The Disagreement with Herding, Market Bubble, and Excess Volatility

Suk-Joon Byun*, Hyun-Sik Jung**

Abstract

We construct a general equilibrium "disagreement with herding" model to identify 1) the joint effect of the disagreement and herding among investors on the price bubble and excess return volatility and 2) whether investors who herd would take advantage of excess volatility they generate. There are two classes of analysts one of which can exploit the information in the public signal. An another class of analysts, on the other hand, do not have an enough ability to refine the public signal to capture the information, and therefore do herd. i.e. tend to revise his opinion by moving toward the other's opinion. As a consequence of the combinational effect of the disagreement and herding, the price bubble and the excess volatility is exaggerated especially when they are both huge in amount.

Keywords: Heterogeneous expectations, Model disagreement, Herding, Bubbles, Excess Volatility

* College of Business, KAIST, Seoul, Korea, Republic of; Tel: +82 2 958 3552; E-mail: sjbyun@business.kaist.ac.kr

** College of Business, KAIST, Seoul, Korea, Republic of; Tel: +82 2 958 3968; E-mail: jhs0850@business.kaist.ac.kr

1. Introduction

Standard economic model with a representative agent has been improved to reflect important interactions among heterogeneous investors. By giving different priors, Hong and Stein (2007) generated heterogeneity among investors who disagree about the value of a stock even when they have the same information sets. Kandel and Pearson (1995) showed the observed volume return relation by introducing heterogeneous investors who have different interpretations about the same public signal. Scheinkman and Xiong (2003) built up the framework in which the agents agree on the model governing the economy but disagree about the public signal. These strands of papers have shown good performance at explaining an excess volatility, identified by Shiller (1981) and LeRoy and Porter (1981). Andrei, Carlin, and Hasler (2014) showed that model disagreement amplifies return volatility and trading volume by inducing agents to have a different parameter for dynamics of the fundamental process. Overconfidence, a kind of behavioral bias, also played a role in generating disagreement and excess volatility when investors agree to disagree about the unobservable parameters (Dumas, Kurshev, and Uppal 2009). In empirical researches, several works find a negative relation between forecasts dispersion and a stock mean return which is called dispersion effect (Diether, Malloy, and Scherbina 2002). A series of literature incorporating diversity among market participants has served as a good ground for the heterogeneous economy that can hardly be rationalized by a single representative model, but has assumed that each participant behaves in the market based on what they truly believe.

In another stream of literature, the seemingly opposite behavior among analysts has been continuously verified, that a portion of analysts tends to mimic the others' opinion. This behavior of analysts is called "Herding" as they appear to flock together in group by following consensus. For every analyst's forecast for each stock's earning per share, Clement and Tse

(2005) classified it as “herding” if analyst revised his own prior forecast toward the consensus. They empirically showed that several firm-level or analyst-level characteristics are verified as determinants for herding forecast. Specifically, it is reported that analysts who have more coverage firms to follow, who belong to smaller brokerage, or who underperformed at the previous period tend to herd. Moreover, Hong and Kubik (2000) empirically proved that analysts with more career concerns thus with less confidence are more likely to herd. Trueman (1994) and Graham (1999) analytically predict that analysts have higher incentives to issue a herding forecast when a firm’s earnings uncertainty is low. Analysts behavior of herding by not fully revealing their opinion has been analyzed to investigate when and how their opinions cohere.

Since the dispersion effect or model disagreement is a story based on how analysts’ opinions are widely dispersed and analysts herding is rather a story about how closely they stick together in reporting their opinions, these two observed phenomena might be closely related inborn. Even empirically a dispersion of opinions for a stock is usually measured as standard deviation of analysts’ EPS forecasts scaled by its mean. For the herding behavior also, analysts’ EPS forecast is a representative value used to determine whether analysts herd or not. Two presumably and intuitively related phenomena have been studied in each stream of research. Therefore, it should be investigated systematically and simultaneously that how a co-existence of two inherently opposite concepts of analysts’ behaviors subsists together and operates in a harmony.

With the purpose of dealing with both disagreement and herding simultaneously, we adopt the “difference-of-opinion” model by Dumas et al. (2009) as a base and introduced herding parameter in it. The difference-of-opinion model could be a good starting point because it supplies a learning environment where heterogeneous expectations between investors can possibly exist. Moreover, the public signal can be a good source to differentiate investors in their ability. Although agents receive the same value of public signal, they have different structure of the dynamics of public signal in their mind as is the case in the original model. Here, the public signal has a true positive correlation with a fundamental process that thus can be useful for filtering out the unobservable current fundamental, whereas the original model has the public signal as a pure noise. As a consequence, the correlation parameter is differently interpreted in our model from what originally analyzed it as a degree of “overconfidence”. With our fruitful public signal process, the correlation parameter represents analysts’ knowledge about the signal or capacity to refine information in the signal so that two types of agents in the economy have asymmetric abilities to figure out the “informativeness” of the signal. As a result, one type of agent cannot obtain any information from the public signal although it is valuable while another agent who might be well experienced perceives the public signal structure as the same as the true structure. This is a plausible assumption reflecting the restricted approachableness among analysts to public signal (Cote and Goodstein 1999). In this asymmetric ability environment, an inferior type of agent is set to decide to herd by taking a superior agent’s forecast into account. For example, he can take weighted average of his own value and the other’s. As a result, the inferior agent does not fully reflect his opinion and refers to the other. This setting coincides with the definition of herding that an analyst’s herding forecast does not fully reveal the analyst’s private information (Trueman 1990). To my knowledge, there have been no such models considering both the disagreement and herding behaviors.

As we previously mentioned, several firm-level or analyst-level characteristics that make analysts more likely to herd are known. Nonetheless, we regard the herding parameter in our model as an exogenous variable not depending on other variables or parameters to focus on our main purpose of comparing possible scenarios with several degrees of the herding. Briefly, each type of agent in this economy filters out the current value of the fundamental via learning process with dividend and own signal processes. A skillful analyst reports their own forecast value but the other takes a weighted average of their own forecast and the other's at the final stage. The latter analyst does not carry their intact private opinion but herds toward the consensus.

With several plausible assumptions, we derived the equilibrium state of the model and verified the herding behavior combined with a disagreement plays a pivotal role in a stock bubble and excess volatility through the market price density. From the fact that the formula of martingale market price density contains the Radon Nykodym derivative which includes a herding parameter as a functional variable, we can infer that a herding could have an effect on the entire economy and all securities. By formulating the price and volatility of each security at equilibrium under the one agent's perspective, we verified the bubble and excess volatility in the stock price. In advance, we investigated the change of the wealth of agent depending on the degree of herding and verified a wealth transfer to the herding analyst. Moreover, portfolio choice of agent tells us that investor's reaction to the fundamental shock could be changed in the presence of herding behavior in the economy.

By introducing herding parameter into the difference of opinion model, the amount of the disagreement just seems to be squeezed at first but the effect of the herding parameter permeates into the whole economy through the state variables. The existence of the herding parameter not only suppresses the pattern that the disagreement produces but reverses the original pattern resulting in the total distortion of disagreement effects on the economy. Specifically, the concave pattern of an equity's exposure to fundamental shock against the disagreement becomes less concave and even changes into convex finally as herding parameter increases. Moreover, a herding behavior reveals the interesting and important points that its existence itself could distort the total economy structure. From this theoretical analyses, it can be inferred that herding behavior by market participants may account for several market wide phenomena with the coexistence of heterogeneity in the economy.

By aggregating all simulation results, we can infer that the effect of an adoption of herding parameter into the disagreement model is prominent when both the disagreement and the degree of herding is huge in amount. Not only did herding parameter expand the price of asset but also it generated excess stock return volatility especially when a herding behavior is prevalent and an intrinsic disagreement is rather considerably large. To our knowledge, this is the first paper considering both disagreements between market participants and the herding behavior. Since this model serves as a good ground for the two behaviors of investors, there could be a room for improvement to incorporate various phenomenon observed in market. Additional settings related to the determinants of the herding or the release of the exogeneity of the herding would be an interesting research for more realistic and complicated dynamics of herding and disagreement in the model. Also, additional empirical works of the joint effects of the disagreement and herding behavior would be a great help to improve our understanding of its functions to the connected entities.

