Time-Varying Market Participation, Consumption Risk-Sharing, and Asset Dynamics*

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Abstract

We propose a general equilibrium model where heterogeneous risk-averse agents endogenously choose to enter or exit the stock market. We characterize the equilibrium in semi-closed form and present a novel conditional CCAPM. The model implies a procyclical variation in stock market participation. This time-variation gives rise to a countercyclical share of dividend in stockholders' consumption, which drives the amount of stockholders' consumption risk countercyclically, as opposed to the well-documented procyclical aggregate consumption risk. The price of consumption risk in our model is not only affected by consumption re-distribution of stockholders, but also by the time-variation in stock market participation. We find, under the assumption of time-invariant individual risk aversion, that the latter effect dominates the former, leading to a procyclical price of consumption risk. We provide empirical evidence for both the amount and price of consumption risk dynamics, supporting our theory. Overall, this article shows that it is the countercyclical stockholders' amount of risk due to time-varying risk-sharing that explains time-varying risk premium, excess volatility, and the price-dividend ratio.

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Keywords: *Time-varying market participation, Consumption Risk, Heterogeneous agents, Conditional consumption-based asset pricing, labor income, recursive utility*

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1 Introduction

Leading representative-agent dynamic asset pricing theories (e.g., Campbell and Cochrane, 1999; Bansal and Yaron, 2004; Barro, 2006) have been successful in explaining salient features of financial markets. In these representative-agent models, the amount of consumption risk is countercyclical or constant. Duffee (2005) and Roussanov (2014), however, document that the amount of aggregate consumption risk is procyclical. This gap between the empirical finding and theory raises the question of how a consumption-based asset pricing theory can explain the countercyclical equity premium and excess volatility while accommodating a weakly countercyclical to a procyclical variation in the amount of aggregate consumption risk. The aim of this article is to address this question and in doing so, we present a new foundation to understand some stylized facts of financial market.

Motivated by the empirical evidence of limited market participation and its time-variation,[1] we propose a general equilibrium model where heterogeneous time-invariant risk-averse agents endogenously choose to enter or exit the stock market. We illustrate the importance of the distinction between the dynamics of consumption risk of stock market participants versus non-participants in reconciling consumption-based asset pricing models with empirical observations. Our article mainly shows that time-varying risk-sharing which stems from moderate and endogenous time-varying market participation is important in driving the dynamics of (i) equity risk premium, (ii) equity excess volatility, and (iii) price-dividend ratio.

Specifically, we consider an economy populated by finite number of investors with recursive preferences. Households differ only in their time-invariant risk aversion and they receive a stochastic non-financial income. In the presence of short-selling constraints, at each point in time, unconstrained investors are stockholders who trade in both a riskless bond and a risky asset, whereas constrained investors are non-stockholders who optimally

[1]As emphasized in the American Finance Association Presidential Address by Campbell (2006), limited market participation is a well-established stylized fact and its evidence is well documented in the literature (e.g., Mankiw and Zeldes, 1991; Gomes and Michaelides, 2008). The 2016 Survey of Consumer Finances shows only 29.7% of US. households hold a stock directly (60.2% accounting for indirect holding). Furthermore, the group of stock market participants is found to be time-varying in the data (e.g., Vissing-Jørgensen, 2002b; Brunnermeier and Nagel, 2008; Bonaparte et al., 2018).

trade only in a riskless bond. Due to time-varying investment opportunities for the optimal portfolio choice, constraints bind intermittently for investors and therefore the optimal decision to enter or exit market is time-varying. Based on this setup, we solve in semi-closed form for optimal investment policies, consumption choices for both stockholders and non-stockholders at each point in time. We provide a complete characterization of equilibrium asset prices, and present a CCAPM under time-varying market participation.

Our conditional consumption-based asset pricing model shows that the equilibrium equity premium is given by the product of two components: the price per unit of consumption risk, which is the consumption-weighted harmonic mean of stockholders' risk aversions, and the amount of stockholders' consumption risk, which is the covariance between the stockholders' consumption growth and stock returns. Hence, our model implies that in an economy populated by both stockholders and non-stockholders, stockholders' risk aversion and consumption risk have the first-order effect on the equity premium. However, non-market participants also affect the equity premium in three ways. First, by simply exiting the market, the risk-sharing of the remaining participants becomes less effective. Second, non-market participants affect the equilibrium parameters. Third, from the consumption clearing condition, their non-financial income affects the stock volatility which is a component of the covariance between equity returns and stockholders' consumption growth.

A simulation of our model shows that the amount of stockholders' consumption risk is strongly countercyclical. In bad times, more risk-averse investors are not willing to take market risk and therefore leave the market. Accordingly, aggregate dividends are shared only by fewer remaining stockholders and account for a larger fraction of these remaining stockholders' consumption. Given this greater share of dividend in stockholders' consumption, a change in consumption of the remaining stock market participants is highly sensitive to dividend shocks and thus equity returns. This dynamics leads to a countercyclical amount of stockholders' consumption risk. This is a unique finding to our article and complement Duffee (2005)'s conjecture that even in a heterogeneous framework, as long as stockholders are endowed with non-financial income, the amount of stockholders' consumption risk is procyclical. This conjecture does not take into account the effect of time-varying risk-sharing implied by endogenous time-varying market participation in equilibrium. We present empirical evidence on the countercyclical stockholders' consumption risk which is depicted in the left panel of Figure 1.

When it comes to aggregate consumption, our model implies that the share of dividend in aggregate consumption is rather procyclical, consistent in this case with the composition effect introduced in Duffee (2005). In bad times, due to a negative dividend shock, dividend accounts for a smaller fraction of aggregate consumption. Given a lower share of dividend in aggregate consumption, a change in aggregate consumption is less sensitive to equity returns. All else equal, this effect results in the amount of aggregate consumption risk varying procyclically. In our setting, the countercyclical covariance between dividend growth and equity returns dominates the procyclical dynamic of the dividend share, leading to a weakly countercyclical amount of aggregate consumption risk.[2] We confirm this dynamics in the data as displayed in the right panel of Figure 1. Another way of examining the dynamic of the amount of consumption risk is to decompose stock returns into the cash flow and the discount rate components, separately. Empirically, the covariance of aggregate consumption growth with the cash flow (discount rate) component is procyclical (countercyclical), driving the weakly countercyclical to procyclical aggregate consumption risk. Our model replicates the dynamics observed in the data for both component of returns. Overall, time-varying market participation seems to reconcile these observed dynamics of both stockholders' and aggregate consumption risk and their respective components.

Endogenizing time-varying market participation in this article also uncovers new implications for the price of consumption risk which can have an important implication for habit-type models. We show that time-varying market participation in our setting leads to a procyclical price of consumption risk, which is surprising given the extant literature. In our model, the price of consumption risk is driven by two counterbalancing effects: time-varying market participation and a time-varying cross-sectional consumption re-distribution effect. The latter is discussed in Chan and Kogan (2002). On the one hand, when the stock market is bad, we find that only risk-tolerant households choose to stay in the market. The exit of

[2]When we use simulated data from our model to infer the conditional aggregate consumption risk, as econometricians would do, instead of using the analytical solution, we recover the well-documented procyclical aggregate consumption risk, while the estimated stockholders' consumption risk is countercyclical. This result is portrayed in Figure OA.1.

the more risk-averse investors drives down the consumption-weighted harmonic mean of risk aversion of stockholders. On the other hand, consumption of the remaining relatively risk-tolerant market participants declines the most because their consumption is damaged by the negative dividend shock given they are the ones who heavily invest in the stock. This decrease in the consumption of risk-tolerant investors drives up the consumption-weighted average risk aversion, holding market participation unchanged. Taken together, we show that the effect of time-varying market participation dominates the cross-sectional consumption re-distribution effect, resulting in a procyclical price of consumption risk with a reasonable average of 3.8.[3]

This finding, however, should be interpreted with caution because a procyclical price of consumption risk in our simulation stems from time-varying market participation combined with our assumption of time-invariant individual risk aversion. We show in the online appendix that assuming individual countercyclical risk aversion may lead to a countercyclical price of consumption risk, but its variation is much less countercyclical than in a full participation economy. Thus, our crucial finding is that it is a countercyclical stockholder amount of risk, due to relatively ineffective risk-sharing in bad times, which essentially explains the countercyclical equity premium. This is in contrast to the previous understanding that the amount of aggregate consumption risk is procyclical and thus consumption-based asset pricing models require a very strong countercyclical price of consumption risk to explain the observed equity premium.[4] Using the Consumer Expenditure data, we find some empirical evidence consistent with our model implication that time-varying market participation and consumption-redistribution drive the price of consumption risk procyclically and countercyclically, respectively. Our finding also shows that the price of risk is procyclical, but

[3]To illustrate the effect of both time-varying market participation and consumption re-distribution, consider an example of three agents with risk aversion of 3, 6, and 9. Suppose in a normal state, the consumption share of each agent is 50%, 30%, and 20%, resulting in the price of risk (stockholders' harmonic mean of risk aversion) of 5.1. In a bad state, the consumption share is re-distributed to 40%, 30%, and 30%, which leads to the price of risk of 5.7, higher than in the normal state. However, if the most risk-averse agent leaves the market, the consumption share is 57% and 43% for the first two agents, resulting in the price of risk of 4.3, lower than in the normal state.

[4]See Duffee (2005), Nagel and Singleton (2011), and Roussanov (2014). Based on simulated data in our economy, our model reproduces the required strongly countercyclical price of risk implied by aggregate consumption as in previous empirical studies using aggregate consumption. We also reproduce the negative risk-return trade-off discussed in those papers. See Section 5.1.3.

not significant because these two competing forces drive the price of consumption risk in opposite directions.

We also examine the implication of time-varying market participation for both the level and dynamics of stock volatility. In our model, stock volatility is tightly linked to two terms: (i) the aggregate dividend share in stockholders' consumption, and (ii) the stockholders' consumption-weighted mean of risky asset share in their total wealth. Excess volatility is generated when the first term is higher than the second. We find that it is countercyclical as the dispersion between the two terms is higher in bad economic times. As discussed before, in bad states, the share of dividend in stockholders' consumption is greater than in good states because the total dividend is shared only by few remaining stockholders. In contrast, the stockholders' consumption-weighted mean of risky asset share in total wealth becomes lower because (i) investors optimally reduce their risky asset holding, and (ii) the consumption of risk-tolerant investors drops the most, leading the consumption-weighted average to be more tilted towards the risky asset share of relatively risk-averse remaining stockholders. This mechanism generates countercyclical equity excess volatility, in contrast to the procyclical variation which the full market participation case of our setup would generate. Therefore, our paper provides a new explanation for both the empirically observed level and dynamics of the stock volatility through time-varying market participation.

Finally, we also examine the price-dividend ratio generated in our model. In the literature, it is challenging to produce a procyclical variation in the price-dividend ratio with the elasticity of intertemporal substitution less than one (e.g., Ju and Miao, 2012; Chabakauri, 2015b) with few exceptions (e.g., Guvenen, 2009). We show that our model produces the empirically observed procyclical price-dividend ratio due to both a procyclical risky asset holding and the implied level of equity excess volatility. We also show that our model generates long-horizon predictability of the equity premium with a quantitatively similar R^2 as in the data. In addition, we conduct the backward-looking test in Bansal et al. (2012). The result shows the price-dividend ratio in our model is forward-looking because lagged consumption growth does not counterfactually forecast future price-dividend ratio.

Overall, this article shows that in a heterogeneous risk-averse agents setup, introducing non-financial income together with short-selling constraints gives rise to a reasonable time-variation in the stock market participation. Consequently, our model generates a procyclical (26.9% in bad states and 31.8% in normal states) and moderate time-variation in market participation rate as portrayed in Figure 2. We shed light on the crucial importance of the distinction between the consumption risk dynamics of stock market participants and non-participants in reconciling dynamic asset pricing models with the empirical observations. Our paper gives support to the conjecture of Brunnermeier and Nagel (2008) stating that time-varying market participation rather than time-varying individual risk aversion drives the time-varying risk premium.

The rest of the paper unfolds as follows: Section 2 reviews the literature which could be skipped by informed readers. Section 3 discusses the economic setup and solves the optimization problems in a general equilibrium framework. Section 4 solves the equilibrium and presents its results. Section 5 simulates the model. Section 6 provides an empirical test of our model. Section 7 concludes.

2 Literature review

Our work directly belongs to the studies that have theoretically examined limited equity market participation to explain broad asset pricing features. One class of these studies exogenously specifies a group of investors excluded from the stock market. For instance, Basak and Cuoco (1998) study asset prices and optimal consumption/investment policies in a restricted economy where one of two investors is the only stockholder and compare it with the unrestricted economy where both agents are stockholders. They show that limited market participation helps resolve the equity premium puzzle. Guvenen (2009) studies the implications of limited market participation for asset pricing dynamics in a setup where two investors differ in their EIS (Elasticity of Intertemporal Substitution) under a real business cycle framework. In his model, an agent with low EIS, assumed to be a non-stockholder, has strong desire to smooth consumption. The other agent with high EIS, assumed to be a stockholder, borrows from the non-stockholder which amplifies their consumption volatility and they demand a large premium for holding aggregate risk. His model generates some empirically observed asset prices as well as wealth inequality between market participants and non-participants. Since there is no dynamics in the market entry or exit in this class of models, these studies did not derive implications of time-varying risk sharing due to time-varying stock market participation for asset pricing, which is one of the contribution of this article.

The other class of studies[5] allows the stock market participation to be determined by an individual utility maximization. These papers focus on the unconditional asset moments, participation rate, or investors' life-cycle behavior, but they are silent on the implication on the asset price dynamics along with market participation dynamics, which is the focus of our paper. For example, Gomes and Michaelides (2008) endogenize limited market participation by considering heterogeneous agents who differ in both risk aversion and EIS, idiosyncratic labor income, borrowing and short-selling constraints, and fixed costs for participation. While they study the effect of limited risk-sharing on unconditional asset moments and unconditional market participation rate, they do not draw implications for the dynamics of asset pricing and market participation. Fagereng et al. (2017) develop a model explaining the shape of the life-cycle profile of the average household's stock market participation.

A few papers endogenize market participation without the life-cycle setup. For example, Calvet et al. (2004) study the effect of financial innovation on asset prices, which in turn, lead to a change in market participation. In two periods model with CARA utility, they consider heterogeneous labor income risks and fixed costs. Our paper is different from their model in that we consider preferences which give rise to a state-dependent portfolio choice through the wealth effect. In the online appendix OA.1, we show that CARA utility does not generate a state-dependent portfolio choice. More recently, Bonaparte et al. (2018) also present a model with time-varying market participation. Their primary focus is to explain the high level of stock market participants' turnover in non-retirement accounts. In their model, investors differ in their income levels, and limited market participation arises from transaction costs, borrowing, and short-selling constraints. While they attempt to match the model-implied unconditional moments to the data, they do not examine either the dynamics

[5] See Allen and Gale (1994), Williamson (1994), Constantinides et al. (2002), Haliassos and Michaelides (2003), Cao et al. (2005), Gomes and Michaelides (2005), Alan (2006), Gomes and Michaelides (2008), and Fagereng et al. (2017).

of the equilibrium asset prices nor consumption risk components as in the present paper.

Our work is directly related to the vast consumption-based asset pricing literature. Among others, Duffee (2005) finds that the amount of aggregate consumption risk is procyclical. This implies that the price of consumption risk is substantially countercyclical in order to explain the countercyclical variation in the equity premium under a consumption-based asset pricing framework. A recent study by Xu (2018) develops a representative-agent model with habit preferences. Her model generates a procyclical amount of aggregate consumption risk and assumes a strongly countercyclical price of consumption risk dynamics to explain the countercyclical equity premium.

Different from this paper, we consider a heterogeneous economy. We advocate the importance of distinguishing market participants from non-participating households. Our key finding is that it is a countercyclical stockholders' amount of risk that explains the countercyclical equity premium rather than the price of consumption risk. In doing so, we show both theoretically and empirically the opposite dynamics of stockholders' consumption risk to aggregate consumption risk, which is a novel finding in the literature. We also show that the assumption of strongly countercyclical price of consumption risk does not hold in our setting.

3 Economy

3.1 Basic setup

Time and Uncertainty structure: We consider a continuous pure-exchange economy over the infinite time horizon. The uncertainty in this economy is represented by a filtered probability space (Ω,$\mathcal{F}$,$\mathbb{P}$). Ω is the set of all possible states. $\mathcal{F} = \{\mathcal{F}_t\}_{t\in\tau}$ is the filtration that represents the investors' information available at time t where $\tau \in [0,\infty)$. The probability measure $\mathbb{P}$ is defined on $(\Omega, \mathcal{F}_\infty)$ where $\mathcal{F}_\infty = \bigcup_{t=0}^{\infty} \mathcal{F}_t$, represents the investors' common beliefs. The filtration $\mathcal{F}$ is generated by two-dimensional standard Brownian motion $W = [W_{d,t}, W_{y,t}]$. The two Brownian motion shocks are correlated (i.e., $dW_{d,t}dW_{y,t} = \rho dt$). All stochastic processes introduced in the remainder of the paper are assumed to be adapted to $\mathcal{F}_t$.

Agents and Preferences: The economy is populated by infinitely lived N investors all having the recursive utility developed in Epstein and Zin (1989) and Duffie and Epstein (1992).[6] Investor i is maximizing

$$V_{i,t} = \mathbb{E}_t[\int_t^\infty f(C_{i,s}, V_{i,s})ds], \quad \forall t \in [0, \infty) \tag{1}$$

where $f(C_{i,t}, V_{i,t})$ is the normalized aggregator for consumption $C_{i,t}$ and indirect utility $V_{i,t}$. For the Epstein-Zin utility, $f(C_{i,t}, V_{i,t})$ is given by

$$f(C_{i,t}, V_{i,t}) = \frac{\delta}{1-\psi^{-1}} \frac{C_{i,t}^{1-\psi^{-1}} - ((1-\gamma_i)V_{i,t})^{\theta_i}}{((1-\gamma_i)V_{i,t})^{\theta_i - 1}} \tag{2}$$

where $\theta_i = \frac{1-\psi^{-1}}{1-\gamma_i}$. ψ is the Elasticity of Intertemporal Substitution (EIS) and γ_i is the coefficient of risk aversion of an investor i. For investors $i = 1, ..., N$, risk aversion coefficient is $\gamma_1, ..., \gamma_N$, respectively, with $0 < \gamma_1 < ... < \gamma_N$.[7] $C_t \in \mathbb{R}^+$ is one perishable consumption good that serves as the numéraire. $\delta > 0$ is the subjective time preference rate. $\mathbb{E}_t$ denotes the expectation taken at time t.

Non-financial income: All investors receive, for simplicity, the same level of stochastic exogenous non-financial income (labor income) $Y_t = Y_{i,t}$ that evolves as[8]: $\frac{dY_t}{Y_t} = \mu_y dt + \sigma_y dW_{y,t}$ where $\mu_y > 0$ is the expected labor income growth rate, $\sigma_y > 0$ is the labor income growth volatility. With this setup, as in the heterogeneous agents literature (e.g., Chabakauri, 2015a; Gârleanu and Panageas, 2015), state variables for agent i's maximization are financial wealth $X_{i,t}$, non-financial income Y_t, $N-1$ consumption shares $w_{j,t} = \frac{c_{j,t}}{\sum_{j=1}^N c_{j,t}}$ which has the following dynamics: $dw_{j,t} = w_{j,t}[\mu_{w_{j,t}} dt + \sigma^d_{w_{j,t}} dW_{d,t} + \sigma^y_{w_{j,t}} dW_{y,t}]$[9] $\forall j = 1, ..., N-1$. Therefore, indirect utility is a function of those $N+1$ state variables: $V_{i,t} = V_i(X_{i,t}, Y_t, \mathbf{w_t})$ where $\mathbf{w_t} = [w_{1,t}, ..., w_{N-1,t}]$. In equilibrium, asset parameters are time-varying because of time-varying investment opportunity that arises from shocks to the economy and a change

[6] In a previous version of our paper, we assumed CRRA utility and show that the main results in this article hold. The choice of the recursive utility is mainly due to generate a reasonable interest rate level.

[7] Heterogeneous risk aversion with the same EIS is considered in the literature (e.g., Coen-Pirani, 2004, 2005; Buss et al., 2013; Chabakauri, 2015b).

[8] In the online appendix OA.5, we extend the model to a setup with an idiosyncratic labor income. The simulation of the extended model shows our results in the baseline model are robust to idiosyncratic labor income. The implication of uninsurable idiosyncratic risk in human capital for asset pricing is well studied in Ai and Bhandari (2018).

[9] $\mu_{w_{j,t}}$, $\sigma^d_{w_{j,t}}$, and $\sigma^y_{w_{j,t}}$ are to be determined in equilibrium.

in the endogenous consumption distribution.

Financial assets: An agent can allocate her wealth to two assets: a riskless asset $\frac{dB_t}{B_t} = r_{f,t}(D_t, Y_t, \mathbf{w_t})dt$ where the parameter $r_{f,t}$ denotes the risk-free rate and a risky asset which is a claim to an exogenous dividend D_t that follows: $\frac{dD_t}{D_t} = \mu_d dt + \sigma_d dW_{d,t}$ where $\mu_d > 0$ is the expected dividend growth rate, and $\sigma_d > 0$ is the dividend growth volatility. Agents cannot short-sell the risky asset in the presence of short-selling constraints. The equilibrium equity returns dynamics has the form:[10]

$$\frac{dS_t + D_t dt}{S_t} = \mu_{s,t}(D_t, Y_t, \mathbf{w_t})dt + \sigma^d_{s,t}(D_t, Y_t, \mathbf{w_t})dW_{d,t} + \sigma^y_{s,t}(D_t, Y_t, \mathbf{w_t})dW_{y,t} \tag{3}$$

where S_t is the stock price, $\mu_{s,t}$ is the expected stock returns, and $\sigma^d_{s,t}$ and $\sigma^y_{s,t}$ are the sensitivity of equity returns with respect to dividend and labor income shocks, respectively, which constitute the stock volatility $\sigma_{s,t} = \sqrt{{\sigma^d_{s,t}}^2 + {\sigma^y_{s,t}}^2 + 2\rho\sigma^d_{s,t}\sigma^y_{s,t}}$. In Section 4, the risk-free rate, expected stock returns, and stock volatility are endogenously determined in equilibrium. They are a function of dividend D_t, non-financial income Y_t, consumption distribution $\mathbf{w_t}$: $r_{f,t}(D_t, Y_t, \mathbf{w_t}), \mu_{s,t}(D_t, Y_t, \mathbf{w_t}), \sigma_{s,t}(D_t, Y_t, \mathbf{w_t})$. To make the notation easier to follow, we omit the argument $(D_t, Y_t, \mathbf{w_t})$ for asset parameters hereafter.

3.2 Individual optimization problem

We impose a short-selling constraint to generate non-market participation.[11] Therefore, individual optimization problem is to search for possible consumption and nonnegative dollar amount of risky asset holding to maximize the lifetime sum of expected utility at each point in time, given the states i.e., $\{C^*_{i,t}, \pi^*_{i,t}\} = \arg\max_{(c,\pi\geq 0)} \mathbb{E}_t[\int_t^\infty f(C_{i,s}, V_{i,s})ds]$. A trading strategy, with short selling constraints, satisfies the following dynamic budget constraints.

[10]This conjecture for the equilibrium stock price dynamics is confirmed in **Proposition 1**.

[11]The importance of short-selling constraints in explaining limited market participation is well studied in Athreya et al. (2018). In the literature, other mechanisms to generate the limited market participation are considered. Fixed setup or transaction costs: Allen and Gale (1994), Williamson (1994), Heaton and Lucas (1996), Vissing-Jørgensen (2002b), Haliassos and Michaelides (2003), Alan (2006), Fagereng et al. (2017); Life-cycle model: Constantinides et al. (2002), Gomes and Michaelides (2005), Alan (2006), Gomes and Michaelides (2008), Fagereng et al. (2017); Model uncertainty: Cao et al. (2005); Borrowing constraint: Allen and Gale (1994), Heaton and Lucas (1996), Constantinides et al. (2002), Haliassos and Michaelides (2003), Alan (2006), Gomes and Michaelides (2008), Fagereng et al. (2017).

$$dX_{i,t} = \pi_{i,t}(\frac{dS_t + D_t dt}{S_t}) + (X_{i,t} - \pi_{i,t})r_{f,t}dt + (Y_t - C_{i,t})dt$$
$$= [\pi_{i,t}(\mu_{s,t} - r_{f,t}) + r_{f,t}X_{i,t} + Y_t - C_{i,t}]dt + \pi_{i,t}(\sigma^d_{s,t}dW_{d,t} + \sigma^y_{s,t}dW_{y,t}) \tag{4}$$

subject to $\pi_{i,t} \geq 0$.

A maximization with portfolio constraints can be solved via the Lagrangian method as in the literature on portfolio choice with constraints (e.g., Yiu, 2004; Chabakauri, 2013). Let $l_{i,t}$ denote the time t Lagrange multiplier for short-selling constraint $\pi_{i,t} \geq 0$. Then, the Hamilton-Jacobi-Bellman (HJB) equation with short-selling constraints for an agent i is

$$\begin{aligned}
0 = &\max_{(c,\pi)\in\mathcal{A}} f(C_{i,t}, V_{i,t}) + [\pi_{i,t}(\mu_{s,t} - r_{f,t}) + r_{f,t}X_{i,t} + Y_t - C_{i,t}]V_{x_i,t} + \frac{1}{2}\pi^2_{i,t}\sigma^2_{s,t}V_{x_ix_i,t} + \mu_y Y_t V_{y,t} \\
&+ \frac{1}{2}\sigma^2_y Y^2_t V_{yy,t} + \rho_{s,t}\sigma_y Y_t \sigma_{s,t}\pi_{i,t}V_{x_iy,t} + \sum_{j=1}^{N-1}\mu_{w_j,t}w_{j,t}V_{w_j,t} + \frac{1}{2}\sum_{j=1}^{N-1}\sigma^2_{w_{j,t}}w^2_{j,t}V_{w_jw_j,t} \\
&+ \sum_{j\neq k}\rho_{w_j,w_k,t}\sigma_{w_{j,t}}\sigma_{w_{k,t}}w_{j,t}w_{k,t}V_{w_jw_k,t} + \sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}\sigma_{s,t}\pi_{i,t}V_{w_jx_i,t} \\
&+ \sum_{j=1}^{N-1}\rho_{w_j,y,t}\sigma_{w_{j,t}}w_{j,t}\sigma_y Y_t V_{w_jy,t} + l_{i,t}\pi_{i,t} \qquad \forall i = 1, ..., N, \forall t \in [0,\infty)
\end{aligned} \tag{5}$$

subject to $E_t[V_{i,T}] \to 0$, as $T \to \infty$ where $\rho_{s,t}$ is the correlation between equity returns and labor income growth $\rho_{s,t} \equiv Corr_t(\sigma^d_{s,t}dW_{d,t} + \sigma^y_{s,t}dW_{y,t}, \sigma_y dW_{y,t}) = \frac{\sigma^d_{s,t}\rho + \sigma^y_{s,t}}{\sigma_{s,t}}$, $\sigma_{w_{j,t}}$ is the volatility of consumption share j dynamics $\sigma_{w_{j,t}} \equiv \sqrt{{\sigma^d_{w_{j,t}}}^2 + {\sigma^y_{w_{j,t}}}^2 + 2\rho\sigma^d_{w_{j,t}}\sigma^y_{w_{j,t}}}$, $\rho_{w_j,w_k,t}$ is the correlation between consumption share j and k dynamics $\rho_{w_j,w_k,t} \equiv Corr_t(\sigma^d_{w_{j,t}}dW_{d,t} + \sigma^y_{w_{j,t}}dW_{y,t}, \sigma^d_{w_{k,t}}dW_{d,t} + \sigma^y_{w_{k,t}}dW_{y,t})$, $\rho_{w_j,s,t}$ is the correlation between consumption share j dynamics and equity returns $\rho_{w_j,s,t} \equiv Corr_t(\sigma^d_{w_{j,t}}dW_{d,t} + \sigma^y_{w_{j,t}}dW_{y,t}, \sigma^d_{s,t}dW_{d,t} + \sigma^y_{s,t}dW_{y,t})$, and, $\rho_{w_j,y,t}$ is the correlation between consumption share j dynamics and labor income growth $\rho_{w_j,y,t} \equiv Corr_t(\sigma^d_{w_{j,t}}dW_{d,t} + \sigma^y_{w_{j,t}}dW_{y,t}, \sigma_y dW_{y,t})$. In the online appendix OA.11, we formally derive the HJB equation with the Lagrange multiplier to confirm (5). Note that the effect of the term $l_{i,t}\pi_{i,t}$ is to penalize the objective function when the short-selling constraint is binding. The first-order necessary conditions for the optimization problem are given by

$$C^*_{i,t} = (\delta V^{-1}_{x_i,t}((1-\gamma)V_{i,t})^{-\theta+1})^{\psi} \tag{6}$$

$$\pi^*_{i,t} = -\frac{(\mu_{s,t} - r_{f,t})V_{x_i,t}}{\sigma^2_{s,t}V_{x_ix_i,t}} - \frac{\rho_{s,t}\sigma_y Y_t \sigma_{s,t} V_{x_iy,t}}{\sigma^2_{s,t}V_{x_ix_i,t}} - \frac{\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_j,t}w_{j,t}\sigma_{s,t}V_{w_jx_i,t}}{\sigma^2_{s,t}V_{x_ix_i,t}} - \frac{l^*_{i,t}}{\sigma^2_{s,t}V_{x_ix_i,t}} \tag{7}$$

The last term in (7) is the adjustment from the constraint and therefore we can rewrite (7) in the form

$$\pi^*_{i,t} = \pi^{w/o}_{i,t} - \frac{l^*_{i,t}}{\sigma^2_{s,t}V_{x_ix_i,t}} \tag{8}$$

where $\pi^{w/o}_{i,t}$ refers to the expression for the risky asset holding without any constraints. Furthermore, the Kuhn-Tucker optimality conditions are

$$l^*_{i,t}\pi^*_{i,t} = 0 \tag{9}$$

$$l^*_{i,t} \geq 0, \pi^*_{i,t} \geq 0 \tag{10}$$

Equation (9) is the complementary slackness condition and is used to solve for $l^*_{i,t}$ whenever $l^*_{i,t} \neq 0$. Therefore, from equation (7), (9), and (10),

$$l^*_{i,t} = \begin{cases} 0 & \text{if } \pi^{w/o}_{i,t} > 0 \\ -(\mu_{s,r} - r_{f,t})V_{x_i,t} - \rho_{s,t}\sigma_y Y_t \sigma_{s,t} V_{x_iy,t} - \sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}\sigma_{s,t}V_{w_jx_i,t} & \text{Otherwise} \end{cases} \tag{11}$$

We plug $C^*_{i,t}$, $\pi^*_{i,t}$, and $l^*_{i,t}$ back into (5) to solve the HJB equation.

