Market and Non-market Variance Risk in Individual Stock Returns

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ABSTRACT

This paper shows that the price and quantity of variance risk of individual stock returns jointly explain future short-term variations of stock returns. The variance process of an individual stock inherits the factor structure of the stock returns. Therefore, the price of variance risk measures the priced second-moment risk of the stock, both due to market and non-market factors. Then, following the beta representation, the stock's own variance beta suggests how its risk premium should be connected to the price of variance risk. Empirically, while stocks with a high negative quantity of variance risk tend to have higher subsequent returns, the spread largely depends on the size and sign of the price of variance risk. For stocks with a high negative price of variance risk, the spread between stocks with a negative and positive quantity of variance risk is -0.92% per month but is 0.33% for stocks whose price of risk is small. This paper suggests that non-market variance risk is also an important source that is priced among individual stocks.

JEL classification: G11, G12, G15 and G17.

Keywords: Option-implied Variance, Variance Risk, Variance Risk Premium, Beta Representation

I. Introduction

Variance risk is one essential source of risk that determines stock prices. For example, Ang, Hodrick, Xing, and Zhang (2006) suggest that market variance risk is priced among individual stocks. Stocks with high negative exposure to variance risk tend to have higher subsequent returns. Also, for the market index, Bollerslev, Tauchen, and Zhou (2009) find that the variance risk premium of the index strongly predicts short-term market returns. Variance risk may affect asset prices for at least two reasons. Variance risk might affect investors' intertemporal portfolio decisions since a positive variance shock may decrease the investment opportunity set. Variance risk may also affect ambiguity averse investors as more variance shocks mean higher parameter uncertainty.[1]

While previous studies mainly focus on the role of market variance risk,[2] it is yet unclear whether shocks to market variance is the only ones that should be priced. This paper uses individual stock returns and options data to provide a unique framework that captures the risk premium of individual stocks due to both market and non-market variance shocks. Focusing on non-market variance risk is a natural choice since it is generally acknowledged that there are other factors than the market factor that is priced among individual stocks. The result of this paper suggests that market variance risk is not the only second-moment risk that affects individual stock returns.

This paper investigates the joint role of the price and quantity of variance risk of individual stock returns in explaining its future returns. The variance process of individual stock returns embeds information about the underlying factor structure. The price of variance risk of the stock is a linear combination of the price of relevant second-moment factor risk. Then, following the 'beta representation,' the quantity of variance risk measures how the risk premium of the stock should be related to its price of variance risk. Hence, the price and the quantity of variance risk should have an interactive role – the price of risk should matter more for stocks

[1] See, for example, Drechsler and Yaron (2011).

[2] Besides Ang, Hodrick, Xing, and Zhang (2006), see also, Adrian and Rosenberg (2008), Chang, Christoffersen, and Jacobs (2013), Conrad, Dittmar, and Ghysels (2013), and Cremers, Halling, and Weinbaum (2015) among others.

that have a high quantity of risk and vice versa. Empirically, forming a trading strategy based on the combination of the price and quantity of variance risk generates a monthly risk-adjusted return that is at least 1.46% per month.

Focusing on variance risk of individual stock returns is useful for both empirical and logical reasons. First of all, empirically, the price of variance risk is tractable. Variance risk is a unique type of risk in that both the price of risk and the underlying risk factor is observable. The price of variance risk can approximately be estimated as the difference between option-implied variance and realized variance of the underlying asset. The underlying factor, i.e., unexpected changes in variance, is also identifiable. The quantity of variance risk can then be estimated using regressions of stock returns on its variance shocks. Therefore, both the price and quantity of variance risk are essentially estimable as long as the underlying asset has tradable options.

Second, the variance process of individual stocks contains information about the latent factor structure. When there is a systematic variance shock in a factor, only the variance of stocks whose prices are exposed to the factor risk will respond. For example, in a market model, the variance of higher market beta stocks are likely to increase more to a positive market variance shock. Since stock variance also depends on other risk sources, we can essentially decompose the stock variance process into two components – one that is induced by market variance shocks and the other. Hence, due to the first component, the variance processes of these stocks are more risky, containing a more negative price of risk. Then, the quantity of variance risk, which measures how stock prices changes to these (market related) variance shocks, suggests how stock returns should be connected to this price of variance risk induced by market variance risk.

A similar argument even follows for multiple factor structures. This is easily observed when a factor has either a significant positive or negative skewness. For example, Daniel and Moskowitz (2016) argue that the variance of momentum strategy is time-varying and the payoff of the momentum strategy is highly negatively skewed. When momentum crashes, both past winners and losers are affected. When momentum variance increases, the returns of both winners and losers will increase due to the factor structure. The price of variance risk would be high for both winners and losers. Also, since high variance is associated with negative momentum returns, winners will have negative returns and losers will have positive returns. Therefore, on average, winners will have a more negative variance beta while past losers are likely to have a positive beta. This argument is connected to that of Daniel and Moskowitz (2016), who argue that past winners have negative exposure to market variance risk. The difference is that they consider the exposure to market variance risk while this paper considers momentum variance risk directly.

Finally, the combination of the price and quantity of risk has a direct interpretation as a risk premium. In particular, this combination measures the one-month risk premium of the stock that is due to its variance shocks. This is likely to embed important information about the stock's risk premium since the variance of stock returns is a natural measure of the risk of a stock. In particular, this combination is likely to contain some key information about the stock's risk premium when the size of correlations between stock returns and its variance shocks are high. For market returns, this is shown by Pyun (2018), who argues that correlations between market shocks and variance shocks are high, a large proportion of market risk premium is explained by the variance risk premium of the market. In this regard, therefore, combining the price and quantity of variance risk allows us to estimate an important fraction of the risk premium.

Empirically, this paper finds that stocks with a negative price of variance risk tend to have higher subsequent returns. This finding is consistent with Bekaert and Hoerova (2014) who find a positive relationship between the difference of implied variance and realized variance and future stock returns. Although statistically insignificant, this paper also confirms that stocks with a high negative quantity of variance risk tend to have higher subsequent returns. However, the main interest of this paper is whether there is an interactive relation between the price and quantity. This paper finds evidence that is consistent with the hypothesis. The price of variance risk matters only for stocks that have negative exposure to its variance risk. Also, the risk exposure matters only when a stock has a high negative price of variance risk. During 1996 – 2015, for stocks that have options traded, the overall spread between stocks with a negative and positive price of variance risk is -0.75% per month. However, the sign and the size of the spread depend largely on how stock returns react to variance shocks. This spread is more negative (-1.61%) when stocks have a negative variance risk exposure than when the variance risk exposure is small or positive (-0.37%). Similarly, the spread between the high minus low portfolio of the quantity-sorted stocks is -0.06%. The spread is negative for stocks with a high negative price of risk (-0.92%) but is positive for stocks with a relatively small price of risk (0.33%). Finally, trading on the price and quantity of variance risk simultaneously yields as much as 1.33% per month. The risk-adjusted returns are 1.46% per month.

This paper provides three pieces of evidence that suggest the exposure to aggregate market variance risk (Ang, Hodrick, Xing, and Zhang 2006) has little role in explaining the interactive relation. First, I control for the aggregate variance risk exposure by controlling for market variance risk. The interactive relation remains both economically and statistically significant even when the risk exposure is estimated after controlling. Second, I construct a measure of the price and quantity of non-market variance risk using a market model. The interactive relation does not disappear even when the stocks are double sorted by the price and quantity of non-market variance risk. Finally, I control for the FVIX factor of Ang, Hodrick, Xing, and Zhang (2006). The variance risk factor return constructed using an interactive relation remains statistically significant even after controlling for the FVIX factor.

One may wonder conjecture that a substantial proportion of stock variance idiosyncratic which should remain unpriced. The existence of an unpriced risk in a stock would at most have a small influence in this framework. First, the price of variance risk would not be affected, as the price of unpriced risk is zero. Second, the quantity of variance risk would also be unaffected if the size of unpriced risk does not vary over time.[3] Third, even when the unpriced variance is time-varying, it is unclear whether *idiosyncratic volatility risk* can be diversified, especially, when idiosyncratic volatilities of different stocks co-move over time. For example, Herskovic, Kelly, Lustig, and Nieuwerburgh (2016) show that the co-movement of idiosyncratic volatility affects the cross-section of individual stock returns. They suggest that co-movements of idiosyncratic volatility may proxy for labor income risk of households.

The price and quantity of non-market variance risk may also be important when investors are under-diversified. While the classical finance theory suggests that households should be entirely diversified, numerous studies suggest that households are actually severely under-diversified

[3]Time-varying idiosyncratic volatility would create a bias towards zero in the risk exposure

(e.g., Blume and Friend (1985)). Several motivations have been proposed. Under-diversion in households portfolio can come from investors' preference for lottery-like payoffs (Barberis and Huang 2008, Mitton and Vorkink 2007), lack of education (Campbell 2006, Calvet, Campbell, and Sodini 2007), wealth constraints (Liu 2014) or familiarity to certain firms or countries (Ivkovic and Weisbenner 2005, Van Nieuwerburgh and Veldkamp 2010). In particular, in the context of familiarity, Uppal and Wang (2003) show that cross-sectional difference of ambiguity on individual firms can be one motive for under-diversification. Therefore, for these under-diversified investors, the variance of non-market factors may also influence their portfolio decisions.

This paper contributes to several streams of literature. First, this paper is connected to studies that analyze the role of variance risk in explaining the cross-section of stock returns. Bali and Hovakimian (2009) and Han and Zhou (2012) report a negative relationship between the price of variance risk and the subsequent stock returns. Han and Zhou (2012) argue that the price of risk is a proxy for the exposure to market variance risk, while Bali and Hovakimian (2009) claim that it proxies for jump risk. Ang, Hodrick, Xing, and Zhang (2006), Chang, Christoffersen, and Jacobs (2013), and Cremers, Halling, and Weinbaum (2015) also study how variance risk is priced in the cross-section, but their focus is on market variance risk. This paper focuses on both market and non-market variance risk. Furthermore, to my knowledge, this paper is also the first paper that studies the interactive effect between the price and quantity of variance risk for individual equities.

Second, previous studies including Goyal and Santa-Clara (2003), Fu (2009), Chen and Petkova (2012), Stambaugh, Yu, and Yuan (2015), and Hou and Loh (2016), among others, argue that idiosyncratic volatility is priced in stock returns. This paper is related, because part of non-market variance risk that is priced maybe diversifiable idiosyncratic variance risk. However, these articles differ from this paper in that they mainly focus on the level of volatility rather than volatility risk. One exception is Herskovic, Kelly, Lustig, and Nieuwerburgh (2016), but they mainly focus on variance risk that is purely idiosyncratic.

Third, this paper also relates to studies using option implied information of individual stocks to explain the cross-sectional variation of stock returns. Xing, Zhang, and Zhao (2010) and Conrad, Dittmar, and Ghysels (2013) find that option-implied skewness and kurtosis is related to future individual stock returns. Buss and Vilkov (2012) and Chang, Christoffersen, Jacobs, and Vainberg (2012) study the role of option-implied forward-looking betas, and Buss, Schoenleber, and Vilkov (2016) argue that correlation risk explains systematic diversification benefits.

Finally, the main model of this paper inherits that of Pyun (2018), who studies the joint relationship between the price and quantity of variance risk for predictability of aggregate stock returns. When using a combination of the price and quantity of market variance risk, he shows that market returns are predictable in a statistically and economically significant manner. This paper is different in that it focuses on the cross-section of stock returns. Also, the framework of this paper can capture non-market variance risk.

The remainder of the paper is organized in the following way. Section II introduces a simple model. Section III describes the estimation methodology and the data used in this article. Section IV provides the empirical result. Section V concludes.

