Stock Return Autocorrelations and the Cross Section of Option Returns*

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March 31, 2019

Abstract

We present a new finding between the cross-section of average returns of equity option and the return autocorrelations of underlying stocks. Extended Black-Scholes model incorporating the presence of stock return autocorrelation suggests that expected returns of both call and put options are increasing in return autocorrelation coefficient of the underlying stock. Consistent with this insight, we find strong empirical support in the cross-section of average returns of equity options. Average returns of calls and puts as well as average returns of straddles all show monotonically increasing relationship with the degree of underlying stock's return autocorrelation coefficient. Additional equity option portfolio analysis shows that the information on stock return autocorrelation helps investors to significantly improve the out-of-sample performance of their portfolios.

JEL Classification: G11, G12, G13

Keywords: stock return autocorrelation, cross-section of option returns, expected option returns, option portfolios

*We are grateful to Peter Christoffersen, Bing Han and Chayawat Ornthanalai for many helpful comments.
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1 Introduction

Since the seminal work of Black and Scholes (1973), the academic literature on the options market has produced tremendous amount of work in the model specification of stock returns. Studies in this literature typically focus on extending the Black-Scholes model by different ways of relaxing the assumption that stock price follows geometric Brownian motion with constant drift.[1] Consequently, the focus of the literature has been mainly placed on the pricing of the options contract rather than the returns from option investments.[2] Recent literature has started to pay attention to the determinants of the cross-sectional differences of average equity option returns. Papers along this direction include Goyal and Saretto (2009), Cao and Han (2013), Vasquez (2017), Cao, Han, Tong, and Zhan (2017), and Cao, Vasquez, Xiao, and Zhan (2018). In this paper, we make an attempt to understand how departures from the traditional assumption of geometric Brownian motion with constant drift as stock price process can be important determinants of expected option returns. Specifically, we focus on the underlying stock's return autocorrelation and establish both theoretical and empirical relationship between the autocorrelation and expected equity option returns.

In this paper, we build upon the insight from the model studied by Lo and Wang (1995) that incorporates non-zero stock return autocorrelation in the option pricing. It is not entirely obvious why autocorrelation can be an important determinant of expected option returns. This is because while non-zero return autocorrelation implies the drift of the stock price process is not constant, it has no impact on the pricing of options as pointed out by Grundy (1991) and Lo and Wang (1995). Nevertheless, expected future payoffs and hence expected returns of options can be heavily influenced by the non-constant drift of the stock price process. In particular, we show that under the model of Lo and Wang (1995), the

[1] Influential papers in this category includes Merton (1976), Heston (1993), Bates (1996), and Duffie, Pan, and Singleton (2000).

[2] Existing literature has mainly focused on the investment problem on index options rather than cross-section of equity options. See, e.g., Liu and Pan (2003) and Faias and Santa-Clara (2017).

expected returns for both calls and puts monotonically increase with the autocorrelation of the underlying stock's return.[3]

The monotonic relationship between the return autocorrelation and expected returns on options is well supported empirically. Using equity option returns data from January 1996 to December 2017, we document the monotonic pattern in quintile portfolio returns sorted by the underlying stock's autocorrelation coefficients. The difference between the highest autocorrelation portfolio and the lowest autocorrelation portfolio delivers a statistically significant monthly return of 4.7% for call options and 5.9% for put options. We also show that these results are not due to the other known drivers of cross-sectional equity option returns and that they are robust to different holding periods, different methods to compute autocorrelation, and different moneyness of the option contract. Analyses based on Fama-MacBeth regressions further suggest that our result is not driven by the known determinants of expected equity option returns. Our measure of stock return autocorrelation remains statistically significant after controlling for idiosyncratic volatility from Cao and Han (2013), realized stock return volatility from Hu and Jacobs (2018), variance risk and illiquidity premium from Goyal and Saretto (2009) and term structure of implied volatilities from Vasquez (2017).

Recent empirical literature has identified some interesting variables that explain the cross-sectional differences of average equity option returns. In particular, several studies have focused on variables that measure the frictions of the underlying stock market and analyzed their impact on the average equity option returns. For instance, Cao and Han (2013) shows the relationship between the underlying stock's idiosyncratic volatility and average delta-hedged equity option returns while Cao, Vasquez, Xiao, and Zhan (2018) documents the volatility uncertainty measured by volatility of volatility is related to average delta-hedged

[3] This is in contrast to Hu and Jacobs (2018) where they find that expected call option return is decreasing in volatility while expected put option return is increasing in volatility. Therefore, our result is unlikely driven by the volatility effect.

equity option returns. In those papers, the cross-sectional differences in average option returns are attributed to market imperfections and financial intermediary constraints. In addition, Cao, Han, Tong, and Zhan (2017) documents interesting findings that many predictors of underlying stock returns can also explain the average delta-hedged equity option returns. Instead of finding variables that capture various aspects of market incompleteness, our study suggests that an often overlooked attribute of stock return dynamics, namely the autocorrelation, can help to explain the cross-sectional differences of expected option returns.

We also contribute to the theoretical literature on how expected returns of options are determined. Relative to the massive literature on pricing of options, there are few studies that analyzed expected return of options. Under the Black-Scholes model, Rubinstein (1984) derives the expected return of an option over a finite holding period. Coval and Shumway (2001) computes the average returns of zero-beta straddles using index option data, where the betas of the options are computed using the Black-Scholes model. Broadie, Chernov, and Johannes (2009) computes expected hold-to-expiration returns of options under various option pricing models, including the Black-Scholes, the Heston (1993) model, and the stochastic volatility jump model suggested by Bates (1996). Boyer and Vorkink (2014) studies the option portfolio returns sorted by the ex-ante skewness of option returns computed from the Black-Scholes model. Xiao and Vasquez (2016) uses the structural model of firm's capital structure to derive an analytical relationship between the firm's leverage and equity option returns. Also, Hu and Jacobs (2018) uses the Black-Scholes and the Heston model to study the relationship between the underlying stock return volatility and expected option returns. Our contribution is to derive the expected holding period return of options for the class of models in Lo and Wang (1995) which incorporates stock return autocorrelation.

Besides explaining the cross-sectional difference of average returns of equity options, stock return autocorrelation can also provide valuable information for investors to improve the performance of their portfolios. We demonstrate this by considering an optimal portfolio problem that involves the risk-free asset, the S&P500 index, and a set of equity options. We show that incorporating the information on stock return autocorrelation in constructing the optimal portfolio leads to a significant improvement in out-of-sample Sharpe ratio and certainty equivalent, even after taking into account of transaction costs.

The remainder of the paper is organized as follows. Section 2 provides the analytical relationship between the stock return autocorrelation and expected option returns building on the models developed in Lo and Wang (1995). Section 3 provides the empirical result using the cross-sectional equity option returns data. Section 4 proposes and implements an investment strategy that incorporates our empirical findings. Section 5 performs various robustness checks and Section 6 concludes.

2 Stock Return Autocorrelation and Expected Option Returns

Geometric Brownian motion with constant drift has been the standard assumption for stock price process used in option pricing, for example, in the Black-Scholes model. However, this stock price process implies that stock returns have zero autocorrelation. In order to accommodate non-zero autocorrelations in returns, Lo and Wang (1995) considers the following trending Ornstein-Uhlenbeck (O-U) process for log stock price:

$$\mathrm{d}\log(S_t) = (-\gamma(\log(S_t) - \mu t) + \mu)\mathrm{d}t + \sigma \mathrm{d}W_t, \tag{1}$$

where S_t is the stock price at time t, μ is the drift coefficient, σ is the diffusion coefficient, $\gamma \geq 0$ is the "speed of adjustment" parameter, and W_t is a standard Weiner process. Unlike the original Black-Scholes model, which assumes that log-prices follow an arithmetic random walk with independently and identically distributed Guassian increments, this log-price process is the sum of a zero-mean stationary autoregressive Gaussian process and a deterministic linear trend.

Defining

$$r_k = \log(S_{t+k}) - \log(S_t) \tag{2}$$

as the k-period continuously compounded return of the underlying stock, it can be readily shown that under the trending O-U process in (1), $r_k \sim N(k\mu, k\sigma_k^2)$, where

$$\sigma_k^2 = \frac{\sigma^2\left(1 - e^{-k\gamma}\right)}{k\gamma}. \tag{3}$$

In addition, the k-period return exhibits a first-lag autocorrelation of

$$\rho_k(1) = -\frac{1}{2}(1 - e^{-k\gamma}), \tag{4}$$

where $\rho_k(1)$ is a monotonic decreasing function of γ.

As noted by Grundy (1991) and Lo and Wang (1995), the risk-neutral dynamics of the stock price remains the same as in the Black-Scholes model even though the stock price follows the trending O-U process under the physical measure. Simple reasoning is that the drift under the risk-neutral measure has to be equal to the risk-free rate r in order to avoid arbitrage. This means that call and put option price C_t and P_t remains the same as in the Black-Scholes model even when the stock price follows the trending O-U process. We denote the corresponding Black-Scholes call and put prices as

$$C^{BS}(S_t, K, r, \tau, \sigma) = S_t\Phi(d_1) - Ke^{-r\tau}\Phi(d_2), \tag{5}$$

$$P^{BS}(S_t, K, r, \tau, \sigma) = Ke^{-r\tau}\Phi(-d_2) - S_t\Phi(-d_1), \tag{6}$$

where K is the strike price, r is the continuously compounded risk-free rate, τ is time-to-maturity,

$$d_1 = \frac{\log\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}, \tag{7}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}, \tag{8}$$

and $\Phi(\cdot)$ is the cumulative distribution function of a standard normal random variable.

It is important to emphasize that we need to use σ, i.e., the instantaneous volatility in the above formulae to obtain the correct option prices. Using the volatility of r_τ (i.e., σ_τ) in the Black-Scholes formula will lead to erroneous prices for the options. While γ or $\rho_\tau(1)$ has no impact on the price of an option today, it plays an important role in determining the expected future price of an option. In the following Proposition, we present the expected future price of a general European derivative on the underlying stock when it follows the trending O-U process in (1).

Proposition 1. *Suppose the stock price follows the trending O-U process defined in (1). Consider a general European derivative with its payoff at time $t+\tau$ being a deterministic function of the stock price at time $t+\tau$. If the derivative has a current price of*

$$H_t = H_t(S_t, \sigma), \tag{9}$$

then its expected price at $t+k$ for $0 < k \leq \tau$ is given by

$$E_t[H_{t+k}] = e^{rk} H_t(S_t^*, \sigma^*), \tag{10}$$

where

$$S_t^* = S_t \exp\left(\left[\mu - r + \frac{\sigma_k^2}{2}\right] k\right), \tag{11}$$

$$\sigma^{*2} = \frac{k\sigma_k^2 + (\tau - k)\sigma^2}{\tau}. \tag{12}$$

Proofs of all propositions are given in the Appendix. The result of this Proposition is quite general; it covers all kinds of European derivatives, including calls, puts, binary options and compound options. It suggests that as long as one has the ability to compute the derivative price today (either analytically or numerically), then one can easily compute its expected price at any time before maturity by replacing S_t and σ in the pricing formula with S_t^* and σ^* and then multiplying the price by e^{rk}. It should be noted that Rubinstein (1984) has derived the expected value of the call and put options at times $t + k$ under the assumption that the stock price follows a geometric Brownian motion. Their result can be obtained as a special case of Proposition 1 when we set $\sigma_k = \hat{\sigma}$.[4]

As a special case of Proposition 1, we can compute the expected payoff of a European derivative at its maturity, and the result is summarized in the following corollary.

