Risk Characteristic of Fat Tails in Return Distributions: A Case of the Korean Stock Market

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KEY WORDS: Stock return distributions, Characteristic of fat tails, Principal component analysis, Portfolio diversification.

***JEL* Classification:** C10, G10, G11

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ABSTRACT

This study uses statistical methods to empirically investigate fat tails in return distributions, focusing on identifying the economic implications of the risk property included in these tails. Through principal component analysis (**PCA**), the risk property in the fat tails of return distributions is found to have the economic meaning of eigenvalues with value greater than 1, as well as systematic risk that cannot be removed through portfolio diversification. These results suggest that the fat tails of return distributions have the properties of common factors that may explain changes in stock returns. We observe that the fatness of the tails in portfolio return distributions shows an asymmetric relationship of the common factors with the tails in the distributions. Specifically, the negative tail in the portfolio return distribution has a much closer relationship with the properties of the common factors than does the positive tail. Our empirical evidence may complement existing studies related to tail risk that is used as a common factor in pricing models.

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1. INTRODUCTION

The extensive losses arising from financial market crashes such as the 1997 Asian financial crisis, the 2008 U.S. credit risk crisis and the 2011 European debt crisis have raised basic questions regarding the effectiveness of risk management based on currently available models. The successful expansion of quantitative models into the area of risk management is dependent on how effectively the models reflect the various properties observed in a financial time series. The returns located in the tails of a return distribution stem from large-scale fluctuations. This means that the risk property included in the tails of return distributions needs to be identified to improve traditional risk management. The variables commonly used to introduce the characteristic of fat tails into a model are skewness and kurtosis. Although these measures might signal deviation from normality in a return distribution, they cannot describe the risk property associated with the fat tails of the distribution. In particular, these measures are unable to explain the downside risk (e.g., Ang et al. 2006) of large losses. On the other hand, current pricing models often struggle to explain the changes of returns using the systematic risks of common factors. Determining whether to include the risk property in the fat tails of return distributions in the models as an added explanatory variable will require preferentially defining the economic implications of the risk property in these tails—that is, whether to have economic meanings of common factors that can explain the changes in returns, and whether to ensure the property of systematic risk that does not be removed through portfolio diversification. Taleb (2007) has proposed the Black Swan Theory to describe an event that is extremely unlikely but has strong influence, i.e., an unexpected event in the past that deviates radically from the expected norm. He argues that such an event is represented in the tails of a return distribution and is the cause of important changes in the market regime. Bhansali (2008) mentioned that fat tails in return distributions have the property of systematic risk based on the strong relationship between market crashes and the liquidity problem in macroeconomic analysis. That is, the risk property in the tails of a return distribution is closely related to macroeconomic exposure. Kelly and Jiang (2014) show that tail risk has a significant impact on aggregate market returns and individual stock returns in both time series and cross sectional cases. However, no consensus has yet been reached about whether the risk property in the fat tails of return distributions systematically affects changes of returns as a common factor. This necessitates research that defines in advance the economic implications of the risk property included in the fat tails of return distributions in order to produce pricing models that properly reflect the characteristic of fat tails.

This study uses statistical methods to empirically investigate the risk characteristic of fat tails in return distributions in order to provide evidence supporting the economic implications of the risk property included in these fat tails. The fatness of the tails is measured by statistical probability, where the number of return values belonging to each of the two end-areas—that is, the areas outside the 99% central section of the distribution—is divided by the total number of return data points in the distribution. We devised empirical designs based on two traditional methods: principal component analysis (PCA) in multivariate statistics as applied by King (1966) and Ross (1976) and the testing method used to test the portfolio diversification effect proposed by Evans and Archer (1968). As reported by Chan et al. (1986), Brown (1989), Plerou et al. (2002), and Eom et al. (2009), an eigenvalue extracted by PCA having a value greater than 1 has important economic meaning insofar as being able to explain changes of return in a pricing model, such as market, industry and macroeconomic factors. We test whether the magnitude of eigenvalues is positively correlated with the fatness of the tails in the distribution of an eigenvalue time series. If so, this is evidence suggesting that the risk property included in the fat tails of a return distribution may have the same economic meaning as eigenvalues with a value greater than 1. Using the test of portfolio diversification described in Elton and Gruber (1977) and in Staman (1987), it is possible to visually determine systematic and unsystematic risk by observing the change pattern of portfolio risk as the number of stocks in a portfolio increases. The tails of portfolio return distributions contain return data reflecting large-scale price fluctuations that deviate substantially from the average value. Consequently, changes in the fatness of the tails in a portfolio return distribution are very closely related to portfolio risk. A continuously decreasing pattern of portfolio risk as the number of stocks in the portfolio is increased indicates unsystematic risk; a pattern of convergence to a certain level of portfolio risk indicates systematic risk. Thus, identifying the type of change pattern in the fatness of the tails in a portfolio return distribution may serve to determine whether the risk property in the fat tails of a return distribution is systematic or unsystematic risk.