II. The Model

The expected return of any asset, under the beta representation, can be expressed as a linear function of multiple betas. The betas are explanatory variables that are specific to the asset in the cross-sectional relation and measures what extent the factors influence the risk premium of the assets. Factors are proxies for the marginal utility of consumption growth. Then, the representation tells us that the common price of the risk factor should be the slope that connects the betas to the expected return of the assets. Empirically, to understand how stock returns are priced, we commonly sort stocks by the betas and evaluate whether there is a positive/negative risk premium. This single sorting methodology is useful if we entirely understand the factor structure as well as the price of risk factors. However, entirely understanding the cross-section of expected returns can be challenging when we do not entirely understand how these factors proxy the marginal utility of investors, especially in a dynamic economy.

Several recent studies suggest that variance risk is an important determinant of stock risk premium. For example, Ang, Hodrick, Xing, and Zhang (2006) find that stocks with a negative exposure to market variance risk tend to outperform those stocks with a positive exposure. Chang, Christoffersen, and Jacobs (2013) find weaker explanatory power if the betas are estimated over a longer horizon, which suggests that the variance betas tend to be highly time-varying. Furthermore, Bollerslev, Tauchen, and Zhou (2009) find that the price of market variance risk, measured as the difference between VIX^2 and the realized variance of the market index, is highly time-varying and predicts future market returns in a significant manner.

A highly time-varying price of market variance risk suggests that there are periods when market variance risk tends to be more important. There are also times when market variance risk should be essentially unpriced. In the context of market return predictability, Pyun (2018) finds that the relation between the price of market risk and market variance risk largely depends on how market prices react to changes in variance. In other words, when market risk and variance risk are related, the price of variance risk and the price of market risk is related, and when they are unrelated, the risk premia are unrelated.

Focusing on the variance of individual stock returns is useful for similar reasons. First of all, individual stock returns are closely tied to the variance of its returns. The variance of its stock returns is a natural measure of risk of the stock. A higher stock return variance means more risk for investors or a higher discount rate for the stock. Therefore, all else being equal, when there is a positive variance shock, the stock price must decrease simultaneously. As a result, stock returns and variance innovations should exhibit a negative relation. This explanation is reminiscent of volatility feedback (Pindyck 1984, French, Schwert, and Stambaugh 1987) and is clearly observed at the aggregate stock level (Bollerslev, Litvinova, and Tauchen 2006).

However, unlike in the case of the well-diversified market portfolio, the sign and the magnitude of the relation between individual stocks returns and variance shocks also depend on many other firm-specific variables. For example, firms may possess one or multiple valuable real options. Barinov (2013) studies the role of growth options in variance risk and value premium. Growth options will naturally grow as variance increases, as will increase firm value. Limited liability will also create an option-like payoff in equity returns. The existence of growth options and limited liability would particularly matter for smaller firms that have a higher likelihood of default and will reduce the magnitude of the negative exposure to its variance risk. Therefore, price and variance shocks of individual stocks can also be positively related.

Furthermore, the relation between stock returns and its variance shocks is determined by the stock's factor exposures, in which both the exposure and the importance of the factor may be time-varying. When the variance of a factor increases, the variance of stocks that are exposed to the factor will increase. The direction of the stock price movement is determined by the sign of the factor risk exposure and factor skewness. Consider a factor which is built on the difference in portfolios returns of some highs and lows. A negatively (positively) skewed factor means that a positive factor variance shock is associated with a negative (positive) return shock. Therefore, in response, high stocks and the low stocks will move in the opposite direction. Hence, marginally, the factor would affect the variance risk exposure of high and low stocks in the opposite direction.[4] The variance risk of a factor may affect stock returns more when the factor is more skewed.

The following simple model describes the relationship between the expected return of a stock and the price of its variance risk. Suppose that the stock i ($S_{i,t}$) follows a geometric Brownian motion which can be decomposed as the sum of the stock-specific systematic ($dY_{i,t}$) and idiosyncratic ($dW_{i,t}^{idio}$) risk:

$$\frac{dS_{i,t}}{S_{i,t}} = a_i dt + b_i dY_{i,t} + \sigma_{idio} dW_{i,t}^{idio}.$$

$dY_{i,t}$, the systematic risk component can, for example, be linear functions of multiple factors and their corresponding slopes, the stock-factor specific betas. If stock i follows a N-factor structure, $dY_{i,t}$ can be represented as $\sum_{n=1}^{N} \beta_{i,n} dF_{n,t}$ for N-independent factors $dF_1, dF_2, \ldots, dF_N$. The second component, $dW_{i,t}^{idio}$, represents unpriced idiosyncratic risk. For now, I assume a constant volatility for this component, but I will later relax the assumptions and discuss the implications of time-varying idiosyncratic volatility.

[4] To see this, let R_H and R_L be the high and low portfolio returns, which forms the basis of a factor. Factor F is built on the difference between these two returns. Let the variance of factor F be σ_F, then the difference in the variance risk exposure of the two portfolios is $Cov(R_H, \sigma_F) - Cov(R_L, \sigma_F) = Cov(R_F, \sigma_F)$. The latter covariance is a transformation of the third moment of factor F.

Let each of the risk factors $dF_{n,t}$ further follow a stochastic process that is correlated with the corresponding factor's variance process. That is, let

$$
\begin{aligned}
dF_{n,t} &= \mu_{n,t}dt + \sqrt{V_{n,t}}(\rho_n dW_{n,t}^v + \sqrt{1-\rho_n^2}dW_{n,t}^o) \\
dV_{n,t} &= \theta_n dt + \sigma_{nv} dW_{n,t}^v,
\end{aligned}
$$

where $V_{n,t}$ is the variance of factor n, and $dW_{n,t}^v$ and $dW_{n,t}^o$ are independent Brownian Motions. The correlation between factor returns and factor variance (ρ_n) is determined exogeneously. If $\rho_n < 0$, an increase in factor variance leads to an instantaneous negative shock in factor returns.

In this stochastic process, variations in each factor $F_{n,t}$ is further decomposed into two parts. First, they can be related to their second-order variance shocks. As the variance of factor returns is a measure of the risk of the factor, changes in factor variance is directly related to the price of the factor risk.[5] If there is a change in the factor risk premium, the factor returns should react immediately. Therefore, the reaction to variance shock is directly connected to the discount rate channel and should be crucial in explaining the risk premium of the factor. Second, there are other shocks, which does not lead to changes in the variance of the factor. This decomposition is always possible. I refer all other risk that does not involve a time-varying level of the second moment as orthogonal shocks. Orthogonal shock also includes all cash-flow shocks, as they are one-time price shocks that do not affect the discount rate or the variance.

In this model, the factor structure of the underlying stock returns determines the relation between stock returns and its variance shocks. For example, the negative relation between returns and variance shocks may be stronger for stocks with a higher beta, large stocks, or stocks relatively little growth options. According to the proposed model, factors that represent these firm characteristics may have stronger leverage effect than others. Therefore, either the firm characteristic or the risk structure of underlying stock returns determines the variance process and the underlying stock process simultaneously, which in the end affects the size of the leverage effect.

[5] This can be observed from the equation $\text{Cov}(-SDF, F) = \rho_{\text{SDF},F}\sigma_{\text{SDF}}\sigma_F$, where SDF is the stochastic discount factor.

Solving the above three equations together and expressing stock returns as a function of factor variances yields[6]

$$\frac{dS_{i,t}}{S_{i,t}} = \{\cdot\}dt + b_i \sum_n (\beta_{n,i}\rho_n \frac{\sqrt{V_{n,t}}}{\sigma_{n,v}})dV_{i,t} + \sum_n B_{i,n} dW^o_{i,t} + dW^{idio}_{i,t},$$

where $B_{n,i} = b_i\sqrt{V_{n,t}(1-\rho_n^2}$ and $\{\cdot\}$ is used for some function that is can be solved explicitly, but is unnecessary to derive the final formula we need.

The connection between the expected stock returns and the price of variance risk of factor n $dV_{n,t}$ follows the argument of Pyun (2018). If we let the discounted marginal utility as Λ_t, the stochastic discount factor (SDF) in continuous time is $\frac{d\Lambda_t}{\Lambda_t}$. Thus, the expected return of a stock can be expressed as

$$\mathrm{Cov}_t(-\frac{d\Lambda_t}{\Lambda_t}, \frac{dS_{i,t}}{S_{i,t}}) = b_i \sum_n (\beta_{n,i}\rho_n \frac{\sqrt{V_{nt}}}{\sigma_{nv}})\mathrm{Cov}_t(dV_{n,t}, -\frac{d\Lambda_t}{\Lambda_t}) + \sum_n B_{n,i}\mathrm{Cov}_t(dW^o_{n,t}, -\frac{d\Lambda_t}{\Lambda_t}).$$

Notice that $V_{n,t}$ is the variance of factor n (dF_n), and also that $b_i\beta_{n,i}\rho_n\frac{\sqrt{V_{n,t}}}{\sigma_{n,v}}$ is the beta we get when we regress individual stock returns on the variance of the n^{th} factor $dV_{n,t}$. The relation follows from

$$\begin{aligned} Cov_t(\frac{dS_{i,t}}{S_{i,t}}, dV_{n,t})/Var(dV_{n,t}) &= Cov_t(b_i \sum_n \beta_{n,i} dF_{n,t}, dV_{n,t})/Var(dV_{n,t}) \\ &= b_i\beta_{n,i}\rho_n\sqrt{V_{n,t}}\sigma_{n,v}/\sigma^2_{n,v} \\ &= b_i\beta_{n,i}\rho_n\sqrt{V_{n,t}}/\sigma_{n,v} \end{aligned}$$

The above equations show that the expected return of stock i can be represented as the sum of the products of the price of variance risk of the factors that the stock is exposed to, and the stock's exposure to the variance shock of the particular factor. The first key result of this paper is represented as follows:

[6] plug-in the third equation into $dW^v_{n,t}$ in the second equation, and put $dF_{n,t}$ into the first equation

Result 1. Assume stock i follows a N-factor structure with factors $\{F_1, \ldots, F_N\}$ and betas $\{\beta_{i,1}, \ldots, \beta_{i,N}\}$ that correspond to the factors. Let $\lambda_{n,i,t}$ be the price of variance risk of the n^{th} systematic risk factor, $\lambda_{o,i,t}$ the linear combination of the price of risk due to orthogonal shocks,[7] and $\beta_{n,i,t}$ the slope of a hypothetical regression of stock i's return on the variance of latent factor n (dF_n). Then, we have

$$E_t[R_{i,t+1}] = \sum \beta_{n,i,t}\lambda_{n,i,t} + \lambda_{o,i,t}, \tag{1}$$

Equation (1) tells us that the expected excess return of a stock can be represented as a linear combination of the product of the price and quantity of variance risk of the latent factors. Estimating these components is not straightforward since a direct estimation of the above equation would require us to assume a certain factor structure and compute the price of variance risk of each of these factors separately. In fact, it is even difficult to determine the factors that should be used.

However, a simple transformation shows that the first component of Equation (1), the risk premium of a stock due to variance shocks of factors, can be estimated quite easily by just computing the price and quantity of variance risk of each stock. The second key result of this paper (proof in Appendix) is given as below.