Corollary 1.1. *Suppose the stock price follows the trending O-U process defined in (1). For a general European derivative with current price of $H_t = H_t(S_t, \sigma)$ and a time-to-maturity of τ, its expected payoff at maturity is given by*

$$E_t[H_{t+\tau}] = e^{r\tau} H_t(\tilde{S}_t, \sigma_\tau), \tag{13}$$

where

$$\tilde{S}_t = S_t \exp\left(\left[\mu - r + \frac{\sigma_\tau^2}{2}\right]\tau\right). \tag{14}$$

With the ability to compute the expected price of a derivative at any time before maturity, we can compute the expected return of an option for different holding periods, including expected hold-to-expiration return. For the European call and put with time-to-maturity τ, we can use Proposition 1 to obtain their expected k-period ($k \leq \tau$) returns under the

[4]Rubinstein's proof relies on direct integration of the Black-Scholes formula to obtain the expected return of the option. It is difficult to generalize his proof to the case of more complex European derivatives.

trending O-U process as

$$E[R_t^C(k)] = \frac{E_t[C_{t+k}]}{C^{BS}(S_t, K, r, \tau, \sigma)} - 1 = \frac{e^{rk} C^{BS}(S_t^*, K, r, \tau, \sigma^*)}{C^{BS}(S_t, K, r, \tau, \sigma)} - 1, \tag{15}$$

$$E[R_t^P(k)] = \frac{E_t[P_{t+k}]}{P^{BS}(S_t, K, r, \tau, \sigma)} - 1 = \frac{e^{rk} P^{BS}(S_t^*, K, r, \tau, \sigma^*)}{P^{BS}(S_t, K, r, \tau, \sigma)} - 1. \tag{16}$$

With the explicit expressions of $E[R_t^C(k)]$ and $E[R_t^P(k)]$ available, we can analyze the impact of stock return autocorrelation on expected option returns. Note that from (3) and (4), we can see that $\rho_k(1)$ and σ_k are monotonic increasing function of each other. Therefore, $\partial E[R_t^C(k)]/\partial \rho_k(1)$ has the same sign as $\partial E[R_t^C(k)]/\partial \sigma_k$. Similarly, $\partial E[R_t^P(k)]/\partial \rho_k(1)$ has the same sign as $\partial E[R_t^P(k)]/\partial \sigma_k$. In the following Proposition, we provide explicit expressions of $\partial E[R_t^C(k)]/\partial \sigma_k$ and $\partial E[R_t^P(k)]/\partial \sigma_k$.

Proposition 2. *Suppose the stock price follows the trending O-U process defined in (1). The partial derivatives of $E[R_t^C(k)]$ and $E[R_t^P(k)]$ with respect to σ_k are given by*

$$\frac{\partial E[R_t^C(k)]}{\partial \sigma_k} = \frac{e^{rk} S_t^* k \sigma_k}{C_t} \left[\Phi(d_1^*) + \frac{\phi(d_1^*)}{\sigma^* \sqrt{\tau}} \right], \tag{17}$$

$$\frac{\partial E[R_t^P(k)]}{\partial \sigma_k} = \frac{e^{rk} S_t^* k \sigma_k}{P_t} \left[-\Phi(-d_1^*) + \frac{\phi(d_1^*)}{\sigma^* \sqrt{\tau}} \right], \tag{18}$$

where $\phi(\cdot)$ is the probability density function of a standard normal random variable and

$$d_1^* = \frac{\log\left(\frac{S_t^*}{K}\right) + \left(r + \frac{\sigma^{*2}}{2}\right)\tau}{\sigma^* \sqrt{\tau}}. \tag{19}$$

From (17), we can easily see that $\partial E[R_t^C(k)]/\partial \sigma_k > 0$, so expected return of a call option is an increasing function of the first order autocorrelation of stock returns. For the case of (18), we show in the proof of Proposition 2 that when $k = \tau$ and $\mu > 0$, a sufficient condition for $\partial E[R_t^P(k)]/\partial \sigma_k > 0$ is $K \le S$ (i.e., the put option is at-the-money or out-of-the-money). In principle, (18) can take negative values when the put is deep in-the-money. However, for reasonable choices of parameters we often encounter, the partial derivative in (18) is positive.

As an example, Figure 1 plots the expected stock return, expected hold-to-expiration returns for at-the-money (ATM) calls, puts and straddles as a function of first-order autocorrelation of stock returns under the trending O-U process, assuming $\mu = 0.1$, $r = 0.05$, $\sigma = 0.2$, and $\tau = 1/12$. The τ-period expected return of the stock is given by $\exp\left(\tau\mu + \frac{\tau\sigma_\tau^2}{2}\right) - 1$. Although σ_τ^2 is an increasing function of the stock return autocorrelation, the impact of stock return autocorrelation on the expected return of the stock is quite minimal, especially for a short horizon like $\tau = 1/12$. In contrast, the expected returns of the ATM calls, puts, and straddles all display a monotonic increasing relation with the stock return autocorrelation coefficient. When we compare the expected option returns between two extreme cases $\rho = 0$, which corresponds to the Black-Scholes model, and $\rho = -0.2$, the difference is 11.23% for call option while it is 11.72% for put options. This suggests that autocorrelation in stock returns indeed has a significant impact on the expected returns of ATM options.

While Figure 1 suggests that autocorrelation of stock return is an important determinant of expected returns of ATM options, it is of interest to understand whether the same pattern continues to hold for options with different levels of moneyness. To this end, we plot expected hold-to-expiration call and put option returns for different levels of moneyness ($K/S = 0.95$, 1, or 1.05). Instead of plotting the expected option return as a function of stock return autocorrelation alone, we plot the expected option return as a function of stock return autocorrelation and stock return volatility. This allows us to judge the relative importance of these two determinants of expected option returns in different scenarios. Figure 2 suggests that stock return autocorrelation is extremely important in determining the expected return of out-of-the-money options (i.e., $K/S = 1.05$ for call and $K/S = 0.95$ for put), especially when the volatility is low. In contrast, for in-the-money options, stock return autocorrelation is an important determinant of their expected returns only when return volatility is high. Hu and Jacobs (2018) suggests that stock return volatility is an important determinant of expected option returns. In particular, they suggest that expected call option return is a decreasing function of stock return volatility whereas the expected put option return is an increasing function of stock return volatility. For the Black-Scholes case (i.e., $\rho = 0$), we indeed observe such a pattern. However, when returns exhibit negative autocorrelation, we find that the expected return of in-the-money put option ($K/S = 1.05$) is in fact a decreasing function of volatility. While we find that stock return volatility is an important determinant of expected option returns across various cases considered in Figure 2, the effect of stock return autocorrelation is just as an important determinant of expected option return as the stock return volatility.

Although the trending O-U process provides a simple analytical tool to embed autocorrelation of stock returns into the option pricing framework, it has a limitation of being only able to generate negative return autocorrelation. In practice, both negative and positive autocorrelation of stock returns are often observed in the cross-section. To overcome this issue, Lo and Wang (1995) proposes a bivariate trending O-U process that can generate positive autocorrelation coefficients. In particular, Lo and Wang (1995) considers the following special case of the bivariate trending O-U process:

$$\mathrm{d}\log(S_t) = (\mu + \lambda X_t)\mathrm{d}t + \sigma \mathrm{d}W_t, \tag{20}$$

$$\mathrm{d}X_t = -\delta X_t \mathrm{d}t + \sigma_x \mathrm{d}W_t^x, \tag{21}$$

where the two Weiner processes W_t and W_t^x are independent to each other. In this model, λ controls the first-order autocorrelation coefficient and plays a similar role to γ in the univariate trending O-U process. For this special case, σ_k^2 and $\rho_k(1)$ have the following expressions

$$\sigma_k^2 = \frac{\sigma^2}{1-\sigma_{qx}}\left[1 - \frac{\sigma_{qx}}{k\delta}(1 - e^{-k\delta})\right], \tag{22}$$

$$\rho_k(1) = \frac{\frac{\sigma_{qx}}{\delta}(1 - e^{-k\delta})^2}{2\tau\left[1 - \frac{\sigma_{qx}}{k\delta}(1 - e^{-k\delta})\right]}, \tag{23}$$

where

$$\sigma_{qx} = \frac{1}{1 + (\frac{\delta}{\lambda})^2 \frac{\sigma^2}{\sigma_x^2}}. \tag{24}$$

For fixed δ and σ_x, it is easy to show that both σ_k^2 and $\rho_k(1)$ are increasing functions of σ_{qx}, whereas the latter is a monotonic increasing function of λ. It turns out that for this bivariate O-U process, we can apply Proposition 1 to obtain the expected future price of a European option by simply replacing the expression of σ_k in (3) with the one in (22).

In Figure 3, we plot the expected stock return and expected hold-to-expiration returns for ATM calls, puts, and straddles as a function of autocorrelation coefficient under the bivariate trending O-U process, assuming $\mu = 0.1$, $r = 0.05$, $\sigma = 0.2$, $\tau = 1/12$, $\delta = 0.2$, and $\sigma_x = 0.1$. The expected stock return is largely unaffected by the stock return autocorrelation, but expected returns of ATM calls and puts again show a clear monotonic relationship with the stock return autocorrelation coefficient for the case of bivariate trending O-U process, similar to the case of univariate trending O-U process in Figure 1.

We plot in Figure 4 the expected hold-to-expiration returns of calls and puts as a function of stock return autocorrelation and stock return volatility for different levels of moneyness ($S/K = 0.95$, 1, or 1.05) under the bivariate trending O-U process. Similar to Figure 2 which is for the case of the univariate trending O-U process, we find that stock return autocorrelation also has an important impact on expected returns of options under the bivariate trending O-U process, especially for those out-of-the-money options on stocks with low return volatility. In most cases, the stock return autocorrelation is an important determinant of expected option returns, and often more so than the stock return volatility.

Overall, the analytical exercise in this section provides an insight that expected option returns should be an increasing function of the first-order autocorrelation coefficient. In the next section, we test this relationship empirically using equity option return data.

3 Empirical Evidence

3.1 Data and Variable Construction

We collect equity options data including best bid, best offer, implied volatility, expiration date, strike price, and realized volatility from Ivy OptionMetrics database. The sample period is from January 1996 to December 2017. When computing monthly option returns, we define the beginning of the holding period of equity options to be the first trading day after the standard monthly option expiration date (i.e., the third Friday of the expiration month), so that we are able to observe the largest portion of equity option trading every month. For each month, we choose the individual equity options with the moneyness closest to 1 within the range between 0.95 and 1.05, and with the time to maturity closest to 30 days within the range between 25 and 40 days. The reason to choose 30-day at-the-money equity options is because they are the most actively traded contracts in the equity option market.[5] Following the existing literature, we exclude observations that apparently violate no-arbitrage conditions (e.g., $S \geq C \geq \max(0, S - Ke^{-rT})$ for call option), have no trading volumes or open interests, and have a quoted mid-price less than \$0.125. The selected options will be held to expiration and the portfolio is rebalanced every month.[6] In our study, we consider three types of option portfolios: the call option portfolio, the put option portfolio, and the straddle portfolio (i.e., a long position in both call and put options with the same

[5]In Section 5 (robustness check), we show that our results also hold for alternative types of options such as in-the-money and out-of-the money options.

