further evaluate the riskiness among the hedging positions associated with the different strategies, the F-test statistics for the equality of variances discussed in Section 3 are reported in Table 3. The F-statistic is the ratio of the two variances, in particular the DCC-GARCH based strategy against any competing hedging strategy. Rejection of the null hypothesis of the equal variances means that the variance ratio is greater than 1, as stated in the alternative hypothesis. This implies a higher degree of volatility of the DCC-GARCH hedge. The significant F-statistics in Table 3 indicate the potential inferior performance of the hedging strategy based on the DCC-GARCH model. The bootstrap method appear to provide a safer hedge with less volatility in returns than the DCC-GARCH in most of the asset classes. The two bootstrap approaches are shown indifferent in providing the better stability in the hedged returns compared to the DCC-GARCH. However, hedging positions in the US dollar show different outcomes, since the variance comparison test fails to conclude that the DCC-GARCH hedged returns are more volatile than their counterparts. For the commodities, we find the hedged returns from the naïve strategy are less volatile than the DCC-GARCH based strategy, but not for the equities and the US dollar index.

Overall, the hedging strategies based on the 50^{th} and 75^{th} percentiles of the wild bootstrap distribution of the optimal hedge ratio are found to outperform those based on the DCC-GARCH and naïve method. This is plausible since the central percentiles are highly likely to be associated with the true value of the optimal hedge ratio. The wild bootstrap based on resampling the pairs of observations appears to be marginally better than the one based on resampling the residuals of the regression model, showing moderately narrower interval width of the percentile-based hedge ratios and the improved stability in the hedged returns. This suggests that the endogeneity issue in the estimation of the optimal hedge ratio may play a role, since the spot and futures returns are both possibly affected by the same shocks.

6. Conclusion

The paper proposes a new method of hedging based on the percentiles of the optimal hedge ratio. The proposed method provides a range of possible hedging strategies within the 95% confidence interval for the optimal hedge ratio. This is more informative than the conventional hedging based on a single point estimate, since it provides a hedger with a clear sense of estimation uncertainty and a range of alternative strategies, with a prescribed level of confidence. In order to estimate the percentiles of the optimal hedge ratio distribution, we employ the wild bootstrap (the one based on residual resampling and the other based on pairs resampling), which is a non-parametric method of approximating the sampling distribution of a statistic based on repeated data resampling. The wild bootstrap percentiles exhibit a range of desirable features, being robust to influential outliers; and robust to non-normality and unknown forms of heteroskedasticity. These hedging strategies are compared to those based on the naïve method and the DCC-GARCH model, adopting 250-day rolling sub-sample windows.

Hedging effectiveness among the alternative approaches is evaluated for a range of assets, i.e. the equities (S&P500 and FTSE100), commodities (gold, oil, and corn) and the US dollar, using the daily spot and futures prices from 1980. The estimation uncertainty of the optimal hedge ratio is presented with a varying width of the bootstrap percentile-based hedge ratios during the historical turbulent periods of financial and commodity market activity. The

DCC-GARCH hedge ratios are found to fluctuate in a highly volatile manner, in comparison with the bootstrap percentile-based alternatives. The high volatility of the DCC-GARCH estimates adversely impact on the hedging position and increase the uncertainty in hedging.

Overall, the hedging strategies based on the wild bootstrap percentiles are found to outperform those based on the DCC-GARCH model and the naïve hedge, in terms of hedging effectiveness, downside risk and hedged return variability. In particular, those based on the central percentiles (the 50th and 75th) show highly desirable hedging performance. The two alternative bootstrap methods show similar performance in hedging effectiveness, but the one based on resampling the pairs provide more stable hedging performance. This may indicate the importance of endogeneity issue in estimation, which is widely neglected in the past studies. While the conventional hedging methods rely solely on the point estimate of the optimal hedge ratio, this paper represents the first study that proposes hedging based on interval or percentile estimation. The latter is associated with a richer information content with a range of alternative hedging strategies, which can lead to safer and more informed risk analysis.

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Table 1: Descriptive statistics for the asset returns

	S&P 500		FTSE 100		USD	
	SPOT	FUTURES	SPOT	FUTURES	SPOT	FUTURES
Mean	0.032 ***	0.032 ****	0.022 *	0.022 *	-0.004	-0.004
Variance	1.246 ***	1.467 ***	1.186 ***	1.334 ***	0.278 ***	0.310 ***
Skewness	-1.28	-2.38	-0.49	-0.59	0.53	0.56
Kurtosis	29.44	84.28	10.00	11.35	1.98	1.88
J-B test	307483.4 ***	2509963 ***	34103.25 ***	43991.54 ***	1254.43 ***	1136.25 ***
ARCH test	679.49 ***	721.33 ***	1621.28 ***	1196.89 ***	156.76 ***	132.99 ***
Pearson Correlation	0.96***		0.96***		0.97***	

	GOLD		OIL		CORN	
	SPOT	FUTURES	SPOT	FUTURES	SPOT	FUTURES
Mean	0.009	0.009	0.011	0.011	0.004	0.004
Variance	1.39 ***	1.434 ***	6.129 ***	5.846 ***	2.997 ***	2.700 ***
Skewness	-0.13	-0.18	-0.76	-0.82	-0.28	-0.70
Kurtosis	7.20	8.43	15.33	15.22	4.49	14.68
J-B test	20011.39 ***	27428.62 ***	75893.53 ***	74956.42 ***	6767.62 ***	71919.36 ***
ARCH test	1167.84 ***	1261.52 ***	321.96 ***	398.72 ***	660.53 ***	57.62 ***
Pearson Correlation	0.99***		0.89***		0.82***	

Note: *Asset return is presented in percentage. J-B test is the Jarque-Bera normality test for the asset returns' distribution with the test statistic following the* χ^2 *distribution. The ARCH test is the Lagrange Multiplier test for conditional heteroskedasticity. *, **, and *** denote the statistical significance at the 10%, 5% and 1% levels, respectively.*