Result 2.. Let the price of variance risk of stock i to be $\lambda_{v,i,t}$, and the stock i's exposure to its variance process be $\beta_{v,i,t}$, respectively. Assuming that idiosyncratic volatility stays constant, we have the following alternative representation of the Result 1, where the price of orthogonal risk $\lambda_{o,i,t}$ remains same as in Result 1.

$$E[R_{i,t}] = \beta_{v,i,t}\lambda_{v,i,t} + \lambda_{o,i,t} \tag{2}$$

The representation of (2) shows that the product of the price and quantity of variance risk of each stock $(\beta_{v,i,t}\lambda_{v,i,t})$ exactly measures the risk premium of the stock that is implied by

[7]Here, $\lambda_{n,i,t} = \mathrm{Cov}_t(dV_{n,t}, -\frac{d\Lambda_t}{\Lambda_t})$ and $\lambda_{o,i,t} = \sum_n B_{n,i}\mathrm{Cov}_t(dW^o_{n,t}, -\frac{d\Lambda_t}{\Lambda_t})$

the second moment shocks – including market and non-market variance shocks– that the stock is exposed to. This representation, in addition to the traditional factor models, is useful for several reasons. First, both the price and quantity of variance risk of individual stock returns are directly measurable. As mentioned, Carr and Wu (2009) show that the payoff of a variance swap is equivalent to the price of variance risk and can be represented as the difference between the risk-neutral expectation of variance and the real-world expectation of variance. At the individual stock level, the price of variance risk can also be estimated using individual stock option prices and high-frequency intraday trading data. The quantity of risk is estimable as well since the first-order difference of the option-implied variance is observable. We can measure the quantity of risk using a regression of stock returns on changes in its variance.

Second, the price-quantity combination of the individual stock is essentially a combination of the price of variance risk of all systematic risk proxies. The exact slope in the representation is determined by the factor variance risk exposure of the stock, but whether or not the variance risk of a particular factor dF is priced in a stock is determined by the factor structure of the underlying return process. In other words, the variance process inherits the underlying factor structure, and the factor structure determines whether the second moment risk of a factor, potentially related to marginal utility proxies, should be priced.

So far, the model assumes that unpriced idiosyncratic variance remains constant over time. One may wonder how the time-varying idiosyncratic volatility affects the relation between the expected stock returns and the price of variance risk. A recent study by Herskovic, Kelly, Lustig, and Nieuwerburgh (2016) suggest that idiosyncratic volatility is time-varying and their co-movement across stocks is priced. They argue that idiosyncratic volatility may affect heterogeneous agents, as idiosyncratic volatility shock may affect income risk faced by households.

The implications of a time-varying idiosyncratic variance are straightforward. If idiosyncratic variance risk has a common component that is priced as in Herskovic, Kelly, Lustig, and Nieuwerburgh (2016), the idiosyncratic risk of these firms could be treated as if there is a latent factor, which has a variance process that resembles the co-movement component. A potential issue arises when there is unpriced time-varying idiosyncratic volatility. A time-varying idiosyncratic variance means $\beta_{v,i,t}$ would be biased downwards, as it would create an error-in-variables problem in the estimation process. The quantity of variance risk of individual stock returns will have a downward bias, and the magnitude of the bias depends on the level of the variance of the unpriced variance.

One limitation of this approach is that the risk premium due to the variance unrelated market shocks, namely the price of orthogonal risk $\lambda_{o,i,t}$, is essentially not estimable in this framework. As mentioned, this type of risk includes systematic shocks that do not lead to changes in the level of variance.

Several empirical implications of the main result are worth mentioning. The risk premium of a stock is likely to be higher for stocks whose price of variance risk is more negative. Intuitively, this is because when the price of variance risk of the stock is highly negative, the variance process of the stock highly covaries with the marginal utility of the investors. Therefore, investors dislike variation in the variance of the stock. For the index, this may be because the variance process measures the investment opportunity set, which affects investors. The model of this paper suggests more than this. The price of variance risk is related to the stock's risk premium in part because stock returns are related to its variance shocks. The price of variance risk is a good measure of the risk premium of the stock only when returns are highly related to variance shocks.

Second, one may wonder why individual stocks have its price of risk. In this framework, the variance risk of each stock has a different price due to the unique exposure to different risk factors. For example, when the price of market variance risk is high, the price of variance risk is high for stocks that have high exposure to market factor. Therefore, the price of variance risk differs across stocks not because they have a unique price of risk, but due to its unique factor exposures (betas).

This paper suggests that the price and quantity of risk have a joint role in determining the short-term risk premium of individual stocks. The interactive effect implied by the model suggests that for stocks that are not exposed to its variance risk, the price of risk would have a smaller influence on the stock's risk premium. While the role of the exposure to variance risk and the price of variance risk have previously been studied separately, the interactive relation in explaining the time-variation of individual stock returns has been commonly ignored for several reasons. First, the focus of the variance risk premium has been different from the interpretation as a price of risk. For various assets, this difference is often regarded as a measure of time-varying risk aversion. Second, the variance risk exposure is time varying and is difficult to estimate.

The following sections empirically study the interaction between the price and the quantity of variance risk. I show the role of market variance risk and investigate the possibility that non-market variance risk is priced in individual stock returns. I also evaluate a possible trading strategy using the interactive relation between the price and quantity of risk.

III. Data and Methodology

The sample of this paper consists of stocks from NYSE, AMEX, and NASDAQ that have options actively traded in the market between 1996 and 2015. The study period is restricted due to data availability in both option pricing data and intra-day high-frequency trading data. The option-implied volatility is obtained from OptionMetrics, high-frequency intraday data from Trade and Quote, and individual stock returns from the Center for Research in Security Prices (CRSP). Among options that expire in the following month, several other filters are applied to the data to minimize possible data errors. First, following previous studies such as Goyal and Saretto (2009), options in which the ask price is lower than the bid price, options whose bid price is equal to zero, and options whose price violates arbitrage bounds are eliminated. Second, options with zero open interest are also deleted. Third, options that have moneyness smaller than 0.95 or higher than 1.05 are removed. Among these options, I choose one call and one put option that is closest to ATM and compute the simple average of the option-implied volatility between the call and the put. The implied variance (IV) of an individual stock is the square of the average. When IV is missing, I let the value to remain unchanged from the previous day.

The option-implied variances are then matched to CRSP using both CUSIP and tickers. Using daily observations of stock returns and IV, I use multiple specifications to estimate a stocks exposure to its variance risk. As the main specification, similar to Ang, Hodrick, Xing, and Zhang (2006), the variance risk exposure is the slope of the two-variable regression

$$R_{i,t} = \beta_{0,i} + \beta_{m,i}\ R_{m,t} + \beta_{v,i}\ [IV_{i,t} - IV_{i,t-1}] + \epsilon_{i,t}$$

where R_m are the excess market returns, and R_i is the excess stock returns of stock i. Here, $\hat{\beta}_{v,i}$ is estimated every month using the past twelve months of data. As alternative specifications, I also consider a single-variable regression with only the variance innovation of the stock, and a three-variable regression with the market factor, variance innovation in VIX^2, and the variance innovation of the stock and show that the key results of this paper holds regardless of the exact specification used to estimate the slope.

For each stock, the price of variance risk is computed as the difference between historical realized variance (RV) and IV. For each month, the price of variance risk ($\lambda_{v,i,t}$) is computed as the difference between the IV at the end of the month and RV averaged over the most recent month. This follows from the fact that

$$Cov_t(-SDF_{t+1}, RV_{i,t+1}) \approx E_t[RV_{i,t+1}] - E_t^Q[RV_{i,t+1}],$$

and option-implied variance proxies for the risk-neutral expectation of variance. The RV is estimated as the sum of 75-minute squared returns over the past month. To deal with possible microstructure noise, I use the average over five subsamples (Hansen and Lunde 2006). RV computed over 75-minutes follows earlier works (e.g. Bollerslev, Li, and Todorov (2016)) on intra-day individual stock returns. The frequency is lower than what is typically used for the market index but reflects the high possibility of microstructure noise at the individual stock level. However, the main results are unaffected by choice of the sampling interval. Furthermore, while realized variance estimated in this manner may be inaccurate on a daily basis as there only six observations per day, they are only used after summing up to the monthly level. To deal with the possibility that the estimate of realized variance for the illiquid stocks is not correct, I drop all observations when trading of the stock (changes in prices during the day) does not occur for at least ten days per month.

It is still possible that the price of variance risk estimated in this manner is biased, as there may be a timing mismatch between the two variance components. While the RV reflects recent realizations, the IV reflects the future expectation of those variations. As a standard alternative, for the market index, a variance forecast model based on the intra-day square of high-frequency returns is used. Although they are more likely to be plausible at the index level, we need to be more cautious applying this alternative methodology to individual stocks. Above all, the sample period of this data is restricted to 20 years due to data availability. A time-varying out-of-sample variance forecast model over 240 observations would contain massive estimation error. In addition, the RV estimates of individual stocks are more subject to microstructure noise. A forecast model based on an already noisy estimate should be problematic. Finally, as Bekaert and Hoerova (2014) argue, relying on a particular variance forecast model can be controversial as the empirical implications may depend on the particular model used. Therefore, I also test the main result where the RV component is based on a variance forecast model. However, notably, Goyal and Saretto (2009) show how the difference between RV and IV, as defined as the main specification of this paper, affects the cross-sectional average of option returns.

Table I summarizes the mean, median and standard deviations of the sample considered in this paper. There is a total of 196,934 stock-month during the sample period. While the entire CRSP sample has an average of 4,948 stocks in a given month during 1996/01-2015/12, on average there are 821 stocks that have options traded actively in the market. There are 278 stocks at the beginning of 1996 and 1,449 stocks at the end of the sample. The median of the IV is 0.011, which is equivalent to 36.3% in annual standard deviations, while the RV has a median of 0.007, equivalent to 29.0% in annual standard deviations. The mean of IV is 0.018, and the mean of RV is 0.014, which suggests that the cross-sectional distribution of the variance has positive skewness. The outliers in stock variance are not necessarily a serious concern for the purpose of this paper, as they are not influential when forming portfolios based on the rankings. The table also suggests that the price of variance is mostly negative. The difference between the RV and IV tends to be negative on average, but a sizable proportion is still positive (19.0% in the panel), which is consistent with other previous research (e.g., Goyal and Saretto). Although I believe that a substantial proportion of stocks with a positive price of risk estimates is due to the estimation error in RV, especially for small stocks, there may also be some downward bias in IV due to using only ATM options.

The variance betas or the variance exposures are, on average, negative, suggesting that for an average stock, the leverage effect or volatility feedback tends to dominate the possible influence of growth or real options. However, for a notable number of stocks (21.3%), returns are positively related to contemporaneously variance movements, which is partly due to growth or real options. Compared to the entire CRSP database, the chosen sample has a similar average market beta. Also, stocks in the sample tend to be larger than the CRSP average. This is natural as the sample systematically excludes options on these stocks that are less likely to be traded.

There is a possibility that the price and quantity of variance risk are strictly related. A high negative price of variance risk means that the variance process is either extremely volatile or highly correlated with the latent marginal utility process of investors. The stock prices of these stocks are more likely react sharply to changes in variance. As a result, these stocks may have both a negative price and quantity of variance risk. However, if we compute the average of the estimated cross-sectional correlation coefficient between the price and quantity of variance risk, it is -0.095 (not reported in the table) which indicates that there are not heavily related.

IV. Empirical Results

The 'beta representation' of expected returns suggests that if the expected return is represented as a linear combination of different prices of risk, the weights of each price of risk are the betas that measure the asset's exposure to the underlying risk factor. The model of the previous section also implies this. If a stock is exposed to different variance shocks, the price of these variance shocks is connected to the stock risk premium, and this can be best measured by the price of variance risk of the own stock returns. Then, the slope that connects the price of variance risk to the risk premium of the stock is the beta of the stock returns on its own variance shocks.

Empirically, when the price of variance risk is highly negative, we expect the quantity of risk to affect future stock returns more than stocks with a less negative price of variance risk. Analogously, the price of variance risk should matter most when it is highly exposed to its variance shocks. If the variance risk exposure is positive, either due to valuable growth options or due to limited liability constraint, those a high negative price of variance risk may even mean lower subsequent returns. In simple terms, the price and quantity of risk would have an interactive relation. This section provides empirical evidence suggesting that both the price and quantity are important in explaining the time-variation of individual stock returns. To test the hypothesis, I first sort the stocks by the price or quantity of variance risk and evaluate the performance of single-sorted quartile portfolios separately. Then, the performance of price-quantity double-sorted portfolios is provided. Finally, I evaluate the role of aggregate variance risk exposure.