[6]Note that the equity options are American style that can be exercised early. However, several studies (see, for example, Broadie, Chernov, and Johannes (2007) and Boyer and Vorkink (2014)) argue that adjusting for early exercise has minimal empirical implications. We therefore ignore the possibility of early exercise in our empirical analysis.

strike price). The returns for each of the three option portfolios are then defined below:

$$R_{i,t}^{C} = \frac{\max(S_{i,t} - K_i, 0)}{C_{i,t-1}} - 1, \tag{25}$$

$$R_{i,t}^{P} = \frac{\max(K_i - S_{i,t}, 0)}{P_{i,t-1}} - 1, \tag{26}$$

$$R_{i,t}^{Straddle} = \frac{\max(S_{i,t} - K_i, 0) + \max(K_i - S_{i,t}, 0)}{C_{i,t-1} + P_{i,t-1}} - 1, \tag{27}$$

where i stands for firm i. One problem of using hold-to-expiration option returns is that there are significant portions of options that expire out-of-the-money, leading to a highly skewed return distribution as many of the return observations are equal to -1. The highly skewed distribution may affect the performance of the statistical tests that rely on asymptotic normality (e.g., t-test and Fama-MacBeth regression). Accordingly, following the empirical options literature, we also look at the straddle portfolio which generates a less skewed return distribution (e.g., fewer observations of -1). As we have shown in Section 2, since the stock return autocorrelation affects both expected call and put option returns in the same direction, theoretically it should be able to explain expected straddle returns as well.

The underlying stock variables, such as stock return, stock price, trading volume, shares outstanding, and share code, are collected from the CRSP database. The Fama-French five factors and the risk-free rate are obtained from Kenneth French's website. At the end of each month, we use a 36-month rolling window to estimate the first-order autocorrelation of underlying stock's return as

$$\hat{\rho}_{i,t} = \frac{\sum_{n=0}^{34}(r_{i,t-n} - \bar{r}_{i,t})(r_{i,t-n-1} - \bar{r}_{i,t})}{\sum_{n=0}^{35}(r_{i,t-n} - \bar{r}_{i,t})^2}, \tag{28}$$

where $\bar{r}_{i,t} = \frac{1}{36}\sum_{n=0}^{35} r_{i,t-n}$. The calculated stock return autocorrelation is then used to sort stocks and options to form the corresponding quintile portfolios. In order to show the robustness of our empirical results, we construct several alternative measures of stock return autocorrelation at both daily and monthly frequency. The results of the robustness checks are discussed in Section 5. A stock (or option) is eligible to be included in the sample at certain month, if it has more than 20 monthly observations during the past 36 months (or 12 daily observations within each month when autocorrelations are measured at daily frequency). Lo and Wang (1995) suggests that investors can misprice an option if they use the unconditional variance in the Black-Scholes formula if returns indeed have non-zero autocorrelation. Different from them, we study how the underlying stock's return autocorrelation can explain the cross section of average equity option returns, rather than option prices.

Table 1 provides the summary statistics. During our sample period, we observe around 676 firms on average for each month that has liquid options contract traded. Panel A reports the average stock return autocorrelation coefficient being slightly negative at -0.022. Within these firms, we see large variations in the stock return autocorrelation that the 25^{th} percentile of the observations is -0.118 while the 75^{th} percentile of the observations is 0.070. Panel B then reports the summary statistics of hold-to-expiration equity option returns in our sample. As we discussed earlier, the option return is often -100% as many options expire out-of-the-money. As options represent highly leveraged position, the positive side is also quite extreme with the maximum return for call, put, and straddle are all close to 2,000%. The average monthly return of ATM call options is 6.1% while the average monthly return of ATM put options is -13.9%, as the stock returns are positive in average.

Table 1 about here

3.2 Portfolio Sorting

At the end of each month, we sort all eligible stocks, having observations of stock returns with call and put equity options, into quintiles based on their stock return autocorrelations. Within each quintile, we compute both equal-weighted and value-weighted (based on the underlying stock's market capitalization) returns and then construct a long-short portfolio between top and bottom quintiles for the following cases: call option returns, put option returns, straddle returns, and stock returns. The reason for including the underlying stock portfolio is to examine whether the explanatory power of the stock return autocorrelation on average option returns comes from its ability to explain the average returns of the underlying stocks. We hold the portfolio until the expiration date of the options and calculate the corresponding holding period returns. Table 2 displays the empirical results. In addition to computing the raw portfolio returns, we also calculate the alphas of the portfolios based on the Fama-French 5-factor model from Fama and French (2015) to check if our sorting only captures different risk exposures from the underlying stocks.

Table 2 confirms our Proposition 2 that expected return of an option is increasing in the stock return autocorrelation. For example, the call options with the lowest underlying stock return autocorrelations (Low) underperform those with the highest underlying stock return autocorrelations (High) by almost 5%/month. Similar evidence applies to put options and straddles. As illustrated in Figures 1 and 3, the expected return of a stock is hardly affected by its return autocorrelation. Consistent with this, we do not see significant difference in the average returns between the portfolios of stocks with low and high return autocorrelation. This suggests that the explanatory power of stock return autocorrelation on average option returns is not due to its explanatory power on average returns of their underlying stocks. In addition, the explanatory power of stock return autocorrelation cannot be fully explained by the volatility effect. If it is all driven by volatility, we should observe opposite effects of stock return autocorrelation on call and put options (Hu and Jacobs (2018)), while in our case, stock return autocorrelation positively explains the average returns of both call and put options. In Table 2 Panel B, we show that our results hold for value-weighted portfolios, although the results are slightly weaker than those for the equal-weighted portfolios, implying that our empirical findings are not driven by options of small firms. In addition to the raw option returns, we also consider the alpha of the long-short option (stock) portfolio based on the Fama-French 5-factor model. The alphas of all long-short option portfolios are statistically significantly positive, implying that our results cannot be explained by the underlying stock's risk factors.

Table 2 about here

One may have concerns that the stock return autocorrelation captures known risk factors that determine option returns such as: volatility (Hu and Jacobs (2018)), variance risk premium (Goyal and Saretto (2009)), liquidity risk (Christoffersen, Goyenko, Jacobs, and Karoui (2018)), or reflects some well-known option mispricings such as idiosyncratic volatility (Cao and Han (2013)), implied volatility term structure (Vasquez (2017)), or lottery preference (Boyer and Vorkink (2014)). In order to avoid the case that the stock return autocorrelation only captures the existing factors documented in the literature (either risk or mispricing), we extend our portfolio analysis through double sorting the stocks by various characteristics first and then by stock return autocorrelation. We then evaluate the performance of the long-short portfolio sorted by stock return autocorrelation within each group. The empirical results are provided in Table 3.

Table 3 confirms that the explanatory power of stock return autocorrelation on average return of options cannot be fully explained by any existing risk factors or mispricing effects. The increasing pattern of the high-low straddle portfolio returns corresponding to the stock return autocorrelation exists in most of the bins sorted by the control variables. The results in Table 3 are robust if we adjust the raw returns to alphas based on the Fama-French 5-factor model, or if we look at call or put option portfolios.

Table 3 about here

3.3 Fama-MacBeth Regression

The Fama-MacBeth regression proposed by Fama and MacBeth (1973) provides an alternative way to test whether the explanatory power of stock return autocorrelation on average option return is statistically significant. For each type of options (call, put, or straddle), we run the following cross-sectional regressions of their returns on stock return autocorrelation and other control variables, which have been linked to option returns in the literature (e.g., risk premium and existing option mispricing):

$$R_{i,t} = \alpha_t + \beta_t \hat{\rho}_{i,t-1} + \sum_{j=1}^{M} \gamma_t^j X_{i,t-1}^j + \epsilon_{i,t}, \quad i = 1, \ldots, N_t, \tag{29}$$

where $R_{i,t}$ is the return of option i at time t, $\hat{\rho}_{i,t-1}$ is the estimated stock return autocorrelation for stock i at time $t-1$, and $X_{i,t-1}^j$ ($j = 1, .., M$) are the control variables. The cross-sectional regression above is ran each month with N_t return observations to obtain the coefficients for the independent variables. After obtaining the time-series coefficients for the independent variables, we conduct the t-test for each coefficient with one-lag correction of Newey and West (1987). The hypothesis of the t-test is: $H_0 : \beta = 0$ vs. $H_a : \beta \neq 0$. The average time-series coefficients and the corresponding t-statistics are reported in Table 4.

The results of Table 4 support our claim that stock return autocorrelation is an important determinant of expected returns of options. The regression results in Table 4 are also consistent with the previous findings in the literature. For example, the stock realized volatility has opposite effects on call and put option returns (Hu and Jacobs (2018)), variance risk premium and liquidity risk premium can strongly predict straddle returns (Goyal and Saretto (2009)), idiosyncratic volatility is positively related to delta-hedged and raw option returns (Cao and Han (2013)), and the term structure of implied volatilities is positively related to straddle returns (Vasquez (2017)). All in all, our regression results are consistent with previous literature's empirical conclusions, and more importantly, confirm that the stock return autocorrelation effect cannot be explained by any of the previous findings in the literature.

Table 4 about here

4 Investment Strategy

In Section 3, we have provided strong statistical evidence that average returns of equity options are increasing function of the stock return autocorrelation. However, our empirical tests in Section 3 do not consider two important aspects of equity option trading: transaction cost and out-of-sample performance. As a result, it is not entirely clear that our empirical results can lead to economic benefits for investors. In particular, transaction cost is a substantial issue for equity option trading because many options have very high relative bid-ask spreads. The average relative bid-ask spread of equity option is around 9% and could have a significant impact on the performance of an investment strategy that involves equity options.[7] The option literature typically incorporate the impact of transaction cost by assuming the effective spread to be a fixed percentage of quoted spread (such as 25%, 50%, and 100%) when computing the return of a long-short option portfolio (e.g., Vasquez (2017), Cao, Han, Tong, and Zhan (2017)). The advantage of this approach is that it can clearly show how the bid-ask spreads affect the performance of a long-short portfolio of all the options. However, investors do not need to invest in every option and they can reduce the impact of transaction costs by concentrating on the more liquid equity options. As a result, the average impact of bid-ask spread on the returns of equity options may not be the most relevant metric for investors. Another issue is that our empirical results are mostly in-sample and they do not measure the out-of-sample benefit for investors who would like to incorporate the autocorrelation effect in their option portfolio allocations. In practice, investors do not know the true expected returns of options and have to estimate them us-

[7]The real transaction cost may not be as high as the observed relative bid-ask spread as argued in Muravyev and Pearson (2016).

ing historical data. As a result, there are estimation errors when constructing an optimal portfolio that involves options and these could lead to poor out-of-sample performance.