The rest of papers proceed as follows: Section 2 explains how the economy works and how agents perceive the economy. We derived the equilibrium market price density and security price in section 3 and 4. Section 5 investigated the price and the return volatility for each security, and wealth of agents at equilibrium. Section 6 concludes.

2. The dynamics of economy

We consider the standard endowment economy of Lucas (1978) in which a single equity pays a dividend continuously. In addition to this stock with a total outstanding share of one unit, there is a bond with a net supply of zero so that each investor forms a portfolio depending on each demand for risk exposure. The expected growth of the dividend process henceforth called the fundamental is not observed by any agent, and thus must be estimated.

A. Stochastic process of the economy

Assumption 1. *(Aggregate endowment of consumption)* The dividend process is assumed to be positive and to follow the stochastic process:

$$\frac{d\delta_t}{\delta_t} = f_t dt + \sigma_\delta dZ_t^\delta \tag{1}$$

where Z_t^δ is a Brownian motion under the physical probability measure, which governs the empirical realizations of dividends. Note that the volatility of the consumption growth, σ_δ, is positive. The dividend, value of δ_t is commonly observed but its expected growth rate denoted f_t, cannot be observed and assumed to follow an Ornstein-Uhlenbeck process.

Assumption 2. *(Fundamental process)* The unobservable fundamental behaves according to mean reverting process:

$$df_t = -\zeta\left(f_t - \bar{f}\right)dt + \sigma_f dZ_t^f \tag{2}$$

where Z_t^f is an another Brownian motion under the physical probability measure. Note that the volatility of the fundamental process, σ_f is assumed to constant and known with enough time has passed so that everyone can detect. The fundamental value mean reverts to its long term mean $\bar{f}$ at the speed of ζ.

For a mean reverting structure and governing parameters, $\{\zeta, \bar{f}, \sigma_f\}$, of the fundamental process is assumed to be known to all types of agents in the economy. In addition to the dividend process, one more source that helps to estimate the fundamental value exists, which is the public signal.

Assumption 3. *(Public signal)* There comes a public signal which is approachable information to every participant in the economy as its name suggests:

$$ds_t = \phi\sigma_s dZ_t^f + \sqrt{1-\phi^2}\sigma_s dZ_t^s \tag{3}$$

where Z^s is the third Brownian motion under the physical probability measure as well. As can be seen in the process, the public signal has a positive correlation ϕ with the fundamental process and has the positive volatility σ_s. σ_s is assumed to be known.

The correlation parameter ϕ represents the "informativeness" of the public signal in the way that the signal with higher correlation contains more valuable information and therefore helps investors to estimate the fundamental better. As will be noted later, this correlation parameter is excavated by only a superior type of agent.

In summary, this economy is characterized by three independent Brownian motions, $\{Z_t^\delta, Z_t^f, Z_t^s\}$ which governs the process of dividend, fundamental, and public signal respectively.

B. Heterogeneous expectations between agents

For the purpose of this paper, we assumed the heterogeneous economy which is populated by two types of agents, A and B with the asymmetric capacity to utilize a common public signal. These two agents considered as analysts have a different accumulation of experiences at forecasting fundamental values so that there are asymmetric abilities between them in exploiting the information from the public signal.

Assumption 4. *(Asymmetric abilities among agents)* Agent A and B have different abilities to figure out the “informativeness” in the public signal. As a result, each agent A and B draw different structure of the public signal in their mind

$$ds_t^A = \phi\sigma_s dZ_{A,t}^f + \sqrt{1-\phi^2}\sigma_s dZ_{A,t}^s \tag{4}$$

$$ds_t^B = \sigma_s dZ_{B,t}^s \tag{5}$$

where the volatility of the signal, σ_s is positive.

Although the public signal can be freely observed by any agent, the only a skillful agent exactly knows the detail about it. Agent in group A, henceforth superior, exploits the correlation hidden in the public signal and utilizes it to filter out the fundamental. On the other hand, an agent in group B, henceforth inferior, has no idea about what is in the public signal as the same as a white noise. This assumption is plausible in that there exists a wide spectrum of the performance by analysts in forecasting earnings (Stickel (1992) and Sinha, Brown, and Das (1997)). Clement (1999) identified systematic and persistent differences in analysts’ earnings forecast accuracy are attributable to analysts’ experience. It is also widely assumed that agents could interpret the public signal differently. In particular, several papers feature a setting in which investors process information differently (Scheinkman and Xiong 2003 or Dumas, Kurshev, and Uppal 2009).

As described in the previous assumption, the parameter ϕ characterizes “informativeness” of the public signal about the fundamental. By the way, it could also be interpreted as the degree of “asymmetric ability” between two parties of agent A and B, in that one analyst outperforms the other in purifying the noisy signal. If ϕ is big enough, agent A is much better at forecasting unobservable fundamental value by utilizing public signal that he knows the correlation with fundamental value. Therefore, the parameter ϕ could be analyzed as both informativeness and asymmetric ability. Not only the public signal, agents could also utilize the aggregate dividend process δ_t to estimate the current value of the fundamental under their respective probability measures.

Because of the unlike structures of the public signal in their minds, they filter out unobservable fundamental, f, differently.

Assumption 5. *(Disagreement)* Under the probability measure of B, the conditional expected values, $\hat{f}_t^A$ and $\hat{f}_t^B$ of f according to individuals obey the following stochastic processes:

$$d\hat{f}_t^A = -\zeta\left(\hat{f}_t^A - \bar{f}\right)dt + \frac{\gamma^A}{\sigma_\delta^2}\left(\frac{d\delta}{\delta} - \hat{f}_t^A dt\right) + \frac{\phi\sigma_f}{\sigma_s}ds \tag{6}$$

$$d\hat{f}_t^B = -\zeta\left(\hat{f}_t^B - \bar{f}\right)dt + \frac{\gamma^B}{\sigma_\delta^2}\left(\frac{d\delta}{\delta} - \hat{f}_t^B dt\right) \tag{7}$$

With original estimates of individual agents, an intrinsic disagreement between agents is defined as:

$$\hat{g}_t^0 = \hat{f}_t^B - \hat{f}_t^A \tag{8}$$

which satisfies the stochastic differential equation:

$$d\hat{g}_t^0 = d\hat{f}_{B,t} - d\hat{f}_{A,t} = -\left(\zeta + \frac{\gamma^A}{\sigma_\delta^2}\right)\hat{g}_t^0 dt + \left(\frac{\gamma^B - \gamma^A}{\sigma_\delta}\right)dW_{\delta,t}^B - \phi\sigma_f dW_{s,t}^B \tag{9}$$

Each agent's estimate of the unobservable fundamental, $\hat{f}_t^A$ and $\hat{f}_t^B$ are computed using standard Bayesian updating methods. Learning is implemented via Kalman filtering following Theorem 12.7 in Lipster and Shiryaev (2001).

The numbers, γ^A and γ^B are the Bayesian uncertainty i.e. posterior variance, reflecting an incomplete understanding of the true expected growth rate. These variances would normally change depending on time. But we assume, as in Scheinkman and Xiong (2003), that there has been an enough time of learning for agents to converge to a steady-state value:

$$\gamma_A = \sigma_\delta^2 \left(\sqrt{\zeta^2 + (1-\phi^2)\frac{\sigma_f^2}{\sigma_\delta^2}} - \zeta \right) \tag{10}$$

$$\gamma_B = \sigma_\delta^2 \left(\sqrt{\zeta^2 + \frac{\sigma_f^2}{\sigma_\delta^2}} - \zeta \right) \tag{11}$$

As pointed out by Scheinkman and Xiong (2003), both posterior variances increase with the volatility of the fundamental σ_f and the volatility of the aggregate endowment σ_δ.

The normalized innovation process of the dividend under each agent's probability measure, $\frac{d\delta}{\delta} - \hat{f}_t^A dt$ and $\frac{d\delta}{\delta} - \hat{f}_t^B dt$ will be noted as $d\widehat{W}_{A,t}^\delta$ and $d\widehat{W}_{B,t}^\delta$ respectively from now on. Note that $d\widehat{W}_{i,t}^\delta$ is a standard Brownian motion from agent i's point of view.