When the dividend growth is perfectly correlated with labor income growth $\rho = 1$, there is a closed form solution for this maximization problem.[12] For the non-perfect correlation case, there is no closed form solution in general for the optimal consumption and portfolio. However, following Koo (1998) and Wang et al. (2016) who, in their settings, solve the optimization problem for the case where the ratio of financial wealth to labor goes to infinity ($\frac{X}{Y} \rightarrow \infty$) in (5), we also provide the closed form solutions in our setup.[13] The following proposition shows the optimal consumption and investment as functions of asset

[12] We solve for the closed form and show that our general solution reduces to this special case when imposing $\rho = 1$. See the online appendix OA.3.

[13] Wang et al. (2016) solve the consumption choice in their setting both analytically in this case with the assumption and numerically when this assumption is not applied and show a non-significant difference especially when $\frac{X}{Y}$ is high (See Figure 1 in their paper).

parameters.

Proposition 1. *An investors' optimal consumption, stock holdings, and the wealth dynamics are given by* $\forall i = 1, ..., N$

$$C^*_{i,t} = \begin{cases} ((r_{f,t} + \frac{\lambda_t^2}{2\gamma_i})(1-\psi) + \delta\psi) \cdot (X_{i,t} + H_{h,t}) & \text{if } \pi^{w/o}_{i,t} > 0 \\ (r_{f,t}(1-\psi) + \delta\psi)(X_{i,t} + H_{n,t}) & \text{Otherwise} \end{cases} \tag{12}$$

$$\pi^*_{i,t} = \begin{cases} \pi^{w/o}_{i,t} = \frac{\lambda_t}{\gamma_i \sigma_{s,t}}(X_{i,t} + H_{h,t}) - \frac{\rho_{s,t}\sigma_y}{\sigma_{s,t}} H_{h,t} & \text{if } \pi^{w/o}_{i,t} > 0 \\ 0 & \text{Otherwise} \end{cases} \tag{13}$$

$$dX^*_{i,t} = \begin{cases} (\pi^*_{i,t}(\mu_{s,t} - r_{f,t}) + r_{f,t}X_{i,t} + Y_t - C^*_{i,t})dt + \pi^*_{i,t}(\sigma^d_{s,t}dW_{d,t} + \sigma^y_{s,t}dW_{y,t}) & \text{if } \pi^{w/o}_{i,t} > 0 \\ (r_{f,t}X_{i,t} + Y_t - C^*_{i,t})dt & \text{Otherwise} \end{cases} \tag{14}$$

where λ_t is the Sharpe ratio, $H_{h,t} \equiv \frac{Y_t}{r_{f,t}+\rho_{s,t}\sigma_y\lambda_t-\mu_y}$, and $H_{n,t} \equiv \frac{Y_t}{r_{f,t}-\mu_y}$

Proof : See Appendix A.1

Regarding the optimal consumption, first, equation (12) shows that the marginal propensity to consume out of labor income is not unity (i.e., $\partial C^*_{i,t}(X_{i,t}, Y_t)/\partial Y_t = \frac{(r_{f,t}+\frac{\lambda_t^2}{2\gamma_i})(1-\psi)+\delta\psi}{r_{f,t}+\rho_{s,t}\sigma_y\lambda_t-\mu_y} \neq 1$), different from heterogeneous CARA utility case or a representative agent setup.[14] Therefore, labor income shocks affect the optimal wealth dynamics in (14) and in turn the stock price in equilibrium. Second, the consumption-to-total wealth ratio ($k_{h,i,t} \equiv \frac{C_{i,t}}{X_{i,t}+H_{h,t}} = (r_{f,t} + \frac{\lambda_t^2}{2\gamma_i})(1-\psi) + \delta\psi$) of stockholders is different from that of non-stockholders as non-stockholders do not face an uncertainty from the risky asset holding.[15] Lastly, our model does not guarantee a stationary cross-sectional distribution of consumption. An extension of the current model featuring overlapping generations as in Gârleanu and Panageas (2015) guarantees a stationary equilibrium. We show that our main results are not affected by non-stationary distribution as our simulation is conducted for 40 years horizon and also the

[14]For more details of CARA, see Appendix OA.1. With a representative agent setup, by the consumption market clearing condition $C_t = D_t + Y_t$, the marginal propensity to consume out of labor income is always unity.

[15]A simulation of the cross-sectional consumption is in Figure OA.2.

presence of non-financial income makes the speed of the dominance significantly slow.

The unconstrained investors' optimal stock holding $\pi_{i,t}^{w/o}$ has an intertemporal hedging demand due to a positive correlation between stock returns and labor income $\rho_{s,t} = \frac{\sigma_{s,t}^d \rho + \sigma_{s,t}^y}{\sigma_{s,t}} > 0$.[16] The short-selling constraint is binding for investors with relatively high risk aversion and they sub-optimally have zero position. Furthermore, given the equation (13), the condition for a positive holding $\pi_{i,t}^{w/o} > 0$ is

$$\frac{X_{i,t}}{Y_t}\lambda_t(r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y) + \lambda_t - \gamma_i\rho_{s,t}\sigma_y > 0 \tag{15}$$

It shows that given $r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y > 0$, the higher the financial wealth-to-labor income $\frac{X_{i,t}}{Y_t}$, the more likely the investor has a positive holding. Therefore, the financial wealth-to-labor income $\frac{X_{i,t}}{Y_t}$ plays a crucial role in investors' dynamic decision on stock market participation. This condition also shows that the higher risk aversion γ_i an investor has, the less likely the investor has a positive holding. The higher expected non-financial income growth μ_y leads to a higher value of human capital which in turn induces investors less likely to have a positive holding due to a greater hedging concern. The level of correlation between equity returns and labor income growth also play a role in market participation decision. In the online appendix OA.10, we conduct the comparative static analysis of the correlation between dividend and labor income growth ρ in the market participation.

4 Equilibrium

This section discusses the equilibrium. Section 4.1 describes the equilibrium. Section 4.2 derives the equilibrium. Section 4.3 examines the characteristics of equilibrium asset parameters and how our model reduces to nested economies studied in the literature. 4.4 presents a novel CCAPM featuring time-varying market participation.

4.1 Description of the equilibrium

Definition 1. *An equilibrium is a set of asset parameters* $\{r_{f,t}(D_t, Y_t, \mathbf{w_t}), \lambda_t(D_t, Y_t, \mathbf{w_t}), \sigma_{s,t}(D_t, Y_t, \mathbf{w_t})\}$*, consumption and investment policies* $\{C_{i,t}^*, \pi_{i,t}^*\}_{i\in 1,\ldots,N}$ *which maximize the*

[16] Guiso et al. (1996), Angerer and Lam (2009), and Betermier et al. (2012) empirically present evidence, consistent with our result that investors optimal stock holdings are negatively associated with the labor income risk.

sum of life time expected utility (1) subject to the dynamic budget constraint (4) for each investor and satisfy the market-clearing conditions:

1. Stock market clears: $\sum_{i=1}^{N} \pi_{i,t}^{*} = S_t$ (16)

2. Bond market clears: $\sum_{i=1}^{N} X_{i,t} - \sum_{i=1}^{N} \pi_{i,t}^{*} = 0$ (17)

3. Consumption market clears: $\sum_{i=1}^{N} C_{i,t}^{*} = \sum_{i=1}^{N} Y_{i,t} + D_t$ (18)

The stock is in unit supply and hence the stock market clearing condition is represented by (16). Since $\sum_{i=1}^{N} X_{i,t}$ is the total demand for both stock and bond, $\sum_{i=1}^{N} X_{i,t} - \sum_{i=1}^{N} \pi_{i,t}^{*}$ represents the total demand on the bond. The zero supply bond market clearing condition is therefore represented by $\sum_{i=1}^{N} X_{i,t} - \sum_{i=1}^{N} \pi_{i,t}^{*} = 0$. This also implies that $S_t = \sum_{i=1}^{N} X_{i,t}$. Lastly, the stock market clearing condition (16) together with the bond market clearing condition (17) implies the consumption market clearing condition (18).[17]

4.2 Derivation of the equilibrium

We solve for the general equilibrium based on the optimal consumption and portfolio choice obtained by solving for the HJB (Hamilton-Jacobi-Bellman) equation. We derive the equilibrium in the following steps. First, from the stock market clearing condition (16), the equation for the equilibrium Sharpe ratio is obtained. Second, by matching the deterministic terms of the dynamics of both left and right-hand side of (18), the equation for the equilibrium risk-free rate is obtained. Third, by matching the diffusion terms of the dynamics of (18), two equations for the equilibrium stock volatility are obtained. Fourth, from the consumption clearing condition (18) and the optimal consumption in (12) together with the fact that $S_t = \sum_{i=1}^{N} X_{i,t}$, the closed form solution for the equilibrium stock price is computed. Finally, the cut-off stockholder h_t^* is determined monotonically such that unconstrained agents do not optimally choose to be constrained, and vice versa. For example, at time t, investors $1, 2, ..., h_t^*$ are stockholders while $h_t^* + 1, ..., N$ are non-stockholders.

[17] This is by the Walras' law. See Appendix A.2 for proof.

For more details on the equilibrium cut-off stockholder, see Appendix OA.9. **Proposition 2** summarizes the set of equations for the equilibrium asset parameters and stock price.

Proposition 2. *In equilibrium defined by* ***Definition 1*** *the set of equations for the Sharpe ratio λ_t, the risk-free rate $r_{f,t}$, the stock volatility $\sigma_{s,t}$ and the stock price are given by:*

$$\lambda_t = \frac{\sigma_{s,t}\sum_{i=1}^{N} X_{i,t} + h_t^* \rho_{s,t}\sigma_y H_{h,t}}{\sum_{i=1}^{h_t^*} \frac{X_{i,t}+H_{h,t}}{\gamma_i}} \tag{19}$$

$$r_{f,t} = \delta + \frac{\mu_d D_t + \mu_y N \cdot Y_t}{D_t + N \cdot Y_t}\frac{1}{\psi} - \sum_{i=1}^{h_t^*} \frac{C_{i,t}}{D_t + N \cdot Y_t}(\frac{\lambda_t^2}{\gamma_i}\frac{1+\psi}{2\psi}) \tag{20}$$

$$\sigma_{s,t} = \sqrt{(\sigma_{s,t}^d)^2 + (\sigma_{s,t}^y)^2 + 2\rho\sigma_{s,t}^d\sigma_{s,t}^y} \tag{21}$$

$$\sigma_{s,t}^d = \frac{\sigma_d D_t}{\sum_{i=1}^{h_t^*} k_{h,i,t}\pi_{i,t}^*} \tag{22}$$

$$\sigma_{s,t}^y = \frac{\sigma_y Y_t N[1 - \frac{1}{N}(\sum_{i=h_t^*+1}^{N} k_{n,t}/(r_{f,t}-\mu_y) + \sum_{i=1}^{h_t^*} k_{h,i,t}/(r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y))]}{\sum_{i=1}^{h_t^*} k_{h,i,t}\pi_{i,t}^*} \tag{23}$$

$$S_t = \frac{D_t + N \cdot Y_t - \sum_{i=1}^{h_t^*} \frac{\lambda_t^2}{2\gamma_i}(1-\psi)(X_{i,t}+H_{h,t})}{r_{f,t} - (\frac{\mu_d D_t + \mu_y N \cdot Y_t}{D_t + N \cdot Y_t} - \sum_{i=1}^{h_t^*} \frac{C_{i,t}}{D_t+N\cdot Y_t}\frac{\lambda_t^2}{\gamma_i}\frac{1+\psi}{2})} - (H_{h,t}h_t^* + H_{n,t}(N - h_t^*)) \tag{24}$$

Proof : See Appendix A.3.

4.3 Description of the equilibrium asset parameters

We briefly compare the endogenous asset parameters (λ_t, $r_{f,t}$, $\sigma_{s,t}$) with our nested cases: (i) a representative agent economy without labor income, (ii) a heterogeneous economy without labor income which in turn characterizes a full participation economy. In doing so, we also confirm that our asset parameters in closed forms reduce to the well-known expressions in nested economies studied in the literature.[18] More discussion on the role of limited and time varying market participation is covered in the simulation of our model in Section 5.

[18]We also solve for the equilibrium asset parameters in the case where there is no labor income and investors preferences are CRRA using the Martingale approach and verify that our general solution with labor income converges to this special case. See Appendix OA.4.

4.3.1 Sharpe ratio

From (19), if we shut down both heterogeneity and labor income, the Sharpe ratio reduces to $\lambda_t = \gamma\sigma_d$. For a heterogeneous economy without labor income income, $\lambda_t = \frac{\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} \frac{C_{i,t}^*}{\gamma_i}}\sigma_d$, that is the consumption-weighted harmonic mean of stockholders' risk aversion multiplied by the dividend growth volatility, which coincides with the expression in Cvitanić et al. (2012). The time-variation of the Sharpe ratio in this case only comes from the cross-sectional consumption re-distribution which generates countercyclical variation as Chan and Kogan (2002) point out. This is because in bad states, the consumption share of risk-tolerant investors who heavily invest in the risky asset drops the most, leading the average risk aversion to be tilted towards risk-averse investors. However, in our economy, there is another source of time-variation in the Sharpe ratio which is time-varying market participation. Time-varying market participation drives $\frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}}$ in a procyclical way. This is because in bad economic times, only risk-tolerant investors optimally stay in the market, which decreases the average risk aversion of stockholders. By contrast, in good times even risk-averse investors are willing to enter the market, increasing the average risk aversion of stockholders. We elaborate on this finding in detail in Secition 5.1.3.

4.3.2 Risk-free rate

From (20), the risk-free rate reduces to the known expression in the simplest representative economy $r_{f,t} = \delta + \frac{\mu_d}{\psi} - \frac{1+\psi}{\psi}\frac{\gamma\sigma_d^2}{2}$. For a heterogeneous economy without labor income, $r_{f,t} = \delta + \mu_d\frac{1}{\psi} - \frac{1+\psi}{2\psi}(\sum_{i=1}^{N} \frac{C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*}\frac{1}{\gamma_i})^{-1}\sigma_d^2$. Putting $\psi = 1/\gamma_i$, this expression is the same as in Cvitanić et al. (2012) which consider CRRA preferences. In our model, as (20) shows, the consumption smoothing demand $\frac{\mu_d D_t + \mu_y N \cdot Y_t}{D_t + N \cdot Y_t}\frac{1}{\psi}$ is time-varying due to the time-varying dividend share in total consumption.

4.3.3 Stock volatility

From (21) and (22), the stock volatility in the representative economy reduces to the dividend volatility. $\sigma_{s,t} = \sigma_d$. The stock volatility in a heterogeneous economy without labor income reduces to $\sigma_{s,t} = \sigma_d \sum_{i=1}^{N} \frac{X_{i,t}}{\sum_{i=1}^{N} X_{i,t}}\frac{1}{\gamma_i}/(\sum_{i=1}^{N} \frac{C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*}\frac{1}{\gamma_i})$. This equation shows that the stock volatility is determined by the ratio of the wealth-weighted average risk tolerance to the consumption-weighted average risk tolerance. Thus, in a full participation economy without labor income, a countercyclical stock volatility can be generated only if the wealth distribution is more unequal than the consumption distribution in bad time than in good time.

In our economy, we have two parameters $\sigma^d_{s,t}$ and $\sigma^y_{s,t}$ associated with the stock volatility, but as we shall show in the simulation, the second parameter $\sigma^y_{s,t}$ contributes to the stock volatility only marginally compared to the first parameter $\sigma^d_{s,t}$. First, from equation (22), the following holds $\frac{\sigma^d_{s,t}}{\sigma_d} = \frac{D_t}{\sum_{i=1}^{h^*_t} C^*_{i,t}} / (\sum_{i=1}^{h^*_t} \frac{C^*_{i,t}}{\sum_{i=1}^{h^*_t} C^*_{i,t}} \frac{\pi_{i,t}}{X_{i,t}+H_{h,t}})$. Therefore, the excess volatility from the first parameter is generated (i.e., $\sigma^d_{s,t} > \sigma_d$) when the dividend share in stockholders' consumption is greater than the risky asset share in total wealth. The intuition is as follows. When the dividend accounts for a large proportion of the stockholders' consumption, a change in the stockholders' consumption is highly sensitive to dividend shocks. However, since the risky asset accounts for only a small proportion of total wealth, a high sensitive change in the stockholders' consumption with respect to dividend shocks translates into the high volatility associated with the dividend shocks $\sigma^d_{s,t}$. We discuss this point in detail in Section 5.1.4. Regarding the second parameter $\sigma^y_{s,t}$ in (23), this can be re-written as $\sigma^y_{s,t} = \frac{\sigma_y Y_t N}{\sum_{i=1}^{h^*_t} k_{h,i,t}\pi^*_{i,t}}(1 - \frac{1}{N}\sum_{i=1}^N \partial C^*_{i,t}(X_{i,t}, Y_t)/\partial Y_t)$. Note that if the average marginal propensity to consume out of labor income across all investors is less than unity (i.e., $\frac{1}{N}\sum_{i=1}^N \partial C^*_{i,t}(X_{i,t}, Y_t)/\partial Y_t < 1$), then $\sigma^y_{s,t} > 0$. In this case, agents invest some fraction of their labor income in the risky asset and therefore the sensitivity of the stock returns with respect to labor income shocks $\sigma^y_{s,t}$ is positive. In particular, for a CARA investor or a representative agent, $\partial C^*_{i,t}(X_{i,t}, Y_t)/\partial Y_t$ is always unity and hence $\sigma^y_{s,t}$ is always zero.

4.3.4 Stock price

The first term in (24) is aggregate wealth in this economy including both financial wealth and human capital. Therefore, the equilibrium stock price is expressed by subtracting total human capital from aggregate wealth. This equilibrium stock price equation shows how non-stockholders and labor income affect the stock price. In a heterogeneous economy without labor income, the stock price reduces to $\frac{D_t - (\sum_{i=1}^N \frac{C_{i,t}}{\sum_{i=1}^N C_{i,t}} \frac{1}{\gamma_i})^{-2} \sigma_d^2 \frac{1-\psi}{2} \sum_{i=1}^N \frac{X_{i,t}}{\gamma_i}}{r_{f,t} - (\mu_d - \frac{1+\psi}{2} (\sum_{i=1}^N \frac{C^*_{i,t}}{\sum_{i=1}^N C^*_{i,t}} \frac{1}{\gamma_i})^{-1} \sigma_d^2)}$. The price-dividend ratio in this case is counterfactually more volatile than the data because the dividend shock is the only fundamental shock. On the contrary, in our economy labor income shock as well as dividend shock affects the price-dividend ratio and given the fact that labor income shock is less volatile than the dividend shock, the volatility of price-dividend ratio matches the data reasonably well, as we will show in Section 5.2. If we further simplify the economy by considering a representative economy, the equilibrium stock price is $S_t = \frac{D_t}{r_f(1-\psi)+\delta\psi+\gamma\sigma_d^2\frac{1-\psi}{2}} = \frac{D_t}{\delta+\mu_d\frac{1-\psi}{\psi}-\gamma\frac{\sigma_d^2}{2}(\frac{1-\psi}{\psi})}$, the same as in the existing studies (e.g., Yan, 2008; Cvitanić et al., 2012). If there is no uncertainty on dividend stream ($\sigma_d = 0$), the equilibrium stock price is the same as in the Gordon's dividend model ($S_t = \frac{D_t}{r_f-\mu_d} = \frac{D_0 exp(\mu_d t)}{r_f-\mu_d}$).

4.4 A Novel Conditional Consumption Capital Asset Pricing Model

In the canonical consumption-based asset pricing model with a representative agent, the conditional equity premium is the price of consumption risk, represented by risk aversion, multiplied by the amount of consumption risk, represented by the conditional covariance between stock returns and consumption growth.

$$E_t[dR_t^e] = \underbrace{\gamma_t}_{\textit{Price of risk}} \cdot \underbrace{Cov_t(dR_t^e, dC_t^*/C_t^*)}_{\textit{Amount of risk}} \tag{25}$$

where $dR_t^e \equiv \frac{dS_t+D_t dt}{S_t} - r_{f,t}dt$ is the total instantaneous excess equity return, $\frac{dC_t^*}{C_t^*}$ is the consumption growth, and γ_t ($\equiv -\frac{C_t^* u''(C_t^*)}{u'(C_t^*)}$) is the coefficient of relative risk aversion. The price of risk is the required compensation for one unit of consumption risk. If the representative investor's preference is power utility, γ_t is constant over time ($\gamma_t = \gamma$), whereas in habit preferences, γ_t is time-varying. In the following proposition, we present a novel CCAPM featuring time-varying stock market participation.

Proposition 3. *In an economy where market participation is time-varying, the equilibrium* *equity premium is given by*[19]

$$E_t[dR^e_t] = \underbrace{\frac{\sum_{i=1}^{h^*_t} C^*_{i,t}}{\sum_{i=1}^{h^*_t} \frac{C^*_{i,t}}{\gamma_i}}}_{\textit{Price of risk}} \cdot \underbrace{Cov_t(dR^e_t, \frac{d\sum_{i=1}^{h^*_t} C^*_{i,t}}{\sum_{i=1}^{h^*_t} C^*_{i,t}})}_{\textit{Amount of risk}} \tag{26}$$

Proof : See Appendix A.4

Proposition 3 shows that among all investors, it is the consumption of stockholders which **directly** determines the equity premium and the consumption of non-stockholders affects the equity premium **indirectly** through the market clearing condition. Since the set of stockholders is time-varying in our economy, time-varying market participation rate h^*_t/N affects time-variation in the equity premium through both the price and amount of risk. As explained above, a procyclical market participation drives the price of risk in a procyclical way as stockholders average risk aversion increases (decreases) in good (bad) states due to the entry (exit) of risk-averse investors. When it comes to the amount of risk, in bad states risk-averse investors leave the market and only few remaining investors bear the entire market risk. Therefore, the amount of risk is not effectively shared-out and it remains high. Through this mechanism, time-varying market participation can generate a countercyclical amount of risk. We simulate both the price and amount of risk in more details in Section 5.

Contrary to the implication of **Proposition 3**, previous empirical studies testing the conditional consumption-based asset pricing model have relied on aggregate consumption. **Lemma 1** helps understand how a large countercyclical and negative price of risk can be implied, as documented in those studies, when full participation is assumed.[20]

Lemma 1. *In an economy where market participation is time-varying, the association between the equilibrium equity premium and the conditional covariance of aggregate consump-*

[19]We also derive the equity premium equation for an individual stock in Appendix OA.12 and for internal habit preferences in Appendix OA.6

[20]Empirical studies document the large countercyclical implied price of risk ranging from -88 to -4 in Duffee (2005), -3000 to 2000 in Nagel and Singleton (2011), and -250 to 600 in Roussanov (2014).

tion growth with stock returns is given by

$$E_t[dR_t^e] = \frac{\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}} Cov_t(dR_t^e, \frac{d\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*}) - \frac{\sum_{i=h_t^*+1}^{N} k_{n,t} H_{n,t} \sigma_y \sigma_{s,t} \rho_{s,t}}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}} \tag{27}$$

Proof : See Appendix OA.14

Note that the above equation has a second term, different from (26) because the consumption of non-stockholders does not affect the equity premium directly. The previous empirical studies which test the conditional consumption-based asset pricing have modeled the equity premium as follows.

$$E_t[dR_t^e] = \alpha + \Gamma_t Cov_t(dR_t^e, \frac{d\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*}) \tag{28}$$

By equating (27) with (28), we can recover what the estimated price of risk Γ_t in (28) from the lenses of our theoretical model:

$$\hat{\Gamma}_t \equiv \frac{\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}} - a_t = \frac{\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{h_t} C_{i,t}^*} \Gamma_t^H - a_t \tag{29}$$

where $a_t \equiv \frac{\sum_{i=h_t^*+1}^{N} k_{n,t} H_{n,t} \sigma_y \sigma_{s,t} \rho_{s,t} + \alpha}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i} Cov_t(dR_t^e, \frac{d\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*})}$ and $\Gamma_t^H \equiv \frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}}$

Although $\hat{\Gamma}_t$ is not interpretable in a formal way as opposed to Γ_t^H (stockholders' average risk aversion), it has the following implications. First, a procyclical market participation leads $\frac{\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{h_t} C_{i,t}^*}$ to vary in a countercyclical way, which in turn leads $\hat{\Gamma}_t$ to vary in a more countercyclical way or at least less procyclical than Γ_t^H. This provides an explanation for the large countercyclical implied price of risk in the empirical literature using aggregate consumption. Second, if the second term a_t is large enough, it will generate a negative implied price of risk as documented in the empirical literature. Section 5.1.3 shows that our model reproduces a large countercyclical and negative price of risk as in previous studies (e.g., Duffee, 2005; Nagel and Singleton, 2011; Roussanov, 2014).

5 Simulation

To simulate the model, we map our economy to the United States. The continuous model is discretized and simulated in monthly time increments for 40 years.[21] For the choice of parameter values, we estimate the parameters using US dividend and non-financial income data from the BEA (Bureau of Economic Analysis) for the longest sample[22] - 1930 to 2016.

Table 1 reports the annualized parameter values used in the simulation. Panel A shows the first and second moments of the real per capita dividend growth, non-financial income growth and their correlation. Our estimates are consistent with the findings in the literature (see Table 1). Our choice of investors' preferences is reported in the Panel B. First, for the subjective time preference rate δ, we choose 0.2%, which is the same as Bansal and Yaron (2004). The EIS ψ is set to 0.5, following the general consensus of the EIS level and critique of the high EIS in the long run risk model.[23] Investors' risk aversion is uniformly distributed from 1 to 50. In equilibrium, this distribution translates into the harmonic mean of stockholders risk aversion 4, which matters for the equity premium. Therefore, our implied equity premium level does not rely on high risk aversion coefficient. For comparison, in Chan and Kogan (2002), the risk aversion distribution ranges from 1 to 100, implying an average risk aversion level of 8.14. Panel C reports initial value of aggregate dividend D_t as a function of normalized per capita non-financial income Y_t.

Throughout the analysis of our model, we use the stock market wealth to aggregate labor income ratio $\frac{\sum X_{i,t}}{\sum Y_{i,t}} = \frac{S_t}{\sum Y_{i,t}}$ as a state variable for the following reasons. First, financial wealth $X_{i,t}$ and labor income $Y_{i,t}$ are state variables in the optimization problem for the portfolio and consumption choice as it is the case also in Koo (1998) and Wang (1996). Second, as in equation (15), it is the financial wealth to aggregate labor income ratio which affects market participation decision. Third, a high level of $\frac{S_t}{\sum Y_{i,t}}$ coincides with a high level of the aggregate consumption. Unreported regression based on simulated data shows

[21]This time horizon is similar to the literature (e.g, Stathopoulos, 2017).

[22]This is similar to Mehra and Prescott (1985), Kandel and Stambaugh (1991), Abel (1999), Bansal and Yaron (2004), and Beeler and Campbell (2012)

[23]See Vissing-Jørgensen (2002a), Trabandt and Uhlig (2011), Jin (2012), Rudebusch and Swanson (2012), and Epstein et al. (2014), among others.

that $\frac{S_t}{\sum Y_{i,t}}$ is positively correlated with aggregate consumption.[24] Lastly, this ratio is used in Gomes and Michaelides (2008) and also closely related to the consumption to wealth ratio in Lettau and Ludvigson (2001), the stock market wealth-consumption ratio in Duffee (2005), the labor income to consumption in Santos and Veronesi (2006). The average level and standard deviation of $\frac{S_t}{\sum Y_{i,t}}$ in simulation are 1.22 and 0.84 respectively, versus 0.98 and 0.58 in the data. Moreover, in our simulation, $\frac{S_t}{\sum Y_{i,t}}$ moves closely with the price-dividend ratio $\frac{S_t}{D_t}$. The correlation between the two in the simulation is close to the data counterpart from 1930 to 2016. Given a strong comovement between the two variables, our simulation result is not changed by the choice of $\frac{S_t}{\sum Y_{i,t}}$ versus $\frac{S_t}{D_t}$ as a state variable.

In what follows, We first examine the conditional equilibrium in Section 5.1. Section 5.2 discusses the unconditional equilibrium. In the online appendix OA.2, for the interested reader, we conduct a comparative static analysis for the asset parameters with particular focus on the role of market participation.

5.1 Equilibrium dynamics

5.1.1 Conditional portfolio and market participation

We first examine the conditional optimal portfolio $\pi^*_{i,t}$ across states. To illustrate our results, we select three investors whose risk aversion ranges from 11 to 15 as an example. The top panel of Figure 3 illustrates one sample path of the optimal portfolio for 40 years. We generate in total 1,000 paths each with 40 years, resulting in 480,000 realization of total monthly observation. Throughout this section, we keep the same path of exogenous shocks to dividend and labor income for comparison. The shaded area denotes the bad states defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ in this sample path. We find that the investors' optimal holdings are monotonic with respect to risk aversion at each point in time. We also find that the conditional optimal portfolio varies procyclically in spite of the countercyclical variation in the Sharpe ratio, shown in the bottom panel of Figure 3, which can induce investors to take more financial risk in bad times than in good times. This is because in good economic times, investors have a larger amount of financial wealth available in the stock market. Also, a countercyclical stock volatility which we shall

[24]The regression coefficient of $log \sum C_{i,t}$ on $log \frac{S_t}{\sum Y_{i,t}}$ is 0.07 with R^2 of 0.22.

present in the following section contributes to this procyclical optimal portfolio.