Results are summarized briefly as follows: The relationship between the magnitude of the eigenvalue and the fatness of the tails in the distribution of the eigenvalue time series is positive. This means that the eigenvalues with higher value tend to have much fatter tails in the distribution of the eigenvalue time series. Furthermore, using the method for testing the portfolio diversification effect, we discovered that the fatness of the tails in portfolio return distributions tends to converge to a certain level. This appears to support the proposition that the risk property included in the fat tails of return distributions involves systematic risk that cannot be removed by portfolio diversification. Another interesting finding is that the negative tail in the portfolio distributions is much fatter than the positive tail as the number of stocks in the portfolio is increased. This result is due to the difference in the strength of the relationship with the common factors in each tail of the portfolio return distribution. Specifically, the negative tail of the distribution has a much closer relationship with the properties of the common factors than does the positive tail. This suggests a potential defect in traditional portfolio investment by highlighting two major limitations—missing the opportunity for large profits in the positive tail and taking large losses in the negative tail. The economic implications of the risk property included in the fat tails of return distributions uncovered in this study complement existing studies (e.g., Kelly and Jiang 2014) of tail risk used as a common factor in pricing models. Therefore, we expect that our findings will provide useful new insights.

The remainder of the paper is organised as follows: Chapter 2 describes the data, the observation periods and the testing methods used in the study. Chapter 3 presents the results regarding the economic implications of the risk property included in the tails of return distributions and discusses the main findings. Chapter 4 provides a summary and reports conclusions.

2. EMPIRICAL DESIGN

2.1. Data and Periods

In this study, we used the returns of the market and individual stocks traded on the Korean stock market from July 2006 to June 2015. We identified three stock groups classified by market capitalization (= price×number of outstanding shares). Since the study of Banz (1981), firm size has been one of the key factors used to explain changes in stock returns. Consequently, we divided the stocks into three groups: an all-stocks group, a large-stocks group (stocks of firms whose market capitalization ranked in the top 40% among all firms), and a small stocks-group (stocks of firms whose market capitalization ranked in the bottom 40% among all firms). The classification of stock groups is based on the average value of monthly firm market capitalization within a specific period.

2.2. Methods

This section describes the methods used to identify the economic implications of the risk property included in the fat tails of return distributions. The empirical distribution observed from the return data has a more peaked central section and much fatter tails than the normal distribution, making it a leptokurtic distribution. The tails of a return distribution reflect large losses and large profits that deviate substantially from the average value. We standardize the stock return data for all periods by subtracting the average value and dividing by the standard deviation. By using the standardized return data, the tails can be defined as the area at each end of the frequency distribution beyond the 99% central section. We use statistical probability as a measure to assess the fatness of the tails. This statistical probability is simply the relative frequency (f_N/f_T) ratio calculated as the number of values (f_N) included in the tails of the distribution divided by the total number of values (f_T). The bin size of the frequency distribution is determined according to Scott (1992). A statistical probability with a value greater than 0.5% is taken as evidence of a fat tail. To identify the economic implications of the risk property associated with the fat tails of a return distribution, we applied methods based on traditional methods of PCA and test of the portfolio diversification effect. PCA is used to explore statistically the common factors based on eigenvalues having a value greater than 1; the portfolio diversification method presents a theoretical basis for pricing models that are strongly reliant on systematic risk.