C. Herding behavior of an inferior agent

As widely verified, analysts tend to herd by not fully reflecting their own opinion but revising toward the consensus. This behavior becomes more dominant when analysts have less experience and more career concerns. To adopt this herding behavior of an analyst in forecasting into the original model, we introduce a new parameter, θ, "herding parameter", representing the extent that how much an inferior agent refers to the other agent's opinion.

Assumption 6. *(Herding)* Agent B can observe the forecast value reported by an agent in group A before he reports his own value. The inferior and unconfident agent in group B determines to herd by revising their own forecast toward the estimate of the other agent A:

$$\hat{f}_{B\prime,t} = \theta \hat{f}_{A,t} + (1-\theta)\hat{f}_{B,t} \tag{12}$$

where the herding parameter θ lies between 0 and 1.

Agent B, an inferior analyst revises his own forecast toward the other's by taking an weighted average of the value reported by agent A and his own forecast. A weight is $(\theta, 1-\theta)$. It is an additional economic setting for a forecasting environment that analyst in group B could refer to the value estimated by the other, agent in group A. We assume that agent B has a moment to refer the value reported by agent A after he filters out. In that moment, an agent in group A reports his forecasting result first, and agent in group B could observe the A's result before reporting his own estimate. As empirically verified, it is a plausible assumption that analysts revise their own forecast value toward the consensus. By this setting, agent B who are less experienced or less confident herd in the degree of θ by taking θ portion of the forecast from agent A and $1-\theta$ portion of his own forecast. This analytical economic setting may align the conventional definition of herding in that analysts move toward the consensus by not fully reflecting their private opinions.

Proposition 1. *(Disagreement with herding)* A disagreement between the expectations of an individual agent is modified due to the herding behavior by an inferior agent B:

$$\hat{g}_t = \hat{f}_{B',t} - \hat{f}_{A,t} = \theta \hat{f}_{A,t} + (1-\theta)\hat{f}_{B,t} - \hat{f}_{A,t}$$

$$= (1-\theta)\left(\hat{f}_{B,t} - \hat{f}_{A,t}\right) = (1-\theta)\hat{g}_t^0 \tag{13}$$

and modified disagreement process with the existence of herding behaves according to:

$$d\hat{g}_t = -\left(\zeta + \frac{\gamma^A}{\sigma_\delta^2}\right)\hat{g}_t dt + (1-\theta)\frac{\gamma^B - \gamma^A}{\sigma_\delta} dW_{\delta,t}^B - (1-\theta)\phi\sigma_f dW_{s,t}^B \tag{14}$$

We see that the disagreement process with the herding behavior also mean reverts and becomes 0 when agent B wholly herd by exactly mimicking agent A's forecast discarding his own forecast value, i.e. θ equal to 1.

Proposition 1 characterizes the dynamics of disagreement with the existence of the degree of herding. The herding parameter, θ, makes our model distinct from previous models of disagreement and thus become an unique and important setting in our theoretical framework.

As an agent in group B herds more, they can freely enjoy the information that agent in group A obtained. Therefore it is always optimal and best to follow agent A's forecast intact, i.e. $\theta = 1$ if they have no restriction to choose the θ. This is because an inferior agent A is dominated by an superior agent B for a learning as long as $\phi > 0$. We excluded, however, the option for agent B to choose the degree of the herding, θ, to focus on our main objective. There could be many reasons why analysts do not completely mimic the others even if the others' forecasts dominate theirs. For example, they should reveal the logics used to build up the forecast value that can be hardly copied. θ is given exogenous. It could also be an interesting research in the game theory framework to explore the relative payoffs depending on the herding parameter with additional payoff structure. For more improved research, the model could include the cost for getting correlation parameter, ϕ, or the loss of utility from a herding activity so that they must be balanced between gains and losses.

The two types of agents have different probability measures due to the asymmetrically perceived signal processes albeit the same. Without loss of generality, we choose agent B's probability measure to view the economy. To do so, we should calculate the bridge to link the views among agents, which is the changes of measure, η, such that

$$\mathbb{E}^A[X] = \mathbb{E}^B[\eta X] \tag{15}$$

for every random variable X.

Proposition 2. For every random variable X, each group of agent has a different probability space but linked to each other through η:

$$\frac{d\eta_t}{\eta_t} = -\hat{g}_t \frac{1}{\sigma_\delta} dW^B_{\delta,t} \tag{16}$$

$$\eta_t = \frac{d\mathbb{P}^A}{d\mathbb{P}^B} | = e^{-\frac{1}{2}\int_0^t \left(\frac{1}{\sigma_\delta}\hat{g}_s\right)^2 ds - \int_0^t \frac{1}{\sigma_\delta}\hat{g}_s d\widehat{W}^\delta_{B,s}} \tag{17}$$

According to the above formula, the herding parameter θ has an effect on reducing the amount of change of η. This technical setting coincides the situation that the disagreement between two parties is decreased, therefore the relative perspective changes less.

So far, we have completely characterized the joint dynamics of the four state variables $\{\delta, \eta, \hat{f}^B, \hat{g}\}$, in the perspective of agent in group B. Two of the four state variables have a direct, immediate effect on the economy. They are the fundamental, δ, and the sentiment η, variables. The other two state variables, $\hat{f}^B$ and $\hat{g}$, have only an indirect effect in that they play a role for building up the first two. The herding parameter, θ, acts its power through the diffusion terms of the last two by scaling its coefficients. There are only two Brownian motions that governs four state variables so that diffusion matrix would be in a size of 4×2:

$$\begin{bmatrix} \delta\sigma_\delta & 0 \\ \frac{\gamma^B}{\sigma_\delta} & 0 \\ -\eta\frac{\hat{g}}{\sigma_\delta}(1-\theta) & 0 \\ \frac{\gamma^B - \gamma^A}{\sigma_\delta}(1-\theta) & -\phi\sigma_f(1-\theta) \end{bmatrix} \tag{18}$$

Note that herding parameter θ permeates to the economy through the coefficients of the diffusion terms of η and $\hat{g}$. In summary, introducing a herding parameter brings about two distinct changes in the dynamics. One is the diffusion vector of sentiment and the other is the channel through the diffusion of diffusion of sentiment.

3. Equilibrium consumption sharing and state price density

To explore the equilibrium, we first set up the optimization problem for each individual agent. It is reasonably assumed that both agents have power utility function with the same risk aversion, $1-\alpha$, and the rate of discount rate ρ. Assuming a complete market, by following Cox and Huang (1989), we can use the static martingale approach to set up the maximization problems.

A. Optimization problem for each individual agent

Each agent faces an optimization problem to maximize the expected lifetime utility from whole consumption flows:

$$\max_{c} \mathbb{E}^{B} \int_{0}^{\infty} \eta_t e^{-\rho t} \frac{1}{\alpha} (c_t^A)^{\alpha} \, dt \tag{19}$$

$$\max_{c} \mathbb{E}^{B} \int_{0}^{\infty} e^{-\rho t} \frac{1}{\alpha} (c_t^B)^{\alpha} \, dt \tag{20}$$

subject to the lifetime budget constraint:

$$\mathbb{E}^{\mathrm{B}} \int_{0}^{\infty} \eta_t \xi_t^B c_t^A \, dt \leq \mathbb{E}^{B} \int_{0}^{\infty} \xi_t^B \delta_t \, dt \tag{21}$$

$$\mathbb{E}^{\mathrm{B}} \int_{0}^{\infty} \xi_t^B c_t^B \, dt \leq \mathbb{E}^{B} \int_{0}^{\infty} \xi_t^B \delta_t \, dt \tag{22}$$

By taking a derivative with respect to the consumption unit, c_t, we obtain the first order conditions:

$$\eta_t e^{-\rho t}(c_t^A)^{\alpha-1} = \lambda^A \xi_t^B \tag{23}$$

$$e^{-\rho t}(c_t^B)^{\alpha-1} = \lambda^B \xi_t^B \tag{24}$$

where λ is the Lagrange multiplier of the budget constraint.