This procyclical variation of $\pi^*_{i,t}$ directly translates into the procyclical market participation. The bottom panel in Figure 4 is one sample path of $\frac{S_t}{\sum Y_{i,t}}$ and the cut-off stockholder level h^*_t defined in Appendix OA.9 for 40 years. It shows that market participation varies procyclically in response to the economic state. For example, when the state reaches the lowest level within the second shaded area, the investor with risk aversion equals 14.5 leaves the market and only 8 investors (27%) end up in the market. To document this dynamic in more detail, the top panel in Figure 4 illustrates the relationship between the cut-off stockholders h^*_t and the economic state $\frac{S_t}{\sum Y_{i,t}}$ based on the simulation of 480,000 months. We plot all ranges of the optimal market participation corresponding to the different states as well as other parameters of the model that affect the market participation. We find that market participation h^*_t in our model is procyclical, in line with the empirical findings we shall present in Section 6 and also in related studies (e.g., Brunnermeier and Nagel, 2008; Bonaparte et al., 2018; Yang, 2018).[25] For example, when the state variable $\frac{S_t}{\sum Y_{i,t}}$ is around 0.5, only 6 investors out of 30 (20%) remain in the stock market, whereas every investor chooses to be a stockholder when the state variable $\frac{S_t}{\sum Y_{i,t}}$ is above 3.5. However, we find that time-variation in the market participation is mild because time-variation is generated by entry or exit of agents from 8th to 15th and the rest of investors are almost always stockholders or non-stockholder as illustrated in Figure 2.

Given the finite number of investors, the optimal market participation h^*_t is increasing approximately as a step function of the state variable. Note that since market participation level depends on other parameters in addition to the state of the economy, there is a region of state variables that command similar level of market participation. Interestingly, the relationship between h^*_t and $\frac{S_t}{\sum Y_{i,t}}$ is approximately convex. When the state level is low, for a marginal non-stockholder (whose risk aversion is slightly higher than the stockholders) to enter the market, there should be a large increase in the state level. By contrast, when the state is high, a small change can induce a marginal non-stockholder to enter the market. This convex relationship between the state and market participation provides an empirically

[25] In Section 6 also empirically shows this fact.

testable hypothesis that we leave for future research.

5.1.2 Amount of Consumption risk

Consumption decomposition: We now study how the amount of consumption risk varies over time along with market participation. Duffee (2005) documents that the amount of aggregate consumption risk varies in a procyclical way. He argues that as the financial income accounts for a larger proportion of consumption in good states, a change in consumption becomes more sensitive to stock returns, resulting in a high covariance between equity returns and consumption growth. He also conjectures that as long as stockholders hold a nontrivial amount of wealth in a form other than stocks, the amount of consumption risk is procyclical due to the effect that he labeled "the composition effect". It is difficult to reconcile his empirical finding with major asset pricing theories in a representative agent setting such as Campbell and Cochrane (1999) because those models do not generate a procyclical amount of risk.

We argue in this article that the distinction between stockholders consumption and aggregate consumption helps to resolve this challenge. On the one hand, stockholder consumption risk is countercylical as in the data. On the other hand, aggregate consumption risk is procyclical or mildly countercyclical as displayed in Figure 1. This is because the proportion of stockholders, whose consumption risk is high, varies procyclically.

In order to understand the composition effect and the dynamics of amount of risk in detail, we first decompose the consumption into dividend and other source of consumption for aggregate households and stockholders separately as follows.

$$Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e) = \frac{D_t}{C_t^G}Cov_t(\frac{dD_t}{D_t}, dR_t^e) + \frac{C_t^{G,D-}}{C_t^G}Cov_t(\frac{dC_t^{G,D-}}{C_t^{G,D-}}, dR_t^e) \;\; \forall G = A, H \quad (30)$$

where G denotes group, $G = A$ denotes *a*ggregate households, and $G = H$ denotes stock*h*olders. $C_t^A = \sum_{i=1}^N C_{i,t}$ is aggregate consumption, $C_t^H = \sum_{i\in h} C_{i,t}$ is aggregate stock*h*olders consumption, and $C_t^{G,D-}$ is the non-dividend part of consumption (i.e., $C_t^G - D_t$) $\forall G = A, H$. We compute the average level of each component above across states: the bad (good) states defined by the lowest (highest) 10% percentile of the state variable. We conduct this analysis separately for both stockholders ($G = H$) and aggregate households

($G = A$) since our model makes a distinction between the two groups.

Table 2 reports the levels of each component over two economic states (bad and good) for both aggregate consumption (Panel A) and stockholders' consumption (Panel B). Consistent with the composition, we find that the share of dividend in aggregate consumption is procyclical in our economy. In bad times, the dividend stream accounts for 6.78% of the total consumption stream that compares to 8.12% in good times. Holding other variables unchanged, this procyclical variation in the share of dividend in aggregate consumption should drive the amount of aggregate consumption risk in a procyclical way. However, the covariance between dividend growth and stock returns exhibits a countercyclical variation. In our simulation, the latter dominates the composition effect. Taken together, the amount of aggregate consumption risk dynamics is mildly countercyclical with average 0.85% in bad times and 0.44% in good times. This result is consistent with our empirical finding of a mildly countercyclical amount of aggregate consumption risk in Section 6.

More importantly, when it comes to stockholders' consumption, which directly affects the stock return moments, the share of dividend in aggregate stockholders' consumption exhibits a countercyclical variation with average 25.9% in bad times and 22.1% in good times. Although dividend stream decreases due to a negative shock, the total dividend is shared only by few remaining investors who bear the entire market. Therefore, the dividend accounts for a larger proportion of the remaining stockholders' consumption. This result implies that the effect of time-varying risk-sharing due to time-varying market participation dominates the composition effect. The countercyclical variation in the dividend share in stockholders' consumption renders the amount of stockholder consumption risk more countercyclical than that of aggregate consumption risk. The average amount of risk is 1.87% in bad times and 0.86% in good times. This novel finding is different from Duffee (2005)'s conjecture, which does not take into account the effect of time-varying risk-sharing in equilibrium. We also confirm in the data that stockholder consumption risk exhibits a strong countercyclical behavior as shown in Figure 1 and Table 8 of Section 6.

The above finding is also illustrated in Figure 5 which plots a simulation path for each component. The top-left figure shows that the dividend share is procyclical for aggregate consumption and countercyclical for stockholders' consumption. Please note that there are large jumps in the dividend share in stockholders' consumption. Given our finite number of investors who enter or exit the market at each time, these jumps suggest that the dividend share is mainly driven by time-varying market participation. Considering the negligible difference between the covariance of aggregate households' non-financial income growth with stock returns and that of stockholders in the bottom-left figure, it is the dividend share in consumption that makes a difference in the amount of consumption risk, shown in the bottom-right figure. The figure illustrates that the amount of stockholder consumption risk is higher and more countercyclical than that of aggregate consumption risk.

Returns decomposition: Another way of examining consumption risk dynamics is to decompose equity returns into the cash flow part and the discount rate as in Xu (2018). She shows that it is the cash flow part of returns which contributes to the procyclical variation in aggregate consumption risk while the non-cash flow part returns varies with aggregate consumption countercyclically. In order to illustrate the importance of separating stockholder consumption risk from aggregate consumption risk, we do this decomposition for both stockholders and aggregate household separately as follows and report the dynamic of the amount of risk as well as its components.

$$Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e) = Cov_t(\frac{dC_t^G}{C_t^G}, \frac{dD_t}{D_t}) + Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e - \frac{dD_t}{D_t}) \quad \forall G = A, H \tag{31}$$

Table 3 reports the results. Panel A shows that our model generates a procyclical variation in the conditional covariance between aggregate consumption growth and dividend growth $Cov_t(\frac{dD_t}{D_t}, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$ as in the data. In good times, due to an entry of investors into the market, stockholders consumption constitutes a larger proportion of aggregate consumption. Given the fact that stockholders' consumption is highly correlated with dividend, the covariance between aggregate consumption growth and dividend becomes higher than in bad times. As Xu (2018) shows, major asset pricing models calibrated to aggregate consumption cannot generate this dynamics. When it comes to the non-dividend part of returns, our model generates the same dynamics for $Cov_t(dR_t^e - \frac{dD_t}{D_t}, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$ as observed in the data. In our calibration, the dynamics of $Cov_t(dR_t^e - \frac{dD_t}{D_t}, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$ dominates $Cov_t(\frac{dD_t}{D_t}, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$ and thus $Cov_t(dR_t^e, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$ is weakly countercyclical.

More importantly, since the equity premium is directly driven by stockholders consumption, we also examine each component of equation (31) for stockholders. Our simulation shows that both cash flow and discount rate parts of returns contribute to the countercyclical covariance between equity returns and stockholders' consumption growth. While the representative-agent model of Xu (2018) explains each consumption risk component for aggregate consumption, her model requires a dramatically countercyclical price of consumption risk even stronger than the habit model of Campbell and Cochrane (1999). This is because the amount of aggregate consumption risk is procyclical in her model. Also, her model fails to fit the dynamics of stockholders' consumption, which matters for the equity premium. We argue that the distinction between stockholders and aggregate households is necessary in explaining empirical moments and covariances in the data.

To summarize, our model shows that the distinction between stockholder consumption and aggregate consumption reconciles the empirical finding of Duffee (2005) with other major theories in a representative agent setting. On the one hand, the composition effect for aggregate consumption contributes to the procyclical (or weakly countercyclical) variation variation in the aggregate consumption risk. On the other hand, the composition effect for stockholders consumption leads to a rather countercyclical stockholder consumption risk due to time-varying market participation, which is necessary to explain the asset pricing dynamics.

5.1.3 Price of Consumption risk

Campbell and Cochrane (1999) habit-formation model generates a countercyclical variation in equity premium by imposing a large countercyclical risk aversion of a representative agent. Chan and Kogan (2002) rationalize the countercyclical price of consumption risk in a heterogeneous time-invariant risk-averse agents setting. Their explanation hinges on the changes in consumption re-distribution across investors. However, it is unclear whether the price of consumption risk still varies countercyclically if these investors are allowed to optimally enter or exit the market. If risk-averse investors leave the market, only risk-tolerant investors remain in the market, lowering the price of consumption risk. We show in this section that the effect of time varying market participation on the price of consumption risk is opposite to that of the consumption re-distribution.

To examine the dynamics of the price of consumption risk, Panel A in Figure 6 shows the relationship between our model implied price of consumption risk Γ_t^H and the economic state $\frac{S_t}{\sum Y_{i,t}}$ based on the simulation of 480,000 months. Panel A summarizes both the effect of consumption re-distribution and time-varying market participation on the level of Γ_t^H. Note that within the same level of market participation, the price of risk is countercyclical due to the consumption re-distribution effect, consistent with Chan and Kogan (2002). However, as long as an investor who is more risk-averse than the existing stockholders enters the market, the price of risk increases, driving upward the average risk aversion of stockholders. Panel B in Figure 6 depicts a sample path of the price of risk Γ_t^H in the time-varying market participation case and Panel C illustrates its dynamic under the nested full participation economy without constraints. First, in full participation case, the time-varying consumption re-distribution effect is the only source of the time-variation for the price of risk, and hence the price of risk is countercyclical. Second, in the time-varying market participation case, the price of risk is procyclical, suggesting that the effect of time-varying market participation on the price of risk dominates that of consumption re-distribution in our calibration. We provide empirical evidence in Section 6, consistent with the existence of both consumption re-distribuion and time-varying market participation effect. We also find that the price of risk is procyclical in the data, but not significant due to two competing forces.

Surprisingly, a procyclical variation in the price of risk does not make it difficult to produce a countercyclical equity premium. As we will show in Section 5.1.5, we generate a countercyclical equity premium as observed in the data. This is because the amount of consumption risk is strongly countercyclical due to the ineffective risk-sharing among remaining stockholders during bad states. We emphasize here that a procyclical price of consumption risk in our calibration stems from time-varying market participation coupled with time-invariant risk aversion preferences. Imposing time-varying individual risk aversion as in Campbell and Cochrane (1999) will drive the price of risk countercyclically in the online appendix OA.6, but the key takeaway is that even if we impose countercyclical risk aversion to each heterogeneous agent, allowing for entry and exit will render the price of risk much less countercyclical than in a full participation economy.

Finally, we examine the price of risk implied by aggregate consumption, assuming full participation, even in the presence of limited and time-varying market participation. Specifically, we test the equation (28) and assess whether our model reproduces negative values of the implied price of risk and strong countercyclical variation as in previous empirical studies using aggregate consumption (e.g., Duffee, 2005; Nagel and Singleton, 2011; Roussanov, 2014). Instead of relying on the analytical solutions to the expected equity returns $E_t[dR_t^e]$ and aggregate consumption risk $Cov_t(dR_t^e, \frac{d\sum_{i=1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*})$, we infer them in the same way as econometricians would do based on the simulated data. We adopt the GMM methodology in Duffee (2005).[26] Figure 7 depicts the implied price of risk using aggregate consumption together with the model-implied price of risk, which is stockholders' harmonic mean of risk aversion for comparison. It shows that the implied price of risk using aggregate consumption has negative values over the large sample distribution and varies in a strongly countercyclical way, ranging from -150 to 220, similar to previous studies, but in contrast to the model-implied price of risk. This finding suggests that since aggregate consumption risk is not directly linked to the equilibrium equity premium, relying on the aggregate consumption would deliver a counterfacutal result for the price of risk: implausible levels and a negative risk-return trade-off.

5.1.4 Stock volatility dynamics

In this section, we explore the conditional stock volatility and its associated parameters ($\sigma_{s,t}^d$, $\sigma_{s,t}^y$). As discussed in Section 4.3.3, the stock volatility parameter associated with dividend shock $\sigma_{s,t}^d$ is linked to the gap between the dividend share in stockholders' consumption and the risky asset share in total wealth (i.e., $\sigma_{s,t}^d/\sigma_d = \frac{D_t}{\sum_{i=1}^{h_t^*} C_{i,t}^*} / \sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*} \frac{\pi_{i,t}^*}{X_{i,t}+H_{h,t}}$). For $\sigma_{s,t}^y$, understanding its dynamics boils down to the average marginal propensity to consume out of labor income across all investors $\sigma_{s,t}^y = \frac{\sigma_y Y_t N}{\sum_{i=1}^{h_t^*} k_{h,i,t}\pi_{i,t}^*}(1 - \frac{1}{N}\sum_{i=1}^N \partial C_{i,t}^*(X_{i,t}, Y_t)/\partial Y_t)$. Therefore, we compute the average level of $\sigma_{s,t}$, $\sigma_{s,t}^d$, $\sigma_{s,t}^y$ as well as $\sigma_{s,t}/\sigma_d$, $\sigma_{s,t}^d/\sigma_d$, $\frac{D_t}{\sum_{i=1}^{h_t^*} C_{i,t}^*}$, $\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*} \frac{\pi_{i,t}^*}{X_{i,t}+H_{h,t}}$, and $\frac{1}{N}\sum_{i=1}^N \frac{\partial C_{i,t}^*(X_{i,t}, Y_t)}{\partial Y_t}$ across states.

Table 4 reports the results. First, we find that both $\sigma_{s,t}^d$ and $\sigma_{s,t}^y$ are countercyclical, but

[26] Other methodologies such as the GARCH-in-mean method in Duffee (2005) generate virtually identical result.

most of the variation of $\sigma_{s,t}$ stems from $\sigma^d_{s,t}$. Second, we also find that the average of $\sigma^d_{s,t}/\sigma_d$ and $\sigma_{s,t}/\sigma_d$ is 3.0 and 2.8 respectively. The latter level compares to around 2 in the data. As shown in the discussion of the amount of consumption risk, the dividend share in the stockholders' consumption $\frac{D_t}{\sum_{i=1}^{h_t^*} C^*_{i,t}}$ is countercyclical. By contrast, the stockholders' consumption weighted mean of risky asset share in total wealth $\sum_{i=1}^{h_t^*} \frac{C^*_{i,t}}{\sum_{i=1}^{h_t^*} C^*_{i,t}} \frac{\pi^*_{i,t}}{X_{i,t}+H_{h,t}}$ is mildly procyclical. This is because (i) investors optimally reduce the risky asset holding in bad times, and (ii) consumption of risk-tolerant investors drops the most, leading the average to be more tilted towards the risky asset share of risk-averse investors. Since $\sigma^d_{s,t}/\sigma_d$ is the ratio of these two terms, the countercyclical $\frac{D_t}{\sum_{i=1}^{h_t^*} C^*_{i,t}}$ together with the procyclical $\sum_{i=1}^{h_t^*} \frac{C^*_{i,t}}{\sum_{i=1}^{h_t^*} C^*_{i,t}} \frac{\pi^*_{i,t}}{X_{i,t}+H_{h,t}}$ leads the excess volatility to vary in a highly countercyclical way. For comparison, the dividend share in aggregate consumption $\frac{D_t}{\sum_{i=1}^{N} C^*_{i,t}} = \frac{D_t}{D_t+\sum_{i=1}^{N} Y_{i,t}}$ is procyclical under the full market participation and therefore it is difficult to explain the countercyclical stock volatility. Finally, with respect to $\sigma^y_{s,t}$, we find that it is countercyclical and negative. This is due to the average marginal propensity to consume out of labor income is procyclical and its level is above one.[27]

5.1.5 Conditional equilibrium parameters

In this section, we discuss the conditional equilibrium parameters. Table 5 summarizes the model-implied dynamic of the equity premium, price of consumption risk, amount of consumption risk as well as stock volatility, Sharpe ratio, stockholder's consumption volatility. In doing so, we compute the average level of conditional moments across states.

Our model generates the observed dynamics of the asset parameters: countercyclical equity premium $E_t(dR^e_t)$: 6.67% versus 3.51% in bad and good times, respectively, stock volatility $\sigma_{s,t}$: 45% versus 27%, Sharpe ratio λ_t: 14.5% versus 12.9%, and consumption volatility $\sigma(\frac{d\mathbf{C}^H_t}{\mathbf{C}^H_t})$ 5.58% versus 4.91%. Most importantly, we emphasize that our model generates the countercyclical equity premium in spite of a procyclical variation in the price of consumption risk. This is because the countercyclical amount of consumption risk is

[27] Please note that $\frac{1}{N}\sum_{i=1}^{N} \frac{\partial C^*_{i,t}(X_{i,t},Y_t)}{\partial Y_t} > 1$ does not mean that the marginal propensity to consume out of labor income is also above one at aggregate level because $\frac{\partial \sum_{i=1}^{N} C_{i,t}}{\partial N \cdot Y_t}$ is always one by the consumption market clearing condition.

strong enough to dominate the procyclical price of consumption risk in our model. Finally, different from major representative-agent asset pricing models, our model generates mild time-variation in the market participation: 26.9% in bad times versus 31.8%, otherwise. This is consistent with the empirical findings of Brunnermeier and Nagel (2008) and Bonaparte et al. (2018) documenting a strong dynamics in the stock market participation and a positive relation between a market entry and investor's wealth.

When it comes to the average level of the price of risk, it is only 3.8, which can translate into the risk aversion coefficient of a representative agent and similar to the coefficient 4 in Barro (2009). Also, it is lower than 8.14 in Chan and Kogan (2002). Therefore, our explanation for the equity premium puzzle (Mehra and Prescott, 1985) hinges on the amount of consumption risk rather than a high price of consumption risk. Under limited market participation, the amount of risk is high due to the fact that the risk-sharing is not effective. The price of risk is low because a risk aversion of the remaining stockholders is low and they do not require a high compensation for bearing the risk.

5.1.6 Price-dividend ratio

In this section, we assess whether the price-dividend ratio in the model is consistent with empirical observations.

It is well-known that the price-dividend ratio is procyclical in the data (e.g., Fama and French, 1989). Theoretically, however, it is challenging to generate a procyclical variation in the price-dividend ratio with the EIS less than one (e.g., Ju and Miao, 2012; Chabakauri, 2015b) with few exceptions (e.g., Guvenen, 2009). Our price-dividend ratio is procyclical as in the data with a correlation of 0.464 with aggregate consumption. The top and middle panel of Figure 8 shows one sample path of the aggregate consumption and the price-aggregate labor ratio, respectively along with the price-dividend ratio. The figure shows that the price-dividend ratio moves closely with aggregate consumption and strongly with the price-labor ratio. The procyclicality of the price-dividend ratio follows from the fact that (1) a procyclical risky asset holding as shown in Section 5.1.1, which leads to a procyclical stock price (2) a higher volatility of stock than that of dividends as shown in Section 5.1.4.

Campbell and Shiller (1988) document that future stock market returns are in part predicted by the price-dividend ratio. The bottom panel of Figure 8 shows one sample path of simulated 10-year rolling cumulative excess returns and return forecast by the long-horizon regression using the log price-dividend ratio. The forecast by our log price-dividend ratio notably fits the 10-year future returns reasonably well with R^2 of 0.64 in this particular sample. More formally, we test the predictability using 1,000 sample path with different long-horizons. Panel A of Table 6 presents the results. Our model-implied price-dividend ratio generates the correct negative sign, implying high valuations indicate low expected returns. Moreover, both the coefficients and R^2 rise with horizons, which is well-documented pattern in the literature. R^2 values also match the data counterpart reasonably well. For example, at the 1-year horizon, R^2 in the model is 8.4% versus 9% in the data. At the 7-year horizons, the model generates R^2 of 36.9% versus 33% in the data. This high predictability is due to high expected returns when the price-dividend ratio is low in bad states which leads to future high realized returns.

Finally, Bansal et al. (2012) point out that the price-dividend ratio in the habit model of Campbell and Cochrane (1999) is not forward-looking as the price-dividend ratio in their model is counterfactually predicted by the lagged consumption growth. We conduct the same test as in Bansal et al. (2012) to examine the behavior of the price-dividend ratio. Panel B of Table 6 shows that R^2 in our model is 2.4% at the 1-year horizons and rises up to 6.3% at the 7-year horizon. R^2 values are reasonably low, compared to around 20% to 40% in the habit model, reported by Bansal et al. (2012). This suggests that our price-dividend ratio is forward-looking. With respect to the unconditional moments of the price-dividend ratio, the level and volatility of our price-dividend ratio with limited market participation match the data well, as we will present in the next section. In summary, the price-dividend ratio in our model produces well-documented patterns observed in the data.

5.2 Unconditional moments of asset returns and consumption growth

In this section, we study the unconditional moments. Panel A of Table 7 reports the unconditional moments of consumption growth and their corresponding counterparts from the U.S. data. The empirically observed consumption growth and dividend growth volatility are 2.2% and 12%, respectively. In spite of the huge difference between the two in the data, in models without labor income, the consumption volatility is counterfactually equal to the dividend volatility. In our model, with labor income as an additional source of consumption, the consumption growth volatility is 4.2%. This is because the labor income growth volatility of 4% dampens the volatility of consumption growth. This result echos the importance of labor income in matching the consumption growth mean and volatility.

Panel B of Table 7 shows the unconditional moments of excess equity returns, risk-free rate, and correlations. First, our model-implied equity premium is 4.7%, lower than 5.7% in the data. The Sharpe ratio in the model is 0.14 versus 0.28 in the data. The model-implied stock volatility $\sigma(r^e)$ is 33.6% and the excess volatility $\sigma(r^e)/\sigma_d$ is 2.8. Therefore, our model explains the excess volatility puzzle (Shiller, 1981). However, our model overstates the excess volatility level, relative to the data. With regards to the log price-dividend ratio, our model fits the level and variability reasonably well. The average log price-dividend ratio is 2.9 in the data versus 2.7 in the model. In the full participation economy, its value is 2.5 due to the short-selling demands. Thus, limited market participation helps to match the average log price-dividend ratio in the data. Moreover, the standard deviation of the log price-dividend ratio is 0.494 in the data versus 0.525 in the model. Our model matches a high volatility of the log price-dividend ratio well with time-varying market participation, compared to other major asset pricing models. For example, the volatility of the log price-dividend ratio in Bansal and Yaron (2004) is 0.18. When it comes to the risk-free rate, the model generates a risk-free rate level around 4% versus 1% in the data. Similarly, the unconditional risk-free rate in Bansal and Yaron (2004) is 4.02% when the EIS ψ equals 0.5 and the predictable component of consumption growth is shut down as in our model.[28] As for the second moment, the risk-free rate is not as volatile as observed in the data and almost constant as in Campbell and Cochrane (1999). This is because the EIS is the same for all investors and heterogeneous risk aversion does not generate an ample variability of the risk-free rate in the recursive utility.

Finally, Panel C shows around 31.3% of agents invest in the stock market in our model which is close to the proportion of direct stock holdings from the SCF data 29.7%. Un-

[28] See the Panel C of Table 2 in Bansal and Yaron (2004).

conditional market participation rate for each investor is illustrated in Figure 2. It shows that time-varying market participation is mainly driven by the dynamic optimal choice of investors from 8th to 19th, suggesting that our model does not generate a large time-variation in the market participation given the risk aversion distribution of investors.

To summarize, while our model provides an explanation on the consumption risk dynamics and asset moments through the distinction between stockholders and aggregate households, our model does not generate around 1% of risk-free rate, 3% of its volatility. Also, the unconditional stock volatility is overstated in our model compared to the data.

6 Empirical analysis

In this section, we empirically test the main findings of our theory using micro-level household data from the Consumer Expenditure Survey (CEX) for the period from January 1984 to January 2017.[29]

Amount of consumption risk: Figure 1 depicts the non-parametric estimates of conditional covariances between stockholders or aggregate consumption and excess market returns over the conditioning variables. While the amount of stockholders' consumption risk notably exhibits a countercyclical variation, that of aggregate consumption risk dynamics is procyclical or mildly countercyclical. In Panel A of Table 8, we also find that both stockholders' and aggregate consumption risk are countercyclical in terms of the NBER recession variable, but the countercyclicality of stockholders consumption risk is around 2.5 times stronger than that of aggregate consumption risk, consistent with our theory.[30] In Panel B of Table 8, we find that the dividend share in aggregate consumption is procyclical, consistent with the composition effect. By contrast, the dividend share in stockholders' consumption is countercyclical, which is consistent with the time-varying market participation effect.

Price of consumption risk: Next, we examine whether there is a time-variation in market

[29]More details on the data are in the online appendix OA.15.

[30]Untabulated regression of the conditional covariances between aggregate consumption and excess market returns on the recession, using the NIPA data, shows that its coefficient is 2.4×10^{-5} with t-statistic of 2.22 based on the robust standard error, suggesting that the amount of aggregate consumption risk is very mildly countercyclical.

participation over the business cycle in the CEX data. Panel C of Table 8 shows that both stockholders' consumption share in aggregate consumption and market participation rate are highly significant and come in with a negative sign, suggesting a procyclical variation as in our theory. When it comes to the price of risk, it is challenging to estimate risk aversion from the data. We assume that risk aversion is proportional to the probability of a household reporting no tolerance for investment risk. Under this assumption, we first estimate a Probit regression of households reporting unwillingness to take financial risk on a set of observable characteristics in the Survey of Consumer Finances (SCF) and use those estimates for the CEX households to measure risk aversion of each CEX household.[31] We are agnostic as to the level of risk aversion because what is crucial in testing our theory is heterogeneity in risk aversion.

After constructing the consumption-weighted harmonic mean of stockholders' risk aversion (price of risk) based on our risk aversion measure, we find that aggregate risk aversion is on average 0.27 (probability of reporting no tolerance for risk) versus 0.10 for stockholders, suggesting that stockholders indeed have lower risk aversion. In Panel D of Table 8, we regress the price of risk on stockholders' consumption share and market participation rate. In the univariate regression, both are positively associated with the stockholders' risk aversion with R^2 of 0.205 and 0.331 for consumption share and participation rate, respectively. This suggests that empirically when a higher proportion of households invests in the stock market, the price of risk rises because of the entry of risk averse investors, in line with our simulation result. Moreover, surprisingly, in the multivariate regression, the sign on stockholders' consumption share changes to a negative. Since the OLS coefficient on the stockholders' consumption share captures the marginal effect on the price of risk unrelated to market participation rate, the negative coefficient implies that within the same level of market participation, an increase in stockholders' consumption share decreases the stockholders' average risk aversion. This is because a positive fundamental shock increases the consumption share of risk-tolerant investors, leading the harmonic mean of risk aversion

[31] This methodology is similar to Malloy et al. (2009) which use the Probit regression of stock ownership on the set of observable characteristics from the SCF and use it to the CEX households to obtain a more sophisticated definition of stockholders.

to be more tilted towards risk-tolerant investors. We indeed confirm in the data that risk-tolerant investors consume more and have a higher amount of financial asset.[32] This finding empirically illustrates the time-varying market participation effect and the consumption re-distribution effect on the price of risk in opposite direction, strongly supporting our theory. Finally, a regression of the price of risk on the recession shows that its coefficient has a negative sign, meaning a procyclical time-variation in the price of risk. However, due to both the consumption re-distribution and market participation effect, the coefficient is not significant at a conventional level.

To summarize, we find the empirical evidence that: (1) a strong countercyclical stockholders' consumption risk versus a procyclical or weak countercyclical aggregate consumption risk, (2) Procyclical (countercyclical) dividend share in aggregate (stockholders) consumption, (3) Procyclical time-varing market participation, and (4) the positive (negative) effect of time-varying market participation (consumption re-distribution) on the price of risk and weakly procyclical price of risk.