In order to deal with these two issues and to better judge the economic benefit of stock return autocorrelation, we consider the following investment problem for an investor. We assume an investor optimizes his allocation to the S&P 500, the risk-free asset, and various equity options. Due to the large number of equity options involved, it is not practical to solve an optimization that involves all the options. Instead, we opt to follow the framework proposed by Brandt, Santa-Clara, and Valkanov (2009) (BSV hereafter). The basic idea of BSV is to utilize a parametric approach to model the optimal individual stock weights as a function of their corresponding characteristics (e.g., size, book-to-market, momentum), because those firm characteristics are documented to be linked to expected returns of stocks. Each type of characteristic is controlled by one parameter so that the number of parameters to be estimated in this portfolio problem is relatively small. This parametric portfolio rule could reduce the problem of estimation risk and provide an improvement in the out-of-sample performance. The BSV framework can be applied in our case, because based on previous option literature and our analytical results in Section 2, the expected option returns can be explained by certain underlying characteristics such as: stock volatility, variance risk premium, and stock return autocorrelation. Similar to the logic of BSV, when constructing the optimal option portfolio, we model the optimal equity option weights as a function of those underlying characteristics. Specifically, we consider the following portfolio problem:

$$\max_{\theta,\phi} E_t[U(r_{p,t+1})] = \max_{\theta,\phi} \frac{1}{T}\sum_{t=0}^{T-1} U\left((1-\phi)r^f_{t+1} + \phi R^S_{t+1} + \sum_{i=1}^{N_t}\left[\frac{1}{N_t}\theta^{\mathrm{T}}\hat{x}_{i,t}\right] r^O_{i,t+1}\right). \tag{30}$$

Because the distribution of option returns are non-normal, following BSV and Faias and Santa-Clara (2017), we assume that the investor's objective function is a CRRA utility function $U(r_{p,t+1}) = \frac{(1+r_{p,t+1})^{1-\gamma}}{1-\gamma}$, which is able to take all higher moments into consideration.[8]

[8] Our results are robust if we use the mean-variance utility instead.

Following the literature, we assume a risk aversion (γ) equal to 3. r^f_{t+1} is the risk-free rate at time $t+1$, and R^S_{t+1} is the return on the S&P500 at time $t+1$. ϕ is the proportion of the total wealth the investor allocates to the S&P500. $\sum_{i=1}^{N_t}\left[\frac{1}{N_t}\theta^{\mathrm{T}}\hat{x}_{i,t}\right]r^O_{i,t+1}$ is the return of a long-short option portfolio whose individual weights are determined by a vector of various characteristics of the stock ($\hat{x}_{i,t}$). Based on past research and our theoretical result in Section 2, we consider the following three characteristics: realized volatility, variance risk premium and stock return autocorrelation. Following BSV, we standardize all the characteristics such that the cross-sectional average of $\hat{x}_{i,t}$ is zero, meaning that investors always hold a zero-cost long-short option portfolio. $\frac{1}{N_t}$ is a normalization term to make sure that the cross-section of individual weights are comparable over time when the number of securities changes. Our parameters of interest are ϕ and $\theta = (\theta_{RV},\ \theta_{VRP},\ \theta_\rho)^{\mathrm{T}}$, which are the coefficients to be estimated.

As discussed before, many options are illiquid and have very high relative bid-ask spread. For an investor, it is difficult to invest in such options and it makes sense to exclude these options when constructing our optimal portfolio. As a result, we only consider options with relative bid-ask spread of 10% or less. When performing portfolio optimization based on historical data, the unconstrained optimization can often lead to huge positions in the risky assets because of estimation risk. This can lead to very poor out-of-sample performance. There are various ways of mitigating this problem but one effective solution is to impose a norm constraint on the weights of the risky assets. We follow the suggestion of DeMiguel, Garlappi, Nogales, and Uppal (2009) and imposes the following L^1-norm constraint on θ:[9]

$$|\theta_{RV}| + |\theta_{VRP}| + |\theta_\rho| < 1. \tag{31}$$

We then run the optimization using an expanding window to estimate ϕ and θ. The

[9] In addition to an upper bound of 1, we also try upper bounds of 0.75 and 1.25, and the results are robust to this change.

starting length of the estimation window is 48 months. The optimized weights (ϕ and $\theta^{\mathrm{T}}\hat{x}_{i,t}$) are then used to construct the optimal portfolio for the next month. The out-of-sample evaluation period is from January 2000 to December 2017 (216 months in total). For the out-of-sample returns of options, we consider the case with and without transaction costs. For the case without transaction costs, the returns of the options are based on their mid-prices. For the case with transaction costs, the returns of the options are based on ask-price when $\theta^{\mathrm{T}}\hat{x}_{i,t} > 0$, and based on bid-price when $\theta^{\mathrm{T}}\hat{x}_{i,t} < 0$. To evaluate the utility improvement by trading option portfolios, besides computing those standard summary statistics such as portfolio return, volatility, skewness, Sharpe ratio, and alpha (based on Fama-French 5-factor model), we also calculate the average certainty equivalent (CE) following Faias and Santa-Clara (2017).

$$\mathrm{CE} = [(1-\gamma)\bar{U}]^{\frac{1}{1-\gamma}} - 1, \tag{32}$$

where $\bar{U}$ is the average utility of the out-of-sample portfolio returns ($\bar{U} = \frac{1}{T}\sum_{t=1}^{T} U_t$), and U_t is the CRRA utility with a risk aversion equal to 3. The certainty equivalent can be understood as the constant risk-free rate per period that will make the investor indifferent (in average utility terms) between it and the stream of risky returns actually earned on our portfolio. Table 5 provides details of our investment strategy. We report the estimated model parameters and out-of-sample performance of our portfolio. In Panel A, we first construct a portfolio without considering transaction cost. The reported parameters ϕ and θ are averages of the monthly optimized parameters from January 2000 to December 2017. Then in Panel B, we report the parameters and out-of-sample performance by taking into account of transaction costs for the options. In order to investigate the additional value from the stock return autocorrelation, we also report the results when only two of the underlying characteristics, realized volatility and variance risk premium, are used.

Table 5 about here

Table 5 suggests that investors are able to benefit from trading equity options based on those underlying characteristics, even after considering the transaction costs (Panel B). For example, the monthly Sharpe ratio of call (put) option portfolio is 0.314 (0.435) without transaction cost and 0.158 (0.210) with transaction cost, while the Sharpe ratio of buy-and-hold S&P500 index during the same period is only 0.056. The utility gain of investors is also significantly higher using equity options in the portfolio. For instance, the monthly certainty equivalent (CE) of call (put) option portfolio is 1.836% (2.911%) without transaction cost and 0.475% (0.707%) with transaction cost, where the CE of buy-and-hold S&P500 index is 0.095%. Given that all the numbers are monthly values, the benefit of having equity options in the portfolio is considerably large. In addition, we also show that both Sharpe ratio and CE of the portfolio including equity options are much smaller if the stock return autocorrelation is omitted from the characteristics used. Without transaction cost, the Sharpe ratio of call (put) option portfolio decreases from 0.314 (0.435) to 0.216 (0.062) if investors neglect the stock return autocorrelation when using BSV model to optimize their portfolio allocation. With transaction cost, the exclusion of stock return autocorrelation also leads to a significant drop of Sharpe ratio from 0.158 to 0.070 for call option portfolio, and from 0.201 to −0.057 for put option portfolio.

Two results are worth mentioning here. First, the parameters (θ's) are significant for all the underlying characteristics and the signs are consistent with our theoretical predictions. For example, θ_ρ for stock return autocorrelation is significantly positive, meaning that investors should buy options with higher stock return autocorrelation and sell options with lower stock return autocorrelations. Second, the transaction cost does have a significant impact on the out-of-sample performance of the optimized portfolio. For example, without considering transaction cost, the monthly Sharpe ratio of the portfolio that makes use of call options is 0.314, compared to 0.056 for the benchmark. However, after considering the transaction cost, the Sharpe ratio of the portfolio with call options drops almost by 50% to 0.158, although there are still significant utility gains for the call option portfolio. Overall, the results in this section demonstrates that the stock return autocorrelation is an important characteristic for an equity option portfolio allocation problem and investors can obtain significant economic gain by utilizing this information.

5 Robustness Checks

5.1 Alternative Measures of Return Autocorrelation, Sorting, and Different Holding Periods

We conduct several robustness checks for our main results reported in Table 2 Panel A (equal-weighted portfolios). We first look at the robustness of our measure for stock return autocorrelations. Specifically, we construct the stock return autocorrelation through two alternative ways: daily return autocorrelations within each month and monthly return autocorrelations using a 48-month rolling window. We find that our portfolio-sorting results hold in both cases (Table 6 Panel A). In addition to this, we alternatively sort the portfolio into deciles, and still find a significant result (Table 6 Panel A). Second, we investigate how persistent our finding is in order to understand whether this is a long-term or short-term effect. To study this question, instead of looking at one-month return, we examine the longer horizon return for each portfolio: three-months, six-months, and twelve-months returns. Since the observations of long-term option (i.e., time to maturity greater than two months) are rare, instead of choosing longer-term equity options (time-to-maturity greater than two months), we keep constructing the portfolios using equity options expiring in one month, but sort the portfolio every three, six, or twelve months. The long horizon portfolio returns are calculated as the average of monthly option returns with a monthly rollover. Our empirical evidence shows that the stock return autocorrelation effect on option returns is a long-term effect. For example, the difference of average returns between the highest autocorrelation straddle portfolio and the lowest autocorrelation straddle portfolio can significantly last for one year (3% every month, tantamount to 36% each year), implying that the stock return autocorrelation effect is less likely driven by option mispricing, but is more likely driven by fundamental factors.

Table 6 about here

5.2 Options with Different Time-to-Maturity, Moneyness, and Return Calculation

In Section 3, we mainly consider at-the-money (ATM) options expiring in one month for our empirical tests. However, based on our theoretical derivation in Section 2, we expect our empirical results should also hold for options with different maturities and moneyness, and in particular the stock return autocorrelation effect is expected to be stronger for out-of-the-money (OTM) options. In this subsection, we re-run our empirical tests in Section 3, but use options with alternative maturities, such as 15 days, 60 days, 90 days, and 120 days, or with alternative moneyness such as OTM and in-the-money (ITM). In addition, the option returns we constructed are mainly from the middle of one month (i.e., the third Friday of the expiration month) to the middle of the next month at expiration. Ni, Pearson, and Poteshman (2005) argues that hold-to-expiration option returns are affected by biases at expiration. To avoid this bias, we follow Cao, Han, Tong, and Zhan (2017) to construct the one-month option returns from the beginning and held to the end of each month. The robustness results are provided in Table 7.

We first look at option returns for different time-to-maturity. Now that we construct portfolios using options with different time-to-maturity, the portfolio rebalancing frequency is not one month as in Section 3. Instead, the portfolio rebalancing frequency is determined by the holding period of options we selected. For example, if we select equity options expiring in 15 days, the portfolio will be rebalanced every 15 days. If we choose equity options expiring in 60 days, the portfolio will be rebalanced roughly every two months. Table 7 Panel A shows that the effect of stock return autocorrelation on option returns is robust for different time-to-maturity.

We then investigate the explanatory power of stock return autocorrelation for different levels of moneyness. We consider two other types of options: OTM options with moneyness (K/S) less than 0.95 for put options and greater than 1.05 for call options, and ITM options with moneyness less than 0.95 for call options and greater than 1.05 for put options. We also construct the portfolio with a combination of call and put options with same moneyness, in order to study how the effect of stock return autocorrelation varies by moneyness. Since call and put options will not have the same strike price if they are ITM or OTM, we cannot call them "straddle" anymore. Instead, we name it as "combination". Consistent with our theoretical results, the positive relation between stock return autocorrelation and average option returns holds for different levels of moneyness. More importantly, the stock return autocorrelation effect is much stronger for OTM call and put options, which is consistent with our model's prediction. For example, the average monthly quintile spread of OTM put option portfolio is around 9%, which is larger than the spreads in other cases.

Finally, we consider alternative ways of computing option returns. Instead of constructing option returns from the middle of each month, we select options at the beginning of each month and choose those ATM options with time to maturities closest to 30 days. Through this way, the option returns are calculated from the beginning to the end of each month. Panel C of Table 7 confirms that our empirical results hold for this alternative definition of option returns.