B. Equilibrium pricing measure

The total consumption of both agents at time t is equal to the dividend paid at time t which becomes market clearing condition as below:

$$\left(\frac{\lambda^A \xi_t^B e^{\rho t}}{\eta_t}\right)^{\frac{1}{\alpha-1}} + (\lambda^B \xi_t^B e^{\rho t})^{\frac{1}{\alpha-1}} = \delta_t \tag{25}$$

By solving above equation, we could obtain the equilibrium state price density:

$$\xi_t^B = e^{-\rho t}\left[\left(\frac{\eta_t}{\lambda^A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{1}{\lambda^B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha} \delta_t^{\alpha-1} \tag{26}$$

It is a direct implementation that θ plays an important role in constructing the structure of the state price of density, ξ, through η since θ directly changes the $\hat{g}$, the disagreement. By substituting ξ_t^B into each consumption, the consumption for each agent at the equilibrium would be:

$$c_t^A = \omega(\eta_t)\delta_t \tag{27}$$

$$c_t^B = \big(1-\omega(\eta_t)\big)\delta_t \tag{28}$$

where

$$\omega(\eta_t) = \frac{\left(\frac{\eta_t}{\lambda^A}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{\eta_t}{\lambda^A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{1}{\lambda^B}\right)^{\frac{1}{1-\alpha}}} \tag{29}$$

Consumption sharing rule tells us that the stochastic allocation is also attributed to the herding parameter θ which lies in the formulation of η.

4. Securities at equilibrium

We analyze the impact of an analyst's herding behavior on the price and return volatility. With calculated price and return volatility using formula above, we show that herding by analysts can cause a serious transformation of the structure of the characteristics of securities as will be displayed in detail.

A. The price and volatility of securities at equilibrium

In this economy, there exist two assets, one of which is a stock, and the other is a perpetual bond paying a coupon of one unit of consumption continuously. With the martingale pricing density ξ_t, all the equilibrium price of securities of which the dividend or coupon processes are known can be formulated. The equilibrium price of a bond at time t, denoted as $B(t)$, can be calculated as below:

$$B(t) = \int_0^\infty \mathbb{E}^B\left[\frac{\xi_u^B}{\xi_t^B}\right] du \tag{30}$$

Similarly, the equilibrium price of the stock is obtained as the sum of all discounted dividends. We denote it as $S(t)$:

$$S(t) = \int_0^{\infty} \mathbb{E}^B \left[\frac{\xi_u^B}{\xi_t^B}\delta_u\right] du \tag{31}$$

Because the consumption in this economy only comes from the dividend of this stock, the total value of the equity could be interpreted as the whole wealth of the economy for the rest of time. The detail formulas are stated in the Appendix

Since there are four state variables, $\{\delta, \hat{f}^B, \eta, \hat{g}\}$ which constitute the channel to affect the securities price and return volatility, all economic shocks are transferred through this set of the state variables. With many shocks coming, security prices fluctuate and return volatility is generated. By denoting state variable vector as $X = \{\delta, \hat{f}^B, \eta, \hat{g}\}$, we obtain the price and return volatility for a stock and a bond:

$$\sigma_{Stock} = \left|\frac{\sigma(X_t)' \frac{\partial S_t}{\partial X_t}}{S_t}\right| \tag{32}$$

$$\sigma_{Bond} = \left|\frac{\sigma(X_t)' \frac{\partial B_t}{\partial X_t}}{B_t}\right| \tag{33}$$

Since there come two kinds of shocks to the state variables, every security also has corresponding components of the exposure or sensitivity to each shock. The signal shock is delivered only through the disagreement channel and the fundamental shock is transferred through every state variable as also can be seen in (18). Importantly, herding parameter plays an important role in transferring fundamental shock and signal shock through η and g. Therefore, herding parameter may cause severe changes in the dynamics of the risk at equilibrium, specifically through a sentiment and a disagreement channel.

B. Multiple scenario illustrations

As many parameters contribute to the economy in various ways, we isolated an individual variable for each scenario and made a variation. Particular variables of which we decided to investigate its effect are consumption share of agent A, a disagreement between two groups of agents, and an informativeness of the public signal. These three variables could be good indices to compare the comprehensive dynamics of herding behavior in various situations in that all variables represent the degree of the heterogeneity between the two groups of agents. The disagreement variables, $\hat{g}_t$, directly tells us that the spread of expectations between two agents. The informativeness parameter ϕ, also reflecting the degree of asymmetric ability, speaks out how big a capacity gap exists between heterogeneous agents. Before we delve into three scenarios for several herding parameters, brief illustrations are summarized below.

Scenario 1. The equilibrium consumption share of agent A increases from 0 to 1. The relative aggregate consumption portion of the analysts who do not herd versus the analysts who do herd could indirectly represent the degree of the heterogeneity in the amount of population with respect to the dimension of wealth.

Scenario 2. The disagreement of the expectations about fundamentals increases from -0.1 to 0.1. As $\hat{g}$ is getting bigger, agent B has relatively optimistic opinion compared to agent A. The most important phenomena come from the variation of disagreement for our main objective, the combinational effect of the disagreement and the herding. The deformity arising from herding is expected to be getting worse when the amount of the disagreement is relatively large.

Scenario 3. The correlation between the public signal and the fundamental process, ϕ, increases from 0 to 1. As previously argued, ϕ can be interpreted as both the "informativeness" of the public signal and the amount of "asymmetry in the ability" between analysts. Therfore herding analysts would take the other's intellectual property away more, when the value hidden in the public signal is higher or the fruition that sincere analysts obtained is more valuable although they may not know the worth of it.

For every graph in this paper, there are four curves representing cases for the different degrees of the herding behavior. A solid line always represents the situation without herding, i.e. $\theta = 0$, that agent B choose to report their own pure forecast about the fundamental, a dashed line draws the economy with a slight herding behavior, $\theta = 0.1$ that agent B carries almost their own result and adds a little portion of agent A's opinion, a dotted line shows us the situation of half-herding, $\theta = 0.5$, that agent B takes the mean of their own forecast and the agent A's estimate as a final forecast, and lastly, a dotted-and-dashed line, describes the heavy herding circumstance in which agent B almost discards their own forecasting outcome and nearly imitates the agent A's statement, $\theta = 0.9$. Briefly, from a solid line to a dashed-and-dotted line, the heavier herding is occurred by the analyst B.

Note that the case of $\theta = 1$, describes the situation of the whole herding in which no disagreements occur and thus no other special events happen. The parameter values that are undertaken for calibrations are adopted from the estimation by Brennan and Xia (2001) or Dumas et al. (2009). Benchmark value for the informativeness parameter ϕ, is fixed as 0.5 for Scenario 1 and Scenario 2.

[Insert Table 1 here]

C. Securities price and bubbles

Figure 1 plots the lines showing the effect of changes in prices of securities at the equilibrium as scenario variables vary and comparing four lines each other as the herding parameter is shifted. The herding parameter presses the price graph downward when the graph has an x axis of the variables of ω and ϕ. Notably, the serious expansion in security price happens when the disagreement between two parties is high. In other words, the bubble is generated when analysts tend to herd even against the significant amount of an original disagreement. This result partly coincides with the research that stock price bubble is associated with speculative behavior of investors when the disagreement is decreased (Froot and Obstfeld 1991).

[Insert Figure 1 here]

D. Volatility and risk exposure of securities

We analyze the joint impact of model disagreement and herding behavior on the stock return volatility and bond return volatility. We show that the herding is conveying generally decreasing effect on the both volatilities of securities when the level of disagreement is relatively small, as can be seen in the top and bottom rows in Figure 2, where the benchmark value of a disagreement $\hat{g}$ is equal to -0.03.

[Insert Figure 2 here]

Interesting change in patterns occurred at the scenario 2 with x-axis of the disagreement level. As verified in Dumas et al (2009) in which no herding exists, stock return volatility shapes a concave curve and bond return volatility forms a convex curve against the degree of the disagreement. However, as analysts in group B herd, both securities' return volatility started to become flatter and especially for the case of a stock, it changed into convex finally. As a consequence, the excess return volatility of a risky asset is generated in the environment of combinational existence of both high disagreement and heavy herding.