Table 2: Statistics for the Hedged portfolio returns and Hedging effectiveness

	S&P 500	**Mean**	**Variance**	**SV**	**HE**	**IQ**	**95% range**
	Unhedged	0.0312	1.26	1.175		0.9769	4.4320
	Naïve hedge	0.00029	0.108	0.332	91.41%	0.2534	1.1376
	DCC hedge	0.00485	0.094	0.311	92.57%	0.2537	1.1047
Residual Bootstrap	**10^{th} Percentile hedge**	0.00596	0.08	0.286	93.69%	0.2583	1.1313
	25^{th} Percentile hedge	0.0055	0.079	0.283	93.78%	0.2545	1.0996
	50^{th} Percentile hedge	0.00436	0.081	0.29	93.62%	0.2516	1.0787
	75^{th} Percentile hedge	0.00391	0.081	0.292	93.56%	0.2499	1.0744
	90^{th} Percentile hedge	0.00354	0.083	0.293	93.46%	0.2498	1.0647
Pairs Bootstrap	**10^{th} Percentile hedge**	0.0054	0.081	0.290	93.62%	0.2570	1.1217
	25^{th} Percentile hedge	0.0050	0.080	0.288	93.70%	0.2541	1.0956
	50^{th} Percentile hedge	0.0045	0.080	0.288	93.67%	0.2515	1.0777
	75^{th} Percentile hedge	0.0040	0.081	0.289	93.63%	0.2502	1.0707
	90^{th} Percentile hedge	0.0036	0.082	0.291	93.53%	0.2504	1.0629

	FTSE 100	**Mean**	**Variance**	**SV**	**HE**	**IQ**	**95% range**
	Unhedged	0.0208	1.19	1.126		1.1010	4.3396
	Naïve hedge	0.0001	0.102	0.326	91.47%	0.2734	1.2188
	DCC hedge	0.00261	0.094	0.332	92.14%	0.2617	1.1080
Residual Bootstrap	**10^{th} Percentile hedge**	0.0041	0.082	0.299	93.16%	0.2610	1.0908
	25^{th} Percentile hedge	0.00387	0.079	0.296	93.35%	0.2600	1.0743
	50^{th} Percentile hedge	0.00351	0.079	0.296	93.38%	0.2583	1.0719
	75^{th} Percentile hedge	0.00323	0.08	0.296	93.33%	0.2578	1.0781
	90^{th} Percentile hedge	0.00294	0.082	0.299	93.13%	0.2583	1.0836
Pairs Bootstrap	**10^{th} Percentile hedge**	0.0041	0.080	0.297	93.29%	0.2613	1.0833
	25^{th} Percentile hedge	0.0038	0.079	0.296	93.35%	0.2589	1.0697
	50^{th} Percentile hedge	0.0035	0.079	0.296	93.37%	0.2581	1.0738
	75^{th} Percentile hedge	0.0032	0.080	0.296	93.34%	0.2582	1.0775
	90^{th} Percentile hedge	0.0029	0.082	0.299	93.17%	0.2576	1.0817

SV: semivariance given in (7); HE: Hedging effectiveness given in (8)
IQ: inter-quartile range; 95% range: difference between 97.5^{th} and 2.5^{th} percentiles

Table 2: Statistics for the Hedged portfolio returns and Hedging effectiveness *(continued)*

	USD	Mean	Variance	SV	HE	IQ	95% range
	Unhedged	-0.00199	0.275	0.536		0.5724	2.1496
	Naïve hedge	0.00001	0.019	0.152	92.99%	0.0656	0.5906
	DCC hedge	-0.00121	0.017	0.150	93.78%	0.0681	0.5485
Residual Bootstrap	**10th Percentile hedge**	-0.00062	0.018	0.147	93.63%	0.0723	0.5519
	25th Percentile hedge	-0.0006	0.017	0.146	93.74%	0.0699	0.5421
	50th Percentile hedge	-0.00044	0.017	0.146	93.81%	0.0671	0.5376
	75th Percentile hedge	-0.00039	0.017	0.147	93.80%	0.0654	0.5396
	90th Percentile hedge	-0.00035	0.017	0.147	93.77%	0.0638	0.5412
Pairs Bootstrap	**10th Percentile hedge**	-0.0006	0.0175	0.1465	93.63%	0.0720	0.5504
	25th Percentile hedge	-0.0005	0.017	0.146	93.74%	0.0699	0.5416
	50th Percentile hedge	-0.0004	0.017	0.146	93.81%	0.0672	0.5369
	75th Percentile hedge	-0.0004	0.017	0.147	93.81%	0.0655	0.5398
	90th Percentile hedge	-0.0003	0.017	0.147	93.78%	0.0638	0.5403

	GOLD	Mean	Variance	SV	HE	IQ	95% range
	Unhedged	0.00795	1.23	1.104		0.9643	4.5533
	Naïve hedge	-0.00018	0.034	0.207	97.24%	0.0160	0.6303
	DCC hedge	-0.00164	0.065	0.337	94.72%	0.0351	0.6813
Residual Bootstrap	**10th Percentile hedge**	-0.00048	0.035	0.212	97.12%	0.0399	0.6694
	25th Percentile hedge	-0.00043	0.035	0.209	97.19%	0.0356	0.6521
	50th Percentile hedge	-0.00032	0.034	0.208	97.24%	0.0295	0.6343
	75th Percentile hedge	-0.0003	0.034	0.207	97.24%	0.0259	0.6372
	90th Percentile hedge	-0.00031	0.034	0.208	97.23%	0.0251	0.6369
Pairs Bootstrap	**10th Percentile hedge**	-0.0004	0.035	0.211	97.14%	0.0399	0.6646
	25th Percentile hedge	-0.0004	0.034	0.209	97.20%	0.0353	0.6488
	50th Percentile hedge	-0.0003	0.034	0.207	97.24%	0.0294	0.6351
	75th Percentile hedge	-0.0003	0.034	0.207	97.24%	0.0259	0.6348
	90th Percentile hedge	-0.0003	0.034	0.208	97.23%	0.0259	0.6374

SV: semivariance given in (7); HE: Hedging effectiveness given in (8)
IQ: inter-quartile range; 95% range: difference between 97.5th and 2.5th percentiles

Table 2: Statistics for the Hedged portfolio returns and Hedging effectiveness *(continued)*