7 Conclusion

In this article, we present a general equilibrium model featuring heterogeneous risk-averse investors with non-financial income. Our model generates procyclical market participation, which leads to novel implications for asset pricing dynamics that are supported empirically. We show that due to relatively ineffective risk-sharing among the remaining few stockholders in bad states, the amount of stockholders' consumption risk is strongly countercyclical. We also show that the amount of aggregate households' consumption risk is weakly countercyclical to procyclical, in line with empirical evidence. With respect to the price of consumption risk, we find that its time-variation is procyclical in our setting because the remaining risk-tolerant stockholders in bad states do not require a high compensation for risk. We also show empirical support for this finding. We highlight that it is the countercyclical amount of stockholders' consumption risk that explains the countercyclical equity premium, not the price of consumption risk nor the amount of aggregate consumption risk.

[32] The univariate panel regression of consumption (log of one plus financial wealth) on risk aversion shows the coefficient -0.092 (-8.793) with t-statistic of -11.84 (-193.14).

This article also offers a new explanation for both the level and dynamics of the stock excess volatility and the price-dividend ratio driven by moderate time-varying market participation. Moreover, the model delivers a new testable hypothesis on the market participation dynamics with respect to economic states as well as a novel CCAPM under time-varying market participation. A natural analysis to carry out is to test the empirical validity of this equation following the mainstream methodologies that have evaluated and dramatically rejected the representative-agent consumption-based asset pricing model (Duffee, 2005; Nagel and Singleton, 2011; Roussanov, 2014). We leave this research project to the future research.

Finally, we provide various extensions and clarifications in the online appendix to address potential concerns of the model. However, further realistic features could be considered to address some notable limitations. For example, we can consider other channels for market exit other than short-selling constraints. Also, an extended model with a heterogeneous-EIS agents, in line with Vissing-Jørgensen (2002b), can likely enhance the low volatility of the risk-free rate in our model. We leave these extensions also for future research.

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Supporting Information

The Online Appendix of this article can be found here: please click here

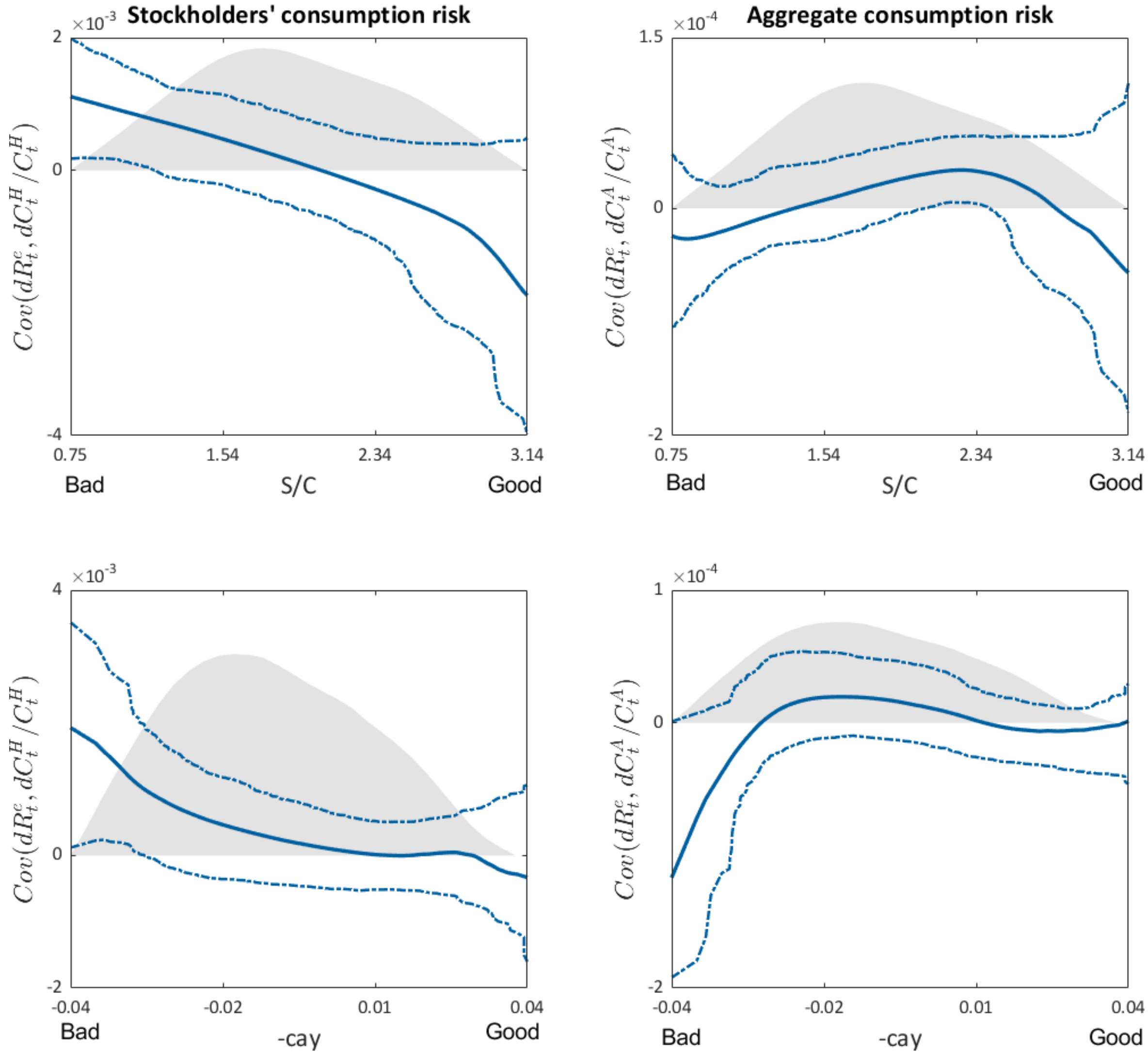

Figure 1: **Conditional amount of consumption risk**

This figure plots the empirically estimated conditional covariance of equity returns with stock*h*olders consumption growth $Cov_t(dR_t^e, \frac{d\mathbf{C}_t^H}{\mathbf{C}_t^H})$ (Left) and *a*ggregate consumption growth $Cov_t(dR_t^e, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$ (Right) using the stock market capitalization-to-aggregate consumption ratio (S/C) by Duffee (2005) (Top) and the consumption-wealth ($\widehat{cay}$) by Lettau and Ludvigson (2001) (bottom). The bold solid lines are the nonparametric estimate of conditional covariance based on the Epanechnikov kernel estimation at monthly frequency. The dash-dotted lines are 95% confidence bounds obtained by stationary bootstrap. The shaded backgrounds are the rescaled kernel density of the conditioning variable. The source of aggregate consumption data is the national income and product accounts (NIPA) by the Bureau of economic analysis and that of stockholders' consumption is the consumer expenditure (CEX) by the Bureau of labor statistics.

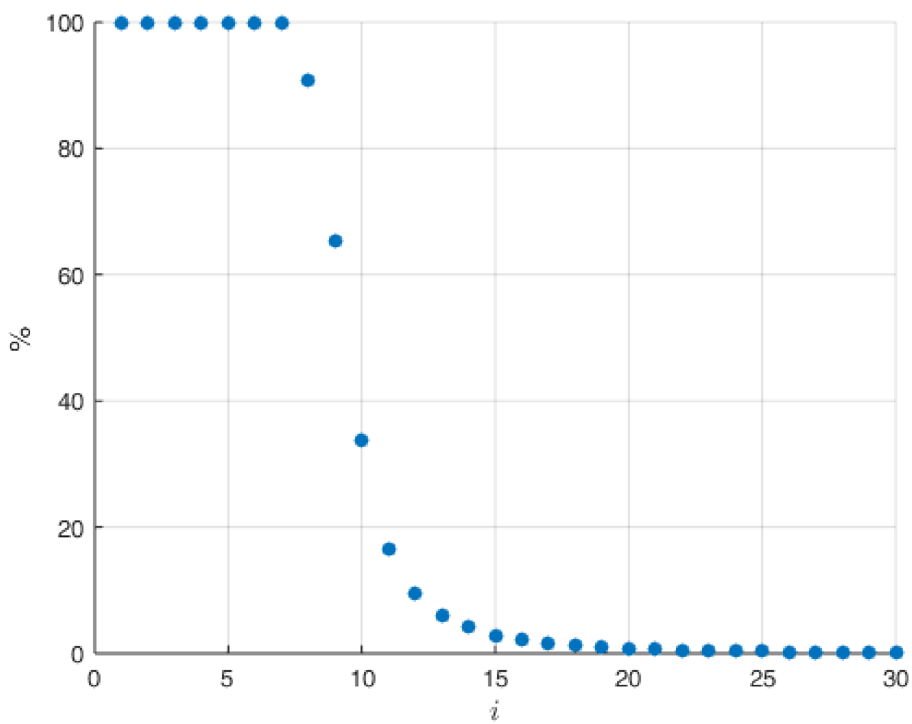

Figure 2: **Market Participation rate**
This figure illustrates market participation rate for each investor in our simulation. To generate this, 1,000 sample paths of economy are simulated. Each path consists of 480 monthly observations (40 years), in total 480,000 months. Parameter values for the simulation are in Table 1.

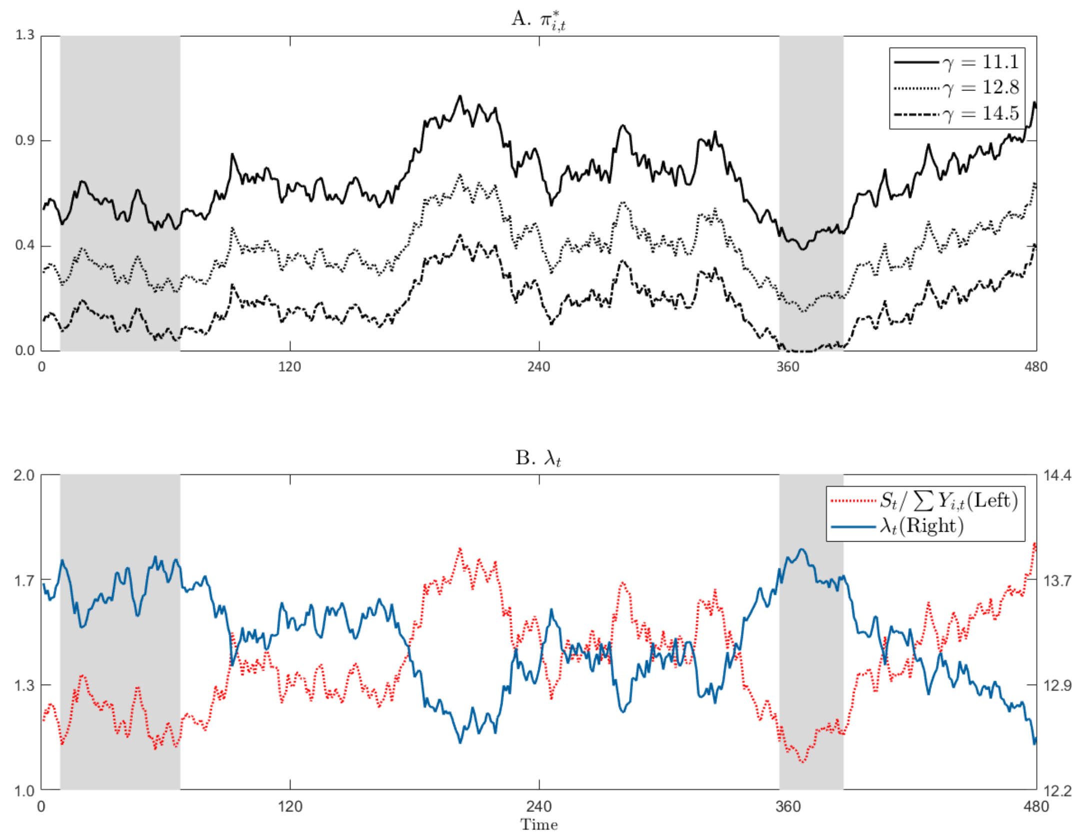

Figure 3: **Time-variation in the optimal portfolio**

The top figure (Panel A) illustrates one sample path of the optimal portfolio $\pi^*_{i,t}$ for investors from $i = 7$ (risk aversion 11.1) to $i = 9$ (risk aversion 14.5) as an example. The bottom figure (Panel B) illustrates one sample path of the Sharpe ratio from simulated data. The solid line is the Sharpe ratio (right y-axis) and the dotted line is the state variable: the stock market wealth-aggregate labor ratio ($\frac{S_t}{\sum Y_{i,t}}$) (left y-axis). The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ based on simulated data. Parameter values for the simulation are in Table 1.

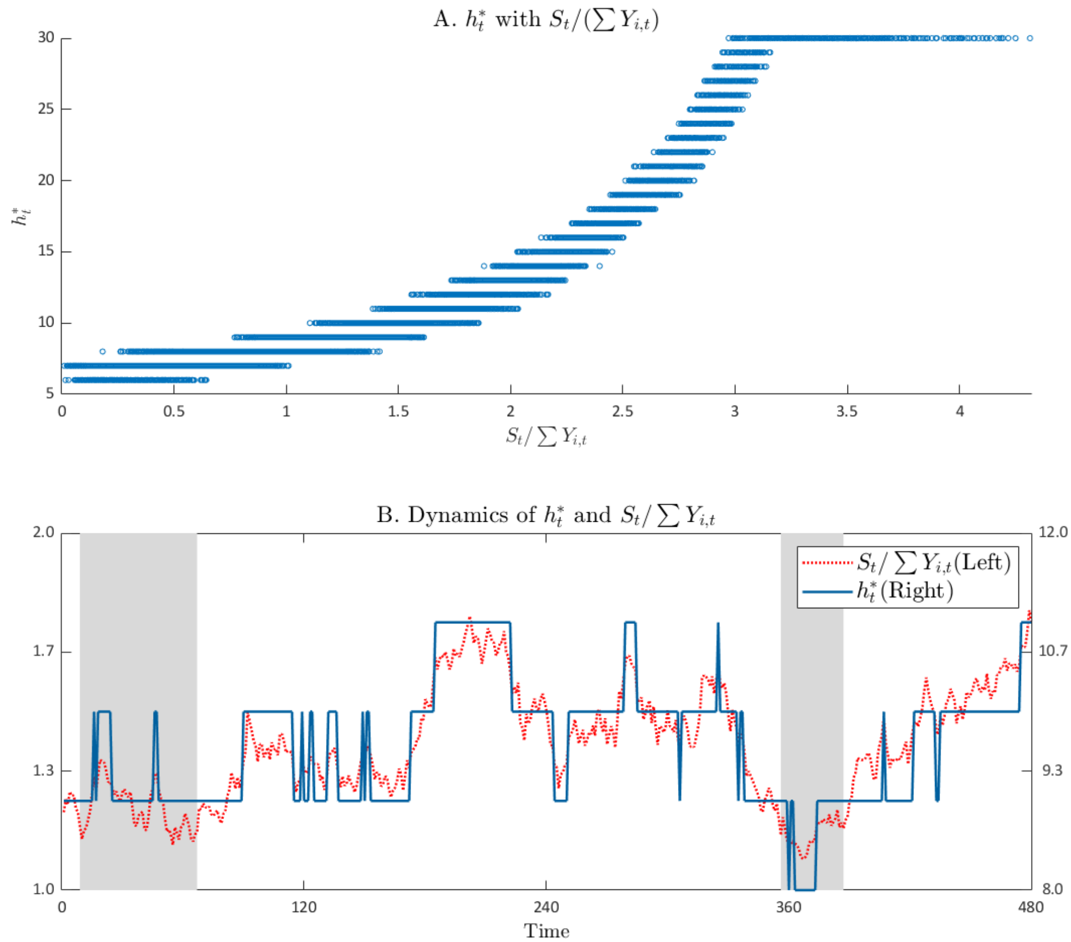

Figure 4: **Dependence of the market participation on the state variable**
The top figure (Panel A) depicts the relationship between the market participation h_t^* and the state variable $\frac{S_t}{\sum Y_{i,t}}$. To generate this figure, 1,000 sample paths of economy are simulated. Each path consists of 480 monthly observations (40 years), in total 480,000 months. The bottom figure (Panel B) illustrates one sample path of time-varying market participation from simulated data. The solid line is market participation (right y-axis) and the dotted line is the state variable: the stock market wealth-aggregate labor ratio ($\frac{S_t}{\sum Y_{i,t}}$) (left y-axis). The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ based on simulated data. Parameter values for the simulation are in Table 1.

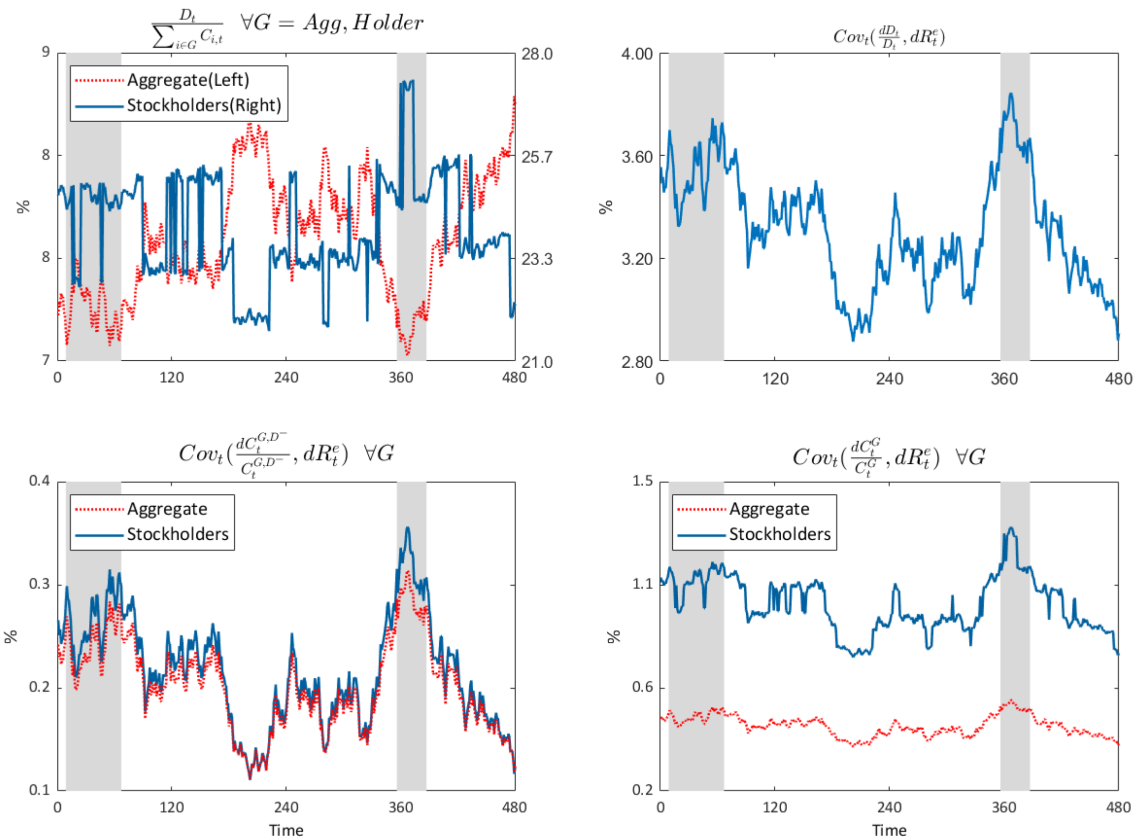

Figure 5: **Consumption decomposition for amount of consumption risk**

This figure illustrates one sample path of dividend share $\frac{D_t}{\sum_{i\in G} C_{i,t}}$ (top left), the covariance of stock returns with dividend growth $Cov_t(\frac{dD_t}{D_t}, dR_t^e)$ (top right), non-financial income part of consumption growth $Cov_t(\frac{dC_t^{G,D^-}}{C_t^{G,D^-}}, dR_t^e)$ (bottom left), and consumption growth $Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e)$ (bottom right) for aggregate consumption ($G = A$) and stockholders' consumption ($G = H$) The dotted line is for aggregate consumption and the solid line is for stockholders' consumption. The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ based on simulated data. Parameter values for the simulation are in Table 1.

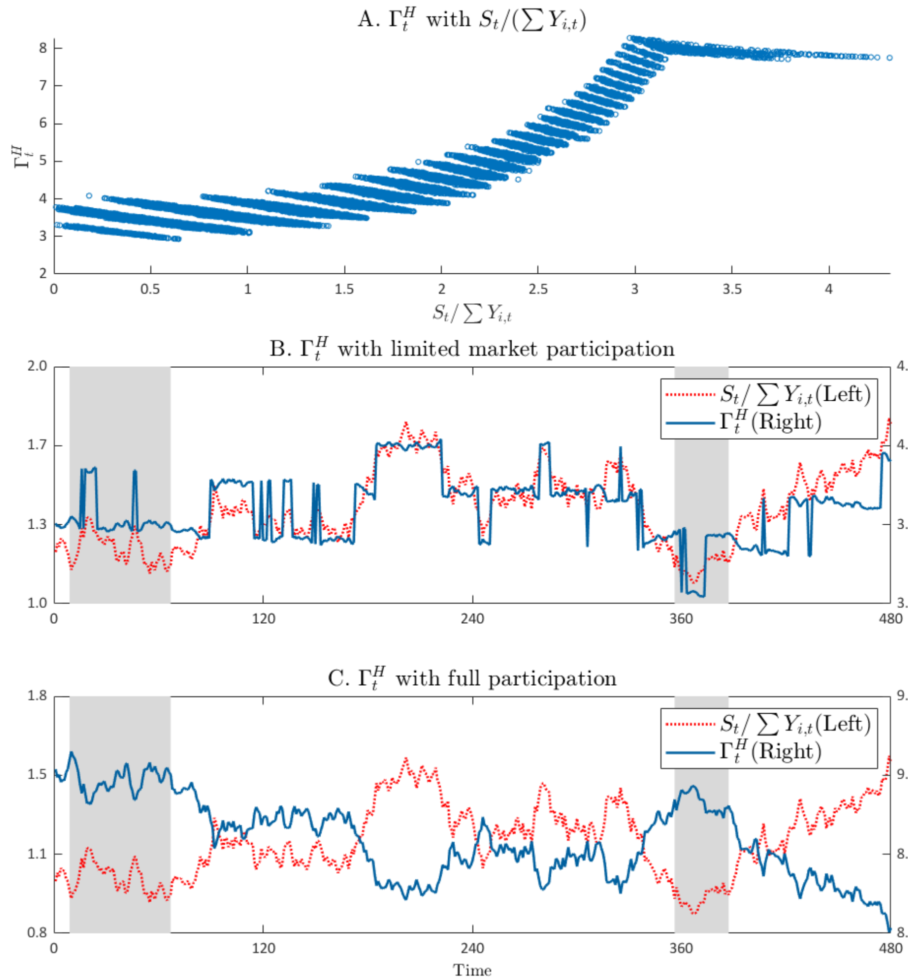

Figure 6: **Time-variation in the price of risk**

Panel A depicts the relationship between the price of risk Γ_t^H with limited market participation and the state variable $\frac{S_t}{\sum Y_{i,t}}$. To generate this plot, 1,000 sample paths of economy are simulated. Each path consists of 480 monthly observations (40 years), in total 480,000 months. Panel B and C illustrate one sample path of the price of risk from simulated data. Panel B shows the price of consumption risk (right y-axis) in an economy where short-selling is not allowed. Panel C shows the price of risk (right y-axis) in an economy where there is no short-selling constraint and therefore market participation is full. The dotted line is the state variable: the stock market wealth-aggregate labor ratio ($\frac{S_t}{\sum Y_{i,t}}$) (left y-axis). The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ based on simulated data. Parameter values for the simulation are in Table 1.

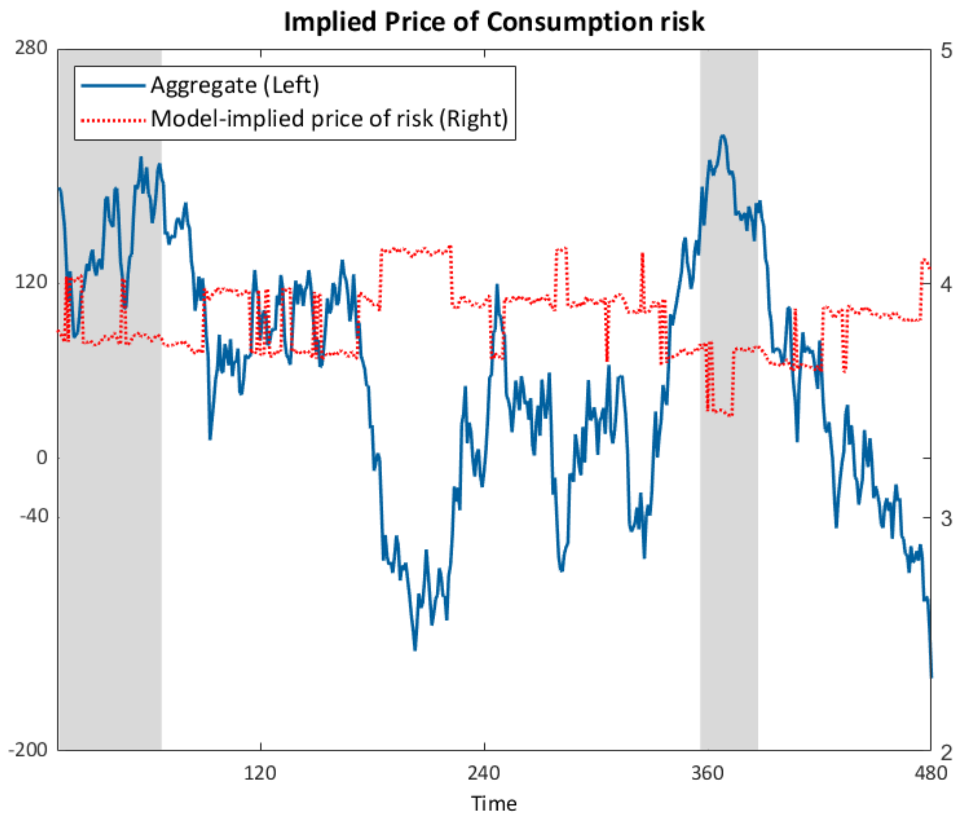

Figure 7: **Implied Price of risk using Simulated data**

This figure illustrates one sample path of implied price of consumption risk (blue straight line, left y-axis) using aggregate consumption. Based on the simulated data, we infer the conditional covariance between aggregate consumption growth and realized equity excess returns based on the GMM methodology by Duffee (2005). For comparison, we also plot the model-implied price of risk (red dashed line, right y-axis). The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$. Parameter values for the simulation are in Table 1.

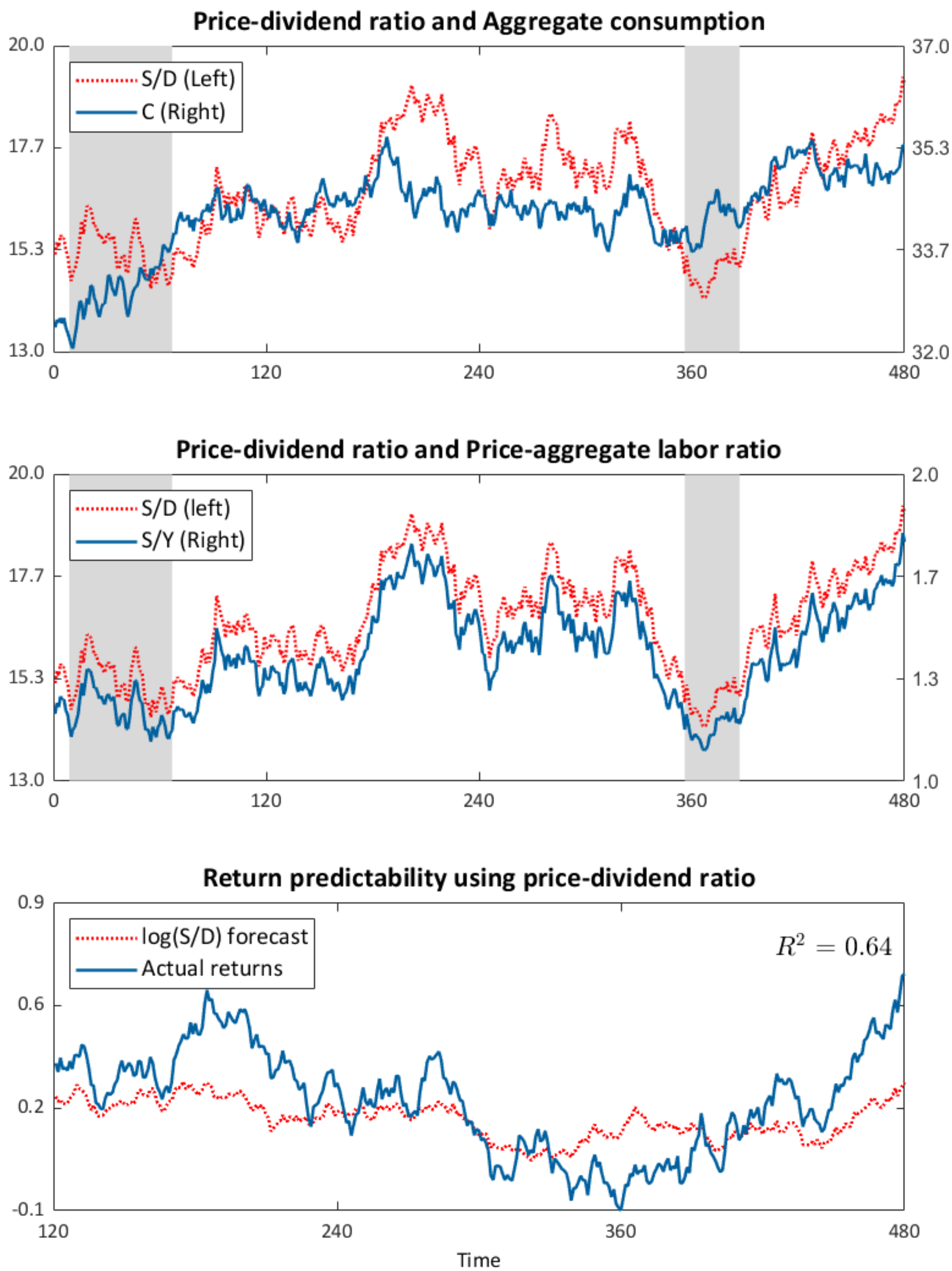

Figure 8: **Price-Dividend ratio**

The top (middle) figure is one sample path of the price-dividend ratio and the aggregate consumption (price-aggregate labor ratio). The bottom figure plots one sample path of 10-year cumulative realized excess returns and log price-dividend ratio forecast from the simulated data. Log price-dividend ratio forecast is based on estimates from the forecasting regression: $r^e_{[t\to t+k]} = \alpha + \beta log(\frac{S}{D})_t + \epsilon_{t\to t+k}, \forall k = 10$ years. This regression uses simulated 1,000 sample paths of economy. Each path consists of 480 monthly observations (40 years), in total 480,000 months. Parameter values for the simulation are in Table 1.