1. The Relation between the Price and Quantity of Variance Risk

Before discussing the performance of the price-quantity double-sorted portfolios, we first evaluate the performance of single-sorted portfolios, where individual stocks are sorted by either the price or quantity of variance risk. Table II summarizes the performance of single-sorted portfolios. To do so, I first estimate the price and quantity of variance risk for each stock. Then, the stocks are sorted either by the price or the quantity of variance risk. The stocks are divided into quartiles based on these estimates, and the simple averages of the price and quantity of variance risk estimates, value-weighted returns, risk-adjusted returns, market beta, and the market capitalization are computed and reported. Risk-adjusted returns, denoted by α_4, are returns controlled for the size, value (Fama and French 1993), and momentum (Carhart 1997). Both returns and risk-adjusted returns are evaluated over the subsequent month after the formation, and the units are in monthly returns.

Panel A summarizes the performances of the price of variance risk sorted portfolios. This panel is a slightly modified version of two previous studies including Bali and Hovakimian (2009) and Han and Zhou (2012). These studies investigate how the difference between the option-implied variance and the realized variance affects future stock returns, but does not interpret this measure as being the price of variance risk of individual stock returns. For example, Han and Zhou (2012) argue that the difference proxies for aggregate variance risk exposures and report a negative relationship between the price of variance risk and subsequent returns. Bali and Hovakimian (2009) interpret this spread as a proxy for jump risk of the stock. The first two columns of Panel A essentially replicate these studies and confirm that stocks with a high negative price of variance risk tend to have higher subsequent returns. The next two columns summarize the average contemporaneous values of the price and quantity of variance risk. Although stocks with a high negative price of variance risk tend to have less variance risk exposure, the relation is not monotone. As mentioned in the previous section, these two estimates are close to unrelated. Lastly, the last few columns provide some additional summary statistics. Stocks with a high negative price of variance risk tend to be smaller, have higher market betas, and have a smaller exposure to variance risk.

Previous studies also suggest that market variance risk is priced in the cross-section of stock returns[8]. These studies focus on the exposure to market variance fluctuations. Here, the focus is the variance risk of individual stock returns. Since the variance of a typical stock return is likely to depend on market variance movements, the market variance beta and the individual stock variance beta may be closely related.

Panel B shows the performances of quantity (variance risk exposure) - sorted portfolios. The first two rows summarize the returns and risk-adjusted returns of the portfolios. There is little difference in performance between stocks with a high-risk exposure and the ones with low-risk exposure, especially in excess returns. For risk-adjusted returns, while being statistically insignificant, stocks with high negative risk exposure slightly outperform stocks with small or positive variance risk exposure. This result is similar to the result of Chang, Christoffersen, and Jacobs (2013) that finds a weak relation between the market variance risk exposure if more

[8]See, for example, Coval and Shumway (2001), Ang, Hodrick, Xing, and Zhang (2006), Bali and Hovakimian (2009), Cremers, Halling, and Weinbaum (2015) and Chang, Christoffersen, and Jacobs (2013).

than a single month of data is used for estimation. The average stock with a high negative exposure to variance risk tends to be bigger and have a slightly lower market beta.

The focus of this paper is on whether there is an interactive or reinforcing relation between the price and quantity of variance risk. If there is an interactive effect, the long-short portfolio of a single sort will be stronger or weaker depending on the level of the other sort. In particular, the long-short portfolio returns of the risk exposure should be much stronger and bigger when the price of risk is highly negative. The long-short portfolio returns concerning the price of risk may be significant only for stocks whose variance risk exposure is highly negative. This possibility is evaluated by analyzing the performance of the double-sorted portfolios.

Panel A of Table III summarizes the performance of the price-quantity double-sorted portfolios, where the price and quantity of variance risk is estimated in the basic specification, as discussed in the previous section. All stocks in the sample are sorted independently by the price and quantity of variance risk and divided into four groups. That is, a total of 16 portfolios are formed based on the estimates of the price and quantity of variance risk. The value-weighted returns along with the four-factor returns of the subsequent month are evaluated. The table summarizes the time-series average of returns and its t-statistics computing using Newey-West methodology. Each row represents different levels of the variance exposure, and the columns represent different levels of the price of variance risk. For both the price and quantity of variance risk, high means highly negative and low means only slightly negative and does include positive.

The price of variance risk has the biggest impact on subsequent returns for stocks that are negatively exposed to variance risk. The spread between stocks with a small or positive price of variance risk and a negative price of risk is -1.61% in monthly excess returns and -1.35% in risk-adjusted returns. When stocks have a small or a positive variance beta, this spread decreases to a value that is close to zero. The difference in the spread of the price of risk sorted portfolios is -0.37% in excess returns and zero in risk-adjusted returns, respectively. Analogously, when the price of variance risk is highly negative, the spread between stocks with a high negative exposure to its variance risk and with small or positive exposure to variance risk is -0.92% in excess returns and -1.46% in risk-adjusted returns. When the price is small or close to zero, the spread also switches its sign but is essentially close to zero, with 0.33% in excess returns and -0.11% in risk-adjusted returns. Most importantly, the difference between the two spreads is highly statistically significant at 1% level for both the excess returns (1.24%) and risk-adjusted returns(1.36%). To summarize, the variance risk exposure mostly affects stock returns whose price of variance risk is highly negative. The price of risk matters most for stocks with a negative exposure to its variance risk. In other words, the interactive effect of the price and quantity is clearly observable.

Panel B summarizes the performance of the double-sorted portfolios using an alternative specification, where the quantity of variance risk is estimated from a single variable regression. This alternative specification is also reasonable because the variance shocks of individual stock can also be related to market variance shocks. This alternative specification somewhat decreases the statistical and economical significance. The difference in spread decreases to 1.04% in excess returns and 1.26% in risk-adjusted returns, but are still marginally statistically significant.

Table IV considers two alternative specifications. First, the price of risk is estimated in an alternative manner. This is because while the risk-neutral expectation of stock variance, estimated using option prices, is a forward-looking measure, RV is a historical measure. Therefore, alternatively, I fit a variance forecast model of

$$\sum_{k=1}^{2} 2RV_{i,t+k} = b_0 + b_1 RV_{i,t} + b_{22} \sum_{k=0}^{21} RV_{i,t-k} + e_{i,t+22}$$

where RV is the realized variance and the regression is estimated every month using the entire panel of stocks on a rolling basis. Furthermore, this alternative measure of the price of variance risk relieves the concern that the price of variance risk may simply a proxy for variance trends.

Panel A shows the performance when this alternative estimate of the price of variance risk is used instead. While the spread between stocks with a high and low price of risk decreases for stocks with a the negative variance risk exposure, this spread remains large both economically and statistically. The spread is 0.90% in excess returns and 1.32% in risk-adjusted returns. Panel B summarizes the performance of the double-sorted portfolios using equally-weighted returns. Overall, the spread decreases somewhat by equally weighting the stocks within each portfolio. This is in part because firm size is related to the sorting variables. Growth options are more likely going to be important for small stocks, and limited liability constraints are more likely to be binding. Also, statistically, small firms tend to have a more negative price of risk. For stocks with a high negative quantity of risk, the spread between low and high price of variance risk stocks decreases to -0.95% in monthly excess returns for stocks and -0.53% in risk-adjusted returns. However, similar to previous panels, the difference between the two groups (difference-in-difference) is still statistically significant at 0.71 % per month in excess returns and 0.58 % in risk-adjusted returns.

2. The Relation to the Market Variance Risk Exposure

The exposure to market variance risk closely relates to the exposure to own variance risk as individual stocks returns are exposed to market risk. One possibility is that market variance risk is the source that is essentially priced among individual stocks, and both the price and quantity of variance risk may be related to the market variance beta. If this is the case, the interactive relation between the price and quantity of variance risk presented in previous tables may simply be a result of it. This subsection tests the feasibility of this hypothesis. In several different ways, I show that individual variance risk that is unrelated to market variance risk is still important and is what essentially drives the interactive relation between the price and quantity of variance risk.

First, instead of using a two-variable regression, a three-variable regression including market excess returns, changes in VIX^2, and changes in implied variance of individual stocks is used to estimate the quantity of variance risk. The regression is given as,

$$R_{i,t} = \beta_{0,i} + \beta_{m,i}\ R_{m,t} + \beta'_{x,i}\ [VIX_t^2 - VIX_{t-1}^2] + \beta'_{v,i}\ [IV_{i,t} - IV_{i,t-1}] + \epsilon_{i,t}.$$

This regression is designed so that a stock's own variance risk exposure captures the risk exposure that is not driven by market variance shocks. To distinguish the variance risk exposure after controlling for market variance risk ($\beta'_{v,i}$) from the one before, I use a prime ($'$) whenever a three-factor model is used to control for market variance risk. I use the term the beta 'controlled' for market variance risk to denote this.

Panel A of Table V summarizes the performance of double-sorted portfolios when the market variance risk controlled beta is used. The spread between high and low variance risk exposure decreases for stocks with a high price of variance risk, but if we focus on the other dimension, for high market variance controlled beta stocks, the spread created by the price of variance risk is higher than before. Overall, the interactive relation is still present and statistically significant, which suggests that aggregate variance risk is not the key component that affects the relationship.

Second, control for market variance risk using a risk factor (FVIX) of Ang, Hodrick, Xing, and Zhang (2006). The risk-adjusted returns are computed as

$$R_{i,t} - (\hat{\beta}_{0,i} + \hat{\beta}_{m,i}\ R_{m,t} + \hat{\beta}_{x,i}\ FVIX_t),$$

where $R_{i,t}$ is the excess return of the stock, $R_{m,t}$ is the excess market returns, FVIX is the aggregate variance risk factor built using factor-mimicking portfolios as in Ang et. al. Panel B of Table V provides the performance of double-sorted portfolios after controlling for the aggregate variance risk factor. The table suggests that even controlling for the aggregate variance risk factor, the price of variance risk matters most for stocks with a negative variance risk exposure, and the risk exposure matters most for stocks with a high negative price of risk.

Finally, using a market model, I decompose both the price and quantity of variance risk into two components – one that represents those driven by market variance risk and the other that represent those from non-market variance risk. The second part, the component due to non-market variance risk, measures how the risk premium of a stock relates to variance of possible factors or systematic shocks excluding the market factor. The decomposition works in the following manner:

Assuming a market model, we have

$$R_{i,t} = a_i + \beta_{m,i} R_{m,t} + \epsilon_{i,t},$$

where returns are excess returns. If individual stock return $R_{i,t}$ follows a N-factor structure ($N > 1$), $a_i + \epsilon_{i,t}$ would capture the sum of the variations due to changes in factors other than the market factor and, also possibly, idiosyncratic shock. The market model also implies a one-factor structure for the variance process.

$$\Delta Var_t(R_{i,t+1}) = \beta_{m,i}^2 \Delta Var_t(R_{m,t+1}) + \Delta Var_t(\epsilon_{i,t}).$$

Similarly, $\Delta Var_t(\epsilon_{i,t})$ would be associated with all non-market variance shocks that affects the variance process of the stock. Thus, the variance risk of a stock i can be represented as the sum of variance risk due to changes in market variance and due to non-market variance shocks.