	OIL	Mean	Variance	SV	HE	IQ	95% range
	Unhedged	0.018	5.73	2.449		2.3018	9.2202
	Naïve hedge	0.00002	1.21	1.073	78.80%	0.3823	3.5886
	DCC hedge	0.0136	1.36	1.135	76.27%	0.4755	3.8056
Residual Bootstrap	**10^{th} Percentile hedge**	0.00729	1.21	1.098	78.94%	0.5058	3.6924
	25^{th} Percentile hedge	0.00564	1.18	1.082	79.37%	0.4729	3.5777
	50^{th} Percentile hedge	0.00402	1.17	1.075	79.52%	0.4408	3.5063
	75^{th} Percentile hedge	0.00307	1.18	1.074	79.44%	0.4177	3.4902
	90^{th} Percentile hedge	0.0022	1.19	1.08	79.22%	0.4131	3.5203
Pairs Bootstrap	**10^{th} Percentile hedge**	0.0070	1.200	1.096	79.04%	0.5018	3.6684
	25^{th} Percentile hedge	0.0056	1.180	1.081	79.40%	0.4682	3.5545
	50^{th} Percentile hedge	0.0040	1.172	1.075	79.52%	0.4380	3.5059
	75^{th} Percentile hedge	0.0030	1.175	1.073	79.47%	0.4183	3.5013
	90^{th} Percentile hedge	0.0022	1.187	1.080	79.28%	0.4105	3.5125

	CORN	Mean	Variance	SV	HE	IQ	95% range
	Unhedged	0.00483	3.06	1.7		1.6907	7.2176
	Naïve hedge	0.00012	1.01	1.031	66.99%	0.5001	3.5860
	DCC hedge	0.0052	1.13	1.079	63.18%	0.6069	3.7265
Residual Bootstrap	**10^{th} Percentile hedge**	-0.00778	1.15	1.164	62.34%	0.7018	4.2345
	25^{th} Percentile hedge	-0.00784	1.1	1.135	64.13%	0.6464	4.0389
	50^{th} Percentile hedge	-0.00138	0.986	1.067	67.83%	0.5636	3.6470
	75^{th} Percentile hedge	-0.00168	0.994	1.075	67.56%	0.5343	3.6293
	90^{th} Percentile hedge	-0.00176	1.01	1.081	67.11%	0.5160	3.6321
Pairs Bootstrap	**10^{th} Percentile hedge**	-0.0038	1.034	1.092	66.26%	0.6784	3.8622
	25^{th} Percentile hedge	-0.0040	1.019	1.084	66.75%	0.6324	3.7911
	50^{th} Percentile hedge	-0.0017	0.990	1.069	67.70%	0.5681	3.6396
	75^{th} Percentile hedge	-0.0022	0.996	1.077	67.49%	0.5391	3.6410
	90^{th} Percentile hedge	-0.0024	1.008	1.084	67.09%	0.5195	3.6280

SV: semivariance given in (7); HE: Hedging effectiveness given in (8)
IQ: inter-quartile range; 95% range: difference between 97.5^{th} and 2.5^{th} percentiles

Table 3: The F-test statistics for the Variance Equality test of the Hedged Returns among Hedging strategies against DCC-GARCH based hedge

	DCC hedge against	S&P 500	FTSE 100	USD	GOLD	OIL	CORN
	Naïve hedge	0.865	0.9215	0.8871	1.9111	1.1196	1.1152
		(1)	(0.9999)	(1)	(0.0000)	(0.0000)	(0.0000)
Residual Bootstrap	**10th Percentile hedge**	1.1778	1.1485	0.976	1.8355	1.1267	0.9777
		(0.0000)	(0.0000)	(0.8533)	(0.0000)	(0.0000)	(0.8387)
	25th Percentile hedge	1.1937	1.1816	0.9932	1.8774	1.1501	1.0263
		(0.0000)	(0.0000)	(0.6154)	(0.0000)	(0.0000)	(0.1272)
	50th Percentile hedge	1.1638	1.1871	1.005	1.9118	1.1587	1.1443
		(0.0000)	(0.0000)	(0.4152)	(0.0000)	(0.0000)	(0.0000)
	75th Percentile hedge	1.1535	1.1789	1.004	1.915	1.1542	1.1351
		(0.0000)	(0.0000)	(0.4322)	(0.0000)	(0.0000)	(0.0000)
	90th Percentile hedge	1.1356	1.1435	0.9987	1.906	1.1421	1.1193
		(0.0000)	(0.0000)	(0.5222)	(0.0000)	(0.0000)	(0.0000)
Pairs Bootstrap	**10th Percentile hedge**	1.1648	1.1704	0.9792	1.8505	1.1323	1.0914
		(0.0000)	(0.0000)	(0.8184)	(0.0000)	(0.0000)	(0.0000)
	25th Percentile hedge	1.1787	1.1814	0.9938	1.8854	1.1517	1.1074
		(0.0000)	(0.0000)	(0.606)	(0.0000)	(0.0000)	(0.0000)
	50th Percentile hedge	1.1749	1.1854	1.0046	1.912	1.159	1.1399
		(0.0000)	(0.0000)	(0.4212)	(0.0000)	(0.0000)	(0.0000)
	75th Percentile hedge	1.1657	1.1795	1.004	1.916	1.1558	1.1326
		(0.0000)	(0.0000)	(0.4316)	(0.0000)	(0.0000)	(0.0000)
	90th Percentile hedge	1.1486	1.1509	0.9992	1.908	1.145	1.1188
		(0.0000)	(0.0000)	(0.5143)	(0.0000)	(0.0000)	(0.0000)

The entries are the F-test statistic for the equality of variance given in (9) and its p-values in the bracket.