Table 7 about here

6 Conclusions

This paper presents a new stock market variable that can explain the cross-sectional difference of expected equity option returns, namely the first-order stock return autocorrelation coefficient. We first show the analytical result using the extended Black-Scholes framework proposed by Lo and Wang (1995) that expected option return is an increasing function of the underlying stock's return autocorrelation. This prediction is strongly supported by the empirical findings where average returns of calls, puts, and straddles are found to be monotonically increasing in the magnitude of their underlying stock's return autocorrelation. These findings are robust to different implementation methods as well as controlling for other known factors that possess explanatory power of cross-sectional difference of average equity option returns.

Our findings contribute to the recent literature on the equity options investment. We identify a new variable that is easy to construct and derive its impact on the cross-section of expected returns on equity options. This approach could be potentially used to study other option pricing models that extends the Black-Scholes formula along different directions although obtaining an analytical formula for expected option return could be challenging for these models. Nevertheless, our results suggest that revisiting the options literature in the perspective of investment could be interesting. Most importantly, we demonstrate that such analysis could lead to superior option investment strategies that offer real benefits for investors even after taking into account of estimation risk and transaction costs.

Appendix

A Proof of Proposition 1

Consider a general European derivative with maturity $t+\tau$ and a payoff at maturity given by

$$H_{t+\tau} = h(S_{t+\tau}), \tag{A.1}$$

where $h(S_{t+\tau})$ is a deterministic function of the underlying stock price at $t+\tau$. As noted in Grundy (1991) and Lo and Wang (1995), the drift of the stock price process is irrelevant for determining the price of the derivative today, and we can use the risk-neutralized process of the stock price to determine the price of the European derivative today. Under the risk neutral measure, the continuously compounded return of $r_{t+\tau} = \log(S_{t+\tau}) - \log(S_t)$ is normally distributed with a mean of $\tau\left(r - \frac{\sigma^2}{2}\right)$ and variance of $\tau\sigma^2$. It follows that the current price of the European derivative is given by

$$\begin{aligned} H_t(S_t,\sigma) &= e^{-r\tau} E_t^{\mathbb{Q}}[h(S_{t+\tau})] \\ &= e^{-r\tau} \int_{-\infty}^{\infty} h\left(S_t e^{(r-\frac{1}{2}\sigma^2)\tau + \sigma\sqrt{\tau}v}\right)\phi(v)\mathrm{d}v. \end{aligned} \tag{A.2}$$

Similarly, the price of the derivative at time $t+k$, where $0 \leq k \leq \tau$, can be obtained as

$$\begin{aligned} H_{t+k}(S_{t+k},\sigma) &= e^{-r(\tau-k)} E_{t+k}^{\mathbb{Q}}[h(S_{t+\tau})] \\ &= e^{-r(\tau-k)} \int_{-\infty}^{\infty} h(S_{t+k} e^{(r-\frac{1}{2}\sigma^2)(\tau-k) + \sigma\sqrt{\tau-k}v})\phi(v)\mathrm{d}v. \end{aligned} \tag{A.3}$$

Under the physical measure, the stock price follows a trending O-U process and its k-period continuously compounded return $r_k = \log(S_{t+k}) - \log(S_t)$ is normally distributed with mean $k\mu$ and variance $k\sigma_k^2$. As a result, we can write S_{t+k} as

$$S_{t+k} = S_t e^{\mu k + \sigma_k\sqrt{k}w}, \tag{A.4}$$

where w is a standard normal random variable. Then, we can compute the expected price of the derivative at time $t+k$ as

$$\begin{aligned} E_t[H_{t+k}] &= \int_{-\infty}^{\infty} H_{t+k}\left(S_t e^{\mu k + \sigma_k\sqrt{k}w}, \sigma\right)\phi(w)\mathrm{d}w \qquad \text{(A.5)} \\ &= \int_{-\infty}^{\infty}\left[e^{-r(\tau-k)}\int_{-\infty}^{\infty} h\left(S_t e^{\mu k + \sigma_k\sqrt{k}w} e^{(r-\frac{\sigma^2}{2})(\tau-k) + \sigma\sqrt{\tau-k}v}\right)\phi(v)\mathrm{d}v\right]\phi(w)\mathrm{d}w, \end{aligned}$$

$$
\begin{aligned}
&= e^{-r(\tau-k)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h\left(S_t e^{\mu k + (r - \frac{\sigma^2}{2})(\tau-k)} e^{\sqrt{\sigma_k^2 k + \sigma^2(\tau-k)}u}\right) \phi_2\left(u, w; \frac{\sigma_k \sqrt{k}}{\sigma^* \sqrt{\tau}}\right) \mathrm{d}w \mathrm{d}u \\
&= e^{-r(\tau-k)} \int_{-\infty}^{\infty} h\left(S_t e^{\mu k + \left(r - \frac{\sigma^2}{2}\right)(\tau-k) + \sigma^* \sqrt{\tau} u}\right) \phi(u) \mathrm{d}u \\
&= e^{rk} H_t\left(S_t e^{\mu k + \left(r - \frac{\sigma^2}{2}\right)(\tau-k) - \left(r - \frac{\sigma^{*2}}{2}\right)\tau}, \sigma^*\right) \\
&= e^{rk} H_t(S_t^*, \sigma^*), \qquad \text{(A.6)}
\end{aligned}
$$

where

$$
S_t^* = S_t e^{\mu k + \left(r - \frac{\sigma^2}{2}\right)(\tau-k) - \left(r - \frac{\sigma^{*2}}{2}\right)\tau} = S_t e^{\left(\mu - r + \frac{\sigma_k^2}{2}\right)k} \qquad \text{(A.7)}
$$

and $\phi_2(\cdot, \cdot; \rho)$ stands for the density function of a standard bivariate normal random variable with correlation ρ. In the above derivation, we make a change of variable of

$$
u = \frac{\sigma_k \sqrt{k} w + \sigma \sqrt{\tau - k} v}{\sqrt{\sigma_k^2 k + \sigma^2(\tau-k)}} = \frac{\sigma_k \sqrt{k} w + \sigma \sqrt{\tau - k} v}{\sigma^* \sqrt{\tau}} \sim N(0,1), \qquad \text{(A.8)}
$$

and we have

$$
\mathrm{Corr}[u, w] = \mathrm{Cov}[u, w] = \frac{\sigma_k \sqrt{k}}{\sigma^* \sqrt{\tau}}. \qquad \text{(A.9)}
$$

This completes the proof.

B Proof of Corollary 1.1

This is a special case of Proposition 1 with $k = \tau$, thus $\sigma^* = \sigma_\tau$ and $S_t^* = S_t e^{(\mu - r + \frac{\sigma_\tau^2}{2})\tau} = \tilde{S}_t$. This completes the proof.

C Proof of Proposition 2

From the Black-Scholes formula, it is easy to show that

$$
\frac{\partial C^{BS}(S_t^*, K, r, \tau, \sigma^*)}{\partial S_t^*} = \Phi(d_1^*), \qquad \text{(A.10)}
$$

$$
\frac{\partial P^{BS}(S_t^*, K, r, \tau, \sigma^*)}{\partial S_t^*} = -\Phi(-d_1^*), \qquad \text{(A.11)}
$$

$$
\frac{\partial C^{BS}(S_t^*, K, r, \tau, \sigma^*)}{\partial \sigma^*} = S_t^* \phi(d_1^*) \sqrt{\tau}, \qquad \text{(A.12)}
$$

$$\frac{\partial P^{BS}(S_t^*, K, r, \tau, \sigma^*)}{\partial \sigma^*} = S_t^* \phi(d_1^*)\sqrt{\tau}, \tag{A.13}$$

where

$$d_1^* = \frac{\log\left(\frac{S_t^*}{K}\right) + \left(r + \frac{\sigma^{*2}}{2}\right)\tau}{\sigma^*\sqrt{\tau}}. \tag{A.14}$$

It follows that

$$\begin{aligned}
\frac{\partial E_t[C_{t+k}]}{\partial \sigma_k} &= e^{rk}\left[\frac{\partial C^{BS}(S_t^*, K, r, \tau, \sigma^*)}{\partial S_t^*}\frac{\partial S_t^*}{\partial \sigma_k} + \frac{\partial C^{BS}(S_t^*, K, r, \tau, \sigma^*)}{\partial \sigma^*}\frac{\partial \sigma^*}{\partial \sigma_k}\right] \\
&= e^{rk}\left[\Phi(d_1^*)S_t^* k\sigma_k + S_t^*\phi(d_1^*)\sqrt{\tau}\frac{k\sigma_k}{\tau\sigma^*}\right] \\
&= e^{rk}S_t^* k\sigma_k\left[\Phi(d_1^*) + \frac{\phi(d_1^*)}{\sigma^*\sqrt{\tau}}\right], \qquad \text{(A.15)} \\
\frac{\partial E_t[P_{t+k}]}{\partial \sigma_k} &= e^{rk}\left[\frac{\partial P^{BS}(S_t^*, K, r, \tau, \sigma^*)}{\partial S_t^*}\frac{\partial S_t^*}{\partial \sigma_k} + \frac{\partial P^{BS}(S_t^*, K, r, \tau, \sigma^*)}{\partial \sigma^*}\frac{\partial \sigma^*}{\partial \sigma_k}\right] \\
&= e^{rk}\left[-\Phi(-d_1^*)S_t^* k\sigma_k + S_t^*\phi(d_1^*)\sqrt{\tau}\frac{k\sigma_k}{\tau\sigma^*}\right] \\
&= e^{rk}S_t^* k\sigma_k\left[-\Phi(-d_1^*) + \frac{\phi(d_1^*)}{\sigma^*\sqrt{\tau}}\right]. \qquad \text{(A.16)}
\end{aligned}$$

We now show that when $k = \tau$ and $\mu > 0$, $\partial E_t[P_{t+k}]/\partial \sigma_k > 0$ for at-the-money and out-of-the-money put options. Note that when $k = \tau$ and $\mu > 0$,

$$S_t \geq K \Rightarrow S_t^* \geq K e^{\left(\mu - r + \frac{\sigma_\tau^2}{2}\right)\tau} \Rightarrow d_1^* \geq \sigma^*\sqrt{\tau}. \tag{A.17}$$

It follows that

$$-\Phi(-d_1^*) + \frac{\phi(d_1^*)}{\sigma^*\sqrt{\tau}} \geq -\Phi(-d_1^*) + \frac{\phi(d_1^*)}{d_1^*} = \frac{\Phi(-d_1^*)}{d_1^*}\left[-d_1^* + \frac{\phi(d_1^*)}{\Phi(-d_1^*)}\right] > 0. \tag{A.18}$$

The last inequality follows from the result of Gordon (1941) regarding inverse Mill's ratio for normal random variable that states for $d_1^* \geq 0$,

$$\frac{\phi(d_1^*)}{1 - \Phi(d_1^*)} > d_1^*. \tag{A.19}$$

This completes the proof.