If we replace the changes in actual variance with option-implied variance, we can estimate the slope of the following regression to obtain the non-market related variance shock of stock i ($\hat{e}_{it}$). Also, the square of the market beta ($\beta_{m,i}^2$), or the variance process of stock i's exposure to market variance risk, can be accomplished by estimating the slope of

$$\Delta IV_{i,t} = \beta_{vx,i} \Delta VIX_t^2 + e_{it}. \tag{3}$$

We can run the above regression using, for example, one year of data, as in other regressions of this paper. The implicit assumption is that $\hat{\beta}_{m,i}^2 = \hat{\beta}_{vx,i}$ would hold, approximately.

Then, non-market variance risk of stock i is $\hat{e}_{i,t}$, and the exposure to non-market variance risk ($\beta_{e,i}$) can be estimated as the slope of a regression of individual stock returns on non-market variance risk.

$$R_{i,t} = \beta_{0,i} + \beta_{m,i}\ R_{m,t} + \beta_{e,i}\ \hat{e}_{i,t} + \epsilon_{i,t} \tag{4}$$

The slope measures how a stock return reacts to changes in variance but that is unrelated to the market variance shocks.

Also, the price of variance risk of individual stock returns can then be represented as

$$\begin{aligned}\mathrm{Cov}_t(-SDF_{t+1}, \Delta Var_t(R_{i,t+1})) = \beta_{vx,i}\mathrm{Cov}_t(-SDF_{t+1}, \Delta\mathrm{Var}_t(R_{m,t+1})) \\ + \mathrm{Cov}_t(-SDF_{t+1}, e_{i,t+1}),\end{aligned}$$

where SDF is the stochastic discount factor. The price of variance risk due to non-market factors can be written as,

$$\widehat{Cov}_t(-SDF_{t+1}, \Delta e_{i,t+1}) = (RV_{i,t} - IV_{i,t}) - \hat{\beta}_{vx,i}(RV_{m,t} - VIX_t^2),$$

which is equivalent to writing as

$$\hat{\lambda}_{e,i,t} = \hat{\lambda}_{v,i,t} - \hat{\beta}_{vx,i,t}\hat{\lambda}_{x,t}, \tag{5}$$

where $\lambda_{x,t}$ is the price of market variance risk. I call $\lambda_{e,i,t}$ as the price of non-market variance risk in the sense that it is driven by second moment of factors other than the market factor. This price of risk is computed by subtracting the price of market variance risk from the price of variance risk of individual stock returns with the relevant scaler. The goal is to evaluate whether the interactive relation is mainly driven either by the price or by quantity that is related to market variance risk.

Table VI shows the performance of the price and quantity of non-market variance risk double-sorted portfolios. This table shows that the high minus low spread for the price-sorted portfolios is still bigger for stocks that have a more negative non-market variance risk exposure. Also, the spread for the quantity-sorted portfolios is bigger for stocks whose price of non-market variance risk is more negative. Comparing it to the earlier table where stocks are double-sorted by the price and quantity of total variance risk, the difference in the spread is slightly smaller with 0.85% in excess returns and 1.13 % in risk-adjusted returns. However, the difference in spreads is still statistically significant, which suggest that the interactive relation is mainly driven by non-market variance risk.

In short, I find no evidence that market variance risk is the only variance factor that affects individual stock returns. Furthermore, the result of this section suggests that even the non-market factor based variance risk is priced and determines the short-term variation of individual stock returns.

3. The Price-Quantity Combination

Earlier tables clearly confirm the interactive and multiplicative relation between the price and quantity of variance risk. Furthermore, the model as well as the 'beta representation' directly implies that the product of the price and the quantity should represent a fraction of the risk premium of the stock. Therefore, a natural next step is to see whether the product directly explains future individual stock returns.

However, multiplying two noisy estimates may create an empirical problem, especially when the estimation error switches the sign of the estimates. As the model suggests, the risk premium implied in individual stock returns is high when both the price of variance risk and the variance risk exposure are negative. The empirical issue is that several small stocks tend to be illiquid and is more likely to be subject to microstructure noise. Therefore, the RV of these stocks will be over-estimated, which means that the price of variance risk is likely to be estimated positively. At the same time, small stocks are more likely to have a positive variance beta. While these stocks have smaller prices of variance risk, they may look as if they are a high risk premium stock if their variance risk exposure is positive.

This is shown in Table VII. The price and quantity of variance risk are estimated as in the basic specification and, similar to previous tables, the returns of the subsequent month are evaluated. Panel A shows the results when portfolios are sorted by the product of the price and quantity. The Panel suggests that stocks with a high variance risk premium (i.e., high positive value of the product) tend to have higher returns. While the difference in the 5-1 spread is statistically significant with a spread of 0.71% in excess returns and 0.59% in risk-adjusted returns.

However, this spread may be misleading if there are stocks with both a high positive price and quantity of risk. Therefore, as an alternative specification, both the price and quantity of variance risk is truncated (set to zero when positive) at zero. Panel B summarizes the performance when two truncated variables are multiplied by each other. Overall, doing so has a very small influence. The 5-1 spreads are similar to 0.71% in excess returns and 0.63

Finally, when both market variance risk and non-market variance risk is priced, we expect that the risk premium due to these two sources of risk to be priced separately. Since we can estimate the price of market variance risk as well as the price of non-market variance risk, we can express the expected return of the stock as a linear combination of the two prices of risk. Both the market variance beta ($\beta_{x,i}$) and the individual variance beta ($\beta_{v,i}$) are from a three-factor model.

Panel C of Table VII summarizes the performance of the portfolios of the risk premium implied in stocks assuming a two-factor structure. Overall, the spread still remains comparable in size (0.64% in excess returns and 0.54% in risk-adjusted returns), and is statistically significant. This is in part because while adding market variance risk should in principal increases the explanatory power, at the same time, adding more estimates increases estimation error. These results confirm that non-market variance risk is important that affects the cross-section of individual stock returns.

V. Understanding the Price and Quantity of Variance Risk

1. Determinants of the Variance Risk Exposure

A simple risk-return tradeoff implies that stock returns should be negatively related to its variance movements. However, the relation can also be positive for stocks of a particular type of firms. An increase in variance can increase the equity value of a firm if it has valuable real options. These companies are likely to be smaller, spend more on research and development, and have higher capital expenditures. Limited liability also implies a convex payoff of the equity. The asymmetry is going to be stronger for firms with high growth options or financial constraints. This subsection investigates the possible determinants of the variance risk exposure. The following variables, retrieved from Compustat North America, are used as potential candidates.

Market Value of Equity The market value of equity is a proxy for firm size and is computed by multiplying the end-of-year stock price (PRCC_F) with the number of shares outstanding (CSHO).

Book Value of Equity The book value of equity is also a proxy for firm size and is the value of common/ordinary equity (CEQ).

Leverage Leverage is a proxy for firm leverage, corresponding to the moneyness of options. It is the ratio of liability to the sum of market value of equity (as defined above) and liability. Liability is computed as the sum of debt in current liabilities (DLC) and long-term debt (DLTT). Whether the leverage should positively or negatively affect the risk exposure is uncertain. Higher leverage would imply stronger leverage effect (negative exposure), but at the same time, it would imply higher moneyness (positive exposure)

Interest Coverage Ratio Interest coverage ratio is used as a proxy for the level of debt. It is the ratio of earnings before interest and tax (EBIT) to the amount of the interest expense (XINT).

Growth Option Growth option is the percentage of firm value arising from grown opportunities as to the total market value and follows the methodology from Cao, Simin, and Zhao (2008) and Trigeorgis and Lambertides (2014). In particular, Trigeorgis and Lambertides show a negative relation between firm's growth options and subsequent stock returns. Cao et. al. explains the time-trend in idiosyncratic volatility using growth options. The growth option of a firm i is computed as

$$GO_{i,t} = 1 - CF_{i,t}/\mathrm{WACC}_{i,t} \times 1/MV_{i,t}$$

where $CF_{i,t}$ is the free cash flow of a firm i, $\text{WACC}_{i,t}$ is the weighted average cost of capital of firm i, and MV is the market value of equity. The value of the asset in place $CF_{i,t}/\text{WACC}_{i,t}$ is the perpetual discount stream of firm cash flow under a no-growth policy.

Capital Expenditure Capital expenditure is the ratio of capital expenditure as a proportion of property, plant, and equipment (PPENT). This is also used as a proxy for firm's growth option. Firms that spend more as a capital expenditure is more likely to grow in the future. These firms are also likely to hold valuable real options.

R&D R&D expense is computed by dividing R&D expenditure by firm's total assets. High R&D expense implies a higher possibility of holding valuable real options.

Profitability Profitability is defined as the earnings before interest and tax (EBIT) divided by total assets (AT).

While it is commonly acknowledged that high variance is what investors dislike, the relation between returns and variance shocks should be negative. However, empirically, many of the individual stock returns are positively related to its variance shocks. This section analyzes the determinants of the variance risk exposure and studies whether they are related to firm-specific financial ratios, as discussed in the previous section. Also, as the variance risk exposure is a transformation of the third moment of the market, evaluating the determinants of the risk exposure may also help understanding why the skewness of individual stock returns is positive for many stocks[9].

To identify the determinants of the variance risk exposure, I run Fama-MacBeth regressions using the variables mentioned in the previous section. That is, I first run a cross-sectional predictive regression of variance risk exposure on various determinants and compute the mean and standard errors for the coefficients. Table VIII summarizes the results of the regressions.

Firstly, the regressions suggest that risk exposure is strongly related to firm size. Large firms with a high market or book value of equity tend to be more negatively exposed to variance risk. The relation between size and risk exposure also explains why the spread in the price sorted

[9] See, for example, Conine and Tamarkin (1981), Mitton and Vorkink (2007), Amaya, Christoffersen, Jacobs, and Vasquez (2015), and Del Viva, Kasanen, and Trigeorgis (2017), among others.

portfolio is much smaller for equally-weighted portfolios. Small firms, which are more likely to have more growth options tend to have a positive exposure to variance risk.

Second, firms' growth options tend to be positively related to the variance risk exposure. The coefficients of growth options, capital expenditures, and R&D expense are all positive and statistically significant. Higher R&D expense and capital expenditures are likely to generate real options, and the value of the option will increase when variance increases. A higher option value will also increase the equity value. As a result, the variance risk exposure on these stocks is more likely to be positive. The growth option and higher capital expenditure may also imply more asymmetry in the equity stakeholder's payoff. As a result of the option-like payoff of the equity, a higher growth option will overall increase the equity value when the economic condition of the firm becomes uncertain. These stocks tend to have positive exposure to variance risk.

Third, there is only weak evidence that suggests financial constraint is related to risk exposure. The equity of firms that are more likely be constrained is likely to have a positive risk exposure because of limited liability. Leverage is potentially a poor variable that measures financial constraints and has little influence on the asymmetry. Interest coverage ratio, which measures a firm's ability to pay interest expenses also has no effect on the variance risk exposure. However, profitability is related to future variance risk exposure. Firms that have high profitability have a more negative variance beta, consistent with the hypothesis. Overall, the results show mixed evidence.

2. Factor Skewness

The model of this paper suggests that when factor returns are skewed, stocks that are exposed to the factor are simultaneously exposed to higher moment risk of the factor. Therefore, variance risk is priced for factors that are more skewed either positively or negatively. In this section, we investigate relations among factor betas, the price of variance risk, the quantity of variance risk, and factor skewness.

In this discussion, we need to distinguish market betas other factor betas because factor betas are not necessarily positive and symmetric– the number of stocks with a positive beta is not much different from stocks with a negative beta. Therefore, how the skewness of market factor influences individual stock returns is much different from other factors. Since stocks have a positive market beta, an increase in market variance will increase individual stock return variance. The increase in individual stock return variance is greater for stocks with a higher market beta. Therefore, when market variance risk is priced, the price of variance risk would be more negative for these stocks. On the other hand, for other factors, an increase in factor variance would have the largest influence on both negative and positive beta stocks, but less to stocks that are unexposed to the risk factor. Therefore, when variance risk of a factor is priced, the price of variance risk would be most negative for stocks on either end of the distribution, but would be closer to zero for stocks that are unexposed to the factor.