Table 1: Model Parameters

Table 1 presents the annualized model parameters used to simulate the model. The moments of dividend and non-financial income are chosen based on the annual U.S. real per capita data from the National Income and Product Account (NIPA) from the BEA (Bureau of Economic Analysis) for the period of 1930 to 2016. Following Jagannathan and Wang (1996), non-financial income (labor income) is defined as the total personal income less the total dividend from the NIPA of the U.S. All nominal values are deflated using the personal consumption expenditures deflator. U.S. population data are also used to obtain the per capita value of both dividend and non-financial income. A detailed description of the data is in the online appendix OA.15.

Parameter	Symbol	Value
Panel A: Dividend and Labor income parameters		
Dividend growth mean (%)	μ_d	2
Dividend growth volatility (%)	σ_d[1]	12
Labor income growth mean (%)	μ_y	2
Labor income growth volatility (%)	σ_y	4
Correlation between dividend and labor income shock (%)	ρ[2]	43
Panel B: Investor-related parameters		
Subjective time preference (%)	δ[3]	0.2
Elasticity of Intertemporal Substitution	ψ[4]	0.5
Lowest risk aversion coefficient	γ_1	1
Highest risk aversion coefficient	γ_N[5]	50
Number of investors	N	30
Panel C: Initial value		
Initial aggregate dividend stream	D_0[6]	$0.08\times N$
Initial per capita non-financial income	Y_0	1

[1] Consistent with Beeler and Campbell (2012), which report the 11.05% of log dividend growth volatility based on the CRSP data.
[2] Consistent with Dittmar et al. (2016), which report 40% based on the Bureau of Economic Analysis and CRSP data.
[3] We follow Bansal and Yaron (2004). They use the value of 0.998 for the time discount factor, which translates into 0.2% for the subjective time preference rate.
[4] We set the EIS to 0.5, consistent with Vissing-Jørgensen (2002a), Trabandt and Uhlig (2011), Jin (2012), and Rudebusch and Swanson (2012).
[5] While Chan and Kogan (2002) consider risk aversion distribution from 1 to 100, we restrict the distribution of risk aversion from 1 to 50.
[6] Initial value of per capita non-financial income (Y_0) is normalized to 1. In the beginning of the sample year (1930) of the NIPA data, per capita dividend share (D_0/N) accounts for 8% of per capita income.

Table 2: Consumption risk with Consumption decomposition

Consumption is decomposed into dividend D_t and non-financial income source of consumption C_t^{G,D^-} for the covariance between equity returns and consumption growth. Panel A is for aggregate consumption ($G = A$) and Panel B is for stockholders consumption ($G = H$). We report the average level of each component across states and its model-implied dynamics. In doing so, we simulate 1,000 sample paths of the economy. Each path consists of 480 monthly observations (40 years), in total 480,000 months. Parameter values for the simulation are in Table 1. The bad (good) states are defined as the lowest (highest) 10% percentiles of the state variable.

$$Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e) = \frac{D_t}{C_t^G} Cov_t(\frac{dD_t}{D_t}, dR_t^e) + \frac{C_t^{G,D^-}}{C_t^G} Cov_t(\frac{dC_t^{G,D^-}}{C_t^{G,D^-}}, dR_t^e) \quad \forall G = A, H$$

	Model-implied Dynamics	Bad (%)	Good (%)	Average (%)
$Cov_t(\frac{dD_t}{D_t}, dR_t^e)$	**Counter**	5.33	3.13	3.95
Panel A: Aggregate consumption				
$\frac{D_t}{C_t^A}$	**Pro**	6.75	8.12	7.42
$Cov_t(\frac{dC_t^{A,D^-}}{C_t^{A,D^-}}, dR_t^e)$	Counter	0.54	0.20	0.33
$Cov_t(dR_t^e, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$	Counter	0.85	0.44	0.59
Panel B: Stockholders' consumption				
$\frac{D_t}{C_t^H}$	**Counter**	25.9	22.1	24.4
$Cov_t(\frac{dC_t^{H,D^-}}{C_t^{H,D^-}}, dR_t^e)$	Counter	0.65	0.21	0.37
$Cov_t(dR_t^e, \frac{d\mathbf{C}_t^H}{\mathbf{C}_t^H})$	Counter	1.87	0.86	1.26

Table 3: Consumption risk with Return decomposition

Equity returns are decomposed into the dividend growth part dD_t/D_t and non-dividend part of returns $dR_t^e - dD_t/D_t$ for the covariance between equity returns and consumption growth. Panel A reports the result for aggregate consumption and Panel B for stockholders consumption. We report the average level of each component across states and its model-implied dynamics. In doing so, we simulate 1,000 sample paths of the economy. Each path consists of 480 monthly observations (40 years), in total 480,000 months. The state variable is the stock market wealth-aggregate labor income ratio ($\frac{S_t}{\sum Y_{i,t}}$). Average values in bad states and good states are reported in brackets [bad good]. The bad (good) states are defined as the lowest (highest) 10% percentiles of the state variable. Parameter values for the simulation are in Table 1. $\mathbf{C}_t^A$ denotes the aggregate consumption including both stockholders and non-stockholders' consumption. $\mathbf{C}_t^H$ denotes the consumption of aggregate stock*h*olders. **Notations** "Counter": Counter-cyclical; "Pro": Pro-cyclical.

	Model-implied Dynamics	Bad (%)	Good (%)	Average (%)
Panel A: Aggregate consumption				
$Cov_t(\frac{dD_t}{D_t}, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$	Pro	0.29	0.30	0.30
$Cov_t(dR_t^e - \frac{dD_t}{D_t}, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$	Counter	0.55	0.13	0.29
$Cov_t(dR_t^e, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$	Counter[1]	0.85	0.44	0.59
Panel B: Stockholders' consumption				
$Cov_t(\frac{dD_t}{D_t}, \frac{d\mathbf{C}_t^H}{\mathbf{C}_t^H})$	Counter	0.55	0.48	0.52
$Cov_t(dR_t^e - \frac{dD_t}{D_t}, \frac{d\mathbf{C}_t^H}{\mathbf{C}_t^H})$	Counter	1.31	0.38	0.74
$Cov_t(dR_t^e, \frac{d\mathbf{C}_t^H}{\mathbf{C}_t^H})$	Counter	1.87	0.86	1.26

[1] Instead of relying on the analytical form as in the current table, when we infer the conditional covariance in the same way econometricians would do based on the simulated data, we generate a procyclical variation in the estimated conditional covariance between aggregate consumption and stock returns. See Figure OA.1.

Table 4: Conditional behavior of the Stock Volatility

Table 4 reports the conditional behavior of the parameters associated with the stock volatility. We simulate 1,000 sample paths of the economy. Each path consists of 480 monthly observations (40 years), in total 480,000 months. Parameter values for the simulation are in Table 1. The bad (good) states are defined as the lowest (highest) 10% percentiles of the state variable.

$$\frac{\sigma_{s,t}^d}{\sigma_d} = \frac{\frac{D_t}{\sum_{i=1}^{h_t^*} C_{i,t}^*}}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*} \frac{\pi_{i,t}^*}{X_{i,t}+H_{h,t}}}, \sigma_{s,t}^y = \frac{\sigma_y Y_t N[1 - \frac{1}{N}\sum_{i=1}^N \partial C_{i,t}^*(X_{i,t}, Y_t)/\partial Y_t]}{\sum_{i=1}^{h_t} k_{h,i,t}\pi_{i,t}^*}, \sigma_{s,t} = \sqrt{{\sigma_{s,t}^d}^2 + {\sigma_{s,t}^y}^2 + 2\rho\sigma_{s,t}^d\sigma_{s,t}^y}$$

	Model-implied Dynamics	Bad	Good	Average
$\sigma_{s,t}$	Counter	44.98	27.01	33.6
$\sigma_{s,t}/\sigma_d$	Counter	3.75	2.25	2.80
$\sigma_{s,t}^d$	Counter	47.6	29.4	36.1
$\sigma_{s,t}^d/\sigma_d$	Counter	3.97	2.45	3.01
$\frac{D_t}{\sum_{i=1}^{h_t^*} C_{i,t}^*}$	Counter	25.9	22.1	24.4
$\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*} \frac{\pi_{i,t}^*}{X_{i,t}+H_{h,t}}$	Pro	7.37	9.03	8.48
$\sigma_{s,t}^y$	Counter	-7.41	-7.57	-7.36
$\frac{1}{N}\sum_{i=1}^N \frac{\partial C_{i,t}^*(X_{i,t},Y_t)}{\partial Y_t}$	Pro	1.04	1.07	1.05

Table 5: Dynamics of equilibrium asset parameters

Table 5 reports the dynamics of equilibrium asset parameters based on simulated data. We simulate 1,000 sample paths of the economy. Each path consists of 480 monthly observations (40 years), in total 480,000 months. The state variable is the stock market wealth-aggregate labor income ratio ($\frac{S_t}{\sum Y_{i,t}}$). Average values in bad states and good states are reported in brackets [bad good]. The bad (good) states are defined as the lowest (highest) 10% percentiles of the state variable. The pooled OLS panel regression of a dependent variable on $\frac{S_t}{\sum Y_{i,t}}$ is reported. Parameter values for the simulation are in Table 1. $\mathbf{C}_t^H$ denotes the consumption of stock*h*olders. **Notations**:"Counter": Countercyclical; "Pro": Procyclical.

	Data Dynamics	Model-implied Dynamics	Bad (%)	Good (%)	Average (%)
$E_t(dR_t^e)$	Counter	Counter	6.67	3.51	4.7
Γ_t^H	Pro[1]	Pro	3.61	4.22	3.8
$Cov_t(dR_t^e, dC_t^H/C_t^H)$	Counter[2]	Counter	1.87	0.86	1.2
$\sigma_{s,t}$	Counter	Counter	44.98	27.01	33.6
$\lambda_{s,t}$	Counter	Counter	0.145	0.129	0.137
$\sigma(dC_t^H/C_t^H)$	Counter	Counter	5.58	4.91	5.26
$E(h_t/N)$	Pro[3]	Pro	26.9	31.8	31.3

[1] See Table 8.
[2] See Figure 1 and Table 8.
[3] See Section 6, Brunnermeier and Nagel (2008), Bonaparte et al. (2018), and Yang (2018).

Table 6: Stock Return Predictability and backward-looking test

Panel A reports coefficients and R^2 from the long-horizon forecasting regression: The k-year cumulative rolling ex post excess returns are regressed on the past log price-dividend ratio using the simulated data. The result for the data is from Guvenen (2009). Panel B reports R^2 from the backward looking price dividend ratio test: log price-dividend ratio is regressed on from 1 to L-year lagged consumption growth. The data is from the U.S. data for the period of 1930 to 2016. A detailed description of the data is in the online appendix OA.15. For simulation, we generate 1,000 sample paths of the economy. Each path consists of 480 monthly observations (40 years), in total 480,000 months.

	Year	1	2	3	5	7
Panel A: $r^e_{[t\to t+k]} = \alpha + \beta log(\frac{S}{D})_t + \epsilon_{t\to t+k}$						
Model	Coeff.	-0.10	-0.20	-0.30	-0.49	-0.66
Data	Coeff.	-0.22	-0.96	-0.47	-0.77	-0.94
Model	R^2	0.084	0.152	0.208	0.30	0.369
Data	R^2	0.09	0.14	0.15	0.26	0.33
Panel B: $log(\frac{S}{D})_t = \alpha + \sum_{j=1}^{L} \beta_j \Delta c_{t-j} + \epsilon_t$						
Model	R^2	0.010	0.021	0.030	0.047	0.063
Data	R^2	0.024	0.013	0.005	0.018	0.022

Table 7: Simulated Unconditional Moments of Consumption growth and Asset Returns

Table 7 presents the annualized consumption, stock returns moments, and market participation rate. We use the longest annual data from 1930 to 2016. We follow Beeler and Campbell (2012) in measuring the asset moments in U.S. data. The aggregate consumption data are from the NIPA (National Income and Product Account). Consumption growth is log growth of the real per capita nondurable and services. Excess returns are log growth of real value of all NYSE/Amex/Nasdaq stocks from the CRSP minus the ex-ante real risk free rate measured as in Beeler and Campbell (2012). A detailed description of the data is in the online appendix OA.15. For the simulation of the model, 1,000 sample paths of economy are simulated. Each path consists of 480 monthly observations (40 years), in total 480,000 months. Δc_t^A denote the log aggregate consumption growth. r^e and r_f denote excess stock returns and risk-free rate, respectively. $E(\Gamma_t^H)$ and $Cov(r^e, \Delta c_t^H)$ are the price and the amount of consumption risk in the model, respectively. $p - d$ is the log price-dividend ratio, and h_t/N denotes the market participation rate. The first moments of consumption growth and excess equity returns are corrected for the Jensen's inequality.

Moment	U.S. data	Model
Panel A: Consumption moments		
$E(\Delta c_t^A)$	2.0	2.0
$\sigma(\Delta c_t^A)$	2.2	4.2
Panel B: Asset returns moments		
$E(r^e)$	5.7	4.7
$\sigma(r^e)$	20.1	33.6
$\sigma(r^e)/\sigma_d$	1.7	2.8
$E(r^e)/\sigma(r^e)$	0.28	0.14
$E(p-d)$	2.9	2.7
$\sigma(p-d)$	0.49	0.52
$E(r_f)$	0.7	4.0
$\sigma(r_f)$	3.2	0.01
Panel C: Market Participation rate		
$E(h_t/N)$	29.7[1]	31.3

[1] This is based on direct holding from 2016 SCF.

Table 8: Empirical test of the model

Table 8 reports the OLS regression results as a test of the theory model. $Cov_t(dC_t^G/C_t^G, dR_t^e)\ \ \forall G = A, H$ is the twelve months rolling covariances between either aggregate or stockholders consumption growth and excess market returns. $D_t/C_t^G\ \ \forall G = A, H$ is dividend share in either aggregate or stockholders consumption. C_t^H/C_t^A is the stockholders consumption share in aggregate consumption. $(\frac{h}{N})_t$ is the market participation rate. $\sum_{i\in G} C_{i,t}/\sum_{i\in G}(C_{i,t}/\gamma_i)\ \ \forall G = A, H$ is the consumption-weighted harmonic mean of stockholders or aggregate risk aversion. For the data, the Consumer Ependiture (CEX) Survey and the CRSP value-weighted NYSE/AMEX/NASDAQ index are used. For risk aversion measure, we assume that risk aversion is the probability of reporting that households have no tolerance for investment risk. To compute the probability, we use the regression estimates of households reporting unwillingness to take financial risk on a set of characteristics from the Survey of Consumer Finances data and apply them to the CEX households. A detailed description of the data is in the online appendix OA.15 and Table OA.1. t-statistics based on robust standard error are in parenthesis. ***,**,* denote the statistical significance at 1%, 5%, and 10%, respectively.

Dependent	Independent variable			Adj. R^2
Variable	Recession	C_t^H/C_t^A	$(\frac{h}{N})_t$	
Panel A: Amount of consumption risk dynamics				
$Cov_t(dC_t^A/C_t^A, dR_t^e)$	3.7×10^{-4}**			0.064
	(3.46)			
$Cov_t(dC_t^H/C_t^H, dR_t^e)$	9.1×10^{-4}***			0.029
	(3.71)			
Panel B: Dividend share in consumption				
D_t/C_t^A	-0.022*			0.075
	(-1.78)			
D_t/C_t^H	0.548*			0.064
	(1.86)			
Panel C: Time-varying market participation				
C_t^H/C_t^A	-0.009***			0.048
	(-5.35)			
$(\frac{h}{N})_t$	-0.006***			0.036
	(-4.66)			
Panel D: Price of consumption risk dynamics				
$\sum_{i\in H} C_{i,t}/\sum_{i\in H}(C_{i,t}/\gamma_{i,t})$		1.045***		0.205
		(11.98)		
$\sum_{i\in H} C_{i,t}/\sum_{i\in H}(C_{i,t}/\gamma_{i,t})$			1.645***	0.331
			(16.31)	
$\sum_{i\in H} C_{i,t}/\sum_{i\in H}(C_{i,t}/\gamma_{i,t})$		-0.624***	2.331***	0.348
		(-2.76)	(8.62)	
$\sum_{i\in H} C_{i,t}/\sum_{i\in H}(C_{i,t}/\gamma_{i,t})$	-0.006			0.005
	(-1.47)			

A. Appendix

A.1 Proof of Proposition 1

Substituting C_i^*, π_i^*, and l_i^* in (6) (7), and (11) back into the equation (5) gives

$$\begin{aligned}
0 \;=\;& \frac{\delta(1-\gamma_i)V_{i,t}}{1-\psi^{-1}}(\delta^{\psi-1}((1-\gamma_i)V_{i,t})^{-\theta\psi+\psi-1}V_{x_i,t}^{1-\psi}\psi^{-1}-1) \\
&+(r_{f,t}X_{i,t}+Y_t)V_{x_i,t}-\frac{\lambda_t^2V_{x_i,t}^2}{2V_{x_ix_i,t}}+\mu_yY_tV_{y,t}+\frac{1}{2}\sigma_y^2Y_t^2V_{yy,t}-\frac{\lambda_tV_{x_i,t}\rho_{s,t}\sigma_yY_tV_{x_iy,t}}{V_{x_ix_i,t}}-\frac{\rho_{s,t}^2\sigma_y^2Y_t^2V_{x_iy,t}^2}{2V_{x_ix_i,t}} \\
&+\sum_{j=1}^{N-1}\mu_{w_{j,t}}w_{j,t}V_{w_j,t}+\frac{1}{2}\sum_{j=1}^{N-1}\sigma_{w_{j,t}}^2w_{j,t}^2V_{w_jw_j,t}-\frac{\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}\sigma_{s,t}V_{w_jx_i,t}(\lambda_tV_{x_i,t}+\rho_{s,t}\sigma_yY_tV_{x_iy,t})}{\sigma_{s,t}V_{x_ix_i,t}} \\
&-\frac{(\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}\sigma_{s,t}V_{w_jx_i,t})^2}{2\sigma_{s,t}^2V_{x_ix_i,t}}+\sum_{j=1}^{N-1}\rho_{w_j,y,t}\sigma_{w_{j,t}}w_{j,t}\sigma_yY_tV_{w_jy,t}+\sum_{j\neq k}\rho_{w_j,w_k,t}\sigma_{w_{j,t}}\sigma_{w_{k,t}}w_{j,t}w_{k,t}V_{w_jw_k,t} \\
&+\frac{{l_{i,t}^*}^2}{2\sigma_{s,t}^2V_{x_ix_i,t}} \qquad\qquad (A.1)
\end{aligned}$$

Due to the nonlinearity of $\pi_{i,t}^*$, the first-order condition together with the HJB equation is a non-linear system. Hence, as in the literature (e.g., Haugh et al., 2006), we first solve the unconstrained HJB equation and solve the constrained HJB equation.

Unbinding constraint: At time t, if the constraint is not binding (i.e., $\pi_{i,t}^{w/o} > 0$), the Lagrange multiplier is zero (i.e., $l_{i,t}^* = 0$) from the complementary slackness condition. Please note that this does not mean that the constraints will never bind at time $s > t$. Constraints can bind at different time in the future depending on the states which are incorporated into the HJB equation as state variables in (5). We can solve the PDE (A.1) in a case where the constraint is not binding with $l_{i,t}^* = 0$. We conjecture the functional form of the value function as follows.

$$V_{i,t} = \frac{(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})(X_{i,t}+b_iY_t)^{1-\gamma_i}}{1-\gamma_i} \equiv \frac{p_{i,t}q_{i,t}^{1-\gamma_i}}{1-\gamma_i} \qquad (A.2)$$

where $p_{i,t} \equiv a_i + \sum_{j=1}^{N-1}c_{j,t}w_{j,t}$ and $q_{i,t} \equiv X_{i,t} + b_iY_t$. This functional form of the value function implies the following partial derivatives with respect to state variables.

$$\begin{aligned}
&V_{x_i,t} = p_{i,t}q_{i,t}^{-\gamma_i}, V_{x_ix_i,t} = -\gamma_ip_{i,t}q_{i,t}^{-\gamma_i-1}, \\
&V_{y,t} = b_ip_{i,t}q_{i,t}^{-\gamma_i}, V_{yy,t} = -\gamma_ib_i^2p_{i,t}q_{i,t}^{-\gamma_i-1}, V_{x_iy,t} = -\gamma_ib_ip_{i,t}q_{i,t}^{-\gamma_i-1}, V_{w_j,t} = \frac{c_jq_{i,t}^{1-\gamma_i}}{1-\gamma_i}, \\
&V_{w_jw_j,t} = 0, V_{w_jw_k,t} = 0, V_{x_iw_j,t} = c_jq_{i,t}^{-\gamma_i}, V_{yw_j,t} = b_ic_jq_{i,t}^{-\gamma_i} \qquad (A.3)
\end{aligned}$$

Substituting expressions in (A.3) into the HJB equation and rearranging terms give

$$
\begin{aligned}
0 \; &= (X_{i,t}+b_iY_t)^2[\frac{\delta}{1-\psi^{-1}}(\delta^{\psi-1}(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})^{-\theta\psi}\psi^{-1}-1)+\frac{\lambda_t^2}{2\gamma_i} \\
&+\frac{(\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}c_j)^2}{2\gamma_i(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})^2}+\frac{\sum_{j=1}^{N-1}\mu_{w_{j,t}}w_{j,t}c_j}{(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})(1-\gamma_i)}+\frac{\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}c_j\lambda_t}{\gamma_i(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})}] \\
&+(r_{f,t}X_{i,t}+Y_t)(X_{i,t}+b_iY_t)-\frac{1}{2}\sigma_y^2Y_t^2\gamma_ib_i^2+\frac{\rho_{s,t}^2\sigma_y^2Y_t^2\gamma_ib_i^2}{2} \\
&+(X_{i,t}+b_iY_t)Y_t[\mu_yb_i-\lambda_t\rho_{s,t}\sigma_yb_i-\frac{\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}c_j\rho_{s,t}\sigma_y\gamma_ib_i}{\gamma_i(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})}+\frac{\sum_{j=1}^{N-1}\rho_{w_j,y,t}\sigma_{w_{j,t}}w_{j,t}\sigma_yb_ic_j}{a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t}}]
\end{aligned}
\tag{A.4}
$$

When the correlation between dividend growth and labor income growth ρ is equal to 1 (implying also that the correlation between equity returns and labor income growth $\rho_{s,t}=1$ is equal to 1), we can solve the above PDE in a closed form solution.[33] For a non-perfect correlation between dividend and labor income growth $\rho \neq 1$, there is no closed form solution. However, as discussed in the body section, we follow the assumption that $X_{i,t}/Y_t$ goes to infinity as used in Koo (1998) and Wang et al. (2016) and solve for this expression in closed form. Each term in (A.4) can be factorized as follows.

$$
\begin{aligned}
0 \; &= X_{i,t}^2(d_{i,t}+r_{f,t}) \\
&+X_{i,t}Y_t[2b_id_{i,t}+r_{f,t}b_i+1+\mu_yb_i-\lambda_t\rho_{s,t}\sigma_yb_i \\
&-\frac{\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}c_j\rho_{s,t}\sigma_y\gamma_ib_i}{\gamma_i(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})}+\frac{\sum_{j=1}^{N-1}\rho_{w_j,y,t}\sigma_{w_{j,t}}w_{j,t}\sigma_yb_ic_j}{a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t}}]+Y_to(z)
\end{aligned}
\tag{A.5}
$$

where $d_{i,t}=\frac{\delta}{1-\psi^{-1}}(\delta^{\psi-1}(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})^{-\theta\psi}\psi^{-1}-1)+\frac{\lambda_t^2}{2\gamma_i}+\frac{(\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}c_j)^2}{2\gamma_i(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})^2}$ $+\frac{\sum_{j=1}^{N-1}\mu_{w_{j,t}}w_{j,t}c_j}{(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})(1-\gamma_i)}+\frac{\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}c_j\lambda_t}{\gamma_i(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})}$, $z \equiv \frac{X_{i,t}}{Y_t}$, and $o(z)$ is a function such that $\lim_{z\to\infty}\frac{o(z)}{z}=0$. After dividing all terms by $X_{i,t}$, in (A.5), as z goes to infinity, the above PDE can be solved by

$$
d_{i,t}=r_{f,t}, \quad b_i^*=\frac{1}{r_{f,t}+\lambda_t\rho_{s,t}\sigma_y-\mu_y}, \quad c_1^*=\cdots=c_{N-1}^*=0 \tag{A.6}
$$

$d_{i,t}=r_{f,t}$ is equivalent to

$$
a_i^*=(\delta^{1-\psi}\psi((-r_{f,t}-\frac{\lambda_t^2}{2\gamma_i})\frac{1-\psi^{-1}}{\delta}+1))^{-\frac{1}{\theta_i\psi}} \tag{A.7}
$$

Then, the value function is

$$
V_i^*(X_{i,t},Y_t)=\frac{(\delta^{1-\psi}\psi((-r_{f,t}-\frac{\lambda_t^2}{2\gamma_i})\frac{1-\psi^{-1}}{\delta}+1))^{-\frac{1}{\theta_i\psi}}}{1-\gamma_i}(X_{i,t}+\frac{Y_t}{r_{f,t}+\rho_{s,t}\sigma_y\lambda_t-\mu_y})^{1-\gamma_i} \tag{A.8}
$$

[33] See the online appendix OA.3.