Figure 1: Spot and Futures Price Plots

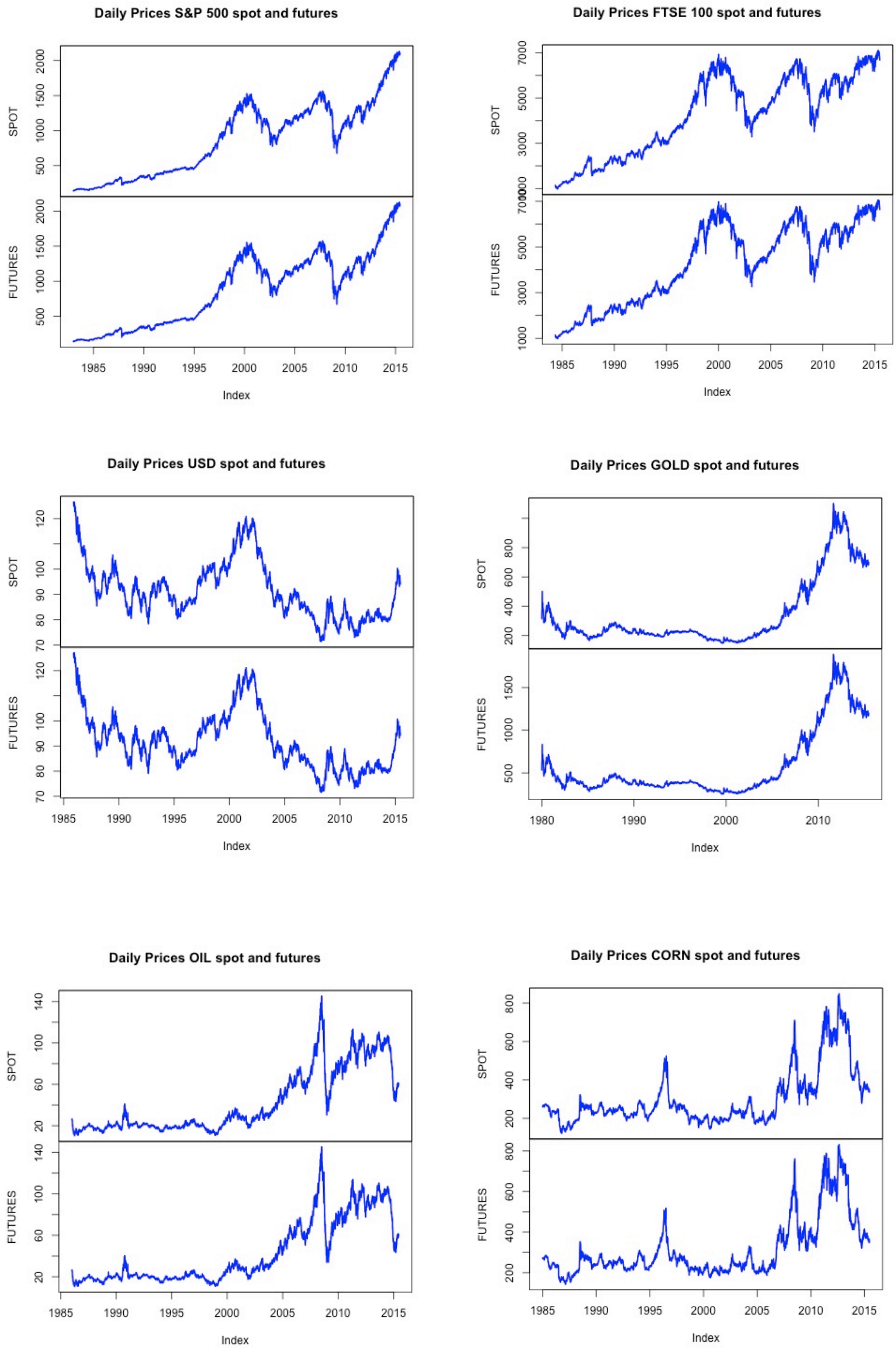


Figure 2: Spot and Futures Return Plots