When a factor variance risk is priced, the stock's exposure to factor variance risk largely depends on the stock's factor beta and factor skewness. A negatively skewed factor, for example, momentum, means that when factor variance increases, the factor returns are negative. The returns of past losers exceed the returns of past winners. Therefore, past winners will have a more negative variance risk exposure than losers. Hence, if we sort stocks by past performance, we will observe a negative relationship between momentum beta and variance beta. A positively skewed factor, for example, market variance shocks, means that an increase in variance-of-variance leads high market variance beta stocks to outperform low market variance beta stocks. The quantity of variance risk will be increasing in market variance beta.

Table IX summarizes how the price and quantity of variance risk differ from the factor beta sorted portfolios. The price and quantity estimates along with the Newey-West standard errors are reported. The results are meant to be indicative since the estimates have strong autocorrelation by construction. Stocks are sorted by factor betas, and decile portfolios are formed. The average of the estimates is reported in the table. Panel A summarizes the statistics for the market beta sorted portfolios. Consistent with the argument, high beta stocks have a more negative price of variance risk. They also have a smaller quantity of variance risk. This is in part expected since stocks with higher beta have more volatile variance movement. When we run regressions, if the independent variable is more volatile by construction, the slope will be smaller in magnitude.

Panel D and E summarize the results for momentum and FVIX factors. Among the factors considered, these are the two factors that are mostly skewed. Skewness is computed every month using daily factor returns, and the average of the skewness along with the t-statistics is reported. For both of these classifications, both stocks with a high negative and positive beta tend to have a more negative price of variance risk. However, the spread in the quantity of variance risk have the opposite signs. For momentum, which is negatively skewed, stocks that have positive momentum betas have more negative quantity of variance risk. For the market variance factor, stocks with positive market variance betas have smaller (positive) quantity of variance risk.

Panel B and C show the results for value and size factors. These two factors are unskewed during the sample period of study. The finding in Table VIII that small firms tend to have a smaller quantity of variance risk is also confirmed in this table. Also, small and growth stocks tend to have a more negative price of variance risk. However, the fact that large or value stocks do not have a negative price of variance risk suggest that variance risk of these two factors may not be priced among individual stocks.

VI. Conclusion

This paper investigates the interactive role of the price and quantity of variance risk. The findings suggest that these two dimensions are interrelated. When stocks are sorted by the quantity of variance risk of its returns, those stocks that are more exposed to its variance risk tend to have higher subsequent returns. However, this effect is substantially stronger for stocks whose price of variance risk is also highly negative. For stocks with a small or positive price, the risk exposure does not matter. Similarly, the price of variance risk matters most for stocks that have high negative exposure to variance risk. Conclusively, the price of individual stock variance risk and the quantity of variance risk have an interactive relation.

This paper finds that the interactive relations are mainly driven by variance risk of factors other than the market factor. The interactive relation exists even when market variance risk is controlled or when stocks are sorted by the price and quantity of non-market variance risk. The results suggest that variance risk of factors other than the market factor does affect individual stock returns. The fact that both the price of variance risk as well as the underlying factor (i.e., variance shocks) is observable makes the new way of representing the risk premium of an asset, as suggested in this paper, a useful way of explaining the time-variation of individual stock returns.

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A. Appendix

We have assumed that the price process of a stock i (S_{it}) be the sum of the stock-specific systematic risk and unpriced idiosyncratic risk.

$$\frac{dS_{i,t}}{S_{i,t}} = a_i dt + b_i dY_{i,t} + \sigma_{idio} dW_{i,t}^{idio},$$

where $dY_{i,t} = \sum_n \beta_{i,n} dF_n$. and

$$\begin{aligned} dF_{n,t} &= \mu_{n,t}dt + \sqrt{V_{nt}}(\rho_n dW_{n,t}^v + \sqrt{1-\rho_n^2} dW_{n,t}^o) \\ dV_{n,t} &= \theta_n dt + \sigma_{nv} dW_{n,t}^v, \end{aligned}$$

We transform the factor returns and variances as two stochastic processes for stock i, Y_i and the variance of Y_i (Called $\bar{V}_i$). Then,

$$dY_{i,t} = b_i \sum_n \beta_{i,n}\mu_{n,t}dt + b_i \sum_n \beta_{i,n}(\sqrt{V_{nt}}(\rho_n dW_{n,t}^v + \sqrt{1-\rho_n^2} dW_{n,t}^o))$$

Let $\bar{\mu}_{i,t} = b_i \sum_n \beta_{i,n}\mu_{n,t}$, and $\bar{V}_{i,t} = \sum_n b_i^2 \beta_{i,n}^2 V_{nt}$. Then, $d\bar{V}_{i,t} = \sum_n b_i^2 \beta_{i,n}^2 dV_{nt}$. Let $\bar{\theta}_{i,t} = b_i^2 \sum_n \beta_{i,n}^2 \theta_n$ and $\bar{\sigma}_i^2 = \sum_n (b_i^2 \beta_{i,n}^2 \sigma_{n,v})^2$. Then, the above equations can be rewritten as,

$$\begin{aligned} dY_{i,t} &= \bar{\mu}_{i,t}dt + \sqrt{\bar{V}_{i,t}} \bar{\rho}_i dW_{i,t}^v + \sqrt{1-\bar{\rho}_i^2} dW_{i,t}^o \\ d\bar{V}_{i,t} &= \bar{\theta}_i dt + \bar{\sigma}_i dW_{i,t}^v, \end{aligned}$$

where $\bar{\rho}_i$ measures the correlation between $dY_{i,t}$ and $d\bar{V}_{i,t}$. Following the same logic as in the main text, we can show that

$$E_t[R_{i,t+1}] = \beta_{Y,i,t}\lambda_{Y,i,t} + \lambda_{o,t},$$

where $\lambda_{Y,i,t}$ is the price of systematic variance risk of stock i, $\lambda_{Y,i,t}$ is the price of risk due to orthogonal shocks, and $\beta_{Y,i,t}$ is the slope of a hypothetical regression of stock i's return on the variance of latent systematic risk dY_i since

$$\begin{aligned} Cov(\frac{dS_{it}}{S_{it}}, dV_{i,t})/Var(dV_{i,t}) &= Cov(b_i dY_{it}, dV_{i,t})/Var(dV_{i,t}) \\ &= b_i \rho_i \sqrt{V_{it}} \sigma_{vi} / \sigma_{vi}^2 \\ &= b_i \rho_i \sqrt{V_{it}} / \sigma_{vi} \end{aligned}$$

This equation is equivalent to the representation of Result 2 since

$$\begin{aligned} \lambda_{v,i,t} \times \beta_{v,i,t} &= Cov(\frac{dS_{it}}{S_{it}}, b_i^2 dV_{i,t})/Var(b_i^2 dV_{i,t}) \times b_i^2 \lambda_{Y,i,t} \\ &= Cov(\frac{dS_{it}}{S_{it}}, dV_{i,t})/Var(dV_{i,t}) \times \lambda_{Y,i,t} \\ &= \beta_{Y,i,t} \lambda_{Y,i,t} \end{aligned}$$

B. Tables

Table I
Summary Statistics

This table provides the summary statistics for several variables of interest. Implied variance (IV) is the average option-implied variance of at-the-money call and put options. Historical realized variance (RV) is the variance of daily stock returns over the past month. The variance risk exposure ($\beta_{v,i}$) is the slope of the regression,

$$R_{i,t} = \beta_{0,i} + \beta_{m,i} R_{m,t} + \beta_{v,i} \left[IV_{i,t} - IV_{i,t-1} \right] + \epsilon_{i,t}$$

where $R_{i,t}$ is the return of stock i over the past month, and $R_{m,t}$ is the value-weighted excess market return. The mean, median, and standard deviations for the sample is computed and the table compares them with those from the entire CRSP database.

	Entire CRSP	Sample				
	Mean	Mean	Median	St Dev	% Negative	% Positive
IV_i	-	0.018	0.011	0.024	-	100
RV_i	-	0.014	0.007	0.021	-	100
$RV_i - IV_i$	-	-0.004	-0.003	0.024	0.810	0.190
$\hat{\beta}_{v,i}$	-	-1.593	-1.180	2.401	0.787	0.213
Market Beta	1.097	1.159	1.101	0.484	0.006	0.994
Market Cap	2.721B	11.904B	2.881B	3.288B	-	100
# of Stocks	4,948	821	815	320	-	-
# of Stock-Month		196,934				

Table II
The Characteristics and Performance of Single Variable Sorted Portfolios

This table summarizes the performance and characteristics of the price and quantity of variance risk sorted portfolios. Excess returns, risk adjusted (value, size, momentum) returns of the subsequent month, the contemporaneous estimates of the price and quantity of variance risk, firm size, and market beta of each of the portfolios are summarized in the table.

A. Portfolios Sorted by the Price of Variance Risk ($\hat{\lambda}_{v,i}$)

	Returns	4-Factor α_4	$\hat{\lambda}_{v,i}$	$\hat{\beta}_{v,i}$	Size	Market Beta
Quartile 1	0.98	0.55	−0.020	−0.990	3.70B	1.443
	(1.68)	(2.12)				
Quartile 2	0.92	0.29	−0.006	−1.706	7.35B	1.116
	(2.49)	(2.46)				
Quartile 3	0.73	0.08	−0.002	−2.209	10.61B	0.941
	(2.53)	(0.90)				
Quartile 4	0.22	−0.20	0.015	−1.826	9.03B	0.945
	(0.63)	(−2.23)				
Q4-Q1	−0.76*	−0.75**	0.035	−0.836	5.33B	−0.498
	(−1.92)	(−2.57)				

B. Portfolios Sorted by the Variance Risk Exposure ($\hat{\beta}_{v,i}$)

	Returns	4-Factor α_4	$\hat{\lambda}_{v,i}$	$\hat{\beta}_{v,i}$	Size	Market Beta
Quartile 1	0.73	0.22	−0.001	−4.772	9.96B	0.932
	(2.51)	(2.60)				
Quartile 2	0.47	0.01	−0.003	−2.205	9.68B	1.146
	(1.13)	(0.14)				
Quartile 3	0.62	−0.03	−0.006	−0.776	6.46B	1.176
	(1.55)	(−0.18)				
Quartile 4	0.67	−0.03	−0.006	0.866	4.67B	1.028
	(1.95)	(−0.32)				
Q4-Q1	−0.06	−0.25	−0.005	5.638	−5.29B	0.096
	(−0.31)	(−1.24)				

Table III
The Performance of Price-Quantity of Variance Risk Double Sorted Portfolios

This table summarizes the value-weighted returns of the quartile portfolios double sorted by the price and quantity of individual stock variance risk. In panel A, the variance risk exposure ($\beta_{v,i}$) is the slope of the regression,

$$R_{i,t} = \beta_{0,i} + \beta_{m,i} R_{m,t} + \beta_{v,i}\,[IV_{i,t} - IV_{i,t-1}] + \epsilon_{i,t},$$

while in panel B, it is the slope of the regression without the market factor. That is,

$$R_{i,t} = \beta''_{0,i} + \beta''_{v,i}\,[IV_{i,t} - IV_{i,t-1}] + \epsilon''_{i,t}.$$

Excess returns and risk adjusted (value, size, momentum) returns of the subsequent month are provided. Portfolios are first constructed after sorting the stocks with respect to the price and then by the risk exposure.