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Table 1
Summary Statistics of the Variables

Summary Statistics	Avg. Obs.	Avg.	Percentile Values Min.	25^{th}	Med.	75^{th}	Max.
	Panel A: Stock Characteristics (Monthly Frequency)						
Return Autocorrelation	676.29	−0.022	−0.438	−0.118	−0.025	0.070	0.443
Return	676.29	0.006	−0.991	−0.059	0.007	0.073	1.891
Realized Volatility	676.29	0.439	0.010	0.230	0.342	0.516	1.998
Implied Volatility	676.29	0.434	0.016	0.254	0.361	0.515	1.991
Variance Risk Premium	676.29	0.004	−1.994	−0.047	0.020	0.082	1.971
	Panel B: Option Returns (Monthly Frequency)						
Call Option Return	676.29	0.061	−1.000	−1.000	−0.804	0.693	19.800
Put Option Return	676.29	−0.139	−1.000	−1.000	−1.000	0.333	19.268
Straddle Return	676.29	−0.025	−1.000	−0.639	−0.200	0.391	19.816

This table provides descriptive statistics for the monthly time-series variables used in the paper. The statistics are calculated by first taking the cross-section average of all eligible firm-level observations and then compute the average over time. The variance risk premium is computed as the difference between the annualized 30-day option implied volatility and the annualized 30-day stock realized volatility. To avoid the effect by outliers, we truncate the cross-section of securities at the 99.9% level before taking the cross-section average. The 0.1% truncation is not considered because the minimum value of call/put options is −1. A stock (or option) is eligible to be included in the sample in certain month, if it has more than 20 observations during the past 36 months. The sample period is from January 1996 to December 2017.

Table 2
Portfolios Sorted by Stock Return Autocorrelation

		Call Option	Put Option	Straddle	Underlying Stock
	Average Autocorrelation	Monthly Return	Monthly Return	Monthly Return	Monthly Return
		Panel A: Equal-Weighted Portfolio			
Low	−0.249	0.006	−0.230	−0.094	0.0073
2	−0.121	0.037	−0.222	−0.072	0.0076
3	−0.039	0.057	−0.213	−0.068	0.0082
4	0.043	0.054	−0.195	−0.064	0.0079
High	0.177	0.053	−0.171	−0.046	0.0087
High−Low	0.426	0.047**	0.059***	0.048***	0.0014
t-stat	-	(2.45)	(2.67)	(2.95)	(0.95)
FF 5-factor alpha	-	0.050**	0.050**	0.042**	0.0004
t-stat	-	(2.48)	(2.15)	(2.46)	(0.65)
		Panel B: Value-Weighted Portfolio			
Low	−0.246	0.010	−0.234	−0.095	0.0078
2	−0.123	0.040	−0.225	−0.072	0.0084
3	−0.042	0.057	−0.214	−0.069	0.0076
4	0.041	0.055	−0.199	−0.066	0.0078
High	0.167	0.052	−0.176	−0.048	0.0082
High−Low	0.413	0.045**	0.058**	0.047***	0.0004
t-stat	-	(2.30)	(2.56)	(2.85)	(0.59)
FF 5-factor alpha	-	0.048**	0.050**	0.042**	0.0001
t-stat	-	(2.32)	(2.09)	(2.39)	(0.61)

This table summarizes the average returns and alphas in monthly frequencies for portfolios sorted by the stock return autocorrelation and hold for one month. Panel A reports the equal-weighted average returns and alphas, while panel B reports the value-weighted (based on the underlying market capitalization) average returns and alphas. The alpha is calculated based on the Fama-French five factor model. The sample period is from January 1996 to December 2017. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 3
Portfolios Double Sorted by Stock Return Autocorrelation and Other Characteristics

Sort by Realized Volatility		**Low**	**2**	**3**	**4**	**High**
Long-Short Portfolio by	High−Low	0.045***	0.029***	0.020**	0.034**	0.017
Autocorrelation	t-stat	(2.85)	(2.83)	(2.23)	(2.16)	(1.56)
Sort by Idiosyncratic Volatility		**Low**	**2**	**3**	**4**	**High**
Long-Short Portfolio by	High−Low	0.042**	0.031***	0.013*	−0.004	0.029**
Autocorrelation	t-stat	(2.55)	(2.94)	(1.76)	(−0.23)	(2.13)
Sort by Variance Risk Premium		**Low**	**2**	**3**	**4**	**High**
Long-Short Portfolio by	High−Low	0.024***	0.021	0.037**	0.059***	0.013
Autocorrelation	t-stat	(3.23)	(1.20)	(2.48)	(3.74)	(0.82)
Sort by ILIQ		**Low**	**2**	**3**	**4**	**High**
Long-Short Portfolio by	High−Low	0.028*	0.035**	0.054***	0.024*	0.020
Autocorrelation	t-stat	(1.75)	(2.56)	(3.36)	(1.67)	(1.26)
Sort by IVTS		**Low**	**2**	**3**	**4**	**High**
Long-Short Portfolio by	High−Low	0.043***	0.014*	0.026**	0.025	0.034**
Autocorrelation	t-stat	(2.78)	(1.86)	(1.98)	(1.61)	(2.23)

In this table, we first sort individual stocks based on several control variables into five quintiles. Within each control variable's quintile, we sort the individual stocks by their stock return autocorrelations into five quintiles and compute the difference of average returns between the high and low stock return autocorrelation quintile. To save space, we only show the results for equal-weighted straddle portfolios, but the results are similar for call and put option portfolios, and also for value-weighted portfolios. The stock realized volatility is computed as the standard deviations based on the past 30-day stock returns and is obtained from OptionMetrics. The idiosyncratic volatility is calculated by running on 48-month rolling window of stock returns on the Fama-French five factor model and taking the standard deviations of the residual terms. The variance risk premium is computed as the difference between the 30-day option implied volatility and the 30-day stock realized volatility. Stock illiquidity (ILIQ) is computed as the average ratio of the daily absolute return to daily trading volume over each month (Amihud (2002)). The term structure of the implied volatility (IVTS) is defined as the difference between the long-term (90-day time to maturity) and short-term implied volatility (30-day time to maturity). The implied volatility is the average of Black-Scholes implied volatilities for at-the-money call and put options from OptionMetrics. The sample period is from January 1996 to December 2017. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 4
Fama-MacBeth Regressions

	Call Option		**Put Option**		**Straddle**		**Underlying Stock**	
	Univariate	Multiple	Univariate	Multiple	Univariate	Multiple	Univariate	Multiple
Intercept	0.105***	0.005	−0.184***	−0.188***	−0.049***	−0.052***	0.008**	0.009**
t-stat	(3.12)	(0.14)	(−3.86)	(−3.93)	(−3.37)	(−3.45)	(2.06)	(2.38)
Autocorrelation	0.118***	0.013***	0.144***	0.009**	0.069***	0.010***	−0.007	0.000
t-stat	(2.72)	(2.79)	(4.41)	(2.08)	(3.91)	(4.13)	(−1.28)	(1.27)
Realized Volatility		−0.056***		0.055***		−0.008		−0.001
t-stat		(−4.00)		(3.75)		(−1.06)		(−0.54)
Idiosyncratic Volatility		0.016**		0.013*		0.015***		−0.0002
t-stat		(2.43)		(1.74)		(3.51)		(−0.33)
Variance Risk Premium		−0.039***		0.030***		−0.008*		0.0006
t-stat		(−4.20)		(3.32)		(−1.68)		(0.47)
ILIQ		−0.014***		−0.016***		−0.012***		−0.011**
t-stat		(−3.35)		(−4.07)		(−5.61)		(−2.34)
Term Structure		0.012***		0.020***		0.015***		−0.005
t-stat		(2.93)		(4.65)		(6.61)		(−1.29)
Past One-Month Return		−0.013**		−0.005		−0.009***		−0.002***
t-stat		(−2.02)		(−0.81)		(−3.29)		(−2.93)
Average adj. R^2	0.26%	2.48%	0.30%	3.07%	0.19%	1.48%	0.61%	6.34%

This table reports the Fama-MacBeth regressions for each portfolio. The dependent variables are the cross-section of individual returns based on the corresponding type of portfolio, and the independent variables are the cross-section of stock return autocorrelation and other control variables in Table 3. The detailed cross-section regression and time-series test are specified in Section 3.3. All portfolios are formed at monthly frequency and held for one month. All dependent and independent variables are expressed as monthly values. The coefficients in the table are calculated by taking the time-series average of the cross-sectional regressions over time. The t-stat reported is the t-test with Newey-West one-lag correction. The sample period is from January 1996 to December 2017. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 5
Performance of Investment Strategies

	Panel A: Performance without Transaction Cost						
	with 3 Characteristics			with 2 Characteristics			Benchmark
Variable	Call	Put	Straddle	Call	Put	Straddle	S&P500
ϕ	0.816***	0.809***	0.932***	0.611***	0.825***	0.897***	
t-stat	(4.33)	(4.46)	(4.47)	(3.97)	(4.35)	(4.43)	
θ_{RV}	−0.411***	0.008	−0.015***	−0.528***	0.176***	−0.030*	
t-stat	(−4.47)	(1.47)	(−3.00)	(−4.47)	(4.37)	(−1.94)	
θ_{VRP}	−0.055***	−0.074***	−0.027***	−0.289***	−0.166***	−0.024	
t-stat	(−3.33)	(−3.18)	(−2.91)	(−4.14)	(−2.71)	(−0.73)	
θ_{ρ}	0.454***	0.844***	0.912***				
t-stat	(4.46)	(4.47)	(4.48)				
$\bar{r}$	0.033	0.042	0.039	0.022	0.005	0.001	0.004
$\sigma(r)$	0.100	0.094	0.068	0.096	0.055	0.046	0.042
SR	0.314	0.435	0.561	0.216	0.062	−0.011	0.056
$SR_3 - SR_2$	0.098**	0.373***	0.571***				
t-stat	(1.96)	(4.78)	(7.61)				
Skewness	0.482	0.093	−0.169	0.238	−0.467	−0.792	−0.580
FF 5-factor alpha	0.025***	0.038***	0.035***	0.013**	−0.001	−0.003	−0.001
t-stat	(3.81)	(5.50)	(7.97)	(2.16)	(−0.43)	(−1.43)	(0.86)
CE	1.836%	2.911%	3.256%	0.843%	−0.002%	−0.262%	0.095%
$CE_3 - CE_2$	0.992%*	2.913%***	3.518%***				
t-stat	(1.94)	(4.41)	(7.20)				
Avg. No. of Options	433	473	312	433	473	312	

Table 5 Continued:

	Panel B: Performance with Transaction Cost						
	with 3 Characteristics			with 2 Characteristics			Benchmark
Variable	Call	Put	Straddle	Call	Put	Straddle	S&P 500
ϕ	0.656***	0.597***	0.884***	0.614***	0.649***	0.653***	
t-stat	(4.05)	(4.22)	(4.46)	(4.02)	(4.09)	(4.16)	
θ_{RV}	−0.380***	0.032***	−0.012**	−0.286***	0.034***	−0.002	
t-stat	(−4.47)	(3.05)	(−2.37)	(−4.44)	(3.18)	(−1.54)	
θ_{VRP}	−0.247***	−0.161***	−0.048***	−0.174***	−0.015*	−0.002	
t-stat	(−4.45)	(−3.95)	(−2.88)	(−4.44)	(−1.92)	(−1.18)	
θ_{ρ}	0.294***	0.719***	0.778***				
t-stat	(4.45)	(4.46)	(4.47)				
$\bar{r}$	0.014	0.018	0.016	0.005	−0.001	0.001	0.004
$\sigma(r)$	0.080	0.084	0.062	0.059	0.036	0.032	0.042
SR	0.158	0.201	0.234	0.070	−0.057	−0.021	0.056
$SR_3 - SR_2$	0.088***	0.257***	0.253***				
t-stat	(2.61)	(3.73)	(3.91)				
Skewness	0.669	−0.184	−0.509	0.263	−1.896	−0.703	−0.580
FF 5-factor alpha	0.008	0.015**	0.011***	0.000	−0.003	−0.001	−0.001
t-stat	(1.52)	(2.48)	(2.91)	(0.04)	(−1.30)	(−0.86)	(0.86)
CE	0.475%	0.707%	0.977%	0.028%	−0.293%	−0.090%	0.095%
$CE_3 - CE_2$	0.447%*	0.999%*	1.067%***				
t-stat	(1.64)	(1.93)	(2.90)				
Avg. No. of Options	433	473	312	433	473	312	