Figure 3 portrays the exposure to the fundamental shock, i.e. dividend shock of each security. We can see that almost all ingredients of the volatility come from its sensitivity to the fundamental shock. Left columns plot the diffusion vector for the stock price and right columns plot ones for the bond price. Notably, a plunging decrease in the exposure of equity seems to be the reason for the convex volatility of the equity against the disagreement. By delving into the diffusion vector deeply, a decreasing trend of the exposure against the disagreement is mainly attributable to increasing price. In summary, exaggerated price, so called a bubble, produced the excess volatility of a risky asset.

[Insert Figure 3 here]

5. Group B's portfolio strategy according to its exposure

Herding behavior, in a new point of view, could detect the analysts' free-riding activity which could cause the wealth transfer between market participants. We now study the fluctuations of the wealth of analysts in group B to detect the amount of additional wealth they get by herding.

A. The wealth of B

The wealth of group B can be obtained by the same way that we used to calculate the security prices. As a current value of an equity is equal to the total wealth of this economy, the value of a security paying dividends exactly equal to the consumption of agent B can be interpreted as the wealth of group B:

$$W^B(t) = \int_0^\infty \mathbb{E}^B\left[\frac{\xi_u^B}{\xi_t^B} c_u^B\right] du \tag{34}$$

We can draw the similar and consistent implementation from Figure 4 that the herding analyst's hijacking the other is prominent when he has a highly optimistic expectation compared to the opponent.

[Insert Figure 4 here]

In an unreported figure, we verified that the same trend is shown for the exposure of wealth as is the case for the security price and its exposure.

B. Portfolio choice

An allocation of a wealth to each asset is a good window to see through the posture that investors have in mind to handle the risk they are exposed to. The risks of B's wealth exposed to dividend and signal shock can be synthesized by forming an appropriate portfolio that replicates the exposure to each shock. We obtain portfolio vector by solving following equation:

$$\frac{\sigma(X_t)' \frac{\partial W_t^B}{\partial X_t}}{W_t^B} = \Omega' \times \left[\frac{\sigma(X_t)' \frac{\partial S_t}{\partial X_t}}{S_t} \quad \frac{\sigma(X_t)' \frac{\partial B_t}{\partial X_t}}{B_t}\right] \tag{35}$$

Proposition 3. *(Portfolio formation of herding analyst)* An analyst in group B demands each security to meet the exposures to which their wealth is exposed:

$$[\Omega_S \;\; \Omega_B] = \left[\frac{\sigma(X_t)' \frac{\partial W_t^B}{\partial X_t}}{W_t^B}\right] \times \left[\frac{\sigma(X_t)' \frac{\partial S_t}{\partial X_t}}{S_t} \quad \frac{\sigma(X_t)' \frac{\partial B_t}{\partial X_t}}{B_t}\right]^{-1} \tag{36}$$

Figure 5 exhibits how herding agent forms a portfolio at the equilibrium. Two interesting points can be drawn from figure 5. First, with the moderate level of disagreement, the herding behavior triggers the agent to demand a risky asset more, and a safe asset less. This happens because even an inferior analyst could utilize the public signal by simply herding the superior analyst' final result. Second, the existence of herding, though small, reverse the structure of preference for securities. Specifically, the combination of the huge amount of disagreement and heavy herding results in a drastic change in demand structure as can be seen in the second row in figure 5.

[Insert Figure 5 here]

6. Conclusion

From the basic idea that a dispersion effect and a herding behavior occur in the same data set of analysts and the forecasts such as EPS, we build up the theoretical framework incorporating both the difference of opinion and the degree of herding simultaneously. Specifically, in our model, two types of agents are equipped with asymmetric abilities at utilizing the public signal and the inferior one herds on the other at the final stage of reporting forecasting values. Moreover, we derived the equilibrium economy and analyzed prices, risk exposures, and return volatilities of securities and the wealth allocated to each agent in the economy.

We first show that the joint effect of the herding behavior and the disagreement is huge. As we explored in previous sections, the degree of a herding not only reduced the tendency of several characteristics of securities but also reversed the direction of the impact, especially against the disagreement. Specifically, the concavely shaped volatility of equity against the disagreement become convex when the degree of herding is high.

Secondly, the existence of the herding associated with the high level of a disagreement generated excess volatility compared to what would have without a herding element. This result infers us that the serious increase in volatility can occur when market participants behave similarly although their intrinsic thoughts or opinions are widely dispersed.

Thirdly, we glanced at a bubble mechanism in that price of securities in the economy rose steeply in the existence of a heavy herding. We supplied a possible bridge that links a herding behavior to asset bubbles as noted in Hott (2009). The combinational dynamics of the disagreement and herding would be a great start point for research about how bubble cumulates in the theoretical framework also.

Lastly, we observed the transfer of wealth to an inferior agent who literally free rides on the other. To prevent rational agents from suffering from unreasonable herding behavior, we should develop the model which can automatically keep herding analyst out of the market by incorporating a new component.

To our knowledge, this is the first model dealing with both the degree of the herding and the disagreement. Analysis of heterogeneity of opinions with the existence of herding behavior based on this model showed us that the vast distortion in the characteristics of securities in the market can happen, which makes this research important and interesting.

Still, several issues are ongoing under the debate. First, we assume that the degree of herding is given exogenous, although multiple characteristics of analysts are known to determine the extent how closely they move toward the consensus. It would be great topics of interest to make the herding parameter endogenous because it is important to understand the detailed mechanism and its serial effects of analysts’ incentives to herd from various components in the environment. In addition, there is a room for improvement in the setting of free-herding though it would seem realistic. In the framework of the economy we built, it would be optimal for an inferior analyst to wholly herd, i.e. discard their own result and mimic the other. By constructing loss to the analysts who herd, we could find a way to prevent them from herding or consequently resolve the perversion of the economy. We left it for the future research.