A. Two Variable Regressions (with Market Factor)

		Variance Risk Exposure ($\hat{\beta}_{v,i}$)									
		Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
		Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4
$\hat{\lambda}_{v,i}$	Q1 (Negative)	1.88	1.30	0.56	0.16	0.79	0.01	0.96	−0.16	−0.92	−1.46***
		(2.94)	(3.43)	(0.90)	(0.54)	(1.24)	(0.02)	(1.77)	(−0.47)	(−1.62)	(−3.11)
	Q2	0.90	0.21	0.70	0.13	0.83	−0.02	0.81	0.08	−0.09	−0.13
		(2.49)	(1.18)	(1.67)	(0.70)	(1.89)	(−0.10)	(2.02)	(0.39)	(−0.33)	(−0.50)
	Q3	0.81	0.21	0.70	0.13	0.83	−0.02	0.81	0.08	−0.09	−0.13
		(2.85)	(1.18)	(1.67)	(0.70)	(1.89)	(−0.10)	(2.02)	(0.39)	(−0.33)	(−0.50)
	Q4 (Positive)	0.27	−0.05	0.14	−0.35	0.57	−0.10	0.59	−0.16	0.33	−0.11
		(0.85)	(−0.41)	(0.34)	(−1.68)	(1.49)	(−0.52)	(1.66)	(−0.64)	(1.07)	(−0.39)
	Q4-Q1	−1.61***	−1.35***	−0.42	−0.50	−0.21	−0.11	−0.37	0.00	1.24**	1.36***
		(−3.03)	(−3.06)	(−0.95)	(−1.49)	(−0.47)	(−0.28)	(−0.86)	(−0.00)	(2.22)	(2.74)

B. One Variable Regression (without Market Factor)

		Single-variable Variance Risk Exposure ($\hat{\beta}''_{v,i}$)									
		Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
		Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4
$\hat{\lambda}_{v,i}$	Q1 (Negative)	1.75	1.21	0.41	0.08	0.83	−0.02	1.08	0.01	−0.68	−1.21**
		(2.54)	(2.97)	(0.62)	(0.25)	(1.41)	(−0.05)	(1.91)	(0.02)	(−1.18)	(−2.35)
	Q2	0.99	0.27	0.70	0.15	0.87	−0.01	0.69	−0.14	−0.30	−0.41
		(2.75)	(1.55)	(1.59)	(0.82)	(1.95)	(−0.04)	(1.71)	(−0.69)	(−1.02)	(−1.55)
	Q3	0.78	0.27	0.70	0.15	0.87	−0.01	0.69	−0.14	−0.30	−0.41
		(2.75)	(1.55)	(1.59)	(0.82)	(1.95)	(−0.04)	(1.71)	(−0.69)	(−1.02)	(−1.55)
	Q4 (Positive)	0.23	−0.08	0.26	−0.21	0.57	−0.36	0.59	−0.02	0.36	0.06
		(0.72)	(−0.70)	(0.61)	(−1.03)	(1.57)	(−1.59)	(1.62)	(−0.12)	(1.18)	(0.24)
	Q4-Q1	−1.52**	−1.30***	−0.15	−0.30	−0.26	−0.34	−0.49	−0.03	1.04*	1.26**
		(−2.54)	(−2.74)	(−0.33)	(−0.77)	(−0.61)	(−0.99)	(−1.16)	(−0.10)	(1.81)	(2.26)

Table IV
The Performance of Price-Quantity of Variance Risk Double Sorted Portfolios

This table summarizes the value-weighted returns of the quartile portfolios double sorted by the price and quantity of individual stock variance risk. In panel A, the price of variance risk is estimated as the difference between the option-implied variance and a variance forecasted formed based on the model

$$RV_{i,t+1,t+22} = b_0 + b_1 RV_{i,t} + b_{22} RV_{i,t-22,t} + e_{i,t+22}.$$

The price of variance risk is then,

$$\hat{\lambda}'_{i,t} = IV_{i,t} - E_t[RV_{t+1,t+22}].$$

The variance risk exposure is estimated using the basic specification, as the slope of

$$R_{i,t} = \beta_{0,i} + \beta_{m,i} R_{m,t} + \beta_{v,i}\left[IV_{i,t} - IV_{i,t-1}\right] + \epsilon_{i,t}.$$

Panel B summarizes the results of the basic specification (Panel A of Table III) using equally-weighted portfolios.

A. Price of Risk Estimated by the Variance Forecast ($\hat{\lambda}'_{v,i}$)

		Variance Risk Exposure ($\hat{\beta}_{v,i}$)									
		Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
		Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4
$\hat{\lambda}'_{v,i}$	Q1 (Negative)	1.21	0.79	0.43	0.10	0.89	0.34	0.41	−0.56	−0.86	-1.27^{**}
		(1.79)	(1.95)	(0.54)	(0.29)	(1.42)	(0.77)	(0.62)	(−1.22)	(−1.52)	(−2.14)
	Q2	1.16	0.63	0.64	0.03	0.81	0.08	0.88	0.00	−0.28	-0.63^{**}
		(2.42)	(2.85)	(1.50)	(0.12)	(1.69)	(0.38)	(2.11)	(0.02)	(−0.74)	(−2.02)
	Q3	0.77	0.63	0.64	0.03	0.81	0.08	0.88	0.00	−0.28	-0.63^{**}
		(2.58)	(2.85)	(1.50)	(0.12)	(1.69)	(0.38)	(2.11)	(0.02)	(−0.74)	(−2.02)
	Q4 (Positive)	0.54	0.04	0.57	−0.06	0.52	−0.43	0.58	0.09	0.04	0.05
		(2.10)	(0.28)	(1.68)	(−0.38)	(1.59)	(−2.93)	(1.91)	(0.56)	(0.19)	(0.29)
	Q4-Q1	−0.70	−0.70	0.14	−0.16	−0.38	−0.77	0.17	0.64	0.90^{*}	1.32^{**}
		(−1.19)	(−1.53)	(0.22)	(−0.39)	(−0.76)	(−1.55)	(0.31)	(1.28)	(1.66)	(2.11)

B. Equally-weighted Portfolios

		Variance Risk Exposure ($\hat{\beta}_{v,i}$)									
		Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
		Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4
$\hat{\lambda}_{v,i}$	Q1 (Negative)	1.41	0.48	1.03	0.46	0.81	0.06	0.91	−0.08	−0.50	-0.56^{*}
		(2.93)	(1.71)	(1.68)	(1.79)	(1.43)	(0.21)	(1.73)	(−0.28)	(−1.45)	(−1.68)
	Q2	1.07	0.25	0.81	0.07	0.91	0.11	0.88	0.01	−0.18	−0.24
		(2.98)	(1.54)	(2.02)	(0.52)	(2.33)	(0.81)	(2.34)	(0.05)	(−1.05)	(−1.38)
	Q3	0.60	0.25	0.81	0.07	0.91	0.11	0.88	0.01	−0.18	−0.24
		(2.08)	(1.54)	(2.02)	(0.52)	(2.33)	(0.81)	(2.34)	(0.05)	(−1.05)	(−1.38)
	Q4 (Positive)	0.46	−0.04	0.59	0.08	0.41	−0.39	0.67	−0.02	0.21	0.03
		(1.41)	(−0.37)	(1.54)	(0.63)	(1.04)	(−2.42)	(1.89)	(−0.12)	(1.17)	(0.14)
	Q4-Q1	-0.95^{**}	−0.53	−0.45	−0.38	−0.40	-0.44^{*}	−0.24	0.06	0.71^{*}	0.58^{*}
		(−2.50)	(−1.65)	(−1.24)	(−1.58)	(−1.24)	(−1.73)	(−0.72)	(0.21)	(1.89)	(1.73)

Table V
Price-Quantity of Variance Risk Double Sorted Portfolios Controlling for Market Variance Risk

This table summarizes the value-weighted returns of the price and quantity of variance risk double-sorted portfolios controlling for market variance risk. Panel A summarizes the results, where the variance risk exposure is estimated using a three factor model

$$R_{i,t} = \beta'_{0,i} + \beta'_{m,i} R_{m,t} + \beta'_{v,i}(IV_{i,t} - IV_{i,t-1}) + \beta'_{x,i}(VIX_t^2 - VIX_{t-1}^2) + \epsilon_t$$

where $R_{m,t}$ is the market excess return, IV_i is the option-implied variance of stock i, and VIX is the volatility index. Panel B summarizes the performance of the basic specification (Panel A of Table III), but the returns are adjusted (α_2) by the market variance risk factor (FVIX) of Ang, Hodrick, Xing, and Zhang (2006). The CAPM α is also provided for comparison.

A. Variance Risk Exposure Estimated using Three Variable Regression

		Variance Risk Exposure ($\hat{\beta}'_{v,i}$)									
		Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
		Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4
	Q1 (Negative)	1.78	1.07	0.61	0.30	0.93	0.27	0.69	−0.32	−1.09**	−1.39***
		(3.08)	(2.71)	(0.82)	(1.03)	(1.61)	(0.67)	(1.28)	(−0.80)	(−2.43)	(−2.85)
	Q2	0.94	0.32	0.53	−0.11	0.87	0.04	0.98	0.20	0.04	−0.12
		(2.60)	(1.72)	(1.23)	(−0.59)	(2.20)	(0.19)	(2.37)	(1.02)	(0.13)	(−0.50)
	Q3	0.72	0.32	0.53	−0.11	0.87	0.04	0.98	0.20	0.04	−0.12
$\hat{\lambda}_{v,i}$		(2.69)	(1.72)	(1.23)	(−0.59)	(2.20)	(0.19)	(2.37)	(1.02)	(0.13)	(−0.50)
	Q4 (Positive)	0.34	0.03	0.10	−0.11	0.37	−0.41	0.57	−0.10	0.24	−0.14
		(1.03)	(0.23)	(0.24)	(−0.65)	(0.90)	(−2.10)	(1.81)	(−0.56)	(0.95)	(−0.64)
	Q4-Q1	−1.44***	−1.04**	−0.51	−0.41	−0.56	−0.68*	−0.11	0.22	1.33***	1.26**
		(−2.91)	(−2.26)	(−1.01)	(−1.31)	(−1.38)	(−1.93)	(−0.29)	(0.55)	(2.80)	(2.52)

B. Market Variance Risk Adjusted Returns

		Variance Risk Exposure ($\hat{\beta}_{v,i}$)									
		Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
		CAPM-α	Var-α_2	CAPM-α	Var-α_2	CAPM-α	Var-α_2	CAPM-α	Var-α_2	CAPM-α	Var-α_2
	Q1 (Negative)	1.22	1.18	−0.20	−0.05	0.03	0.16	0.11	0.27	−1.11**	−0.90*
		(2.72)	(2.73)	(−0.65)	(−0.17)	(0.09)	(0.49)	(0.38)	(0.90)	(−2.15)	(−1.76)
	Q2	0.18	0.16	0.06	0.10	0.12	0.13	0.03	0.14	−0.15	−0.02
		(1.15)	(1.00)	(0.36)	(0.56)	(0.74)	(0.75)	(0.15)	(0.62)	(−0.51)	(−0.07)
	Q3	0.17	0.16	0.06	0.10	0.12	0.13	0.03	0.14	−0.15	−0.02
$\hat{\lambda}_{v,i}$		(1.45)	(1.00)	(0.36)	(0.56)	(0.74)	(0.75)	(0.15)	(0.62)	(−0.51)	(−0.07)
	Q4 (Positive)	−0.10	−0.17	−0.33	−0.42	0.08	−0.01	−0.03	−0.07	0.07	0.11
		(−0.75)	(−1.25)	(−1.65)	(−2.09)	(0.43)	(−0.04)	(−0.13)	(−0.25)	(0.25)	(0.37)
	Q4-Q1	−1.32**	−1.35***	−0.13	−0.37	0.06	−0.17	−0.14	−0.34	1.18**	1.01*
		(−2.59)	(−2.74)	(−0.36)	(−1.03)	(0.16)	(−0.48)	(−0.44)	(−0.99)	(2.17)	(1.86)

Table VI
Price- Quantity of Non-market Variance Risk Double Sorted Portfolios

This table summarizes the value-weighted returns of the price and quantity of non-market variance risk double-sorted portfolios. The price of non-market variance risk ($\lambda_{o,t}$) is estimated as

$$\lambda_{o,t} = \lambda_{v,i,t} - \hat{\beta}_{vx,i}\lambda_{x,t}$$

, and the quantity ($\beta_{i,o}$) is estimated from

$$R_{i,t} = \alpha + \beta_{m,i}R_{m,t} + \beta'_{v,i}\hat{e}_t + \epsilon_t$$

where

$$IV_{i,t} - IV_{i,t} = \beta_{vx,0} + \beta_{vx,i}(VIX_t^2 - VIX_{t-1}^2) + e_t,$$

$\lambda_{x,t}$ is the price of market variance risk, $IV_{i,t}$ is the option implied variance of stock i, and VIX is the square of the market volatility index.