The first of the applied methods is based on the PCA approach taken by King (1966) and Ross (1976), who found PCA to be a useful tool to extract potential common factors from the return data of all stocks. (A common factor is a factor that commonly affects changes in returns.) The number of eigenvalues (λ_k, $\mathrm{k} = 1,2, \ldots \mathrm{N}$) extracted by PCA using the returns of the all-stocks group is equal to N, the number of stocks. As reported by Chan et al. (1986), Brown (1989), Plerou et al. (2002), and Eom et al. (2009), the eigenvalues with a value greater than 1 based on the Kaiser (1960) criterion have economic meaning as market, industry, and macroeconomic factors. We empirically examine the relationship between the magnitude of the eigenvalues and the fatness of the tails in the distribution of the eigenvalues time series. If the magnitude of the eigenvalues is positively correlated with the fatness of the tails in the distribution of the eigenvalue time series, this is evidence suggesting that the risk property in the fat tails of return distributions has economic meaning similar to that of eigenvalues with a value greater than 1. The testing procedure is as follows: We extract all eigenvalues using PCA, and produce the time series ($R^{\lambda}_{k,t}$) of each eigenvalue. The time series of each eigenvalue is created by multiplying eigenvectors (v_k) assigned to the k-th stock to the stock return ($R_{j=k,t}$) in time t; that is, $R^{\lambda}_{k,t} = \sum_{j=1}^{N} v_k R_{k,t}$, $t = 1,2,\dots,T$. We standardize the eigenvalue time series by subtracting the average value and dividing by the standard deviation. Using standardized data, we calculate the statistical probability (f_N/f_T) as a measurement of the fatness of the tails in the distribution. We then test the relationship between the magnitude of the eigenvalue and the statistical probability in the tails of the eigenvalue distribution using correlation analysis.

The second method applied in the study is based on a test of the portfolio diversification effect. The effect of portfolio diversification, according to Evans and Archer (1968), is defined as the reduction in portfolio risk as the number of stocks in a portfolio increases. The risk of a well-diversified portfolio is determined by its systematic risk. The tails of a portfolio return distribution are the product of large-scale price fluctuations and are composed of values that deviate substantially from the average value in the distribution. As a result, the fatness of the tails in a portfolio return distribution is very closely related to portfolio risk. We investigate the changing pattern of the fatness in the tails of portfolio return distributions using a testing method devised to test the portfolio diversification effect. The logic is essentially this: If the fatness of the tails in the portfolio return distribution clearly converges to a certain level as the number of stocks in the portfolio increases, then this is evidence suggesting that the risk property in the fat tails of portfolio return distributions is a systematic risk that cannot be removed through portfolio diversification. On the other hand, if the fatness in the tails of portfolio return distribution disappears, this is evidence suggesting that the risk property in the fat tails of a portfolio return distribution is an unsystematic risk that can be reduced by portfolio diversification. The specific testing procedure is as follows: We set the number of stocks in a portfolio using the range from at least two to a maximum of 50 based on the previous studies of Evans and Archer (1968), Elton and Gruber (1977) and Staman (1987). For each stock, we calculate portfolio returns in each of 100 cases of the portfolio constructed by randomly selecting the stocks using sampling without replacement. All portfolio returns are standardized by subtracting the average value and dividing by the standard deviation. Using the standardized portfolio returns, we calculate the statistical probability (f_N/f_T) as a measure of the fatness of the tails in the portfolio return distribution; the average value of the 100 statistical probabilities is then calculated for each portfolio. We then investigate the pattern of change in the average value of the statistical probability resulting from increasing the number of stocks in the portfolio.

3. RESULTS

3.1. Implication of fat tails in risk management

In this section, we present our results regarding the economic implications of the risk property included in the fat tails of a return distribution. In the field of finance, to determine whether or not to include a new explanatory variable in a pricing model, it is necessary to assess whether the new variable has an economic meaning that affects changes in returns. We also need to determine whether the risk property of the new variable is systematic or unsystematic risk. We use two traditional methods to accomplish this: PCA and a recognised test of the portfolio diversification effect. The results from each method are presented in **Figures 1** and **2**.