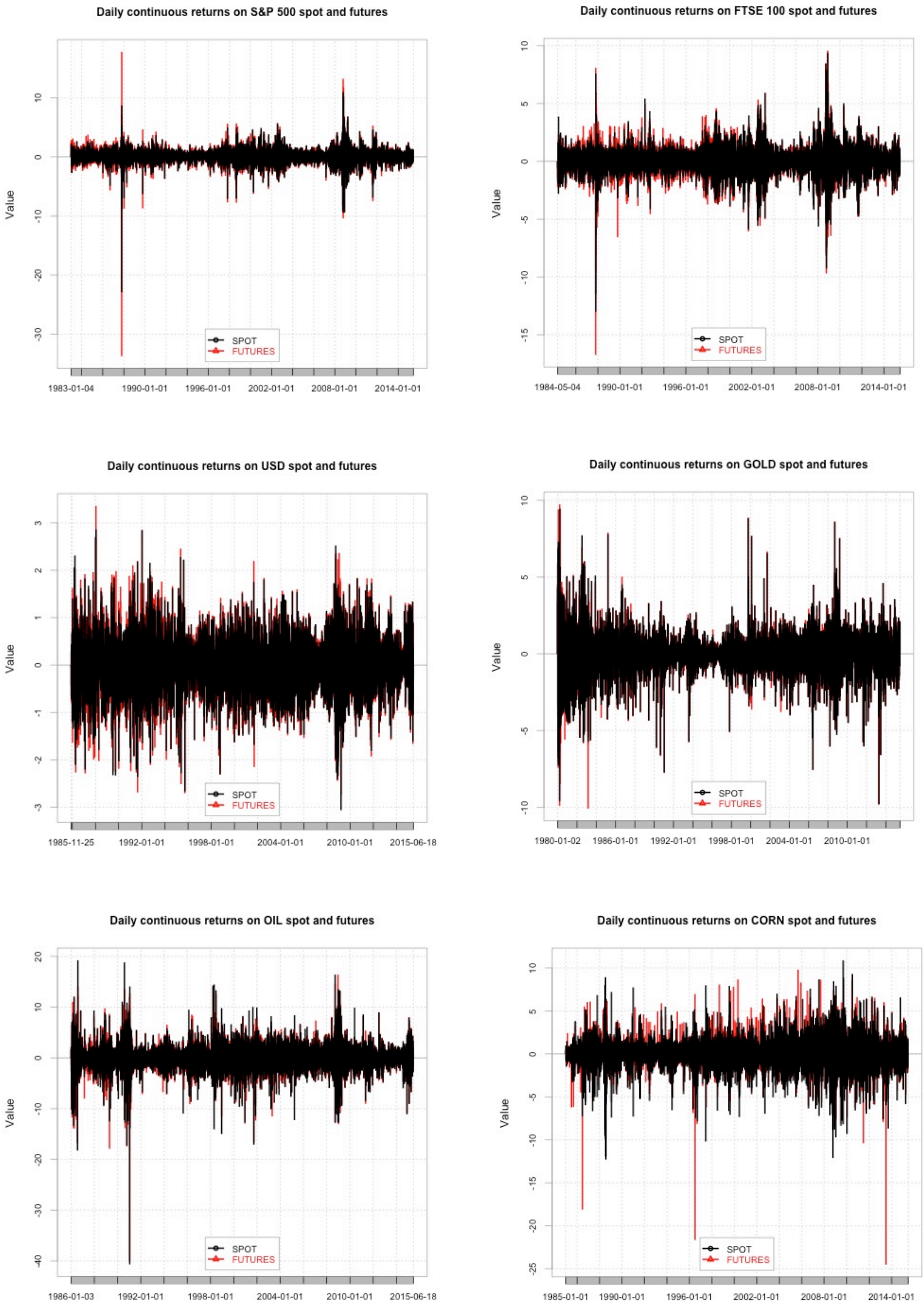
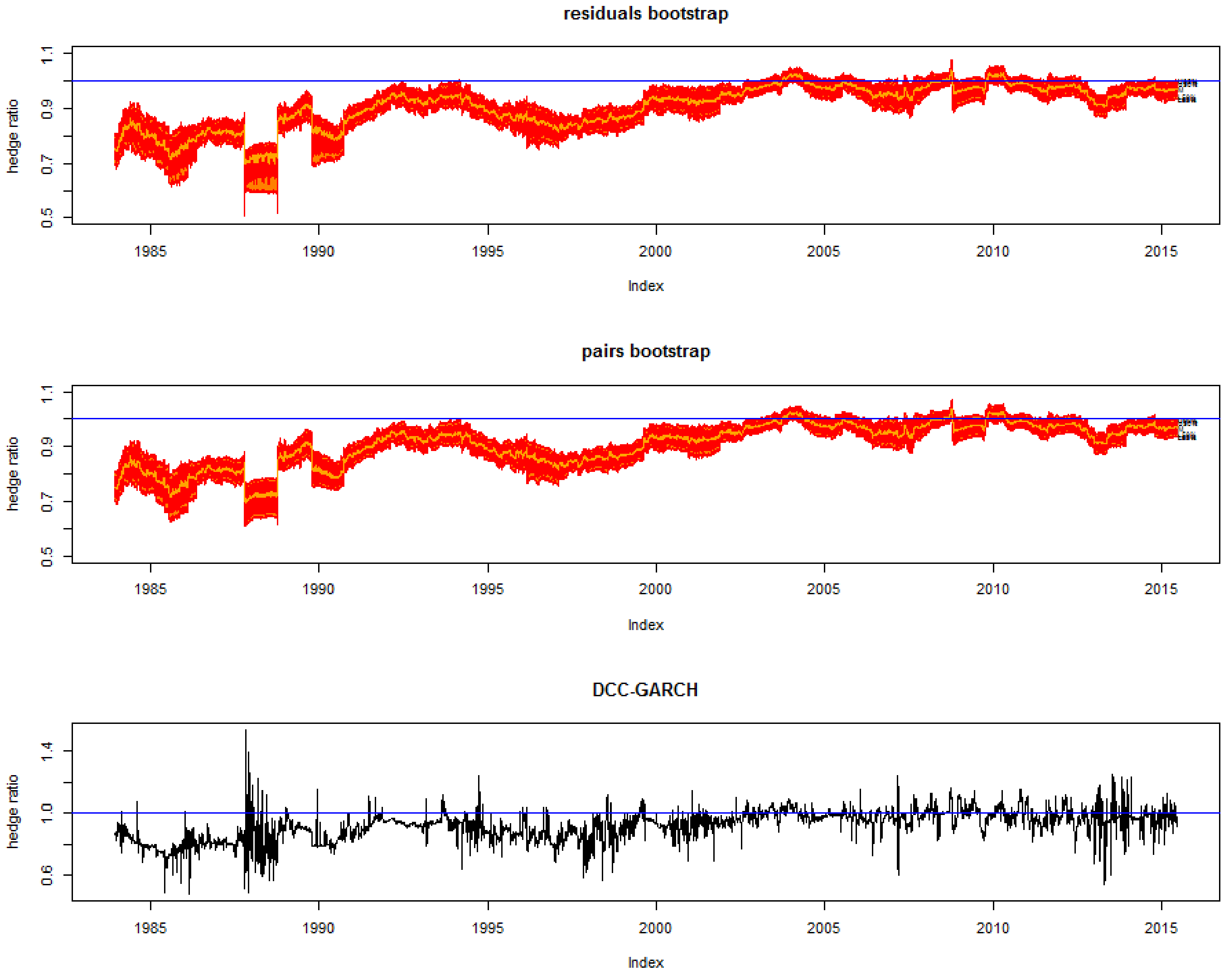
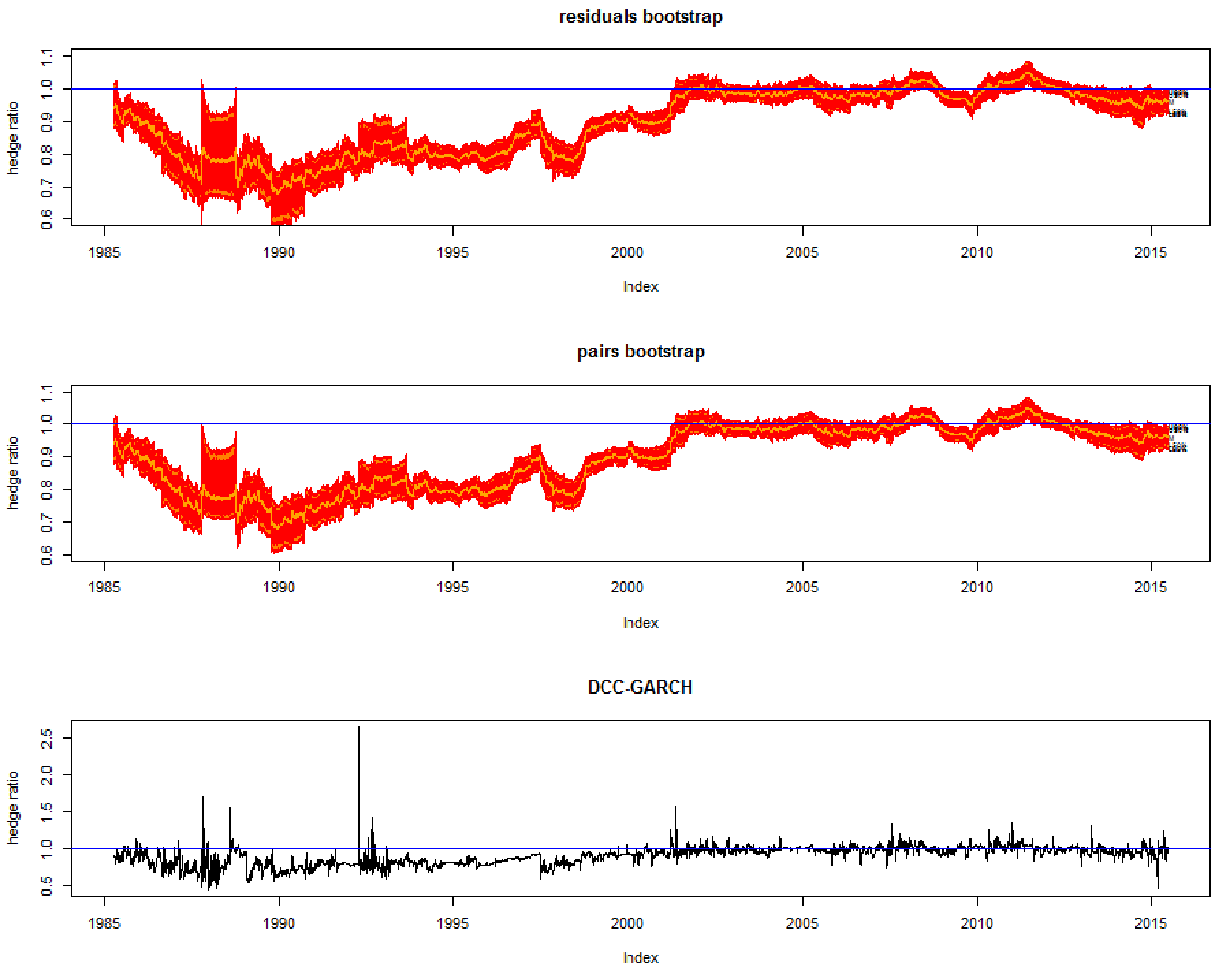