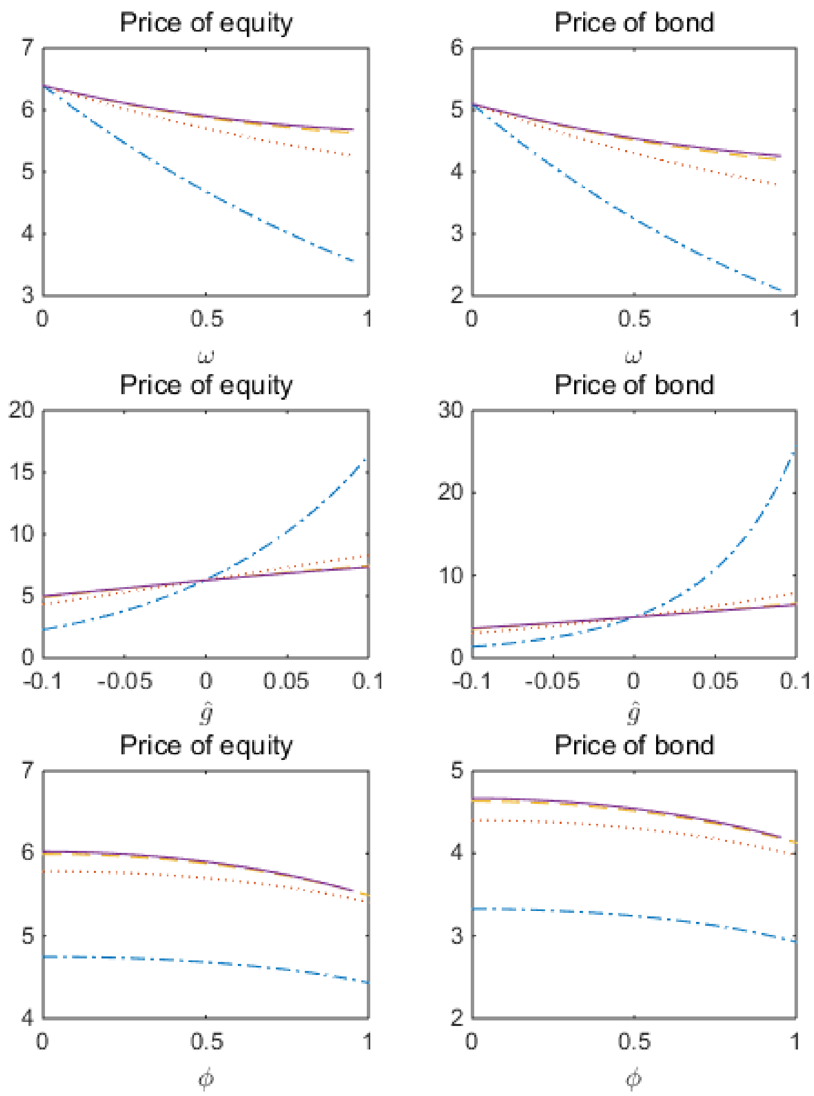

Figure 1. The price of equity and bond. Each graph on the left column plots the price of an equity and each graph on the right column shows the price of a bond at equilibrium. The graphs in the first row has a $\boldsymbol{x}$-axis of the consumption share of agent in group A with the benchmark setting, $\widehat{\boldsymbol{g}}$ = -0.03 and $\boldsymbol{\phi}$ = 0.5. In the second row, $\boldsymbol{x}$-axis represents the degree of the disagreement between two types of agents, $\widehat{\boldsymbol{g}}_{\boldsymbol{t}}$ with the consumption share of agent A and the informativeness parameter are fixed equally 0.5. The graphs in the last row tell us that how vertical variables change upon the correlation between public signal and fundamental process, $\boldsymbol{\phi}$, with $\boldsymbol{\omega}$ = 0.5 and $\widehat{\boldsymbol{g}}$ = -0.03. Each subplot in the figure contains four curves with solid line representing the case in which Group B has own forecast value, no herding, $\boldsymbol{\theta = 0}$, dashed line representing the case in which Group B take the weighted average of their own value and forecast of group A with weight (0.1,0.9), $\boldsymbol{\theta = 0.1}$, dotted line representing the case in which Group B takes the mean of their original forecast and Group A's as a final forecast value to report, $\boldsymbol{\theta = 0.5}$, and the dashed-and-dotted line representing the case in which Group B takes 90% of the value forecasted by the other agent A, $\boldsymbol{\theta = 0.9}$.

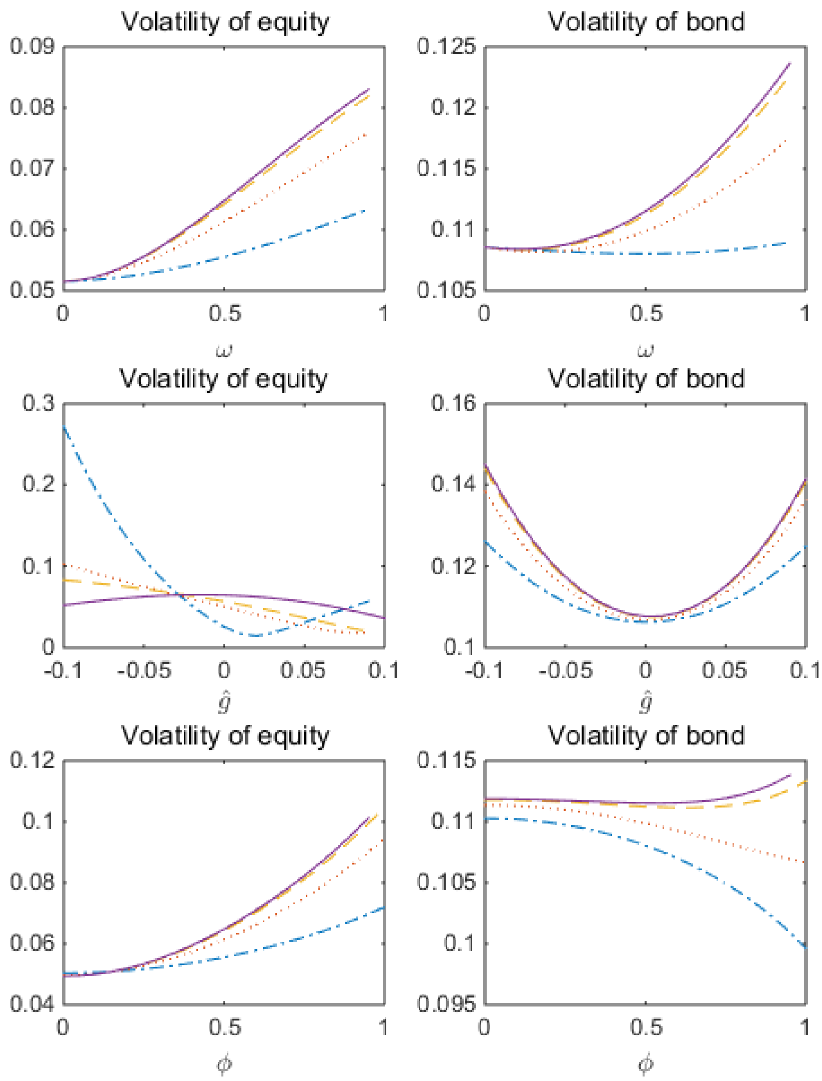

Figure 2. The volatility of equity and bond. Each graph on the left column plots the volatility of an equity and each graph on the right column shows the volatility of a bond at equilibrium. The graphs in the first row has a $\boldsymbol{x}$-axis of the consumption share of agent in group A with the benchmark setting, $\hat{\boldsymbol{g}}$ = -0.03 and $\boldsymbol{\phi}$ = 0.5. In the second row, $\boldsymbol{x}$-axis represents the degree of the disagreement between two types of agents, $\hat{\boldsymbol{g}}_{\boldsymbol{t}}$ with the consumption share of agent A and the informativeness parameter are fixed equally 0.5. The graphs in the last row tell us that how vertical variables change upon the correlation between public signal and fundamental process, $\boldsymbol{\phi}$, with $\boldsymbol{\omega}$ = 0.5 and $\hat{\boldsymbol{g}}$ = -0.03. Each subplot in the figure contains four curves with solid line representing the case in which Group B has own forecast value, no herding, $\boldsymbol{\theta} = \mathbf{0}$, dashed line representing the case in which Group B take the weighted average of their own value and forecast of group A with weight (0.1,0.9), $\boldsymbol{\theta} = \mathbf{0.1}$, dotted line representing the case in which Group B takes the mean of their original forecast and Group A's as a final forecast value to report, $\boldsymbol{\theta} = \mathbf{0.5}$, and the dashed-and-dotted line representing the case in which Group B takes 90% of the value forecasted by the other agent A, $\boldsymbol{\theta} = \mathbf{0.9}$