We first present results uncovering the risk property in the fat tails of return distributions using PCA. Determining whether or not the eigenvalues extracted from PCA have an economic meaning is strongly connected to the magnitude of the eigenvalues. It is for this reason that we investigate the relationship between the magnitude of the eigenvalues and the fatness of the tails in the distribution of the eigenvalue time series. A positive relationship indicates that eigenvalues with a higher value tend to have fatter tails in the eigenvalue distribution, which, statistically, is evidence suggesting that the risk property in the fat tails of return distributions has an economic meaning equivalent to eigenvalues with a value greater than 1. The results are shown in **Figure 1**. The figure shows N=524 stocks belonging to the all-stocks group over the period from July 2006 to June 2015. Thus, the number of eigenvalues extracted from PCA is K=524, and the time series data with the property of each of the eigenvalues has the same length of the sub-period from July 2006 to June 2015. The statistical probability is used to measure the fatness of the tails in the distribution of the eigenvalue time series. The X-axis indicates the magnitude of the eigenvalues and the Y-axis denotes the statistical probability for the tails of the eigenvalue distributions. **Figure 1(a)** and **1(b)** present results for the negative tail; **Figure 1(c)** and **1(d)** do the same for the positive tail. Importantly, we control for the effect of the largest eigenvalue, which can substantially affect results given that the difference between the largest eigenvalue and the second largest eigenvalue is very high, as pointed out in Brown (1989) and Eom et al. (2009). **Figure 1(a)** and **1(c)** show the results of the relationship including the largest eigenvalue using a double-log plot, while **Figure 1(b)** and **1(d)** show the relationship excluding the largest eigenvalue using a scatter plot.

[Insert FIGURE 1 about here.]

According to the results, the magnitude of the eigenvalues has a clearly positive relationship with the fatness of the tails in the distribution of the eigenvalue time series. In **Figure 1(a)** and **1(b)**, the strength of the relationship for the negative tail is 32.17% and 43.01%, respectively, and the strength of relationship for the positive tail in **Figure 1(c)** and **1(d)** is 16.31% and 57.64%, respectively. The relationship for the case in which the largest eigenvalue is excluded has a higher value than for the case in which the largest eigenvalue is included. The positive relationship is obvious in the figures. The results for the large-stocks group ((a)&(b), 37.47% & 34.34%; (c)&(d), 19.14% & 66.43%) and the small-stocks group (41.87% & 22.99%; 14.04% & 49.02%) are not reported in this paper, but they also show the same positive relationship. These results suggest that the risk property included in the fat tails of return distributions has, statistically, the same or similar economic meaning as eigenvalues with a value greater than 1.

We next present results from our use of a portfolio diversification test to determine whether the risk property in the fat tails of return distributions is systematic or unsystematic risk. Our approach is based on the principle that a well-diversified portfolio can eliminate unsystematic risk. The returns located in the tails of return distributions reflect large-scale price fluctuations. As the number of stocks in a portfolio increases, observing the changes in the fatness of the tails in the portfolio return distribution is very closely related to observing the changes in portfolio risk. Therefore, we empirically investigate the changes in the fatness of the tails in portfolio return distributions resulting from an increase in the number of stocks in the portfolio. A pattern converging to a particular level indicates that the fat tails of the portfolio return distribution have the property of systematic risk. The results are shown in **Figure 2**. The figure presents results using all the return data of the three stock groups over the period from July 2006 to June 2015. The measure of the fatness of the tails in the portfolio return distribution is the statistical probability. The X-axis indicates the number of stocks in a portfolio within the range from 2 to 50; the Y-axis shows the average values of all statistical probabilities calculated from 100 iterated simulations for each portfolio constructed by randomly selecting the stocks using sampling without replacement.

[Insert FIGURE 2 about here.]

As shown in **Figure 2**, the statistical probability of the fatness of the tails in the portfolio return distribution tends to converge to a certain level as the number of stocks in the portfolio increases, regardless of stock group. This means that the risk property included in the tails of the portfolio return distribution is not completely removed through portfolio diversification, indicating the property of systematic risk. Notably, the changing patterns in the fatness of the two tails in the portfolio return distribution show opposite behaviour; that is, the changes in the negative and positive tails move in opposite directions. As the number of stocks in the portfolio increases, the statistical probability in the positive tail shows a decreasing pattern converging to a certain level, while the statistical probability in the negative tail has an increasing pattern converging to a certain level. The decreasing pattern in the positive tail means that traditional portfolio diversification misses the opportunity for large profits from frequent large-scale price changes in the financial market, whereas the increasing pattern in the negative tail means that traditional portfolio diversification fails to avoid the possibility of large losses from large-scale price fluctuations in market crashes. This finding thus suggests that traditional portfolio diversification suffers from a two-fold limitation: not effectively controlling for the possibility of large losses or exploiting the opportunity for large profits.