Figure 3: Optimal Hedge Ratios for S&P 500: 250-day rolling sub-sample window



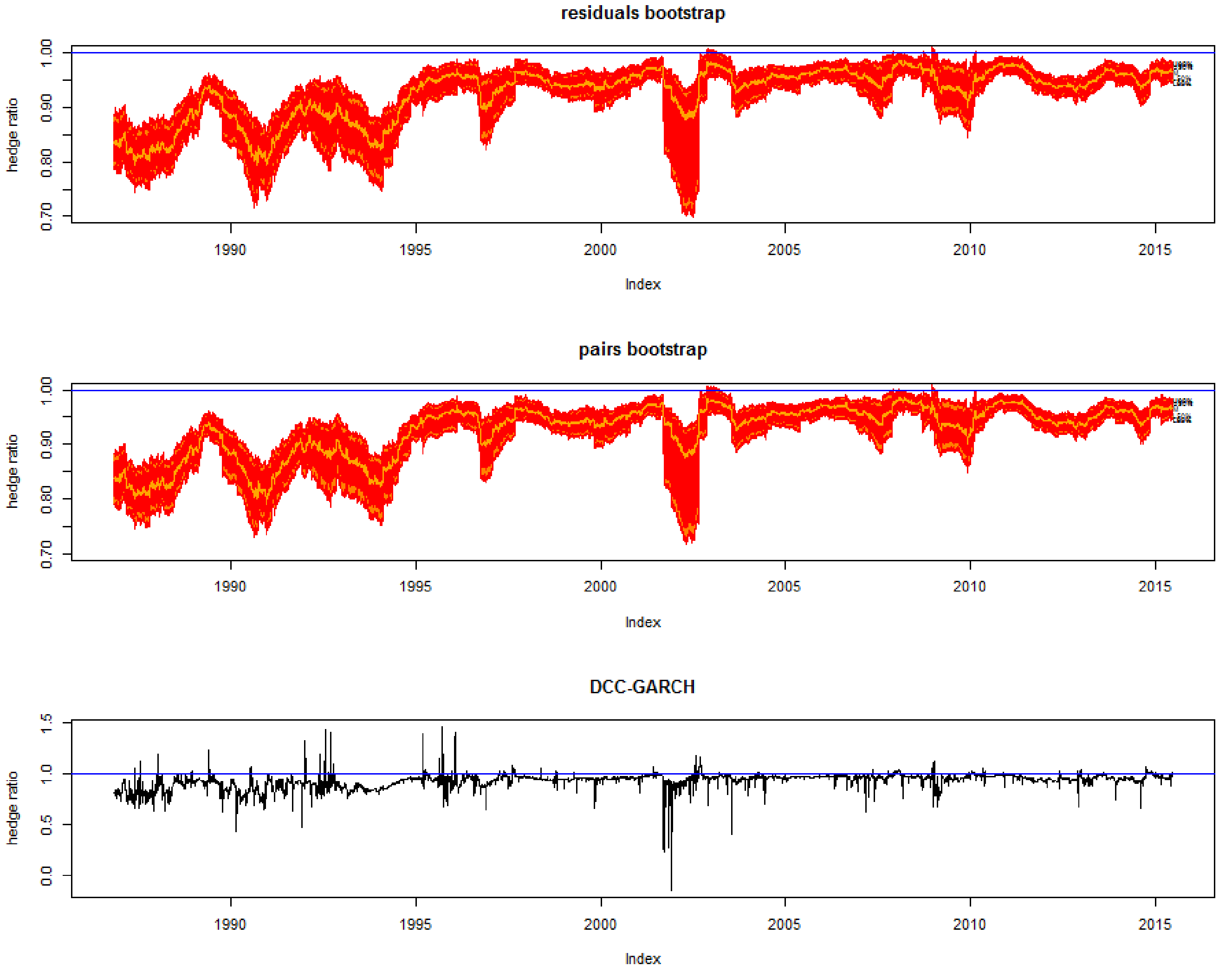
Note: The plots for bootstrap present 95% confidence band for the optimal hedge ratio. The blue horizontal line indicates the hedge ratio of 1.