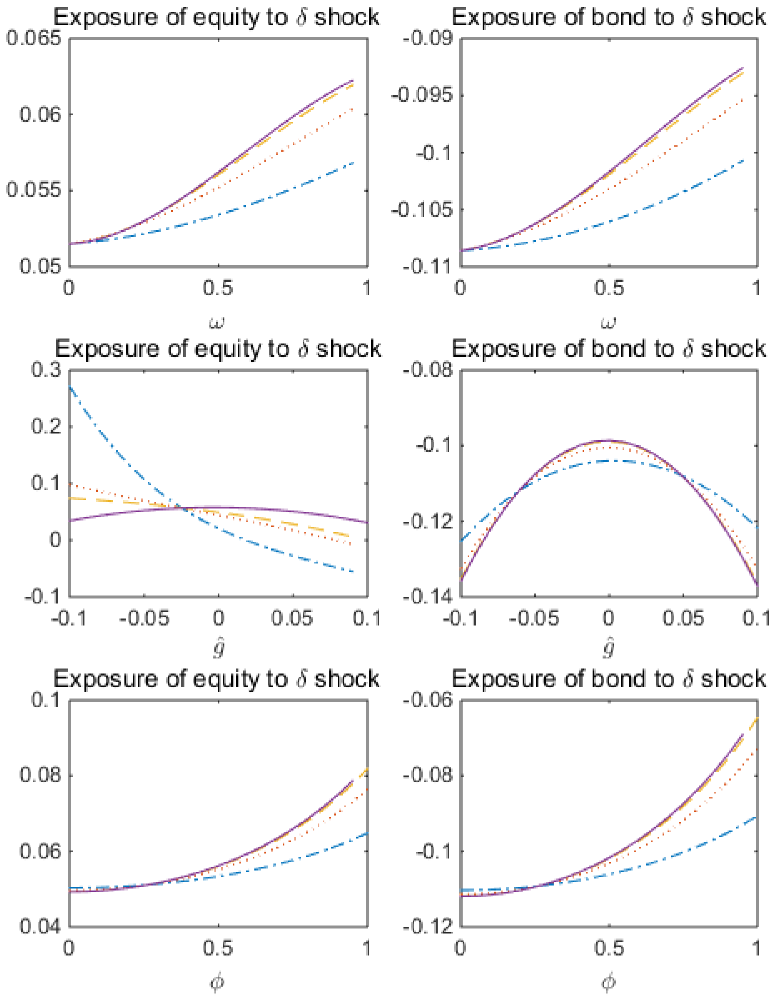

Figure 3. Diffusion of equity and bond. Each graph on the left column plots the exposure of an equity to the fundamental shock, and each graph on the right column shows the sensitivity of a bond to the fundamental shock at equilibrium. The graphs in the first row has a $\boldsymbol{x}$-axis of the consumption share of agent in group A with the benchmark setting, $\hat{\boldsymbol{g}}$ = -0.03 and $\boldsymbol{\phi}$ = 0.5. In the second row, $\boldsymbol{x}$-axis represents the degree of the disagreement between two types of agents, $\hat{\boldsymbol{g}}_{\boldsymbol{t}}$ with the consumption share of agent A and the informativeness parameter are fixed equally 0.5. The graphs in the last row tell us that how vertical variables change upon the correlation between public signal and fundamental process, $\boldsymbol{\phi}$, with $\boldsymbol{\omega}$ = 0.5 and $\hat{\boldsymbol{g}}$ = -0.03. Each subplot in the figure contains four curves with solid line representing the case in which Group B has own forecast value, no herding, $\boldsymbol{\theta = 0}$, dashed line representing the case in which Group B take the weighted average of their own value and forecast of group A with weight (0.1,0.9), $\boldsymbol{\theta = 0.1}$, dotted line representing the case in which Group B takes the mean of their original forecast and Group A's as a final forecast value to report, $\boldsymbol{\theta = 0.5}$, and the dashed-and-dotted line representing the case in which Group B takes 90% of the value forecasted by the other agent A, $\boldsymbol{\theta = 0.9}$

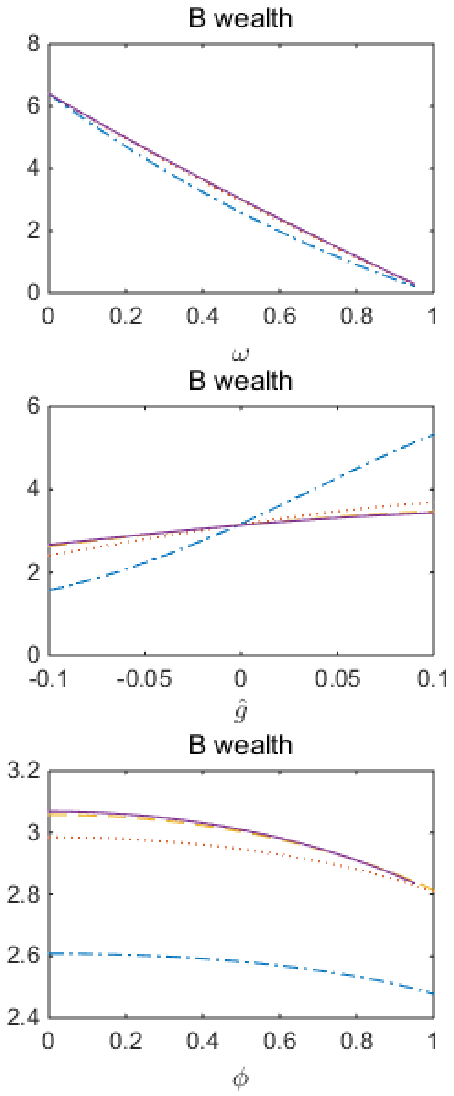

Figure 4. Wealth of agent B The graphs in this figure show the wealth of agent B. The graphs in the first row has a $\boldsymbol{x}$-axis of the consumption share of agent in group A with the benchmark setting, $\hat{\boldsymbol{g}}$ = -0.03 and $\boldsymbol{\phi}$ = 0.5. In the second row, $\boldsymbol{x}$-axis represents the degree of the disagreement between two types of agents, $\hat{\boldsymbol{g}}_{\boldsymbol{t}}$ with the consumption share of agent A and the informativeness parameter are fixed equally 0.5. The graphs in the last row tell us that how vertical variables change upon the correlation between public signal and fundamental process, $\boldsymbol{\phi}$, with $\boldsymbol{\omega}$ = 0.5 and $\hat{\boldsymbol{g}}$ = -0.03. Each subplot in the figure contains four curves with solid line representing the case in which Group B has own forecast value, no herding, $\boldsymbol{\theta = 0}$, dashed line representing the case in which Group B take the weighted average of their own value and forecast of group A with weight (0.1,0.9), $\boldsymbol{\theta = 0.1}$, dotted line representing the case in which Group B takes the mean of their original forecast and Group A's as a final forecast value to report, $\boldsymbol{\theta = 0.5}$, and the dashed-and-dotted line representing the case in which Group B takes 90% of the value forecasted by the other agent A, $\boldsymbol{\theta = 0.9}$.