3.2. Robustness and Discussion

This study empirically verified that the fat tails of a return distribution have the same economic meaning as eigenvalues having a value higher than 1, statistically, and that they have the property of systematic risk that cannot be removed through portfolio diversification. The evidence reveals that the property included in the fat tails of return distributions has a close relationship with the properties of common factors. We also conducted an additional test to determine the reliability of our findings. The testing hypothesis is as follows: If the property of the fat tails in return distributions is closely related to the properties of the common factors, the results are significantly dependent on whether or not to include the properties of the common factor in the return data. Based on the empirical design established for the research goal, we devised a method for testing the hypothesis: The number of eigenvalues having the properties of the common factors is identified from the return data, and then, through the number of common factors, both the return data having only properties of the common factors in the original return data and the return data removing only properties of the common factors in the original return data are separately generated. The same testing procedure as that described in **Figure 2** was performed using the two types of generated return data. The number of eigenvalues having the properties of the common factors was identified using random matrix theory (RMT, Mehta (1995)), and the two types of return data, with and without the properties of the common factors from the original return data, were generated by singular vector decomposition (SVD, Leon (2002)). All three methods (PCA used for the second research goal and RMT and SVD for the additional test) share the use of eigenvalues to control for the various properties included in the return data. Each method also has a unique comparative advantage. RMT can mathematically define the eigenvalues that deviate from the range of eigenvalues having random properties in the distribution of eigenvalues estimated from the return data. In other words, eigenvalues that have a higher value than an eigenvalue having the maximum value among the random eigenvalues are well known to have economic meaning as common factors. SVD can generate the new return data that have only the properties of the eigenvalues included within the pre-specified range from the original return data. That is, this method can generate the return data having only the properties of the eigenvalues identified as common factors through RMT, It can also generate return data removing only the properties of the eigenvalues having the properties of the common factors. The specific details of each method are not presented here due to space considerations. The main testing procedures for the testing hypothesis are described briefly as follows: We check the number of common factors through RMT, and then generate the two types of return data, with and without the property of common factor, through SVD based on the RMT results. Using each of the two types of generated return data, we perform the same testing process as reported in **Figure 2** for the portfolio diversification effect. The results are presented in **Figure 3**. The figure uses the return data of the all-stocks group in the period from July 2006 to June 2015 for the purpose of comparing with **Figure 2**. **Figure 3(a)** shows the result for the return data having only the properties of the common factors; **Figure 3(b)** shows the results for the return data removing only the properties of the common factors. The X-axis indicates the number of stocks comprising the portfolio; the Y-axis shows the average values of the statistical probabilities as the measurement quantifying the fatness of the tails in the return distribution.

[Insert Figure 3 about here]

From **Figure 3**, we empirically verify evidence to support the testing hypothesis. In other words, the properties included in the tails of the return distributions are closely related to the properties of the common factors. In particular, the strength of the relationship with the properties of the common factors is much higher in the negative tail than in the positive tail of the portfolio return distributions. The result shown in **Figure 3(a)** is very similar to the finding that is confirmed in **Figure 2**. With the increasing number of stocks in a portfolio, the statistical probability for the tails of the portfolio return distributions shows a constant level without a specific change. This is clearly the pattern of systematic risk that is not removed through portfolio diversification, due to the fact that we used return data having only the properties of the common factors. In addition, as shown in **Figure 2**, the statistical probability for the negative tail of the distribution has a much larger value than the positive tail. On the other hand, the result of **Figure 3(b)** is clearly different from **Figure 2**. As the number of stocks in the portfolio increases, the statistical probability for the tails of the portfolio return distribution has a decreasing pattern and approaches a certain level. The pattern of the statistical probability for the positive tail in the portfolio return distribution does not differ from **Figure 2**. However, the pattern of statistical probability for the negative tail contrasts sharply with the results in **Figure 3(a)** as well as **Figure 2**. In other words, the statistical probability in the negative tail of the portfolio return distribution has a much smaller value than that in the positive tail. This result is strongly dependent on using the return data that do not have only the properties of the common factors. Consequently, **Figure 3** is evidence supporting the hypothesis that the properties included in the tails of return distributions are closely related to the properties of the common factors, i.e., evidence of robustness for the results from **Figure 2**. Moreover, the properties of the common factors have a closer relationship with the negative tail in return distributions, as compared to the positive tail. That is the asymmetric relationship of the common factor in the tails of the return distributions.