In this table, we follow the framework proposed by Brandt, Santa-Clara, and Valkanov (2009) as specified in Section 4. We report the average value of the parameters over time and their corresponding t-statistics. The t-statistics are adjusted by Newey and West (1987) with 10-lag corrections. We also report the performance statistics out-of-sample: average return ($\bar{r}$), volatility ($\sigma(r)$), Sharpe ratio (SR), Skewness, alpha based on the Fama-French five factor model, and certainty equivalent (CE). All variables are expressed as monthly values. We run the optimization for the three option portfolios separately. In order to show the improvement from considering stock return autocorrelation in the portfolio allocation, we also report the portfolio performance using only two characteristics (without θ_ρ). $SR_3 - SR_2$ ($CE_3 - CE_2$) measures the difference of the SR (CE) between the optimal portfolio using all three characteristics and the optimal portfolio not using the stock return autocorrelation as a characteristic. The last column is the performance of the benchmark, which is a buy-and-hold strategy for S&P500 Index. In Panel B, we report the result by incorporating the transaction cost for the options. The evaluation period is from January 2000 to December 2017. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 6
Robustness Check for Portfolio Sorted by Stock Return Autocorrelation

		Call Option	Put Option	Straddle	Underlying Stock
		Monthly Return	Monthly Return	Monthly Return	Monthly Return
	Panel A: Alternative Measures of Stock Return Autocorrelation				
Daily Stock Returns	High−Low	0.035**	0.043***	0.019***	0.0004
	t-stat	(2.45)	(2.81)	(2.63)	(0.48)
48-month Rolling Window	High−Low	0.033**	0.049***	0.022***	0.0001
	t-stat	(2.08)	(3.94)	(2.77)	(0.55)
Decile Portfolio	High−Low	0.050**	0.076***	0.049**	−0.0003
	t-stat	(2.03)	(2.68)	(2.46)	(−0.61)
	Panel B: Alternative Holding-Period				
3 month	High−Low	0.053***	0.052***	0.049***	0.0001
	t-stat	(3.35)	(3.18)	(4.07)	(1.17)
6 month	High−Low	0.051***	0.035***	0.045***	0.0003
	t-stat	(3.70)	(2.64)	(4.22)	(1.46)
12 month	High−Low	0.072***	0.017	0.027***	−0.0003*
	t-stat	(6.60)	(1.37)	(3.14)	(−1.69)

This table summarizes several robustness checks for the predictive power of stock return autocorrelation. The sorting process is as same as that in Table 2, In Panel A, we first construct two alternative measures for stock return autocorrelation: daily return autocorrelations within each month and monthly return autocorrelations using a 48-month rolling window. In the last row of Panel A, we sort the portfolio into deciles based on the stock return autocorrelation calculated by a 36-month rolling window. In Panel B, similar to Table 2, we keep constructing the portfolios using equity options expiring in one month, but we sort the portfolio every three, six, or twelve months respectively. The corresponding portfolio returns are calculated as the average of monthly option returns with a monthly rollover of the options. All variables in this table are expressed as monthly values, and all portfolios are equally weighted. The sample period is from January 1996 to December 2017. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Table 7
Portfolio Returns Using Different Option Maturity and Moneyness

		Panel A: Options with Different Time to Maturities			
		Call Option	Put Option	Straddle	Stock
15 days	High−Low	0.036*	0.072***	0.034***	−0.0005
	t-stat	(1.89)	(2.64)	(3.72)	(−0.40)
60 days	High−Low	0.039**	0.052***	0.029***	0.0012
	t-stat	(2.16)	(3.05)	(3.86)	(0.95)
90 days	High−Low	0.010*	0.059***	0.066***	0.0008
	t-stat	(1.66)	(3.52)	(2.79)	(0.95)
		Panel B: Options with Different Moneyness			
		Call Option	Put Option	Combination	Stock
OTM	High−Low	0.050**	0.090***	0.056***	0.0007
	t-stat	(2.30)	(3.23)	(2.78)	(0.95)
ITM	High−Low	0.019***	0.030***	0.009*	0.0007
	t-stat	(3.08)	(3.28)	(1.92)	(0.95)
		Panel C: Alternative Return Calculation			
		Call Option	Put Option	Straddle	Stock
1 Month	High−Low	0.012	0.020**	0.020*	0.0007
	t-stat	(1.50)	(2.47)	(1.74)	(0.95)
3 Month	High−Low	0.028***	0.028***	0.025**	0.0004
	t-stat	(4.75)	(4.69)	(2.26)	(1.17)
6 Month	High−Low	0.032***	0.034***	0.016***	0.0005
	t-stat	(6.11)	(6.82)	(3.24)	(1.46)

This table reports the robustness check using alternative types of options. In Panel A, when sorting portfolios, instead of using one-month at-the-money (ATM) options, we use ATM options with alternative time to maturities, such as: 15, 60, and 90 days. In Panel B, instead of using ATM options, we use options with different moneyness such as out-of-the-money (OTM) options with moneyness less than 0.95 for put options and greater than 1.05 for call options, and in-the-money (ITM) options with moneyness less than 0.95 for call options and greater than 1.05 for put options. The moneyness is defined as the strike price divided by the underlying stock price. The call and put combination is the portfolio consisting of both call and put options with the same moneyness. In Panel C, we consider alternative ways of computing options returns. Instead of constructing the option returns from the middle of each month, we use options at the beginning of each month and choose those ATM options with time to maturities closest to 30 days. We then hold it to the end of each month. The sample period is from January 1996 to December 2017. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

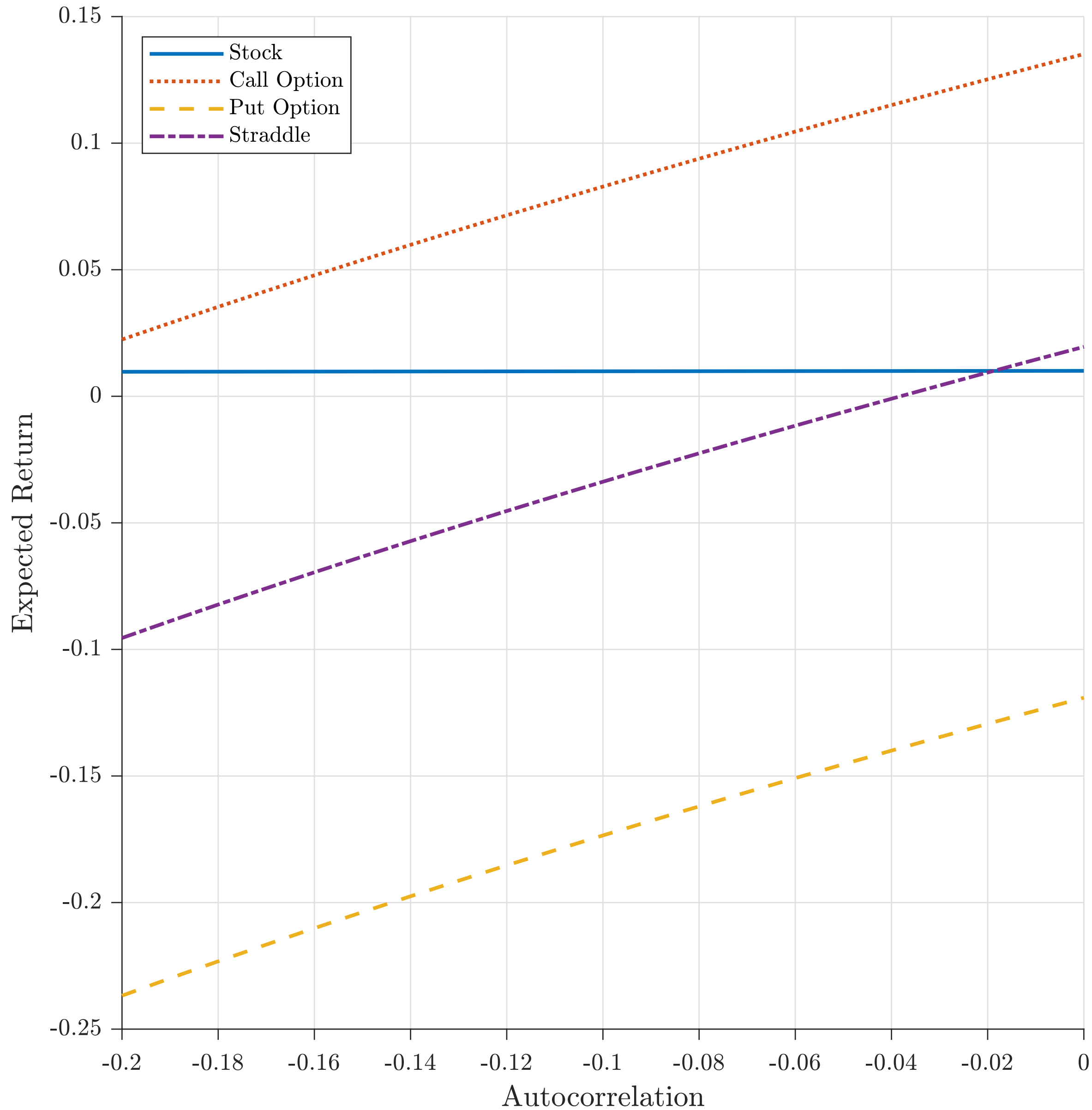

Figure 1
Expected Stock and Option Returns under the Trending O-U Process
This figure plots the expected stock return, expected hold-to-expiration option and straddle returns as functions of first-order autocorrelation of stock returns under the trending O-U process. All options are at-the-money options with the following parameters: $\mu = 0.10$, $r = 0.05$, $\tau = 1/12$, and $\sigma = 0.2$.