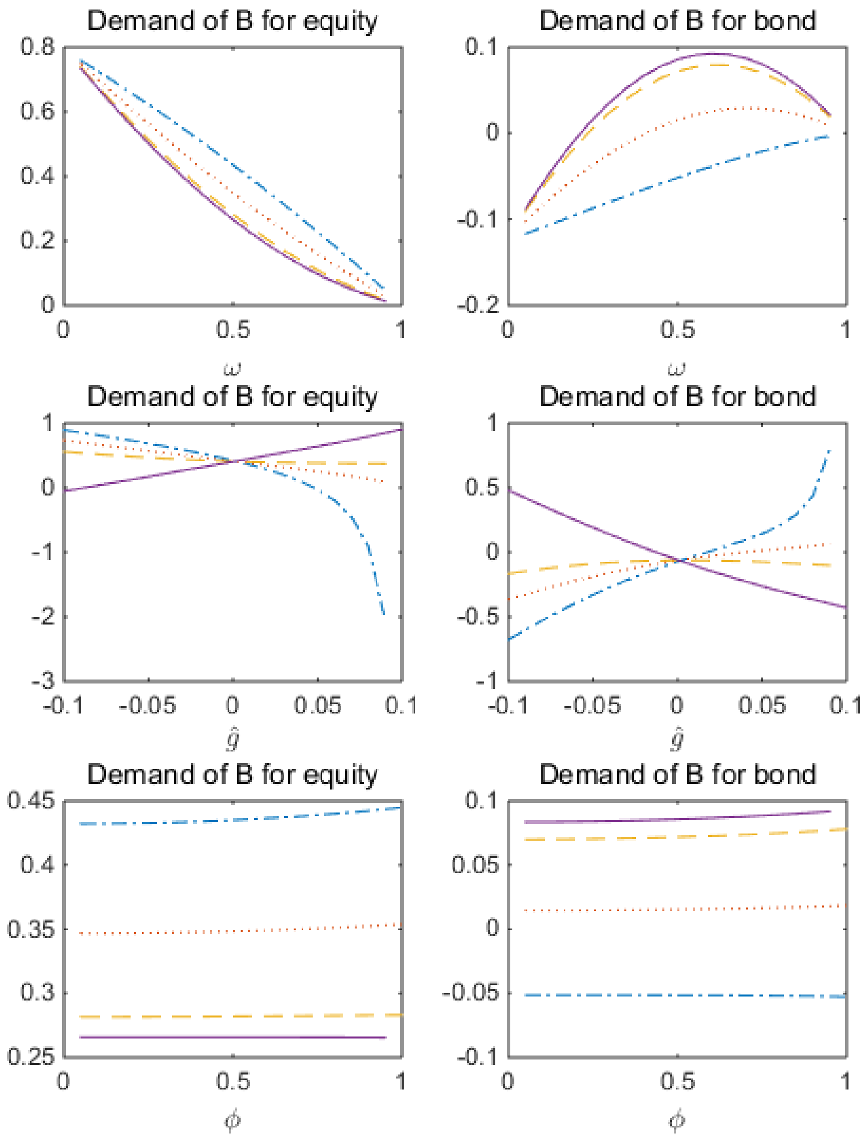

Figure 5. Portfolio allocation of agent B Each graph on the left column plots the demand for an equity by agent B and each graph on the right column shows the demand for a bond by agent B at equilibrium. The graphs in the first row has a $\boldsymbol{x}$-axis of the consumption share of agent in group A with the benchmark setting, $\hat{\boldsymbol{g}}$ = -0.03 and $\boldsymbol{\phi}$ = 0.5. In the second row, $\boldsymbol{x}$-axis represents the degree of the disagreement between two types of agents, $\hat{\boldsymbol{g}}_{\boldsymbol{t}}$ with the consumption share of agent A and the informativeness parameter are fixed equally 0.5. The graphs in the last row tell us that how vertical variables change upon the correlation between public signal and fundamental process, $\boldsymbol{\phi}$, with $\boldsymbol{\omega}$ = 0.5 and $\hat{\boldsymbol{g}}$ = -0.03. Each subplot in the figure contains four curves with solid line representing the case in which Group B has own forecast value, no herding, $\boldsymbol{\theta = 0}$, dashed line representing the case in which Group B take the weighted average of their own value and forecast of group A with weight (0.1,0.9), $\boldsymbol{\theta = 0.1}$, dotted line representing the case in which Group B takes the mean of their original forecast and Group A's as a final forecast value to report, $\boldsymbol{\theta = 0.5}$, and the dashed-and-dotted line representing the case in which Group B takes 90% of the value forecasted by the other agent A, $\boldsymbol{\theta = 0.9}$.

Table 1. Choice of Parameter Values This table lists the parameter values used for all calculation in this paper. These values are very similar to the values estimated by Brennan and Xia (2001).

Name of the variable	Symbol	Value
Parameter for aggregate dividend		
Long-term average growth rate of aggregate dividend	$\bar{f}$	0.015
Volatility of expected growth rate of dividend	σ_f	0.03
Volatility of aggregate dividend	σ_δ	0.13
Mean reversion parameter	ζ	0.2
Parameters for the agents		
Time preference parameter for both agents	ρ	0.1
Relative risk aversion for both agents	$1-\alpha$	3
Benchmark values of the state variables		
The consumption share of agent A	ω	0.5
The level of aggregate dividends	δ	1
The change from B's measure to A's measure	η	1
Group B's belief about expected rate of growth	$\hat{f}_t^B$	$\bar{f}$
The difference in opinions: $\hat{f}_t^{B\prime} - \hat{f}_t^A$	$\widehat{g}'$	-0.03

Appendix

A1. Diffusion vector composition

$$\begin{bmatrix} diffS \\ diffB \end{bmatrix} = \begin{bmatrix} \frac{\partial S}{\partial \delta} & \frac{\partial S}{\partial \hat{f}^B} & \frac{\partial S}{\partial \eta} & \frac{\partial S}{\partial \hat{g}} \\ \frac{\partial B}{\partial \delta} & \frac{\partial B}{\partial \hat{f}^B} & \frac{\partial B}{\partial \eta} & \frac{\partial B}{\partial \hat{g}} \end{bmatrix} \begin{bmatrix} \delta\sigma_\delta & 0 \\ \frac{\gamma^B}{\sigma_\delta} & 0 \\ -\eta \frac{\hat{g}}{\sigma_\delta}(1-\theta) & 0 \\ \frac{\gamma^B - \gamma^A}{\sigma_\delta}(1-\theta) & -\phi\sigma_f(1-\theta) \end{bmatrix}$$

A2. The component in the integral for the price of an equity and a perpetual bond

$$\mathbb{E}^B\left[\frac{\xi_u^B}{\xi_t^B}\right] = e^{-\rho(u-t)} H_f\left(\hat{f}^B, t, u, \alpha - 1\right)$$
$$\times [1-\omega(\eta)]^{1-\alpha} \sum_{j=0}^{1-\alpha} \frac{(1-\alpha)!}{j!\,(1-\alpha-j)!} \left[\frac{\omega(\eta)}{1-\omega(\eta)}\right]^j H_g\left(\hat{g}, t, u, \alpha - 1, \frac{j}{1-\alpha}\right)$$

$$\mathbb{E}^B\left[\frac{\xi_u^B}{\xi_t^B}\frac{\delta_u}{\delta_t}\right] = e^{-\rho(u-t)} H_f\left(\hat{f}^B, t, u, \alpha\right)$$
$$\times [1-\omega(\eta)]^{1-\alpha} \sum_{j=0}^{1-\alpha} \frac{(1-\alpha)!}{j!\,(1-\alpha-j)!} \left[\frac{\omega(\eta)}{1-\omega(\eta)}\right]^j H_g\left(\hat{g}, t, u, \alpha, \frac{j}{1-\alpha}\right)$$

$$H_f\left(\hat{f}^B, u, t, \varepsilon\right) = exp\{\varepsilon[\bar{f}(u-t) + \frac{1}{\zeta}\left(\hat{f}^B - \bar{f}\right)\left[1 - e^{-\zeta(u-t)}\right] + \frac{1}{2}\varepsilon(\varepsilon-1)\sigma_\delta^2(u-t)$$
$$+ \frac{\varepsilon^2\gamma^B}{2\zeta^2}\left[1 - e^{-\zeta(u-t)}\right]^2 + \frac{\varepsilon^2\sigma_f^2}{4\zeta^3}\left[2\zeta(u-t) - 3 + 4e^{-\zeta(u-t)} - e^{-2\zeta(u-t)}\right]\}$$

$$H_g(\hat{g}, u, t, \varepsilon, \chi) = exp\{A_1(\chi, u-t) + \varepsilon^2 A_2(\chi, u-t) + \varepsilon\hat{g}B(\chi, u-t) + \hat{g}^2 C(\chi, u-t)$$

A3. The wealth of agent B

$$W_t^B(t) = \int_t^\infty \mathbb{E}^B\left[\frac{\xi_u^B}{\xi_t^B} c_u^B\right] du$$

$$= \delta \int_t^\infty e^{-\rho(u-t)} H_f\left(\hat{f}^B, t, u, \alpha\right) \times [1-\omega(\eta)]^{1-\alpha}$$

$$\times \sum_{j=0}^{-\alpha} \frac{(-\alpha)!}{j!\,(-\alpha-j)!}\left[\frac{\omega(\eta)}{1-\omega(\eta)}\right]^j H_g\left(\hat{g}, t, u, \alpha, \frac{j}{1-\alpha}\right) du$$

A4. Equation for Demand of agent B for exposure in detail

$$\begin{bmatrix} \frac{\partial W^B}{\partial \delta} & \frac{\partial W^B}{\partial \hat{f}^B} & \frac{\partial W^B}{\partial \eta} & \frac{\partial W^B}{\partial \hat{g}} \end{bmatrix} \begin{bmatrix} \delta\sigma_\delta & 0 \\ \frac{\gamma^B}{\sigma_\delta} & 0 \\ -\eta\frac{\hat{g}}{\sigma_\delta}(1-\theta) & 0 \\ \frac{\gamma^B-\gamma^A}{\sigma_\delta}(1-\theta) & -\phi\sigma_f(1-\theta) \end{bmatrix}$$

$$= \Omega^T \begin{bmatrix} \frac{\partial S}{\partial \delta} & \frac{\partial S}{\partial \hat{f}^B} & \frac{\partial S}{\partial \eta} & \frac{\partial S}{\partial \hat{g}} \\ \frac{\partial B}{\partial \delta} & \frac{\partial B}{\partial \hat{f}^B} & \frac{\partial B}{\partial \eta} & \frac{\partial B}{\partial \hat{g}} \end{bmatrix} \begin{bmatrix} \delta\sigma_\delta & 0 \\ \frac{\gamma^B}{\sigma_\delta} & 0 \\ -\eta\frac{\hat{g}}{\sigma_\delta}(1-\theta) & 0 \\ \frac{\gamma^B-\gamma^A}{\sigma_\delta}(1-\theta) & -\phi\sigma_f(1-\theta) \end{bmatrix}$$

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