These results may also provide evidence to explain the potential defect of traditional portfolio investment. Under the risk-return relationship, a portfolio constructed by including multiple stocks can be expected to produce a lower return in exchange for avoiding higher risk. Portfolio investment always achieves a profit lower than the profit realized by some of the individual stocks in the portfolio. The fatness of the positive tail in the portfolio return distribution shows a decreasing pattern as the number of stocks in the portfolio increases. The positive tail in the portfolio return distribution might have a closer relationship to the properties of individual stocks than with the properties of the common factors. On the other hand, during periods of large losses in the stock market, such as in a market crash, most of the stocks in a portfolio will tend to show a synchronized pattern with the decreasing trend of the market. Since most of the stocks in a portfolio simultaneously experience large losses in a declining market, it is difficult to successfully avoid this large loss through traditional portfolio investment. Thus, the negative tail in the portfolio return distribution might have a substantially closer relationship with the properties of common factors. Therefore, the fatness in the negative tail in the portfolio return distribution shows a constant pattern of having a higher value than the positive tail. Consequently, these results might reveal the potential defect of traditional portfolio investment in that it misses potential opportunities to avoid large losses or, at the other extreme, to achieve large profits. This suggests that investors may suffer large losses in a market crash despite having a well-diversified portfolio. Hence, we expect future research to systematically analyse the relationship between the fat tails in the return distribution and the properties of the common factors with the goal of removing or reducing the impact of this potential defect in portfolio investment.

4. CONCLUSIONS

This study investigated the presence and properties of fat tails in return distributions in order to identify the economic implications of the risk property included in these fat tails. Data from the Korean stock market over the period from July 2006 to June 2015 were used. Results can be summarised as follows: The risk property of the fat tails in return distributions has an economic meaning equivalent to eigenvalues with a value greater than 1, statistically. It also has the property of systematic risk—risk that cannot be removed by portfolio diversification. In other words, the risk property associated with the fat tails of return distributions has the economic implications of common factors that may explain changes in returns in pricing models. Interestingly, these common factor properties have a closer relationship to the negative tail in the distribution than to the positive tail, meaning there is an asymmetric relationship of the common factors with the tails in the return distribution. This finding suggests a potential defect in traditional portfolio investment whereby opportunities to avoid large losses or to earn large profits are missed. Our findings are robust regardless of stock group (large or small). Accordingly, the economic implications of the risk property uncovered in the fat tails of return distributions may complement existing studies that use tail risk as a common factor in pricing models. Hence, we expect these findings to provide new insight into improving portfolio investment.

Acknowledgement

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References

Ang, A., J. Chen, and Y. Xing. "Downside Risk." *Review of Financial Studies* 19 (4) (2006): 1191-1239.

Banz, R. W. "The Relationship between Return and Market Value of Common Stocks." *Journal of Financial Economics* 9 (1981): 3-18.

Bhansali, V. "Tail Risk Management." *Journal of Portfolio Management* 34 (2008): 68-75.

Brown S. T. "The Number of Factors in Security Returns." *Journal of Finance* 44 (5) (1989): 1247-1262.

Chen, N., R. Roll, and S. A. Ross. "Economic Factors and the Stock Market." *Journal of Business* 59 (3) (1986): 383-403.

Elton, E. J., and M. J. Gruber. “Risk Reduction and Portfolio Size: An Analytical Solution.” *Journal of Business* 50 (1977): 145-437.

Eom, C., W. Jung, T. Kaizoji, and S. Kim. “Effect of Changing Data Size on Eigenvalues in the Korean and Japanese Stock Markets.” *Physica A: Statistical Mechanics and its Applications* 388 (22) (2009): 4780-4786.

Evans, J. L., and S. H. Archer. “Diversification and the Reduction of Dispersion: An Empirical Analysis.” *Journal of Finance* 23 (5) (1968): 761-767.

Kaiser, H. F. “The Application of Electronic Computers to Factor Analysis.” *Educational and Psychological Measurement* 20 (1960): 141-151.

Kelly, B., and H. Jing. “Trail Risk and Asset Prices.” *Review of Financial Studies* 27 (10) (2014): 2841-2871.