Figure 4: Hedge Ratio Plots for FTSE 100: 250-day rolling sub-sample window



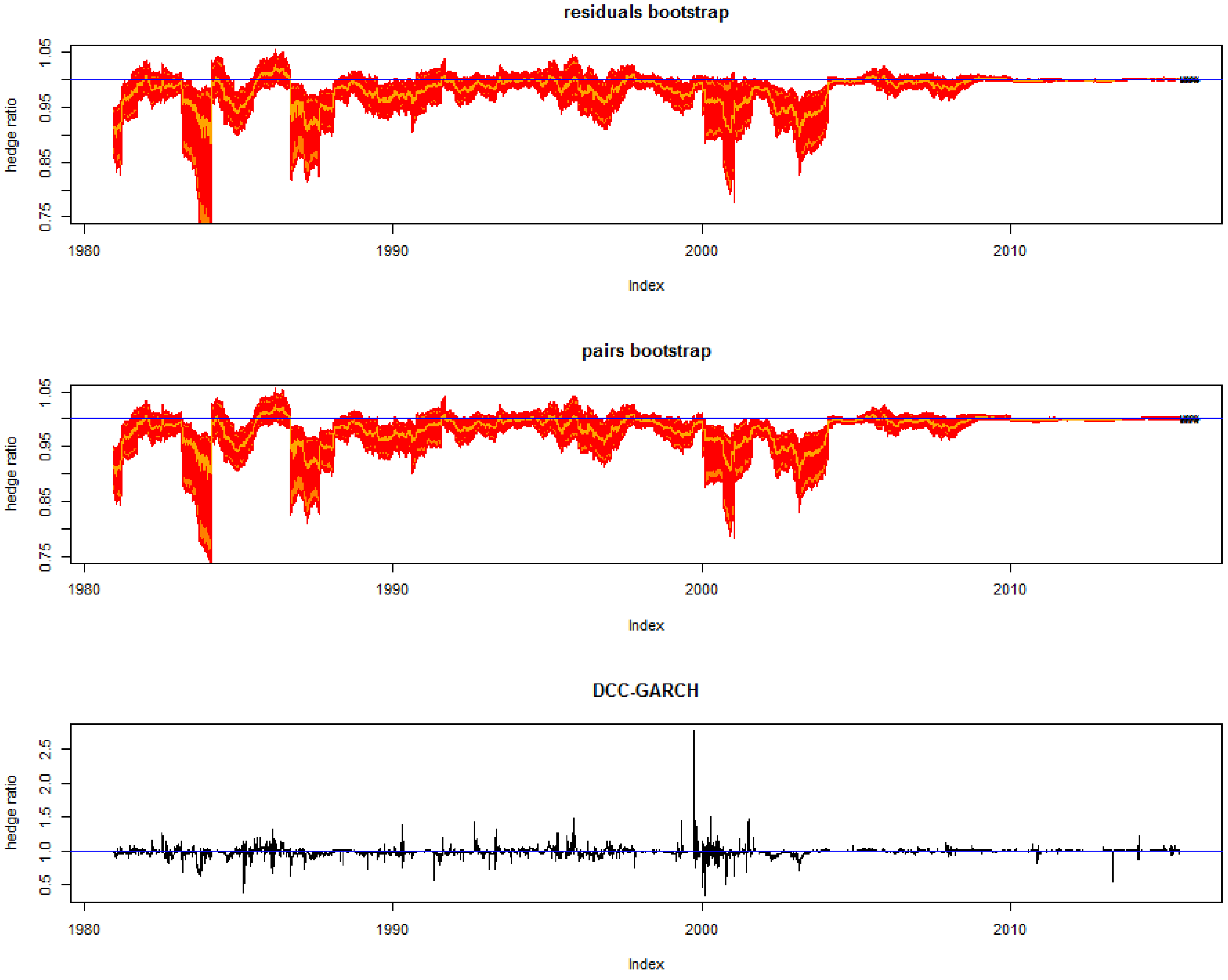
Note: The plots for bootstrap present 95% confidence band for the optimal hedge ratio. The blue horizontal line indicates the hedge ratio of 1.

Figure 5: Hedge Ratio Plots for the US dollar index: 250-day rolling sub-sample window



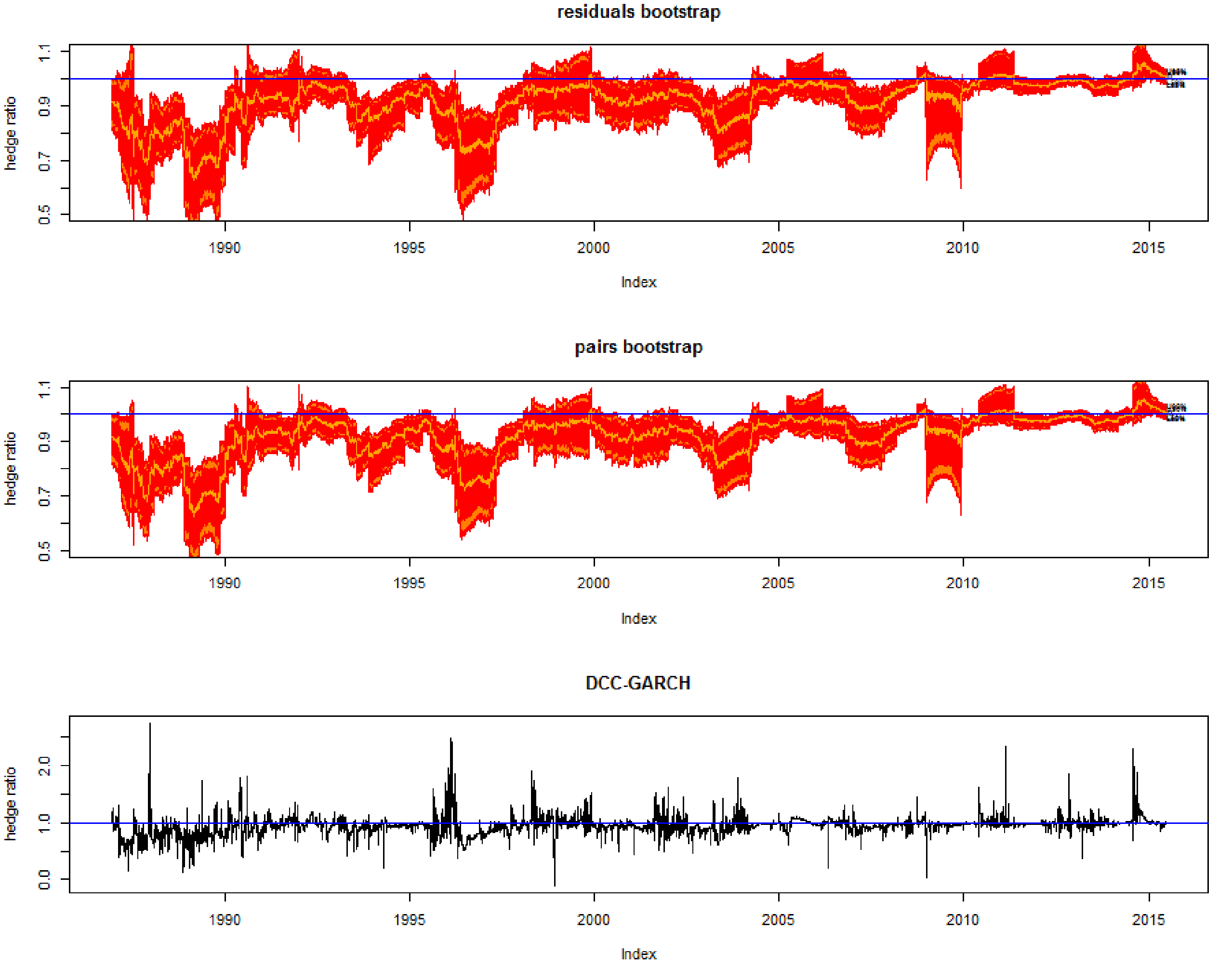
Note: The plots for bootstrap present 95% confidence band for the optimal hedge ratio. The blue horizontal line indicates the hedge ratio of 1.