The optimal policies are given by

$$C^*_{i,t} = (\delta^{\psi} a_i^{-\theta_i \psi}) p_{i,t} = ((r_{f,t} + \frac{\lambda_t^2}{2\gamma_i})(1-\psi) + \delta\psi)(X_{i,t} + \frac{Y_t}{r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y}) \tag{A.9}$$

$$\pi^*_{i,t} = \frac{\lambda_t}{\gamma_i \sigma_{s,t}}(X_{i,t} + \frac{Y_t}{r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y}) - \frac{\rho_{s,t}\sigma_y}{\sigma_{s,t}} \frac{Y_t}{r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y} \tag{A.10}$$

$$l^*_{i,t} = 0 \tag{A.11}$$

Binding constraint: At time t, if the constraint is binding (i.e., $\pi^{w/o}_{i,t} \leq 0$), the Lagrange multiplier is nonzero and from the equation (11), its value is $l^*_{i,t} = -(\mu_{s,r} - r_{f,t})V_{x_i,t} - \rho_{s,t}\sigma_y Y_t \sigma_{s,t} V_{x_i y,t} - \sum_{j=1}^{N-1} \rho_{w_j,s,t}\sigma_{w_j,t} w_{j,t}\sigma_{s,t} V_{w_j x_i,t}$. Substituting $l^*_{i,t}$ into the equation (A.1) and rearranging terms gives

$$\begin{aligned} 0 \ &= \frac{\delta(1-\gamma_i)V_{i,t}}{1-\psi^{-1}}(\delta^{\psi-1}((1-\gamma_i)V_{i,t})^{-\theta_i\psi+\psi-1} V_{x_i,t}^{1-\psi}\psi^{-1} - 1) + (r_{f,t}X_{i,t} + Y_t)V_{x_i,t} \\ &+ \mu_y Y_t V_{y,t} + \frac{1}{2}\sigma_y^2 Y_t^2 V_{yy,t} + \sum_{j=1}^{N-1} \mu_{w_j,t} w_{j,t} V_{w_j,t} + \frac{1}{2}\sum_{j=1}^{N-1} \sigma^2_{w_{j,t}} w_{j,t} V_{w_j w_j,t} \\ &+ \sum_{j=1}^{N-1} \rho_{w_j,y,t}\sigma_{w_{j,t}} w_{j,t}\sigma_y Y_t V_{w_j y,t} + \sum_{j\neq k} \rho_{w_j,w_k,t}\sigma_{w_{j,t}}\sigma_{w_{k,t}} w_{j,t} w_{k,t} V_{w_j w_k,t} \end{aligned} \tag{A.12}$$

In the same way of unbinding constraint case, we conjecture the functional form and solve the HJB equation with $\frac{X_{i,t}}{Y_t} \to \infty$ The value function is then given by

$$V^*_i(X_{i,t}, Y_t) = \frac{(\delta^{1-\psi}\psi(-r_{f,t}\frac{1-\psi^{-1}}{\delta} + 1))^{-\frac{1}{\theta_i\psi}}}{1-\gamma_i}(X_{i,t} + \frac{Y_t}{r_{f,t} - \mu_y})^{1-\gamma_i} \tag{A.13}$$

Based on the above value function, the optimal consumption and stock-holding are

$$C^*_{i,t} = (r_{f,t}(1-\psi) + \delta\psi)(X_{i,t} + \frac{Y_t}{r_{f,t} - \mu_y}) \tag{A.14}$$

$$\pi^*_{i,t} = 0 \tag{A.15}$$

$$l^*_{i,t} = \frac{(\delta^{1-\psi}\psi((-r_{f,t} - \frac{\lambda_t^2}{2\gamma_i})\frac{1-\psi^{-1}}{\delta} + 1))^{-\frac{1}{\theta_i\psi}}}{(X_{i,t} + H_{h,t})^{\gamma_i}}(-(\mu_{s,r} - r_{f,t}) + \frac{\rho_{s,t}\sigma_y\sigma_{s,t}\gamma_i H_{h,t}}{X_{i,t} + H_{h,t}}) \tag{A.16}$$

where $H_{h,t} = \frac{Y_t}{r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y}$, $H_{n,t} = \frac{Y_t}{r_{f,t} - \mu_y}$ ■

A.2 Proof of (18)

The bond market clearing condition is

$$\sum_{i=1}^{h_t} X_{i,t} - S_t + \sum_{i=h_t+1}^{N} X_{i,t} = 0 \tag{A.17}$$

This condition is guaranteed by setting the initial value and dynamics such that $\sum_{i=1}^{h_0} X_{i,0} - S_0 + \sum_{i=h_0+1}^{N} X_{i,0} = 0$ and $d\sum_{i=1}^{h_t} X_{i,t} - dS_t + d\sum_{i=h_t+1}^{N} X_{i,t} = 0$. Therefore,

$$\begin{aligned}
& d\sum_{i=1}^{h_t} X_{i,t} - dS_t + d\sum_{i=h_t+1}^{N} X_{i,t} \\
&= \sum_{i=1}^{h_t} [\pi_{i,t}^*(\mu_{s,t} - r_{f,t}) + r_{f,t}X_{i,t} + Y_t - C_{i,t}^*]dt + \sigma_{s,t}^d \sum_{i=1}^{h_t} \pi_{i,t}^* dW_{d,t} + \sigma_{s,t}^y \sum_{i=1}^{h_t} \pi_{i,t}^* dW_{y,t} \\
&\quad - (\mu_{s,t}S_t - D_t)dt - S_t\sigma_{s,t}^d dW_{d,t} - S_t\sigma_{s,t}^y dW_{y,t} \\
&\quad + \sum_{i=h_t+1}^{N} [r_{f,t}X_{i,t} + Y_t - C_{i,t}^*]dt = 0
\end{aligned} \tag{A.18}$$

The stock market clearing condition is $\sum_{i=1}^{h_t} \pi_{i,t}^* = S_t$ and bond market clearing condition implies $\sum_{i=1}^{h_t} X_{i,t} + \sum_{i=h_t+1}^{N} X_{i,t} = S_t$. Applying these equations to (A.18) and rearranging terms yield the consumption clearing condition.

$$\sum_{i=1}^{N} C_{i,t}^* = \sum_{i=1}^{N} Y_{i,t} + D_t \quad \blacksquare \tag{A.19}$$

A.3 Proof of Proposition 2

Sharpe ratio

Using (A.10), the optimal stock holding can be written as:

$$\begin{aligned}
& \pi_{i,t}^* = \frac{\lambda_t}{\gamma_i \sigma_{s,t}}(X_{i,t} + \frac{Y_t}{r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y}) - \frac{1}{\sigma_{s,t}}\frac{\rho_{s,t}\sigma_y Y_t}{r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y} \\
& \forall X_{i,t} > 0, Y_t > 0, i = 1, 2, ..., h_t^*
\end{aligned} \tag{A.20}$$

The stock market clearing condition is equivalent to the following equation.

$$\sum_{i=1}^{h_t^*}(\frac{\lambda_t}{\gamma_i \sigma_{s,t}}(X_{i,t} + \frac{Y_t}{r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y}) - \frac{1}{\sigma_{s,t}}\frac{\rho_{s,t}\sigma_y Y_t}{r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y}) = \sum_{i=1}^{N} X_{i,t} \tag{A.21}$$

Solving for the Sharpe ratio λ_t gives

$$\lambda_t = \frac{\sigma_{s,t}\sum_{i=1}^{N} X_{i,t} + h_t\rho_{s,t}\sigma_y H_{h,t}}{\sum_{i=1}^{h_t}\frac{X_{i,t}+H_{h,t}}{\gamma_i}} \quad \text{where} \quad H_{h,t} = \frac{Y_t}{r_{f,t} + \rho_{s,t}\sigma_y\lambda_t - \mu_y} \tag{A.22}$$

If we consider no labor income $Y_t = 0$, (A.22) becomes

$$\lambda_t = (\sum_{i=1}^{N} \frac{C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*}\frac{1}{\gamma_i})^{-1}\sigma_d \tag{A.23}$$

This is the same as the one in Cvitanić et al. (2012) without heterogeneity in terms of belief and time discount rate and also in Chabakauri (2013) without constraint.

Risk-free rate

From (A.9), the optimal consumption is

$$C^*_{i,t} = (\delta^{\psi} a_i^{-\theta_i\psi}) p_{i,t} = (\delta^{\psi} a_i^{-\theta_i\psi})(X_{i,t} + b_i Y_t) \quad \forall i = 1, .., N \tag{A.24}$$

Then, the dynamics of (A.19) is given by

$$\sum_{i=1}^{h_t^*} (\delta^{\psi} a_i^{-\theta_i\psi})(dX_{i,t} + b_i dY_t) + \sum_{i=h_t^*+1}^{N} (\delta^{\psi} a_i^{-\theta_i\psi})(dX_{i,t} + b_i dY_t) = dD_t + N \cdot dY_t \tag{A.25}$$

We can obtain the optimal dynamics of the financial wealth by plugging the optimal consumption and portfolio into (4).

$$dX^*_{i,t} = \begin{cases} (\pi^*_{i,t}(\mu_{s,t} - r_{f,t}) + r_{f,t} X_{i,t} + Y_t - C^*_{i,t})dt + \pi^*_{i,t}(\sigma^d_{s,t} dW_{d,t} + \sigma^y_{s,t} dW_{y,t}) & \text{if } \pi^{w/o}_{i,t} > 0 \\ (r_{f,t} X_{i,t} + Y_t - C^*_{i,t})dt & \text{Otherwise} \end{cases} \tag{A.26}$$

Collecting the deterministic terms of (A.25) yields

$$\sum_{i=1}^{h_t^*} (\delta^{\psi} a_i^{-\theta_i\psi})[\frac{\lambda_t^2 (X_{i,t} + H_{h,t})}{\gamma_i} - \lambda_t \rho_t \sigma_y H_{h,t} + r_{f,t} X_i + Y_t \tag{A.27}$$

$$- ((r_{f,t} + \frac{\lambda_t^2}{2\gamma_i})(1-\psi) + \delta\psi)(X_{i,t} + H_{h,t}) + \mu_y H_{h,t}]$$

$$+ \sum_{i=h_t^*+1}^{N} (\delta^{\psi} a_i^{-\theta_i\psi})[r_{f,t} X_{i,t} + Y_t - (r_{f,t}(1-\psi) + \delta\psi)(X_{i,t} + H_{n,t}) + \mu_y H_{n,t}] = \mu_d D_t + \mu_y N \cdot Y_t$$

Rearranging the terms gives the following equation.

$$\sum_{i=1}^{h_t^*} (\delta^{\psi} a_i^{-\theta_i\psi})(X_{i,t} + H_{h,t})(\frac{\lambda_t^2}{\gamma_i}(1 - \frac{1}{2}(1-\psi)) + r_{f,t}\psi - \delta\psi) \tag{A.28}$$

$$+ \sum_{i=h_t^*+1}^{N} (\delta^{\psi} a_i^{-\theta_i\psi})(X_{i,t} + H_{n,t})(r_{f,t}\psi - \delta\psi) = \mu_d D_t + \mu_y N \cdot Y_t$$

Solving the above equation for $r_{f,t}$ yields the closed form solution for the risk-free rate.

$$r_{f,t} = \delta + \frac{\mu_d D_t + \mu_y N \cdot Y_t}{D_t + N \cdot Y_t} \frac{1}{\psi} - \sum_{i=1}^{h_t} \frac{C_{i,t}}{D_t + N \cdot Y_t} (\frac{\lambda_t^2}{\gamma_i} \frac{1+\psi}{2\psi}) \tag{A.29}$$

If we consider no labor income $Y_t = 0$, (A.29) becomes

$$r_{f,t} = \delta + \mu_d \frac{1}{\psi} - \frac{1+\psi}{2\psi} (\sum_{i=1}^{N} \frac{C^*_{i,t}}{\sum_{i=1}^{N} C^*_{i,t}} \frac{1}{\gamma_i})^{-1} \sigma_d^2 \tag{A.30}$$

When the preferences are the CRRA, then $\psi = 1/\gamma_i$, the risk-free is

$$r_{f,t} = \delta + \mu_d D_t (\sum_{i=1}^{N} \frac{c^*_{i,t}}{\gamma_i})^{-1} - (\sum_{i=1}^{N} \frac{c^*_{i,t}}{\gamma_i})^{-3} \sum_{i=1}^{N} \frac{1}{2} \frac{c^*_{i,t}}{\gamma_i} (\frac{1}{\gamma_i} + 1)(\sigma_d D_t)^2 \tag{A.31}$$

This is the same as the one in Cvitanić et al. (2012) without heterogeneity in terms of belief and time discount rate and also in Chabakauri (2013) without constraint.

Stock volatility

Collecting the diffusion terms of (A.25) yields

$$\sum_{i=1}^{h_t^*}(\delta^{\psi}a_i^{-\theta_i\psi})[\pi_{i,t}^*(\sigma_{s,t}^d dW_{d,t}+\sigma_{s,t}^y dW_{y,t})+\sigma_y H_{h,t}dW_{y,t}] \tag{A.32}$$

$$+\sum_{i=h_t^*+1}^{N}(\delta^{\psi}a_i^{-\theta_i\psi})\sigma_y H_{n,t}dW_{y,t}=\sigma_d D_t dW_{d,t}+\sigma_y N\cdot Y_t dW_{y,t}$$

This gives the following two equations for the stock volatility.

$$\sum_{i=1}^{h_t^*}(\delta^{\psi}a_i^{-\theta_i\psi})\pi_{i,t}^*\sigma_{s,t}^d=\sigma_d D_t \tag{A.33}$$

$$\sum_{i=1}^{h_t^*}(\delta^{\psi}a_i^{-\theta_i\psi})(\pi_{i,t}^*\sigma_{s,t}^y+\sigma_y H_{h,t})+\sum_{i=h_t^*+1}^{N}(\delta^{\psi}a_i^{-\theta_i\psi})\sigma_y H_{n,t}=N\cdot Y_t\sigma_y \tag{A.34}$$

Then,

$$\sigma_{s,t}^d=\frac{\sigma_d D_t}{\sum_{i=1}^{h_t^*}(\delta^{\psi}a_i^{-\theta_i\psi})\pi_{i,t}^*} \tag{A.35}$$

$$\sigma_{s,t}^y=\frac{\sigma_y Y_t[N-\sum_{i=h_t^*+1}^{N}(\delta^{\psi}a_i^{-\theta_i\psi})/(r_{f,t}-\mu_y)-\sum_{i=1}^{h_t^*}(\delta^{\psi}a_i^{-\theta_i\psi})/(r_{f,t}+\rho_{s,t}\sigma_y\lambda_t-\mu_y)]}{\sum_{i=1}^{h_t^*}(\delta^{\psi}a_i^{-\theta_i\psi})\pi_{i,t}^*} \tag{A.36}$$

Finally, the equilibrium stock volatility is

$$\sigma_{s,t}=\sqrt{(\sigma_{s,t}^d)^2+(\sigma_{s,t}^y)^2+2\rho\sigma_{s,t}^d\sigma_{s,t}^y} \tag{A.37}$$

If we consider no labor income $Y_t=0$,

$$\sigma_{s,t}=\sigma_d\Big(\sum_{i=1}^{N}\frac{C_{i,t}^*}{\sum_{i=1}^{N}C_{i,t}^*}\frac{1}{\gamma_i}\Big)^{-1}\sum_{i=1}^{N}\frac{X_{i,t}}{\sum_{i=1}^{N}X_{i,t}}\frac{1}{\gamma_i} \tag{A.38}$$

Stock price

Consumption clearing condition is

$$\sum_{i=1}^{h_t^*}((r_{f,t}+\frac{\lambda_t^2}{2\gamma_i})(1-\psi)+\delta\psi)(X_{i,t}+H_{h,t})+\sum_{i=h_t^*+1}^{N}(r_{f,t}(1-\psi)+\delta\psi)(X_{i,t}+H_{n,t}) \tag{A.39}$$

$$=D_t+N\cdot Y_t$$

By taking $r_{f,t}$ from summation and considering $\sum_{i=1}^{N}X_{i,t}=S_t$. We can obtain the following equation.

$$(r_{f,t}(1-\psi)+\delta\psi)S_t = D_t + N\cdot Y_t - \sum_{i=1}^{h_t^*}\frac{\lambda_t^2}{2\gamma_i}(1-\psi)(X_{i,t}+H_{h,t}) - (r_{f,t}(1-\psi)+\delta\psi)(H_{h,t}h_t + H_{n,t}(N-h_t^*)) \tag{A.40}$$

By solving for S_t and rearranging term, S_t can be expressed as

$$S_t = \frac{D_t + N\cdot Y_t - \sum_{i=1}^{h_t^*}\frac{\lambda_t^2}{2\gamma_i}(1-\psi)(X_{i,t}+H_{h,t})}{r_{f,t}(1-\psi)+\delta\psi} - (H_{h,t}h_t^* + H_{n,t}(N-h_t^*)) \quad \blacksquare \tag{A.41}$$

A.4 Proof of Proposition 3

Consider the conditional covariance between stock returns and stockholders' consumption growth.

$$Cov_t(dR_t^e, \frac{d\sum_{i=1}^{h_t^*}C_{i,t}^*}{\sum_{i=1}^{h_t^*}C_{i,t}^*}) \tag{A.42}$$

For convenience, we only need to consider the following diffusion terms.

$$dR_t^e - E_t[dR_t^e] = \sigma_{s,t}^d dW_{d,t} + \sigma_{s,t}^y dW_{y,t} \tag{A.43}$$

$$d\sum_{i=1}^{h_t^*}C_{i,t}^* - E_t[d\sum_{i=1}^{h_t^*}C_{i,t}^*] = \sum_{i=1}^{h_t^*}(\delta^\psi a_i^{-\theta_i\psi})[\pi_{i,t}^*(\sigma_{s,t}^d dW_{d,t} + \sigma_{s,t}^y dW_{y,t}) + \sigma_y H_{h,t}dW_{y,t}] \tag{A.44}$$

Plugging (A.43) and (A.44) into (A.42) yields

$$Cov_t(dR_t^e, \frac{d\sum_{i=1}^{h_t^*}C_{i,t}^*}{\sum_{i=1}^{h_t^*}C_{i,t}^*}) = \frac{\sum_{i=1}^{h_t^*}(\delta^\psi a_i^{-\theta_i\psi})(\pi_{i,t}^*\sigma_{s,t}^d(\sigma_{s,t}^d + \rho\sigma_{s,t}^y) + \pi_{i,t}^*\sigma_{s,t}^y(\rho\sigma_{s,t}^d + \sigma_{s,t}^y) + \sigma_y H_{h,t}(\rho\sigma_{s,t}^d + \sigma_{s,t}^y))dt}{\sum_{i=1}^{h_t^*}C_{i,t}^*} \tag{A.45}$$

Substituting $\pi_{i,t}^*$ into the above equation gives

$$= \frac{\sum_{i=1}^{h_t^*}(\delta^\psi a_i^{-\theta_i\psi})(\frac{\sigma_{s,t}\lambda_t}{\gamma_i}(X_{i,t}+H_{h,t}) - \sigma_{s,t}\rho_{s,t}\sigma_y H_{h,t} + \sigma_y H_{h,t}(\rho\sigma_{s,t}^d + \sigma_{s,t}^y))dt}{\sum_{i=1}^{h_t^*}C_{i,t}^*} \tag{A.46}$$

After rearranging and canceling out some terms, the equation becomes

$$= \frac{\lambda_t\sigma_{s,t}\sum_{i=1}^{h_t^*}(\frac{C_{i,t}^*}{\gamma_i})dt}{\sum_{i=1}^{h_t^*}C_{i,t}^*} \tag{A.47}$$

Solving for $\lambda_t\sigma_{s,t}$ in the (A.47) gives

$$\lambda_t\sigma_{s,t}dt = E_t[dR_t^e] = \frac{\sum_{i=1}^{h_t^*}C_{i,t}^*}{\sum_{i=1}^{h_t^*}\frac{C_{i,t}^*}{\gamma_i}}Cov_t(dR_t^e, \frac{d\sum_{i=1}^{h_t^*}C_{i,t}^*}{\sum_{i=1}^{h_t^*}C_{i,t}^*}) \quad \blacksquare \tag{A.48}$$

Online Appendix to: Time-Varying Stockholders Consumption Risk-Sharing and Asset Dynamics

Contents

OA.1 Preferences: CARA utility case and consumption sensitivity to labor

In this section, we solve the equilibrium of an economy populated by heterogeneous CARA (Constant Absolute Risk Aversion) investors.[34] The purpose of this appendix is to explain why we need to abstract away from using the CARA preferences. First, we are going to show that the marginal propensity to consume out of labor income is always one $\frac{\partial C_{i,t}}{\partial Y_{i,t}} = 1$. Second, due to the unit marginal propensity to consume, the stock price is not affected by labor income shocks. Finally, there is no time-variation in market participation, which leads to time-invariant asset parameters and therefore there is no stochastic dynamics in this economy.

OA.1.1 The basic setup

As in the main section, there is a single riskless bond such that $\frac{dB_t}{B_t} = r_{f,t}dt$ in zero net supply and risky asset in unit net supply, which is a claim to a dividend D_t that follows Arithmetic Brownian Motion: $dD_t = \mu_d dt + \sigma_d dW_{d,t}$. All stockholders receive the same level (systematic) of stochastic exogenous income Y_t that evolves as: $dY_t = \mu_y dt + \sigma_y dW_{y,t}$ where $dW_{d,t}dW_{y,t} = \rho dt > 0$. The equilibrium stock price dynamics has the following form[35] :

$$dS_t = (S_t r_{f,t} + \mu^e_{s,t} - D_t)dt + \sigma_{s,t} dW_{d,t} \tag{OA.49}$$

where $\mu^e_{s,t}$ denotes the total expected excess return over the risk-free rate and $\sigma_{s,t}$ is the (absolute) price volatility. Thus, $\lambda_{s,t} = \mu^e_{s,t}/\sigma_{s,t}$ is the Sharpe ratio. The economy is populated by infinitely lived *N* (types of) investors and all having exponential utility with different risk aversion. Investor *i* is maximizing $\forall t \in [0, \infty)$

$$\mathbb{E}_t[\int_t^\infty -e^{-\delta(s-t)} e^{-a_i C_{i,s}} ds] \tag{OA.50}$$

$\forall i = 1, 2, ..., h_t, ..., N$ whose absolute risk aversion coefficient is $a_1, a_2, ..., a_{h_t}, ..., a_N$, respectively, with $0 < a_1 < a_2 < ... < a_{h_t} < ... < a_N$.

OA.1.2 The individual investor's problem

An investor i's financial wealth dynamics is

$$dX_{i,t} = (r_{f,t} X_{i,t} + \pi_{i,t}\mu^e_{s,t} + Y_t - C_{i,t})dt + \pi_{i,t}\sigma_{s,t} dW_{d,t} \tag{OA.51}$$

[34] The setting in this section of the online appendix is similar to Christensen et al. (2012). But, while Christensen et al. (2012) study the full participation case with finite time horizon and idiosyncratic labor income, we solve the equilibrium of an economy where there are non-stockholders which arises from the short-selling constraint over the infinite time horizon.

[35] We show that this is the correct conjecture. One can prove that if the stock price is modeled as a function of the labor income shocks $dW_{y,t}$ as well as the dividend shocks (i.e., $dS_t = (S_t r_{f,t} + \mu^e_{s,t} - D_t)dt + \sigma^d_{s,t} dW_{d,t} + \sigma^d_{y,t} dW_{y,t}$), the sensitivity of the stock price with respect to the labor income shocks must be zero in equilibrium (i.e., $\sigma^d_{y,t} = 0$). The proof can be provided upon request.

where $\pi_{i,t}$ represents the number of units of the risky asset owned by the investor at time t. With the value function $V_{i,t}(x,y) = \max_{(c_{i,t},\pi_{i,t})\in\mathcal{A}} E_t[\int_t^\infty -e^{-a_i C_{i,s}} ds]$. The HJB equation is

$$0 = \max_{(c_{i,t},\pi_{i,t})\in\mathcal{A}} -e^{-a_i C_{i,t}} - \delta V + [\pi_{i,t}\mu^e_{s,t} + r_{f,t}X_{i,t} + Y_t - C_{i,t}]V_x + \frac{1}{2}\pi^2_{i,t}V_{xx}\sigma^2_{s,t} + l_{i,t}\pi_{i,t} \tag{OA.52}$$

where $l_{i,t}$ is the Lagrange multiplier for the short-selling constraints. After solving the above HJB equation, The investors' optimal portfolio is

$$\pi^*_{i,t} = \begin{cases} \pi^{w/o}_{i,t} = \frac{\lambda_{s,t}}{a_i r_{f,t}\sigma_{s,t}} - \frac{\rho\sigma_y}{r_{f,t}\sigma_{s,t}} & \text{if } \pi^{w/o}_{i,t} > 0 \\ 0 & \text{Otherwise} \end{cases} \tag{OA.53}$$

Equation (OA.53) shows that the only source of time-variation in the optimal holding $\pi^*_{i,t}$ is time-variation in the equilibrium asset parameters ($\lambda_{s,t}$, $r_{f,t}$, and $\sigma_{s,t}$), and neither dividend nor labor income does appear in $\pi^*_{i,t}$, contrary to power or recursive preferences. The following is the optimal consumption.

$$C^*_{i,t} = \begin{cases} r_{f,t}X_{i,t} + Y_t + \frac{1}{a_i r_{f,t}}(\delta - r_{f,t} - a_i\rho\sigma_y\lambda_{s,t} + \frac{\lambda^2_{s,t}}{2} + \mu_y a_i - \frac{\sigma^2_y a^2_i(1-\rho^2)}{2}) & \text{if } \pi^{w/o}_{i,t} > 0 \\ r_{f,t}X_{i,t} + Y_t + \frac{1}{a_i r_{f,t}}(\delta - r_{f,t} + \mu_y a_i - \frac{\sigma^2_y a^2_i}{2}) & \text{Otherwise} \end{cases} \tag{OA.54}$$

The optimal consumption in (OA.54) shows that the marginal propensity to consume out of labor income is unity (i.e., $\frac{\partial C^*_{i,t}}{\partial Y_t} = 1$), suggesting that investors do not invest part of their labor income in financial asset. Therefore, investors' optimal financial wealth dynamics is independent of labor income as follows.

$$dX_{i,t} = \begin{cases} \frac{1}{a_i r_{f,t}}(-(\delta - r_{f,t}) + \frac{\lambda^2_{s,t}}{2} - \mu_y a_i + \frac{\sigma^2_y a^2_i(1-\rho^2)}{2})dt + \pi^*_{i,t}\sigma_{s,t}dW_{d,t} & \text{if } \pi^{w/o}_{i,t} > 0 \\ \frac{1}{a_i r_{f,t}}(-(\delta - r_{f,t}) - \mu_y a_i + \frac{\sigma^2_y a^2_i}{2})dt & \text{Otherwise} \end{cases} \tag{OA.55}$$

OA.1.3 Equilibrium

From the stock market clearing condition, the Sharpe ratio is identified. Also, by matching terms from the dynamics of the consumption clearing condition equation ($\sum_{i=1}^N C^*_{i,t} = D_t + N \cdot Y_t$), the risk-free rate and stock volatility are determined.

$$\lambda_{s,t} = (\sum_{i=1}^{h_t} \frac{1}{a_i})^{-1}(\sigma_d + \rho\sigma_y h_t) \tag{OA.56}$$

$$r_{f,t} = \delta + (\mu_d + \mu_y N)(\sum_{i=1}^N \frac{1}{a_i})^{-1} - \frac{\sigma^2_d}{2}(\sum_{i=1}^{h_t} \frac{1}{a_i})^{-1}(\sum_{i=1}^N \frac{1}{a_i})^{-1} - \sigma_d\rho\sigma_y h_t(\sum_{i=1}^{h_t} \frac{1}{a_i})^{-1}(\sum_{i=1}^N \frac{1}{a_i})^{-1}$$
$$- \frac{\sigma^2_y}{2}(\sum_{i=1}^N \frac{1}{a_i})^{-1}[(\sum_{i=1}^{h_t} a_i)(1-\rho^2) + \sum_{i=h_t+1}^N a_i + \rho^2 h^2_t(\sum_{i=1}^{h_t} \frac{1}{a_i})^{-1}] \tag{OA.57}$$

$$\sigma_{s,t} = \frac{\sigma_d}{r_{f,t}} \tag{OA.58}$$

$$S_t = \frac{D_t}{r_{f,t}} + \frac{\sigma_y^2}{2r_{f,t}^2}[\sum_{i=1}^{h_t} a_i(1-\rho^2) + \sum_{i=h_t+1}^{N} a_i] - \frac{\lambda_{s,t}^2}{2r_{f,t}^2}\sum_{i=1}^{h_t}\frac{1}{a_i}$$
$$- \frac{(\mu_y - \rho\sigma_y\lambda_{s,t})h_t}{r_{f,t}^2} - \frac{\mu_y(N-h_t)}{r_{f,t}^2} - \frac{(\delta - r_{f,t})\sum_{i=1}^{N}\frac{1}{a_i}}{r_{f,t}^2} \tag{OA.59}$$

Since labor income shocks do not affect investors' financial wealth, the equilibrium stock price in (OA.59) is independent of labor income. Most importantly, fundamental shocks ($dW_{d,t}, dW_{y,t}$) do not affect the equilibrium parameters ($\lambda_{s,t}$, $r_{f,t}$, and $\sigma_{s,t}$). Therefore, $\pi^*_{i,t}$ do not vary in response to fundamental shocks and hence market participation is not time-varying in this economy (i.e., $h_t = h \ \ \forall t > 0$). Since the remaining time variation in the equilibrium asset parameters (λ_t, $r_{f,t}$, $\sigma_{s,t}$) only stems from the time-varying market participation h_t which is time-invariant ($h_t = h$), all equilibrium asset parameters are also time-invariant (i.e., $\lambda_{s,t} = \lambda_s, r_{f,t} = r_f, \sigma_{s,t} = \sigma_s \ \ \forall t > 0$).

To summarize, since there is no wealth effect in the CARA investor, there is no stochastic dynamics of the equilibrium asset parameters in this economy, and hence it is impossible to study the conditional asset pricing using the CARA preference. Note that in Christensen et al. (2012), by considering a finite horizon, they generate a deterministic dynamics only which is perfectly predictable.

OA.2 Comparative Statics of equilibrium moments

For the comparative statics exercise, We **exogenously** change the group of stockholders and investigate how r_f, EP (Equity Premium), λ, σ_s, Cov (amount of risk), and Γ (Price of risk) change accordingly. We start this exercise by imposing the least risk-averse investor as a cut-off stockholder $h = 1$, then we repeat this exercise by moving the cut-off stockholder one by one up until the point where every investor is a stockholder ($h = N$). For ease of exposition, we suppress time index throughout this exercise.

Figure OA.3 plots $r_f(h)$, $EP(h)$, $\lambda(h)$, $\sigma_s(h)$, $Cov(h)$, $\Gamma(h)$ in a given level of state $\frac{S_t}{\sum Y_i}$. In Panel A, the risk-free rate is increasing at the low market participation level. This is because there is a greater selling demand on the bond, as risk-tolerant investors, who are willing to borrow money to invest in the risky asset, are included in the market. As more risk-averse investors are included in the market, the risk-free rate is decreasing. The reason is as follows. First, more risk-averse investors are more willing to invest in the bond. Second, as discussed in Section 4.3.2, the decreasing risk-free rate is also attributed to the increasing precautionary saving demand, as more agents become stockholders and hence more exposure to a future uncertainty.

Panel B shows that as we impose more investors to stay in the market, the equity premium is decreasing and turning to increasing. To pin down the source of the variation in the equity premium, we decompose the equity premium with respect to the market risk and consumption risk in Panel C and D, respectively. In Panel C, note that when the least risk-averse investor is the only stockholder, the market price of risk λ is the highest possible level. This is because there should be a substantial compensation in order to induce this investor to bear the market risk alone. As more investors are assumed to be in the market, λ is decreasing with more buying demand. From a certain point, λ is turning to increasing as the investors who want to optimally short-sell the stock are assumed to be in the market. An increasing selling demand requires the market to compensate more to induce investors to hold the market. As for the amount of market risk - stock volatility σ_s, it has the exact same shape as the Sharpe ratio. We delve into and discuss this finding in Figure OA.4.

Panel D decomposes the equity premium into the amount $Cov(dR_t^e, \frac{d\sum_{i=1}^h C_{i,t}^*}{\sum_{i=1}^h C_{i,t}^*})$ and price of consumption risk $\Gamma^H \equiv \frac{\sum_{i=1}^h C_{i,t}^*}{\sum_{i=1}^h \frac{C_{i,t}^*}{\gamma_i}}$ as in **Proposition 3**. When it comes to the price of risk, Γ^H is increasing with market participation. This is because the more risk-averse investors we include in the market, the higher the stockholders' harmonic mean of risk aversion, and the higher the required compensation. By contrast, the amount of risk is decreasing as more investors are in the market. The intuition behind this finding is as more investors bear the market risk together, the risk is effectively shared-out (improving risk-sharing) among stockholders and the amount of risk decreases. Please note that the risk-sharing is improving at a decreasing rate because new agents included in the market are more risk-averse than the existing ones and they are not willing to take the risk as much as risk-tolerant investors. Therefore, their contribution of sharing the risk is only marginal. For more details on the consumption risk-sharing, please see Appendix OA.8.