King, B. “Market and Industry Factors in Stock Price Behavior.” *Journal of Business* 39 (1) (1966): 139-190.

Leon, S. J. *Linear Algebra with Application.* 6th ed., Prentice Hall, 2002.

Mehta M. *Random Matrices*. Academic Press, New York, 1995.

Plerou V., P. Gopikrishnan, B. Rosenow, L. A. N. Amaral, T. Guhr, and H. E. Stanley. “Random Matrix Appoach to Cross Correlations in Financial Data.” *Physical Review E*, 65 (2002): 066126.

Ross, S. “The Arbitrage Theory of Capital Asset Pricing.” *Journal of Economic Theory* 13 (3) (1976): 341-360.

Scott, D. W., 1992. *Multivariate Density Estimation: Theory, Practice, and Visualization*, New York: John Wiley.

Statman, M. “How Many Stocks Make a Diversified Portfolio?” *Journal of Financial and Quantitative Analysis* 22 (3) (1987): 353-363.

Taleb, N. N. *The Black Swan: The Impact of the Highly Improbable*, Random House (NY), 2007.

Figure 1. Economic implications using eigenvalues through the PCA method

The figure shows the results regarding the relationship between the magnitude of eigenvalue and the fatness of the tails in the distribution of the eigenvalue time series for the all-stocks group (524 stocks over the period from July 2006 to June 2015). The X-axis indicates the magnitude of the eigenvalues; the Y-axis denotes the statistical probability in the tails of the eigenvalue distribution. Figure 1(a) and 1(b) show the negative tail; Figure 1(c) and 1(d) show the positive tail. In addition, to control for the excessive effect of the largest eigenvalue (☆) on the results, Figure 1(a) and 1(c) show the results for the case in which the largest eigenvalue is included, using a double-log plot; Figure 1(b) and 1(d) show the results for the case in which the largest eigenvalue is excluded, using a scatter plot.

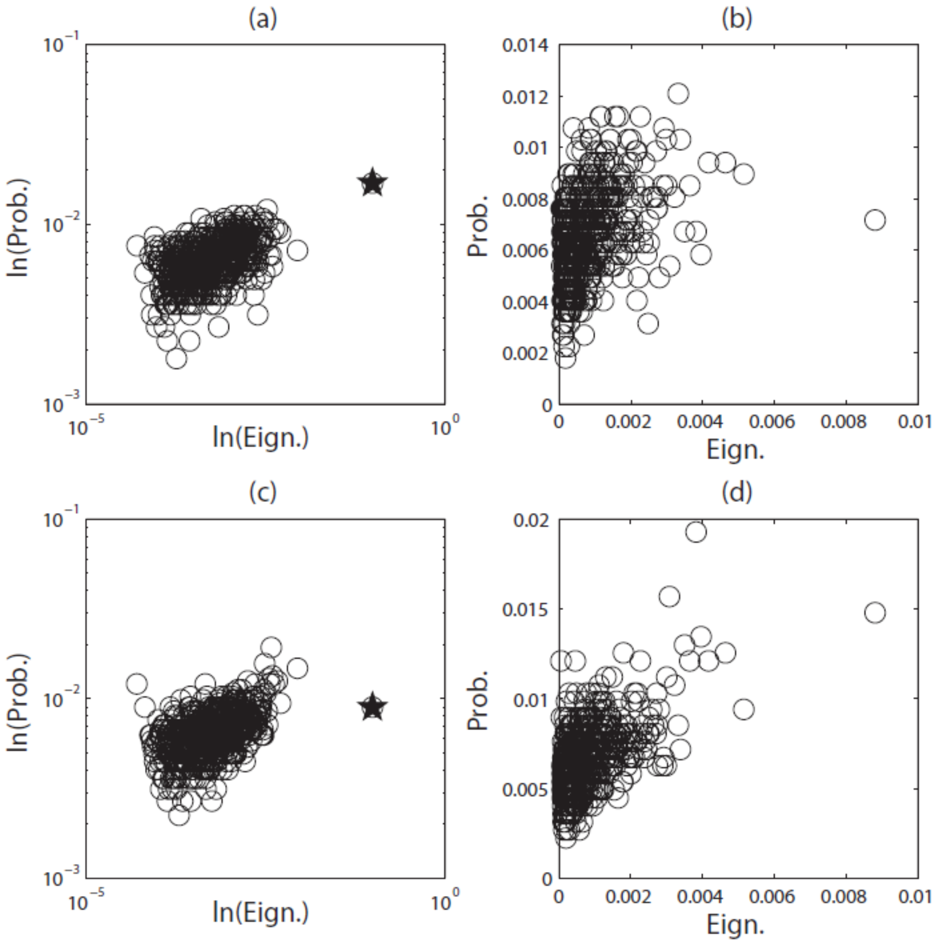

Figure 2. Economic implications using the testing method of portfolio diversification effect