Figure 6: Hedge Ratio Plots for GOLD: 250-day rolling sub-sample window



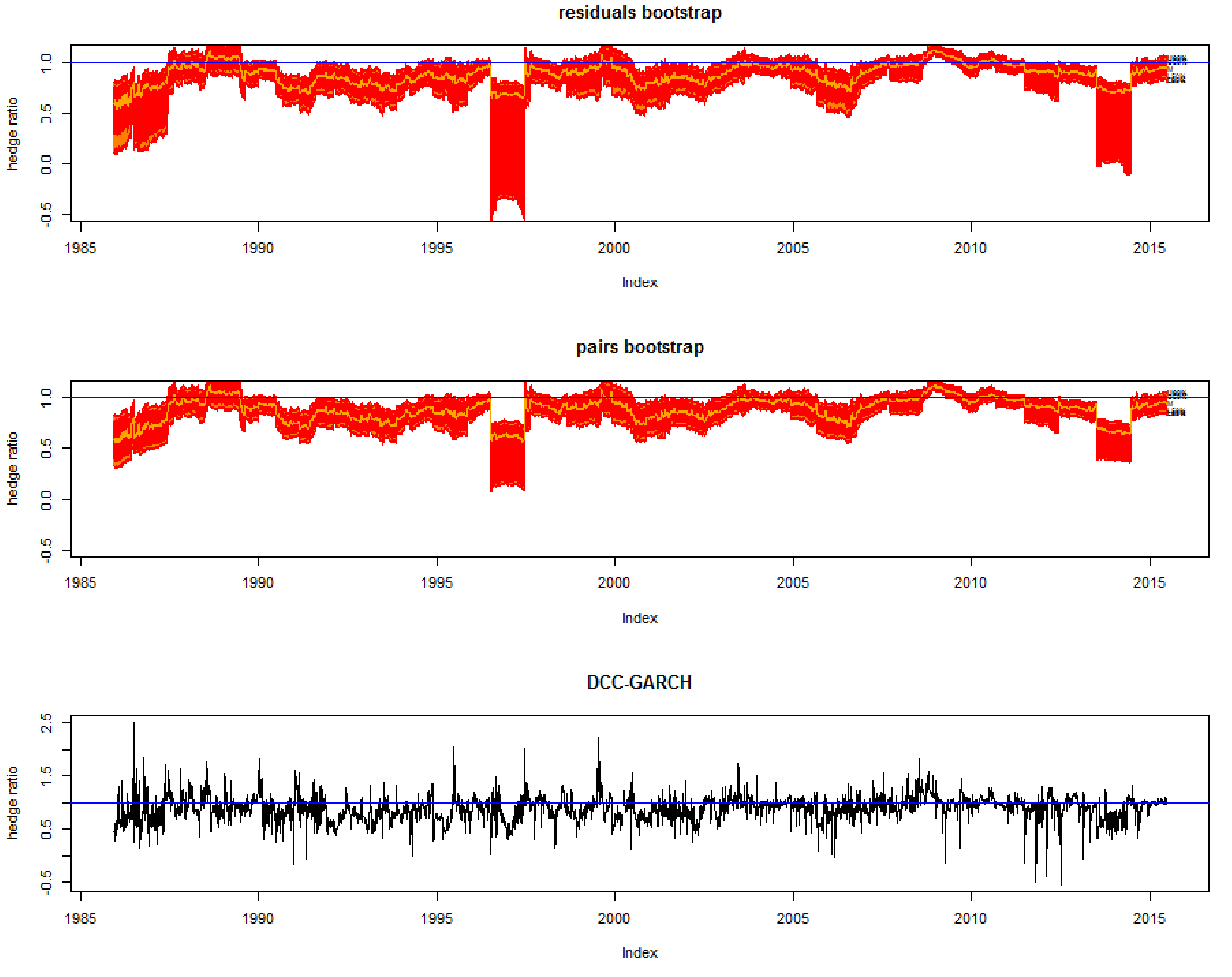
Note: The plots for bootstrap present 95% confidence band for the optimal hedge ratio. The blue horizontal line indicates the hedge ratio of 1.

Figure 7: Hedge Ratio Plots for Oil: 250-day rolling sub-sample window



Note: The plots for bootstrap present 95% confidence band for the optimal hedge ratio. The blue horizontal line indicates the hedge ratio of 1.

Figure 8: Hedge Ratio Plots for CORN: 250-day rolling sub-sample window



Note: The plots for bootstrap present 95% confidence band for the optimal hedge ratio. The blue horizontal line indicates the hedge ratio of 1.