While $EP(h)$ in this comparative statics increases for $h > h_B^*$, this result does not translate into the relationship between the equity premium and market participation across different equilibria. For example, in our base state (B) the equity premium is 4% ($EP(h_B^*) = 4\%$ and $h_B^* = 9$). In a better state (G), the endogenous market participation is $h_G^* = 11$ ($> h_B^* = 9$). The equilibrium equity premium is 3%, lower than 4% in our base state $EP(h_G^*) = 3\% < EP(h_B^*) = 4\%$ even with the inclusion of more investors in the market.

To further understand the shape of $\sigma_s(h)$ with market participation in Panel C of Figure OA.3, we explore the two parameters that govern the stock volatility as a function of the market participation (i.e., $\sigma_s^d(h)$ and $\sigma_s^y(h)$). Panel A of Figure OA.4 illustrates that it is the parameter associated with the dividend shocks $\sigma_s^d(h)$ which drives the shape of $\sigma_s(h)$, whereas $\sigma_s^y(h)$ works in the opposite way. As discussed in Section 4.3.3, σ_s^d/σ_d can be expressed as the dividend share in the stockholder's consumption divided by the stockholders' consumption-weighted mean of risky asset share in total wealth $\frac{D}{\sum_{i=1}^h C_i} / \sum_{i=1}^h \frac{C_i}{\sum_{i=1}^h C_i} \frac{\pi_i}{X_i + H_h}$. Each term is illustrated in Panel B of Figure OA.4.

First, $\frac{D}{\sum_{i=1}^h C_i}$ is decreasing as more investors are assumed to be in the market. This is because the same amount of dividend D_t is shared out by more investors. Also, as the amount of risk is decreasing at a decreasing rate, so does $\frac{D}{\sum_{i=1}^h C_i}$. The reason is that a newly included investor is more risk-averse than the existing stockholders. Due to a high precautionary saving motive, the new investor's consumption level is low (see Figure OA.2.) and thus her contribution of sharing dividend is only marginal. Second, $(\sum_{i=1}^{h} \frac{C_i}{\sum_{i=1}^{h} C_i} \frac{\pi_i}{X_i+H_h})^{-1}$ is positively linked to the price of consumption risk in Panel D of Figure OA.3. As more risk-averse investors are assumed to be in the market, $\sum_{i=1}^{h} \frac{C_i}{\sum_{i=1}^{h} C_i} \frac{\pi_i}{X_i+H_h}$ is decreasing because the inclusion of more risk-averse investor whose optimal portfolio is relatively low drives down the overall average. Thus, $\sum_{i=1}^{h} \frac{C_i}{\sum_{i=1}^{h} C_i} \frac{\pi_i}{X_i+H_h}$ inversely capture the consumption-weighted mean of stockholders' risk aversion. As the increasing price of consumption risk dominates the decreasing amount of consumption risk from h_B^* in Panel D of Figure OA.3, the increasing $(\sum_{i=1}^{h} \frac{C_i}{\sum_{i=1}^{h} C_i} \frac{\pi_i}{X_i+H_h})^{-1}$ dominates the decreasing $\frac{D}{\sum_{i=1}^{h} C_i}$ from h_B^* in Panel B. This leads to non-monotonic relationship for $\sigma_s^d(h)$ and in turn $\sigma_s(h)$.

Lastly, since $\sigma_s^y(h)$ is mainly driven by the average marginal propensity to consume out of labor income across all investors $\frac{1}{N}(\sum_{i=h+1}^{N} \partial C_i^*(X_i, Y)/\partial Y + \sum_{i=1}^{h} \partial C_i^*(X_i, Y)/\partial Y)$ in (23), we explore the marginal consumption with respect to labor income for comparative statics. Panel D of Figure OA.4 shows the decomposition of this term. On the one hand, the first component, non-stockholders' marginal consumption $\partial C_i^*(X_i, Y)/\partial Y = \frac{r_f(1-\psi)+\delta\psi}{r_f-\mu_y}$ (dotted line) is mainly due to the risk free rate in Panel A of Figure OA.3. If the risk-free rate goes down, non-stockholders value their future income highly and therefore the marginal consumption with respect to labor goes up. On the other hand, the second component, stockholders' marginal consumption with respect to labor income $\partial C_i^*(X_i, Y)/\partial Y = \frac{(r_f+\frac{\lambda^2}{2\gamma_i})(1-\psi)+\delta\psi}{r_f+\rho_s\sigma_y\lambda-\mu_y}$ depends on the Sharpe ratio λ due to the trade-off between investment and consumption. As such, the shape of the Sharpe ratio λ in Panel C of Figure OA.3 mimics that of the stockholders' marginal consumption with respect to labor income. Taken together, the two components shape Panel C of Figure OA.4, which in turn explains the effect of market participation on the level of $\sigma_s^y(h)$.

Note that as in the case of the equity premium, this comparative statics of increasing $\sigma_s^d(h)$ with market participation does not translate into the equilibrium result. In equilibrium, if the state changes to a better state (G), new market participation level $h_G^* = 11$ leads to $\sigma_s^d(h_G^*) = 28\%$, lower than $\sigma_s^d(h_B^*) = 33\%$ even with more investors in the market.

OA.3 Optimization problem when $\rho = 1$ to confirm our closed form

In this section, we solve the individual optimization problem, which is formulated as the HJB equation in (5) for the special case where the dividend growth is perfectly correlated with labor income growth i.e., $\rho = 1$. Since $\rho = 1$, the correlation between equity returns and labor income growth is also perfect $\rho_{s,t} = Corr_t(\sigma_{s,t}^d dW_{d,t} + \sigma_{s,t}^y dW_{y,t}, \sigma_y dW_{y,t}) = \frac{\sigma_{s,t}^d \rho + \sigma_{s,t}^y}{\sigma_{s,t}} = \frac{\sigma_{s,t}^d + \sigma_{s,t}^y}{\sqrt{{\sigma_{s,t}^d}^2 + {\sigma_{s,t}^y}^2 + 2\sigma_{s,t}^d \sigma_{s,t}^y}} = 1$. Then, the HJB equation for unconstrained investors in (A.4) is

$$
\begin{aligned}
0 \ &= (X_{i,t}+b_iY_t)^2[\frac{\delta}{1-\psi^{-1}}(\delta^{\psi-1}(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})^{-\theta\psi}\psi^{-1}-1)+\frac{\lambda_t^2}{2\gamma_i} \\
&+\frac{(\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}c_j)^2}{2\gamma_i(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})^2}+\frac{\sum_{j=1}^{N-1}\mu_{w_{j,t}}w_{j,t}c_j}{(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})(1-\gamma_i)}+\frac{\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}c_j\lambda_t}{\gamma_i(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})}] \\
&+(r_{f,t}X_{i,t}+Y_t)(X_{i,t}+b_iY_t)-\frac{1}{2}\sigma_y^2Y_t^2\gamma_ib_i^2+\frac{\sigma_y^2Y_t^2\gamma_ib_i^2}{2} \\
&+(X_{i,t}+b_iY_t)Y_t[\mu_yb_i-\lambda_t\sigma_yb_i-\frac{\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}c_j\sigma_y\gamma_ib_i}{\gamma_i(a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})}+\frac{\sum_{j=1}^{N-1}\rho_{w_j,y,t}\sigma_{w_{j,t}}w_{j,t}\sigma_yb_ic_j}{a_i+\sum_{j=1}^{N-1}c_{j,t}w_{j,t}}]
\end{aligned}
\tag{OA.60}
$$

After the term $\frac{\sigma_y^2Y_t^2\gamma_ib_i^2}{2}$ cancels out, the above PDE can be solved by

$$
\begin{aligned}
a_i^* &= (\delta^{1-\psi}\psi((-r_{f,t}-\frac{\lambda_t^2}{2\gamma_i})\frac{1-\psi^{-1}}{\delta}+1))^{-\frac{1}{\theta_i\psi}} \\
b_i^* &= \frac{1}{r_{f,t}+\lambda_t\sigma_y-\mu_y} \\
c_1^* &= \cdots = c_{N-1}^* = 0
\end{aligned}
\tag{OA.61}
$$

Then, the value function is

$$
V_i^*(X_{i,t},Y_t)=\frac{(\delta^{1-\psi}\psi((-r_{f,t}-\frac{\lambda_t^2}{2\gamma_i})\frac{1-\psi^{-1}}{\delta}+1))^{-\frac{1}{\theta_i\psi}}}{1-\gamma_i}(X_{i,t}+\frac{Y_t}{r_{f,t}+\sigma_y\lambda_t-\mu_y})^{1-\gamma_i} \tag{OA.62}
$$

This solution is the same as the value function (A.8) in closed form with putting $\rho_{s,t}=1$

OA.4 Martingale approach with CRRA

In this section, we solve the equilibrium for the case where investors are not endowed with stochastic labor income and their preferences are CRRA. The purpose of this section is to show that solutions from this approach verify solutions from the HJB approach in our paper. In this case, the agent is facing a dynamically complete market and therefore the optimality of $c_{i,t}$ is equivalent to the marginal utility process $e^{-\rho t}u_i^{'}(c_{i,t})$ being proportional to the equilibrium state price density as in Basak and Cuoco (1998), that is,

$$
e^{-\rho t}u_i^{'}(c_{i,t})=\psi_i\xi_t \tag{OA.63}
$$

for some $\psi_i>0$ and where ξ_t is the state price density and its dynamic process is $d\xi_t/\xi_t = -r_{f,t}dt-\lambda_tdW_{d,t}$ Since we consider the power utility function, the above equation can be rearranged as $c_{i,t}^*=(e^{\rho t}\psi_i\xi_t)^{-\frac{1}{\gamma_i}}$. And, the differential of the optimal consumption is $dc_{i,t}^*=-\frac{1}{\gamma_i}(e^{\rho t}\psi_i\xi_t)^{-\frac{1}{\gamma_i}-1}(\rho e^{\rho t}\psi_i\xi_tdt+e^{\rho t}\psi_id\xi_t)+\frac{1}{2}\frac{1}{\gamma_i}(\frac{1}{\gamma_i}+1)(e^{\rho t}\psi_i\xi_t)^{-\frac{1}{\gamma_i}-2}e^{2\rho t}\psi_i^2d\xi_td\xi_t$. This can be re-written as

$$
dc_{i,t}^*=-\frac{c_{i,t}^*}{\gamma_i}(\rho dt+\frac{d\xi_t}{\xi_t})+\frac{1}{2}\frac{c_{i,t}^*}{\gamma_i}(\frac{1}{\gamma_i}+1)\frac{d\xi_td\xi_t}{d\xi_t^2} \tag{OA.64}
$$

Aggregating the above differentials across investors yields: $\sum_{i=1}^{N} dc_{i,t}^{*} = -\sum_{i=1}^{N} \frac{c_{i,t}^{*}}{\gamma_i}(\rho dt + \frac{d\xi_t}{\xi_t}) + \sum_{i=1}^{N} \frac{1}{2}\frac{c_{i,t}^{*}}{\gamma_i}(\frac{1}{\gamma_i}+1)\frac{d\xi_t d\xi_t}{d\xi_t^2}$. The consumption market clearing condition implies that

$$\sum_{i=1}^{N} dc_{i,t}^{*} = -\sum_{i=1}^{N} \frac{c_{i,t}^{*}}{\gamma_i}(\rho dt + \frac{d\xi_t}{\xi_t}) + \sum_{i=1}^{N} \frac{1}{2}\frac{c_{i,t}^{*}}{\gamma_i}(\frac{1}{\gamma_i}+1)\frac{d\xi_t d\xi_t}{d\xi_t^2}$$
$$= \mu_d D_t dt + \sigma_d D_t dW_{d,t} \tag{OA.65}$$

By matching the diffusion terms of (OA.65) in each side, the market price of risk is

$$\lambda_t = (\sum_{i=1}^{N} \frac{c_{i,t}^{*}}{\gamma_i})^{-1}\sigma_d D_t \tag{OA.66}$$

Also, by matching the deterministic terms of (OA.65), the risk-free rate is

$$r_{f,t} = \rho + \mu_d D_t(\sum_{i=1}^{N} \frac{c_{i,t}^{*}}{\gamma_i})^{-1} - (\sum_{i=1}^{N} \frac{c_{i,t}^{*}}{\gamma_i})^{-3}\sum_{i=1}^{N} \frac{1}{2}\frac{c_{i,t}^{*}}{\gamma_i}(\frac{1}{\gamma_i}+1)(\sigma_d D_t)^2 \tag{OA.67}$$

We verify that these solutions are the same as in other papers (e.g., Cvitanić et al., 2012) which study this economy and our endogenous equilibrium parameters in **Proposition 2** when $Y_t = 0$ and $\psi_i = 1/\gamma_i$.

OA.5 Idiosyncratic non-financial income

In this section, we extend the baseline model by introducing idiosyncratic non-financial income.

OA.5.1 Basic setup

The model setup is the same as in the baseline model except that investors no longer receive the same level of stochastic non-financial income. Each investors' non-financial income evolves as

$$\frac{dY_{i,t}}{Y_{i,t}} = \mu_y dt + \sigma_y dW_{y_{i,t}} \quad \forall i = 1, ..., N \tag{OA.68}$$

where $dW_{y_{i,t}}$ is idiosyncratic non-financial income shock for each investor and its correlation structure is modeled flexibly as follows $dW_{y_{i,t}} = \rho_d dW_d + \rho_y dW_y + \sqrt{1-\rho_d^2-\rho_y^2}dW_{i,t}$ where dW_d, dW_y, and $dW_{i,t}$ are independent Brownian motions. ρ_d governs the correlation between dividend and labor income and ρ_y governs the correlation among non-financial income shocks.[36] Then, the correct conjecture for the equilibrium equity returns dynamics is:

$$\frac{dS_t + D_t dt}{S_t} = \mu_{s,t}dt + \sigma_{s,t}^{d}dW_{d,t} + \sum_{i=1}^{N}\sigma_{s,t}^{y_i}dW_{y_{i,t}} \tag{OA.69}$$

[36]The correlation between dividend shock and non-financial income shock i is $dW_d dW_{y_{i,t}} = \rho_d dt$ and the correlation between non-financial income shock i and j is $dW_{y_{i,t}} dW_{y_{j,t}} = (\rho_d^2 + \rho_y^2)dt$. Depending on the value of ρ_y, $\rho_d^2 + \rho_y^2$ can be greater than ρ_d.

Note that both stockholders and non-stockholders labor income shocks are priced in equilibrium as before due to the all markets clearing condition. Then, the correlation between stock returns and an investor i's labor income growth is $\rho_{s_{i,t}} \equiv Corr_t(\sigma^d_{s,t} dW_{d,t} + \sum_{i=1}^N \sigma^{y_i}_{s,t} dW_{y_{i,t}}, \sigma_y dW_{y_{i,t}}) = \frac{\sigma^d_{s,t}\rho_d + \sum_{j\neq i}\sigma^{y_j}_{s,t}(\rho_d^2+\rho_y^2)+\sigma^{y_i}_{s,t}}{\sigma_{s,t}}$ and the stock volatility is $\sigma_{s,t} = \sqrt{{\sigma^d_{s,t}}^2 + \sum_{i=1}^N {\sigma^{y_i}_{s,t}}^2 + 2\rho_d\sigma^d_{s,t}\sum_{i=1}^N \sigma^{y_i}_{s,t} + (\rho_d^2+\rho_y^2)\sum_{i\neq j}\sigma^{y_i}_{s,t}\sigma^{y_j}_{s,t}}$

OA.5.2 Optimal policies

After solving the HJB equation as before in this setup, the optimal polices are

$$C^*_{i,t} = \begin{cases} ((r_{f,t} + \frac{\lambda_t^2}{2\gamma_i})(1-\psi) + \delta\psi)\cdot(X_{i,t} + H_{h_{i,t}}) & \text{if } \pi^{w/o}_{i,t} > 0 \\ (r_{f,t}(1-\psi) + \delta\psi)(X_{i,t} + H_{n_{i,t}}) & \text{Otherwise} \end{cases} \tag{OA.70}$$

$$\pi^*_{i,t} = \begin{cases} \pi^{w/o}_{i,t} = \frac{\lambda_t}{\gamma_i\sigma_{s,t}}(X_{i,t} + H_{h_{i,t}}) - \frac{\rho_{s_{i,t}}\sigma_y}{\sigma_{s,t}} H_{h_{i,t}} & \text{if } \pi^{w/o}_{i,t} > 0 \\ 0 & \text{Otherwise} \end{cases} \tag{OA.71}$$

$$dX^*_{i,t} = \begin{cases} (\pi^*_{i,t}(\mu_{s,t} - r_{f,t}) + r_{f,t}X_{i,t} + Y_{i,t} - C^*_{i,t})dt + \pi^*_{i,t}(\sigma^d_{s,t}dW_{d,t} + \sum_{i=1}^N \sigma^{y_i}_{s,t} dW_{y_{i,t}}) & \text{if } \pi^{w/o}_{i,t} > 0 \\ (r_{f,t}X_{i,t} + Y_{i,t} - C^*_{i,t})dt & \text{Otherwise} \end{cases} \tag{OA.72}$$

where $H_{h_{i,t}} \equiv \frac{Y_{i,t}}{r_{f,t} + \rho_{s_{i,t}}\sigma_y\lambda_t - \mu_y}$, and $H_{n_{i,t}} \equiv \frac{Y_{i,t}}{r_{f,t} - \mu_y}$

OA.5.3 Equilibrium

After solving the equilibrium as in the main section, the set of equations for the Sharpe ratio λ_t, the risk-free rate $r_{f,t}$, the stock volatility $\sigma_{s,t}$ and the stock price are given by:

$$\lambda_t = \frac{\sigma_{s,t}\sum_{i=1}^N X_{i,t} + \sigma_y \sum_{i\in h_t^*} \rho_{s_{i,t}} H_{h_{i,t}}}{\sum_{i\in h_t^*} \frac{X_{i,t} + H_{h_{i,t}}}{\gamma_i}} \tag{OA.73}$$

$$r_{f,t} = \delta + \frac{\mu_d D_t + \mu_y \sum_{i=1}^N Y_{i,t}}{D_t + \sum_{i=1}^N Y_{i,t}} \frac{1}{\psi} - \sum_{i\in h_t^*} \frac{C_{i,t}}{D_t + \sum_{i=1}^N Y_{i,t}} (\frac{\lambda_t^2}{\gamma_i} \frac{1+\psi}{2\psi}) \tag{OA.74}$$

$$\sigma_{s,t} = \sqrt{{\sigma^d_{s,t}}^2 + \sum_{i=1}^N {\sigma^{y_i}_{s,t}}^2 + 2\rho_d\sigma^d_{s,t}\sum_{i=1}^N \sigma^{y_i}_{s,t} + (\rho_d^2+\rho_y^2)\sum_{i\neq j}\sigma^{y_i}_{s,t}\sigma^{y_j}_{s,t}} \tag{OA.75}$$

$$\sigma^d_{s,t} = \frac{\sigma_d D_t}{\sum_{i\in h_t^*} k_{h,i,t}\pi^*_{i,t}} \tag{OA.76}$$

$$\sigma^{y_i}_{s,t} = \begin{cases} \sigma_y Y_{i,t}[1 - k_{h,i,t}/(r_{f,t} + \rho_{s_{i,t}}\sigma_y\lambda_t - \mu_y)]/\sum_{i\in h^*_t} k_{h,i,t}\pi^*_{i,t} & \text{if } i \in h^*_t \\ \sigma_y Y_{i,t}[1 - k_{n,t}/(r_{f,t} - \mu_y)]/\sum_{i\in h^*_t} k_{h,i,t}\pi^*_{i,t} & \text{Otherwise} \end{cases} \tag{OA.77}$$

$$S_t = \frac{D_t + \sum_{i=1}^N Y_{i,t} - \sum_{i\in h^*_t} \frac{\lambda_t^2}{2\gamma_i}(1-\psi)(X_{i,t} + H_{h_{i,t}})}{r_{f,t} - (\frac{\mu_d D_t + \mu_y \sum_{i=1}^N Y_{i,t}}{D_t + \sum_{i=1}^N Y_{i,t}} - \sum_{i\in h^*_t} \frac{C_{i,t}}{D_t + \sum_{i=1}^N Y_{i,t}} \frac{\lambda_t^2}{\gamma_i}\frac{1+\psi}{2})} - (\sum_{i\in h^*_t} H_{h_{i,t}} + \sum_{i\notin h^*_t} H_{n_{i,t}}) \tag{OA.78}$$

We simulate this setup using the same parameter values reported in Table 1 with $\rho_d = 0.43$, $\rho_y = 0.7$. Figure OA.5 illustrates one sample path of time-varying market participation, the amount of aggregate or stockholder consumption risk, and the price of consumption risk for both baseline model and idiosyncratic labor income setup. It shows that two economies generate the similar equilibrium dynamics. However, idiosyncratic labor income leads to a less time-variation in time-varying market participation. This is because idiosyncratic labor income shocks make market participation decision less systematic. For example, an investor, who would leave the market during the bad states in the baseline setup, can continue to hold a stock in this case depending on her own labor income shock. This result is clearly depicted in the second recession of the top figure of Panel B. A less volatile time-variation in the market participation in turn leads to a less procyclical and less volatile variation in the price of consumption risk as shown in the bottom figure of Panel B, most notably in the second recession of the one simulated sample path.

OA.6 Extension to habit formation utility

OA.6.1 Basic setup and Optimization

In this section, we provide an extension of the baseline model featuring the heterogeneous internal habit preferences. As in the baseline model, agents endowed with stochastic non-financial income allocate her wealth to two assets: a riskless asset and a risky asset in the presence of short-selling constraints. Investors have the internal habit preferences with different risk aversion. Investor *i* is maximizing

$$V_{i,t} = \mathbb{E}_t[\int_t^\infty e^{-\delta s}\frac{(C_{i,s} - H_{i,s})^{1-\gamma_i}}{1-\gamma_i}ds], \quad \forall t \in [0,\infty) \tag{OA.79}$$

where $H_{i,t}$ denotes the internal habit level not the human capital as used in the body section. Its dynamic follow $dH_{i,t} = (bC_{i,t} - aH_{i,t})dt$. Then, the curvature is $\eta_{i,t} = \frac{\gamma_i}{(C_{i,t} - H_{i,t})/C_{i,t}}$. Then, the Hamilton-Jacobi-Bellman (HJB) equation with short-selling constraints for an agent i

is

$$0=\max_{(c,\pi)\in\mathcal{A}}\frac{(C_{i,t}-H_{i,t})^{1-\gamma_i}}{1-\gamma_i}-\delta V_{i,t}+[\pi_{i,t}(\mu_{s,t}-r_{f,t})+r_{f,t}X_{i,t}+Y_t-C_{i,t}]V_{x_i,t}+\frac{1}{2}\pi_{i,t}^2\sigma_{s,t}^2V_{x_ix_i,t}$$
$$+\mu_yY_tV_{y,t}+\frac{1}{2}\sigma_y^2Y_t^2V_{yy,t}+\rho_{s,t}\sigma_yY_t\sigma_{s,t}\pi_{i,t}V_{x_iy,t}+(bC_{i,t}-aH_{i,t})V_{H,t}+\sum_{j=1}^{N-1}\mu_{w_{j,t}}w_{j,t}V_{w_j,t}$$
$$+\frac{1}{2}\sum_{j=1}^{N-1}\sigma_{w_{j,t}}^2w_{j,t}^2V_{w_jw_j,t}+\sum_{j\neq k}\rho_{w_j,w_k,t}\sigma_{w_{j,t}}\sigma_{w_{k,t}}w_{j,t}w_{k,t}V_{w_jw_k,t}+\sum_{j=1}^{N-1}\rho_{w_j,s,t}\sigma_{w_{j,t}}w_{j,t}\sigma_{s,t}\pi_{i,t}V_{w_jx_i,t}$$
$$+\sum_{j=1}^{N-1}\rho_{w_j,y,t}\sigma_{w_{j,t}}w_{j,t}\sigma_yY_tV_{w_jy,t}+l_{i,t}\pi_{i,t}\qquad\forall i=1,...,N,\forall t\in[0,\infty)\tag{OA.80}$$

After solving the above HJB equation as before, the optimal polices are

$$C_{i,t}^*=\begin{cases}H_{i,t}+(r_{f,t}+\frac{\delta-r_{f,t}}{\gamma_i}-\frac{\lambda_t^2(1-\gamma_i)}{2\gamma_i^2})(\frac{r_{f,t}+a-b}{r_{f,t}+a})(X_{i,t}+\frac{Y_t}{r_t+\rho_{s,t}\sigma_y\lambda_t-\mu_y}-\frac{H_{i,t}}{r_{f,t}+a-b}) & \text{if } \pi_{i,t}^{w/o}>0\\ H_{i,t}+(r_{f,t}+\frac{\delta-r_{f,t}}{\gamma_i})(\frac{r_{f,t}+a-b}{r_{f,t}+a})(X_{i,t}+\frac{Y_t}{r_t-\mu_y}-\frac{H_{i,t}}{r_{f,t}+a-b}) & \text{Otherwise}\end{cases}\tag{OA.81}$$

$$\pi_{i,t}^*=\begin{cases}\pi_{i,t}^{w/o}=\frac{\lambda_t}{\gamma_i\sigma_{s,t}}(X_{i,t}+\frac{Y_t}{r_t+\rho_{s,t}\sigma_y\lambda_t-\mu_y}-\frac{H_{i,t}}{r_{f,t}+a-b})-\frac{\rho_{s,t}\sigma_y}{\sigma_{s,t}}\frac{Y_t}{r_t+\rho_{s,t}\sigma_y\lambda_t-\mu_y} & \text{if } \pi_{i,t}^{w/o}>0\\ 0 & \text{Otherwise}\end{cases}\tag{OA.82}$$

If $Y_t=0$, we confirm that our expressions reduce exactly to the expression in Constantinides (1990).

OA.6.2 Equilibrium

Based on the above optimal polices, we impose the market clearing conditions as in the baseline model to solve for the general equilibrium. A novel consumption-based asset pricing model featuring time-varying market participation with heterogeneous habit preferences is given by

$$E_t[dR_t^e]=\frac{\sum_{i=1}^{h_t^*}C_{i,t}^*}{\sum_{i=1}^{h_t^*}\frac{C_{i,t}^*-H_{i,t}}{\gamma_i}}\cdot Cov_t(dR_t^e,\frac{d\sum_{i=1}^{h_t^*}C_{i,t}^*}{\sum_{i=1}^{h_t^*}C_{i,t}^*})\tag{OA.83}$$

Without habit preference $H_{i,t}$, (OA.83) reduces to the equity premium expression in our baseline model (26). In a representative economy, the price of consumption risk reduces to the curvature of the representative agent $\frac{C_{i,t}}{\frac{C_{i,t}-H_{i,t}}{\gamma_i}}=\frac{\gamma_i}{\frac{C_{i,t}-H_{i,t}}{C_{i,t}}}=\eta_{i,t}$. With the introduction of habit preferences, both the effect of consumption re-distribution and habit formation lead the price of consumption risk to vary in a countercyclical way, while time-varying market participation effect works in opposite direction. In bad states, due to time-varying market participation, only risk-tolerant investors remain in the market. However, the consumption of remaining stockholders becomes closer to their habit levels, driving up the price of consumption risk. In short, the introduction of habit preferences clearly shows that even assuming time-varying individual risk aversion, time-varying market participation renders the price of risk less countercyclical than in full participation economy.

OA.7 Unequal initial wealth distribution

In our baseline model, we consider an equal distribution of the initial wealth. In this section, we introduce an wealth inequality, given the empirically well documented large skewness of households wealth. Specifically, following the empirical evidence that a negative relation between risk aversion and financial wealth,[37] we assume that investors' initial wealth is inversely related to their risk aversion.

$$X_{i,0} = \frac{k}{i} \tag{OA.84}$$

since $\gamma_1 < \gamma_2 < \cdots < \gamma_N$, the above rule allocates more wealth to a lower index investors whose risk aversion is relatively low. k in (OA.84) is endogenously determined from the equilibrium, because the sum of aggregate financial wealth is the stock price which is endogenous (i.e., $\sum_{i=1}^{N} X_{i,0} = S_0$). Figure OA.6 presents one sample path of time-varying market participation, the amount of aggregate or stockholder consumption risk, and the price of consumption risk for both baseline model and idiosyncratic labor income setup. The results are summarized as follows. First, as shown in the bottom panel figure, the price of consumption risk under unequal distribution is lower than under equal distribution (baseline model). This is because the consumption share of risk-tolerant investors is higher in this case than in the baseline model, which lowers the stockholders' average risk aversion. Second, the amount of stockholder' consumption risk is slightly lower in this case than in the baseline model since the wealthier stockholders consume more than in the baseline model and thus the risk-sharing is more effective. Third, both the lower price and amount of consumption risk imply a lower equity premium, which directly translates into the higher stock price as shown in the level of the stock market wealth ratio in the top panel. Fourth, since the wealth is more concentrated at the risk-tolerant investors, the average market participation level is lower than in the baseline, as shown in the top panel because risk-averse investors are not only reluctant to take risk, but also they do not have enough wealth to be invested in the stock. Most importantly, the overall result shows that two economies generate similar equilibrium dynamics, suggesting that our baseline results are robust to unequal wealth distribution.

OA.8 Further analysis on consumption sharing implied by our model

In our model, the risk-sharing among the stockholders is limited due to limited market participation. In this section, we analyze the risk-sharing mechanism in a detailed manner. In Panel A of Figure OA.7, we plot the amount of risk (Panel A1) with the **exogenous** inclusion of more investors as in Section OA.2, the stockholders' consumption volatility

[37] See King and Leape (1998), Riley and Chow (1992), Donkers et al. (2001), Guiso and Paiella (2008), and Bucciol and Miniaci (2011), for example, among others.

(Panel A2) and the correlation between stockholders' consumption growth and stock returns (Panel A3). The result shows that as more investors are assumed to be in the market, stockholders' consumption is less volatile and less correlated with stock returns, indicating the stockholders' decreasing exposure to the consumption risk. If only one investor is the stockholder, the investor's marginal utility is highly sensitive to the shocks to the stock price, which is represented by the high amount of consumption risk, consumption volatility, and correlation with stock returns. However, as more investors are imposed to stay in the market, the risk is effectively shared out, decreasing the amount of risk. If every investor is assumed to be a stockholder, then the lowest possible amount of consumption risk is attained.