The figure shows the results indicating the changing pattern of the fatness of the tails in the portfolio return distribution as the number of stocks in a portfolio increases, using the three stock groups over the period from July 2006 to June 2015. The results are based on the stock return data for the all-stocks group (○), the large-stocks group (□), and the small-stocks group (△). Statistical probability is used as the quantitative measure of the fatness of the tails in the portfolio return distribution. The X-axis denotes the number of stocks in the portfolio, from at least 2 to a maximum of 50; the Y-axis indicates the average values of the statistical probabilities calculated from 100 simulations. The results are separately presented for the negative tail (●, ■, ▲) and positive tail (○, □, △).

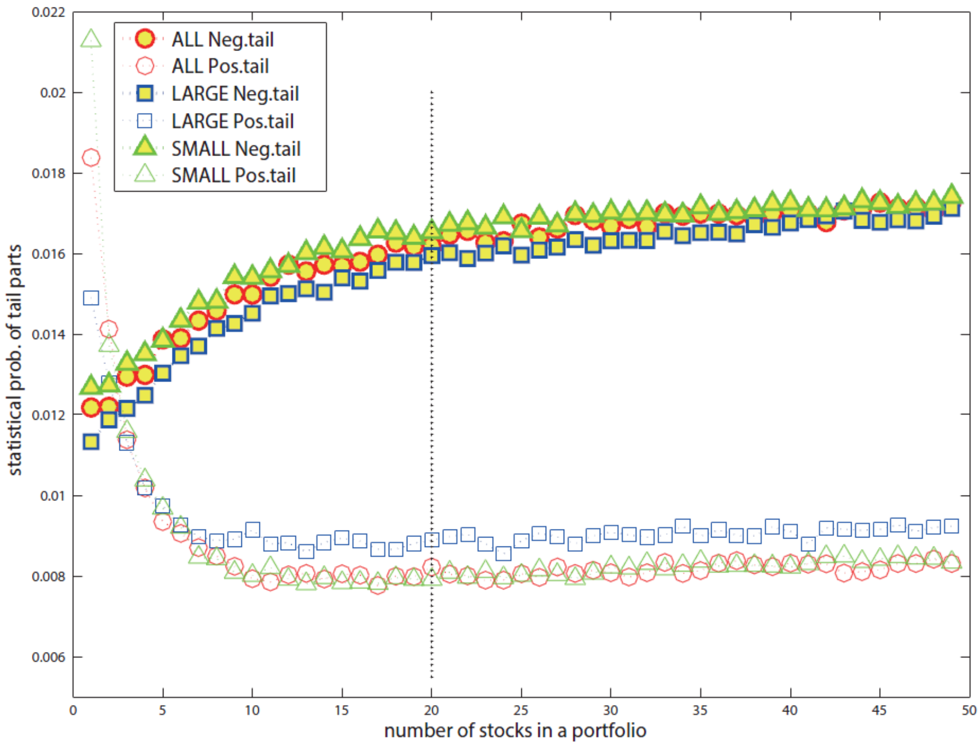

Figure 3. Additional test for the effect of common factors on portfolio diversification

The figure shows the results related to the effect of the common factors on the portfolio diversification effect using the two types of return data— return data (□, ■ in Figure 3(a)) having only the properties of the common factors from the original return data (☆, ★), and return data (○, ● in Figure 3(b)) removing only the properties of the common factors from the original return data. The negative tail (■, ●, ★) and the positive tail (□, ○, ☆) in the portfolio return distribution are separately presented. This figure uses the stocks of the all-stocks group over the period from July 2006 to June 2015. The X-axis denotes the number of stocks in the portfolio, from at least 2 to a maximum of 50; the Y-axis indicates the average values of the statistical probabilities calculated from 100 simulations.