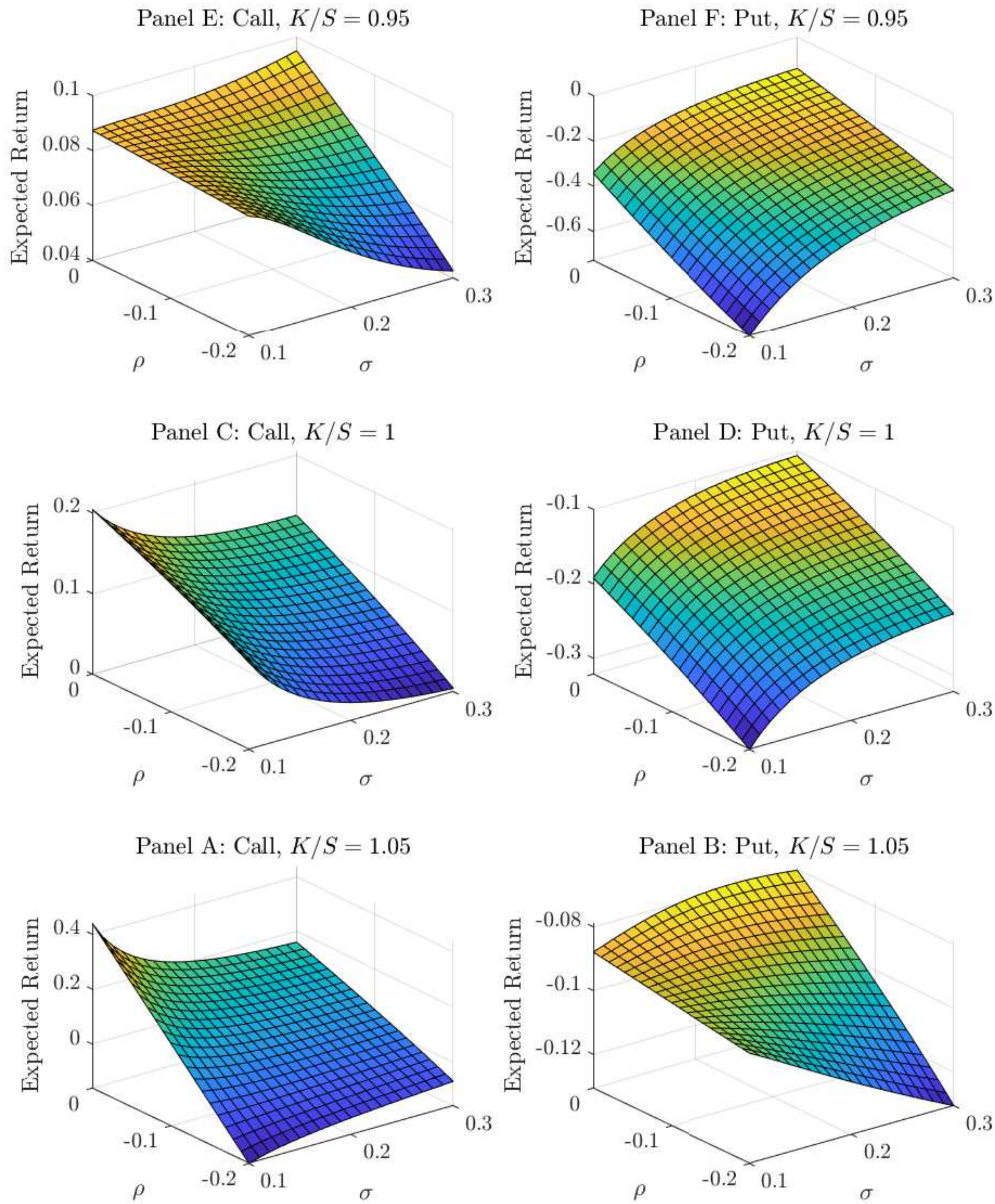

Figure 2
Expected Option Returns under the Trending O-U Process for Different Moneyness

This figure plots the expected hold-to-expiration call and put option returns as functions of volatility (σ) and first-order autocorrelation of stock returns (ρ) under the trending O-U process for three different levels of moneyness and with the following parameters: $\mu = 0.10$, $r = 0.05$, and $\tau = 1/12$.

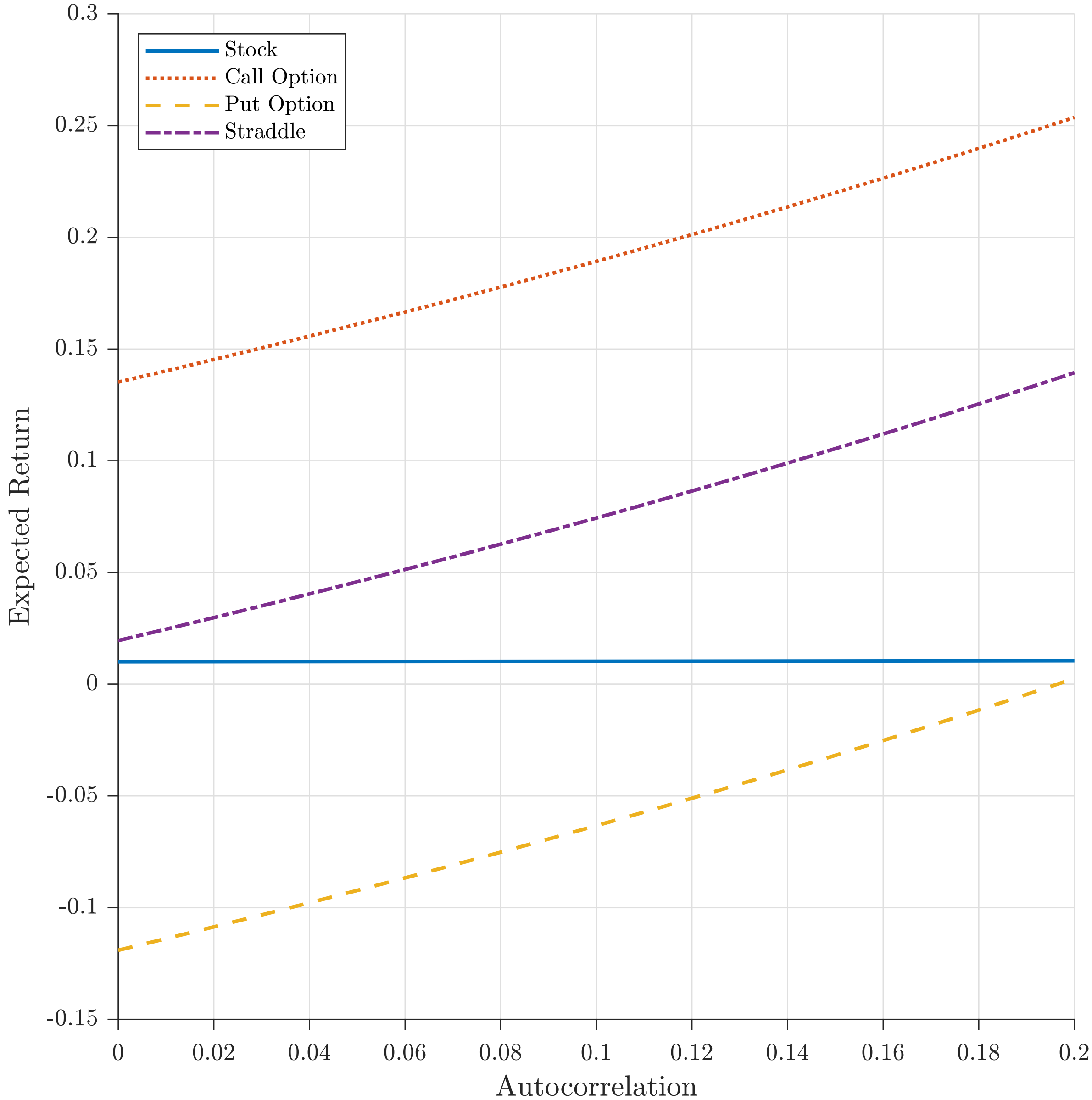

Figure 3

Expected Stock and Option Returns under the Bivariate Trending O-U Process

This figure plots the expected stock return, expected hold-to-expiration option and straddle returns as functions of first-order autocorrelation of stock returns under the bivariate trending O-U process. All options are at-the-money options with the following parameters: $\mu = 0.10$, $r = 0.05$, $\tau = 1/12$, $\sigma = 0.2$, $\sigma_x = 0.1$, and $\delta = 0.2$.

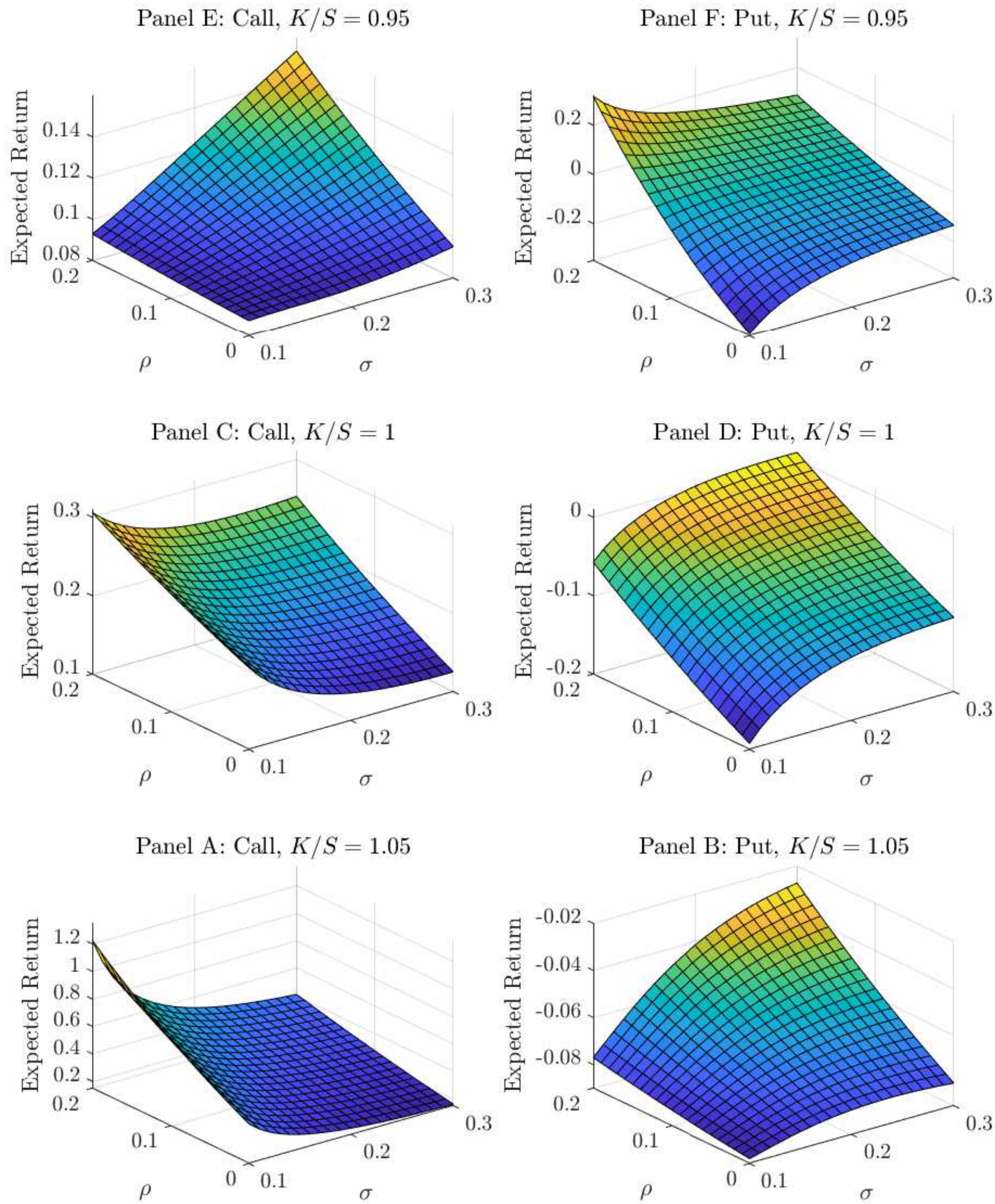

Figure 4
Expected Option Returns under the Bivariate Trending O-U Process for Different Moneyness

This figure plots the expected hold-to-expiration call and put option returns as functions of volatility (σ) and first-order autocorrelation of stock returns (ρ) under the bivariate trending O-U process for three different levels of moneyness and with the following parameters: $\mu = 0.10$, $r = 0.05$, $\tau = 1/12$, $\sigma = 0.2$, $\sigma_x = 0.1$, and $\delta = 0.2$.