		Non-market Variance Risk Exposure ($\hat{\beta}_{e,i}$)									
		Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
		Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4	Returns	α_4
	Q1 (Negative)	1.34	0.99	0.69	0.29	1.13	0.31	0.67	-0.40	-0.67	-1.39^{***}
		(2.18)	(2.58)	(0.95)	(0.87)	(1.68)	(0.84)	(1.21)	(-1.01)	(-1.31)	(-2.69)
	Q2	0.93	0.23	0.57	-0.25	0.94	0.13	0.76	0.03	-0.17	-0.20
		(2.58)	(1.51)	(1.39)	(-1.60)	(2.37)	(0.80)	(1.83)	(0.15)	(-0.61)	(-0.71)
$\hat{\lambda}_{o,i}$	Q3	0.60	0.23	0.57	-0.25	0.94	0.13	0.76	0.03	-0.17	-0.20
		(2.24)	(1.51)	(1.39)	(-1.60)	(2.37)	(0.80)	(1.83)	(0.15)	(-0.61)	(-0.71)
	Q4 (Positive)	0.30	0.05	0.24	-0.15	0.57	-0.33	0.48	-0.21	0.18	-0.26
		(0.94)	(0.31)	(0.75)	(-0.75)	(1.50)	(-1.94)	(1.50)	(-1.06)	(0.72)	(-1.06)
	Q4-Q1	-1.04^{*}	-0.94^{**}	-0.44	-0.44	-0.55	-0.64	-0.19	0.19	0.85^{*}	1.13^{*}
		(-1.86)	(-2.01)	(-0.80)	(-1.11)	(-1.12)	(-1.65)	(-0.41)	(0.41)	(1.71)	(1.91)

Table VII
Portfolios Sorted by the Price-Quantity Combination

This table summarizes the value-weighted returns of the single-sorted portfolios. Stocks in the sample are sorted by the combination of the price and quantity of variance risk. Panel A summarizes the result when stocks are sorted by the product of the price and quantity of variance risk. Panel B shows the performance when stocks are sorted by the product, but both the price and quantity are truncated at zero. Finally, Panel C summarizes the results when stocks are sorted by the sum of the price-quantity combination of the market and non-market variance risk.

A. Sorted by $\hat{\beta}_{v,i} \times \hat{\lambda}_{v,i}$

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5	5-1
Returns	0.31	0.44	0.60	0.67	1.02	0.71***
	(0.80)	(1.34)	(1.90)	(2.07)	(2.57)	(3.25)
α_4	−0.19	−0.06	−0.03	0.10	0.40	0.59***
	(−2.00)	(−0.66)	(−0.38)	(0.96)	(2.62)	(3.38)

B. Sorted by $\min(\hat{\beta}_{v,i}, 0) \times \min(\hat{\lambda}_{v,i}, 0)$

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5	5-1
Returns	0.55	0.36	0.64	0.62	1.07	0.71***
	(1.56)	(0.97)	(2.14)	(1.93)	(2.73)	(3.41)
α_4	−0.22	−0.11	−0.02	0.08	0.45	0.63***
	(−1.85)	(−0.79)	(−0.22)	(0.78)	(2.99)	(3.44)

C. Sorted by $\hat{\beta}'_{v,i}\hat{\lambda}_{e,i} + \hat{\beta}'_{x,i}\hat{\lambda}_{x}$

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5	5-1
Returns	0.35	0.45	0.73	0.59	1.00	0.64**
	(0.98)	(1.47)	(2.42)	(1.82)	(2.23)	(2.38)
α_4	−0.14	0.00	0.17	−0.14	0.39	0.54**
	(−1.50)	(−0.04)	(1.42)	(−1.36)	(2.04)	(2.28)

Table VIII
Determinants of the Variance Risk Exposure

This table summarizes the results of Fama-MacBeth procedure for the panel data. As a first step, I run a cross-sectional one-step-ahead predictive regression of the variance risk exposure on different firm variables. Then, the time-series average of the coefficients along with the Fama-MacBeth standard errors with Newey-West correction are computed. Implied variance is the monthly implied variance. Market value is computed as the price of share multiplied by number of shares outstanding. Leverage is the Debt-to-market value of equity ratio. Interest coverage ratio is earnings before interest and tax divided by interest expense. Growth option is the percentage of firm value arising from grown opportunities as to the total market value. Capital expenditure is the capital expenditure of firm divided by firm's fixed asset. R&D is R&D expense divided firm's assets.

	Dependent Variable : $\hat{\beta}_{v,i,t+1}$								
Explanatory Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
log(MV)	-1.13^{***}	-1.15^{***}	-1.16^{***}	-1.15^{***}	-1.16^{***}	-1.12^{***}	-1.06^{***}	-1.05^{***}	-1.05^{***}
	(−20.77)	(−23.08)	(−21.62)	(−22.23)	(−21.65)	(−20.76)	(−23.50)	(−23.88)	(−22.49)
Leverage		−0.04			−0.02				
		(−0.23)			(−0.11)				
Interest Coverage Ratio (× 100)			−0.044		0.001				
			(−0.85)		(−0.13)				
Profitability				-0.46^{*}	-0.55^{**}			-0.51^{*}	
				(−1.77)	(−2.02)			(−1.90)	
Growth Option						0.37^{***}			0.23^{*}
						(3.27)			(1.87)
Capital Expenditure							1.34^{***}	1.38^{***}	1.29^{***}
							(5.34)	(5.50)	(5.41)
R&D							1.78^{***}	1.32^{**}	1.37^{**}
							(2.86)	(2.05)	(2.29)

Table IX
Understanding the Price and Quantity of Variance Risk from Factors Skewness

This table summarizes the price and quantity of variance risk estimates along with Newey-West t-statistics of the decile portfolios when stocks are sorted by factor betas. Panel A, B, C, D, and E summarize the results when the sorting variable is the market, value, size, momentum, and changes in VIX betas, respectively.

A. Market Factor (Skewness: -0.19, Tstat: -2.74)

	Dec. 1	Dec. 2	Dec. 3	Dec. 4	Dec. 5	Dec. 6	Dec. 7	Dec. 8	Dec. 9	Dec. 10	10-1
$\hat{\beta}_{v,i,t}$	−3.90	−3.59	−3.42	−3.11	−2.91	−2.99	−2.86	−2.61	−2.34	−1.80	2.09***
	(−10.33)	(−11.98)	(−14.03)	(−15.36)	(−13.65)	(−13.39)	(−13.91)	(−16.61)	(−18.39)	(−15.49)	(5.11)
$100\,\hat{\lambda}_{v,i,t}$	0.07	−0.02	0.00	0.05	0.05	−0.03	−0.05	−0.17	−0.29	−0.69	−0.76***
	(2.29)	(−0.63)	(−0.13)	(1.14)	(0.87)	(−0.55)	(−0.64)	(−2.56)	(−3.04)	(−4.26)	(−5.01)

B. Value (Skewness: 0.16, Tstat: 1.64)

	Dec. 1	Dec. 2	Dec. 3	Dec. 4	Dec. 5	Dec. 6	Dec. 7	Dec. 8	Dec. 9	Dec. 10	10-1
$\hat{\beta}_{v,i,t}$	−2.57	−3.23	−3.25	−3.46	−3.49	−3.18	−3.06	−2.81	−2.65	−2.66	−0.09
	(−16.60)	(−16.45)	(−15.30)	(−13.91)	(−13.77)	(−12.63)	(−11.31)	(−12.39)	(−13.41)	(−12.21)	(−0.43)
$100\,\hat{\lambda}_{v,i,t}$	−0.41	−0.14	−0.08	0.00	−0.01	−0.02	−0.02	−0.04	−0.01	−0.01	0.41***
	(−3.84)	(−1.88)	(−1.82)	(−0.04)	(−0.34)	(−0.73)	(−0.49)	(−1.06)	(−0.08)	(−0.06)	(2.91)

C. Size (Skewness: -0.07, Tstat: -0.94)

	Dec. 1	Dec. 2	Dec. 3	Dec. 4	Dec. 5	Dec. 6	Dec. 7	Dec. 8	Dec. 9	Dec. 10	10-1
$\hat{\beta}_{v,i,t}$	−3.73	−3.49	−3.08	−2.75	−2.62	−2.22	−2.01	−1.85	−1.57	−1.18	2.55***
	(−15.06)	(−15.84)	(−15.01)	(−12.10)	(−9.11)	(−12.51)	(−16.13)	(−14.64)	(−13.34)	(−13.49)	(11.15)
$100\,\hat{\lambda}_{v,i,t}$	0.02	−0.05	−0.06	−0.06	−0.09	−0.25	−0.27	−0.42	−0.59	−0.97	−0.98***
	(0.40)	(−1.45)	(−1.59)	(−0.98)	(−1.44)	(−4.41)	(−3.70)	(−4.66)	(−5.17)	(−6.39)	(−7.06)

D. Momentum (Skewness: -0.37, Tstat: -5.69)

	Dec. 1	Dec. 2	Dec. 3	Dec. 4	Dec. 5	Dec. 6	Dec. 7	Dec. 8	Dec. 9	Dec. 10	10-1
$\hat{\beta}_{v,i,t}$	−2.61	−2.79	−2.83	−3.01	−3.14	−3.32	−3.29	−3.32	−3.41	−3.47	−0.87***
	(−11.70)	(−13.34)	(−13.28)	(−12.47)	(−13.58)	(−12.21)	(−11.66)	(−13.99)	(−16.02)	(−14.72)	(−2.93)
$100\,\hat{\lambda}_{v,i,t}$	−0.22	−0.05	−0.02	−0.02	−0.02	−0.05	−0.07	−0.04	−0.17	−0.24	−0.03
	(−1.99)	(−0.93)	(−0.24)	(−0.49)	(−0.38)	(−0.93)	(−1.75)	(−0.90)	(−3.32)	(−2.98)	(−0.22)

E. Market Variance FVIX (Skewness: 0.52, Tstat: 4.57)

	Dec. 1	Dec. 2	Dec. 3	Dec. 4	Dec. 5	Dec. 6	Dec. 7	Dec. 8	Dec. 9	Dec. 10	10-1
$\hat{\beta}_{v,i,t}$	−3.45	−3.60	−3.64	−3.31	−3.23	−3.19	−2.73	−2.47	−2.28	−1.75	1.70***
	(−12.47)	(−13.70)	(−15.11)	(−16.60)	(−14.52)	(−14.60)	(−14.16)	(−11.75)	(−12.66)	(−14.14)	(6.27)
$100\,\hat{\lambda}_{v,i,t}$	−0.19	0.02	−0.06	−0.05	−0.06	−0.02	0.00	−0.08	−0.25	−0.61	−0.42***
	(−4.06)	(0.56)	(−1.80)	(−1.60)	(−1.65)	(−0.34)	(−0.03)	(−0.95)	(−2.74)	(−4.15)	(−3.24)