The amount of consumption risk plotted in Panel A is based on the ascending order of inclusion (h=1,2,...,30) with the risk aversion boundary from 1 to 50. To understand the risk-sharing further, we consider the following variants of the baseline case. In Case 2 (Panel B), the inclusion of investors is first the most risk-tolerant investor ($i = 1$) followed by the most risk-averse investor ($i = 30$) and the second most risk-averse investor ($i = 29$) and so on. In Case 3 (Panel C), the lowest risk aversion is 1.1 ($\gamma_1 = 1.1$). Finally, in Case 4 (Panel D), the highest risk aversion is 10 ($\gamma_N = 10$). Panel B, C, and D of Figure OA.7 show the result. First, Panel B shows that the order of the inclusion does not change the amount of consumption risk. This implies that once the most risk-tolerant investors are in the market, the degree of risk-sharing does not depend on risk aversion of investors who follow the most risk-tolerant investor. Panel C shows that even though the lowest risk aversion marginally changes from 1 to 1.1 ($\gamma_1 = 1.1$), the risk-sharing is ineffective than the baseline case. This is because risk-averse investors are not willing to take the risk and thus their contribution of risk-sharing is lower than risk-tolerant investors. However, the dramatic difference of risk-sharing between the baseline case and the Case 3 is quickly decreasing with the inclusion of more investors. Therefore, the lower bound of risk aversion is important for risk-sharing especially when the market participation rate is low. Finally, Panel D shows that if the highest risk aversion changes from 50 to 10 ($\gamma_N = 10$), the risk-sharing is slightly more effective than the baseline case at each point of the inclusion. This is because investors in Case 4 are more risk-tolerant than investors in the baseline case. Thus, investors in Case 4 are more willing to take the risk and hence their contribution of risk-sharing is high. However, in terms of the magnitude, the amount of consumption risk is virtually identical to the baseline case. This implies that a change in the upper boundary of risk aversion from 50 to 10 does not significantly change the degree of risk-sharing.

To summarize, the improving risk-sharing with the inclusion of investors are represented by decreasing covariance or correlation between stockholders consumption growth and stock return and decreasing stockholders' consumption volatility. Also, the risk aversion of the most tolerant investor (lower boundary of risk aversion) is the most important for the degree of risk-sharing because she is willing to take the risk the most among all investors and this makes it possible to share out the risk effectively.

OA.9 Cut-off Stockholder (Nash equilibrium setting)

In this section, we discuss the methodology to determine the cut-off stockholder h_t. Given fixed economic states $(D_t, Y_t, \mathbf{w_t})$, all endogenous asset parameters are a function of the cut-off stockholder $(\lambda_t(h_t), r_{f,t}(h_t), \sigma_{s,t}(h_t))$. Also, since each investor's optimal stock holding is a function of these endogenous asset parameters $\pi^*_{i,t}(\lambda_t, r_{f,t}, \sigma_{s,t})$, $\pi^*_{i,t}$ is also a function of the cut-off stockholder $\pi^*_{i,t}(h_t)$. Hence, an investor i's decision to be a cut-off stockholder ($h_t = i$) changes not only i's optimal stock holding but also every other agent's optimal stock holding. In this nature of the problem, we therefore restrict the overall equilibrium to a Nash Equilibrium to preclude each investor from optimally deviating from a stockholder to non-stockholder or vice versa, given the cut-off stockholder (h_t).

Definition 2. *An equilibrium is a set of processes* $\{r_{f,t}(h^*_t), \lambda_t(h^*_t), \sigma_{s,t}(h^*_t)\}$ *and consumption and investment policies* $\{C^*_{i,t}(h^*_t), \pi^*_{i,t}(h^*_t)\}_{i\in 1,\ldots,h^*_t}$ *and* $\{C^*_{i,t}(h^*_t)\}_{i\in h^*_t+1,\ldots,N}$ *which maximize the sum of life time expected utility (1) for each investor and satisfy the securities market-clearing conditions (16) and (17) such that short-selling (negative holding) is not allowed and* h^*_t *satisfies the following.*

1. $\pi^*_{i,t}(h_t; h_t = h^*_t) \geq 0 \qquad \forall i = 1, ..., h^*_t$ (OA.85)
2. $\pi^*_{i,t}(h_t; h_t = i) < 0 \qquad \forall i = h^*_t + 1, ..., N$ (OA.86)

The first condition in (OA.85) states that given the cut-off stockholder $h_t = h^*_t$, the investors who are less risk-averse than the investor h^*_t have positive stock holdings and therefore, they do not optimally deviate to having negative stock holdings. The second condition in (OA.86) guarantees that when an investor who is more risk-averse than the investor h^*_t enters the stock market and becomes the cut-off stockholder, her optimal stock holding is negative and therefore, she cannot be a stockholder given short-selling constraint. **Proposition 4** shows how h^*_t who satisfies the **Definition 2** can be determined.

Proposition 4. *The investor* h^*_t *is*

$$h^*_t \equiv \arg\min_i \pi^*_{i,t}(h_t; h_t = i) \;\; \text{s.t. } \pi^*_{i,t}(h_t; h_t = i) > 0 \tag{OA.87}$$

To find h^*_t which is defined in **Proposition 4**, at each point in time, we first consider the first agent as a cut off stockholder $h_t = 1$ and compute the optimal stock holding of the first agent given she is the cut-off stockholder $\pi^*_{1,t}(h_t; h_t = 1)$. We move on to the second agent and compute the optimal stock holding of the second agent given the second agent is the cut-off stockholder $\pi^*_{2,t}(h_t; h_t = 2)$. We repeat this procedure until we find an investor h^*_t whose optimal holding is the lowest among agents whose optimal holding is positive.

We now discuss the reason why this investor is the cut-off stockholder in the Nash Equilibrium. First of all, h^*_t has the positive stock holding. By the monotonicity of $\pi^*_{i,t}(h_t; h_t = h^*_t)$ with respect to risk aversion, every investor whose risk aversion is lower than h^*_t (i.e., $i = 1, 2, ..., h^*_t - 1$) has a higher stock holding than h^*_t. This means that the optimal stock holdings of agents $i = 1, 2, ..., h^*_t$ are positive, satisfying the condition in (OA.85). Second, for the investor whose risk aversion is higher than h^*_t (i.e., $i = h^*_t + 1$), her optimal portfolio given she is the cut-off stockholder is negative $\pi^*_{h^*_t+1,t}(h_t; h_t = h^*_t + 1) < 0$ by the definition of h_t^*. In the presence of short-selling constraint, she cannot be the cut-off stockholder, satisfying the condition in (OA.86).

Figure OA.8 also visually confirms that h_t^* defined in **Proposition 4** guarantees the Nash Equilibrium. Investors whose risk aversion is lower than h_t^* have the positive optimal stock holding after fixing the cut-off stockholder $h_t = h_t^*$. Investors whose risk aversion is more than h_t^* have the negative optimal stock holding after fixing the cut-off stockholder $h_t = h_t^*$.

OA.10 Market participation rate and ρ

In this section, we examine the relation between market participation level and the correlation between dividend growth and non-financial income growth ρ. ρ essentially determines the correlation between equity returns and non-financial income growth and in turn the optimal stock holding in (13). Therefore, ρ is one of the important determinants of the market participation. In our body section, we report the correlation between dividend and non-financial income growth for the period of 1930 to 2016 of 43% in Table 1 which leads to 30% of participation rate at time $t = 0$. We vary the correlation level from 20% to 60% and examine the equilibrium effect on the market participation rate. Figure OA.9 plots the result. When $\rho = 0.2$, every investor is a stockholder because financial income is less correlated with non-financial income. As ρ increases, market participation level declines. As a result, when $\rho = 0.6$ only 20% of total investors hold the stock. This finding provides an empirically testable hypothesis for future research.

OA.11 Proof of the HJB equation with Lagrange multiplier

In this section, we formally derive the Hamilton-Jacobi-Bellman equation with the Lagrange mutliplier for the dynamic programming under constraints.

OA.11.1 Structure of stochastic control problem

The uncertainty and information are represented by a filtered probability space (Ω,$\mathcal{F} = \{\mathcal{F}_t\}_{t\in\tau}$,$\mathbb{P}$). $\forall\tau \in [0, \infty)$. State variables $X = (X_t)$, a subset of $\mathbb{R}^m$, are $\mathcal{F}$-adapted stochastic process representing the evolution of the variables describing the system. In our paper, state variables are financial wealth, labor income, and consumption shares of $N - 1$ investors. A $\mathcal{F}$-adapted process $\alpha = \alpha_t$, a subset of $\mathbb{R}^n$, is a control law whose value is chosen at time t as a function of the state variables X_t. In a portfolio-consumption choice problem, $\alpha_t = (c_{i,t}, \pi_{i,t})$. The control law α_t satisfies the integrability conditions. There can be a constraint for the control law: $g(\alpha) \geq m$ where $g(\cdot)$ is a function from $\mathbb{R}^n$ into $\mathbb{R}$ and $m \in \mathbb{R}$. In our paper, we restrict the set of admissible controls to be non-negative i.e., $\alpha \in \mathcal{A} = \{(c, \pi) \mid c \geq 0 \ \ \& \ \ \pi \geq 0\}$. Consider a Brownian motion W and functions $\mu : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^m$ and $\sigma : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^{+m}$. The dynamics of the state variables in $\mathbb{R}^m$ are given by

$$dX_t = \mu(X_t, \alpha_t)dt + \sigma(X_t, \alpha_t)dW_t \tag{OA.88}$$

Given a function f from $\mathbb{R}^m \times \mathbb{R}^n$ into $\mathbb{R}$, we define the objective function:

$$J(t,x,\alpha) = \mathbb{E}[\int_t^\infty f(X_s,\alpha_s) + \lambda_s(g(\alpha_s)-m)ds], \qquad \forall (t,x) \in [0,\infty) \times \mathbb{R}^m, \quad \alpha \in \mathcal{A} \tag{OA.89}$$

where $\lambda_s \geq 0$ is the Lagrange multiplier and $\lambda_s(g(\alpha_s)-m)$ penalizes the objective function when the constraint is violated. We re-define the objective function $y(X_s,\alpha_s) \equiv f(X_s,\alpha_s) + \lambda_s(g(\alpha_s)-m)$ and the control law $\beta \equiv (\alpha,\lambda) \in \mathbb{R}^{n+1}$. Then, the value function is defined as follows.

$$\hat{J}(t,x) = \sup_{\beta \in \mathcal{A}} J(t,x,\beta) = J(t,x,\hat{\beta}) \tag{OA.90}$$

OA.11.2 Dynamic programming principle and the HJB

The dynamic programming principle implies that for every stopping time $\theta \in \tau_{(t,\infty)}$, it holds that

$$\hat{J}(t,x) = \sup_{\beta \in \mathcal{A}} E[\int_t^\theta y(s,X_s^\beta,\beta_s)ds + \hat{J}(\theta,X_\theta^\beta)] \tag{OA.91}$$

For $\beta \in \mathcal{A}$ and a controlled state variables X_t^β, apply Itô lemma to $\hat{J}(s,X_s^\beta)$ between $s=t$ and $s=t+h$.

$$\hat{J}(t+h,X_{t+h}^\beta) = \hat{J}(t,X_t^\beta) + \int_t^{t+h} \hat{J}_t(s,X_s^\beta) + \mathcal{L}^\beta \hat{J}(s,X_s^\beta)ds + \int_t^{t+h} \hat{J}_x(s,X_s^\beta)\sigma_s^\beta dW_s \tag{OA.92}$$

where $\mathcal{L}^\beta$ is the differential operator associated to the diffusion X with control law β

$$\mathcal{L}^\beta \hat{J} = \mu(x,\beta)D_x\hat{J} + \frac{1}{2}tr(\sigma(x,\beta)\sigma'(x,\beta))D_{xx}\hat{J} \tag{OA.93}$$

By the martingale property of the stochastic integral, taking the expectation of (OA.92) gives

$$E(\hat{J}(t+h,X_{t+h}^\beta)) = \hat{J}(t,X_t^\beta) + E(\int_t^{t+h} \hat{J}_t(s,X_s^\beta) + \mathcal{L}^\beta \hat{J}(s,X_s^\beta)ds) \tag{OA.94}$$

Plugging this into the Dynamic Programming Principle (OA.91) gives

$$\sup_{\beta \in \mathcal{A}} E[\int_t^{t+h} y(s,X_s^\beta,\beta_s) + \hat{J}_t(s,X_s^\beta) + \mathcal{L}^\beta \hat{J}(s,X_s^\beta)ds] = 0 \tag{OA.95}$$

By dividing by h and $h \to 0$ and we obtain that

$$\hat{J}_t(t,X_t^\beta) + \sup_{\beta \in \mathcal{A}} y(t,X_t^\beta,\beta_t) + \mathcal{L}^\beta \hat{J}(t,X_t^\beta) = 0 \tag{OA.96}$$

This can be re-written as

$$\hat{J}_t(t,X_t^\alpha) + \sup_{\alpha \in \mathcal{A}} f(X_t,\alpha_t) + \lambda_t(g(\alpha_t)-m) + \mathcal{L}^\alpha \hat{J}(t,X_t^\alpha) = 0 \tag{OA.97}$$

The HJB equation in our paper (5) is an application of the above HJB equation.

OA.12 Conditional CCAPM for individual firms (consumption-beta)

In this section, we provide a novel equation for the conditional consumption-based Capital Asset Pricing Model. When there are K number of individual stocks, the optimal stock holding for the stock k is

$$\pi^*_{i,k,t} = \frac{\mu^e_{k,t}}{\gamma_i \sigma^2_{k,t}}(X_{i,t} + H_{h,t}) - \frac{\sum_{k \neq j} \pi_{i,j,t}\sigma_{k,j,t}}{\sigma^2_{k,t}} - \frac{H_{h,t}\sigma_{k,y,t}}{\sigma^2_{k,t}} \tag{OA.98}$$

where $\mu^e_{k,t}$ is the equity premium of stock k, $\sigma_{k,j,t}$ is the covariance between stock returns k and j, and $\sigma_{k,y,t}$ is the covariance between stock returns k and the labor income growth y. In (OA.98), $\pi^*_{i,k,t}$ has two intertemporal hedging terms. The investors not only care about the intertemporal hedging motive arising from the labor income, but also care about hedging against the other stocks. In the meantime, the covariance of stockholders' consumption growth with a stock returns k is

$$Cov_t(dR^e_{k,t}, \frac{d\sum_{i=1}^{h^*_t} C^*_{i,t}}{\sum_{i=1}^{h^*_t} C^*_{i,t}}) = \frac{\sum_{i=1}^{h^*_t} \frac{C^*_{i,t}}{X_{i,t}+H_{h,t}}(\sum_{k \neq l} \pi_{i,j,t}\sigma_{k,j,t} + \pi_{i,k,t}\sigma^2_{k,t} + H_{h,t}\sigma_{k,y,t})}{\sum_{i=1}^{h^*_t} C^*_{i,t}} dt \tag{OA.99}$$

By substituting (OA.98) for $\pi_{i,k,t}$ to obtain,

$$Cov_t(dR^e_{k,t}, \frac{d\sum_{i=1}^{h^*_t} C^*_{i,t}}{\sum_{i=1}^{h^*_t} C^*_{i,t}}) = \mu^e_{k,t} \frac{\sum_{i=1}^{h^*_t} \frac{C^*_{i,t}}{\gamma_i}}{\sum_{i=1}^{h^*_t} C^*_{i,t}} dt \tag{OA.100}$$

Finally, the equilibrium excess returns of stock k is

$$E_t[dR^e_{k,t}] = \mu^e_{k,t}dt = \frac{\sum_{i=1}^{h^*_t} C^*_{i,t}}{\sum_{i=1}^{h^*_t} \frac{C^*_{i,t}}{\gamma_i}} Cov_t(dR^e_{k,t}, \frac{d\sum_{i=1}^{h^*_t} C^*_{i,t}}{\sum_{i=1}^{h^*_t} C^*_{i,t}}) \tag{OA.101}$$

Using the **Proposition 3** in (26), it can be re-written as

$$E_t[dR^e_{k,t}] = \underbrace{\frac{Cov_t(dR^e_{k,t}, \frac{d\sum_{i=1}^{h^*_t} C^*_{i,t}}{\sum_{i=1}^{h^*_t} C^*_{i,t}})}{Cov_t(dR^e_{m,t}, \frac{d\sum_{i=1}^{h^*_t} C^*_{i,t}}{\sum_{i=1}^{h^*_t} C^*_{i,t}})}}_{Consumption\ beta} E_t[dR^e_{m,t}] \tag{OA.102}$$

(OA.103)

where $dR^e_{m,t}$ denotes the market excess returns. This result implies that a consumption beta should be computed only from stockholders' consumption. If there is no labor income, the ratio of covariances in (OA.102) reduces to the standard CAPM beta ($= \frac{Cov_t(dR^e_{k,t}, dR^e_{m,t})}{\sigma^2_{m,t}}$).

OA.13 Sample path for the results in Table 3 and 4

OA.13.1 Return decomposition for the consumption risk

Figure OA.10 illustrates the result in Table 3. It displays one sample path of the covariance of consumption growth with cash flow component of returns $Cov_t(\frac{dC^G_t}{C^G_t}, \frac{dD_t}{D_t})$ (left), discount rate component of returns $Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e - \frac{dD_t}{D_t})$ (middle), and total returns $Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e)$ (right) for aggregate consumption ($G = A$) and stockholders' consumption ($G = H$), respectively. The left figure shows that the covariance between aggregate consumption growth and dividend growth is procyclical, while the covariance is countercyclical for stockholders consumption. Most notably, time-variation for both aggregate and stockholders consumption is shaped by the dividend share in consumption, shown in the top-left panel of Figure 5. Due to the time-invariant dividend growth volatility, time-varying dividend share in consumption exclusively drives time-variation in the covariance between consumption growth and dividend growth. Also, the middle figure shows that the covariance between discount rate component of returns and consumption growth is countercyclical for both aggregate and stockholders consumption, contributing to the countercyclical conditional amount of risk for aggregate and stockholders consumption as shown in the right figure.

OA.13.2 Stock volatility

The result in Table 4 is also illustrated in Figure OA.11, which plots a sample path of the aggregate dividend share in the stockholders' consumption $\frac{D_t}{\sum_{i=1}^{h_t} C_{i,t}}$, the consumption-weighted mean of risky asset share in total wealth $\sum_{i=1}^{h_t} \frac{C_{i,t}}{\sum_{i=1}^{h_t} C_{i,t}} \frac{\pi_{i,t}}{X_{i,t}+H_{h,t}}$, and the corresponding conditional stock volatility $\sigma_{s,t}$ in our economy. First, in terms of the level, the dividend share in stockholders' consumption is always higher than the stockholders' consumption-weighted mean of risky asset share in total wealth, generating the unconditional excess volatility observed in the data. Second, a countercyclical dividend share in the stockholders' consumption (numerator) together with a procyclical consumption-weighted mean of stockholders' risky asset share in total wealth (denominator) generates a countercyclical stock volatility: 32% in a bad time and 25% in a good time.

OA.14 Proof of Lemma 1

Let us consider the conditional covariance between stock returns and aggregate consumption growth. The aggregate consumption can be decomposed into the consumption of stockholders and that of non-stockholder.

$$
\begin{aligned}
&Cov_t(dR_t^e, \frac{d\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*}) \\
&= \frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*} Cov_t(dR_t^e, \frac{d\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*}) + \frac{\sum_{i=h_t^*+1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*} Cov_t(dR_t^e, \frac{d\sum_{i=h_t^*+1}^{N} C_{i,t}^*}{\sum_{i=h_t^*+1}^{N} C_{i,t}^*}) \quad \text{(OA.104)}
\end{aligned}
$$

In the same way as before, we only need to consider the diffusion terms from the dynamics of the non-stockholders' consumption.

$$
d\sum_{i=h_t^*+1}^{N} C_{i,t}^* - E_t[d\sum_{i=h_t^*+1}^{N} C_{i,t}^*] = \sum_{i=h_t^*+1}^{N} (\delta^{\psi} a_i^{-\theta_i \psi}) \sigma_y H_{n,t} dW_{y,t} \quad \text{(OA.105)}
$$

Substituting (A.43), (A.47), and (OA.105) into (OA.104) yields

$$Cov_t(dR_t^e, \frac{d\sum_{i=1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*})$$
$$= \frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} \frac{\lambda_t \sigma_{s,t} \sum_{i=1}^{h_t^*}(\frac{C_{i,t}^*}{\gamma_i})dt}{\sum_{i=1}^{h_t^*} C_{i,t}^*} + \frac{\sum_{i=h_t^*+1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} \frac{\sum_{i=h_t^*+1}^N (\delta^\psi a_i^{-\theta_i \psi})(\sigma_y H_{n,t}(\rho \sigma_{s,t}^d + \sigma_{s,t}^y))dt}{\sum_{i=h_t^*+1}^N C_{i,t}^*} \tag{OA.106}$$

After rearranging terms, the equation becomes

$$= \frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} \frac{\lambda_t \sigma_{s,t} \sum_{i=1}^{h_t^*}(\frac{C_{i,t}^*}{\gamma_i})dt}{\sum_{i=1}^{h_t^*} C_{i,t}^*} + \frac{\sum_{i=h_t^*+1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*} \frac{\sum_{i=h_t^*+1}^N (\delta^\psi a_i^{-\theta_i \psi}) H_{n,t} \sigma_y \sigma_{s,t} \rho_{s,t} dt}{\sum_{i=h_t^*+1}^N C_{i,t}^*} \tag{OA.107}$$

Solving (OA.107) for $\lambda_t \sigma_d dt$ yields

$$\lambda_t \sigma_{s,t} dt = E_t[dR_t^e] =$$
$$\frac{\sum_{i=1}^N C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}} Cov_t(dR_t^e, \frac{d\sum_{i=1}^N C_{i,t}^*}{\sum_{i=1}^N C_{i,t}^*}) - \frac{\sum_{i=h_t^*+1}^N (\delta^\psi a_i^{-\theta_i \psi}) H_{n,t} \sigma_y \sigma_{s,t} \rho_{s,t} dt}{\sum_{i=1}^{h_t^*} C_{i,t}^*} \quad \blacksquare \tag{OA.108}$$

OA.15 Data

In this article, the U.S. dividend, non-financial income, financial market, and consumption data are used for the simulation in Section 5 and empirical analysis in Section 6. In this section, we describe the data we use.

OA.15.1 Dividend and non-financial income data

Dividend and non-financial income data for the longest period from 1930 to 2016, similar to Mehra and Prescott (1985), Kandel and Stambaugh (1991), Abel (1999), Bansal and Yaron (2004), and Beeler and Campbell (2012) are used for the choice of parameter values in Table 1. Both data are collected from the National Income and Product Account (NIPA) of the U.S. by the Bureau of Economic Analysis (BEA). Non-financial income is defined as the difference between the total personal income and the total dividend, following Jagannathan and Wang (1996). Nominal values are deflated using the personal consumption expenditures deflator. U.S. population data are also used to obtain per capita value.

OA.15.2 Excess equity returns and risk-free rate

Equity returns and risk-free rate from 1930 to 2016 are used in Table 7. We construct stock returns by log growth of real value of all NYSE/Amex/Nasdaq stocks from the CRSP. To construct ex-ante real risk free, we follow the methodology in Beeler and Campbell (2012). We create a proxy for the ex-ante risk-free rate by forecasting the ex-post quarterly real return on three-month Treasury bills with past one-year inflation and the most recent available three-month nominal bill yield. The detail on the methodology is described in the online appendix in Beeler and Campbell (2012).

OA.15.3 Consumption data

Aggregate consumption data from the NIPA by the BEA for the period from 1930 to 2016 are used in Table 7. Consumption is defined as the sum of nondurable and services as durable is not closely linked to consumers' intertemporal choice of consumption and portfolio. Nominal consumption values are deflated using the personal consumption expenditures deflator. We construct the log per capita consumption growth based on the population data.

OA.15.4 Households Survey data

CEX data: In this article, We highlight the importance of distinction between aggregate consumption and stockholders' consumption. In Section 6, we use stockholders' consumption from the Consumer Expenditure (CEX) by the Bureau of Labor Statistics (BLS) from January 1984 to January 2017 to test the key implications of our theoretical model. The way the interview is conducted is the BLS interviews a selected family every 3 months over four times. After the last interview (fourth), the sample family is dropped from the survey and a new sample family is introduced. Therefore, the composition of interviewed households in a month is different from the next month, and thus, we can calculate the quarterly consumption growth at a monthly frequency. Finance asset holding information is collected in the last interview.[38] As a definition of consumption, we use items in CEX which match the definition of nondurables and services in the NIPA. We exclude housing expenses (but not costs of household operations), medical care costs, and education costs due to its substantial durable components. For the sample choice. We apply the same rules as in Malloy et al. (2009). We drop household-quarters in which a household reports negative consumption. Extreme outliers having consumption growth ($C_{i,t+1}/C_{i,t}$) more than 5.0 and less than 0.2 are drop. Moreover, nonurban households and households residing in student housing are dropped.

To identify the stockholders, we refer to the question of "As of today, what is the total value of all directly-held stocks, bonds, and mutual funds?". Our definition of stockholders is the intersection of the positive holdings of "stocks, bonds, mutual funds and other such securities" and a predicted probability of owning stocks at least 0.5 as in the sophisticated definition of stockholders as in Malloy et al. (2009). In order to compute the probability of owning stocks for CEX households, we use the Survey of Consumer Finances (SCF) as described below where one can accurately observe holdings of stocks and mutual funds, following Malloy et al. (2009). By running a probit regression of whether a household holds stocks or mutual funds on a set of characteristics using the SCF, we obtain coefficients of characteristics and apply them to the CEX households.

SCF data: The SCF is a cross-sectional survey of U.S. families conducted by the Federal Reserve Board every three years. The survey data cover a wide variety of information on families' balance sheets, pensions, income, and demographic characteristics. Unlike

[38]For a more detailed information, see https://www.bls.gov/opub/hom/cex/data.htm

CEX data, the SCF directly asks households whether respondents have any stock (Variable name:hstocks) or mutual funds excluding MMMFs (hnmmf). However, since the survey is conducted on a triennial basis, it is difficult to use the data for the conditional asset pricing test. Using the SCF data from 1989, 1992, 1995, 1998, 2001, 2004, 2007, 2010, 2013, and 2016, We run a probit regression on a set of observable characteristics that are also available in the CEX: age, age squared, number of kids, an indicator for high school and more than college education for household, an indicator for race not being white, the log of income before taxes (set to zero if income = 0), an indicator for income =0, the log of checking and savings accounts (set to zero if checking and savings = 0), an indicator for checking and savings account = 0, an indicator for positive dividend income, year dummies, and a constant. The regression is a cross-sectional regression as a household appears in SCF only once. We also use the SCF data to estimate risk aversion of each household. From the SCF data, we run a probit regression of a dummy variable which takes one if a household reporting no tolerance for financial risk on the same set of independent variables to compute the probability of owning stocks in addition to the log of one plus financial asset holdings. The estimates of the coefficients from the Probit model in the SCF data are applied to the CE data to obtain the probability of reporting no tolerance for financial risk, which is assumed to be risk aversion of each household. All Probit regression results are reported in Table OA.1.

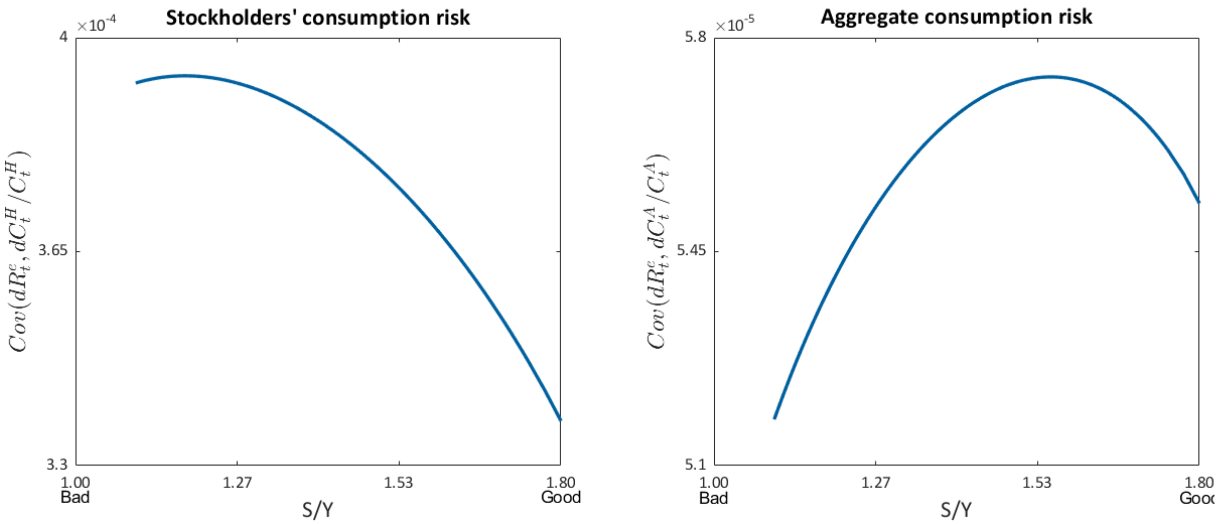

Figure OA.1: **Conditional amount of consumption using simulated data**

This figure plots the empirically estimated conditional covariance of equity returns with stock*h*olders consumption growth $Cov_t(dR_t^e, \frac{d\mathbf{C}_t^H}{\mathbf{C}_t^H})$ (Left) and *a*ggregate consumption growth $Cov_t(dR_t^e, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$ (Right) using the stock market capitalization-to-aggregate labor income ratio (S/Y) based on the simulated data. The conditional covariances are estimated by the Epanechnikov nonparametric kernel estimation. Parameter values for the simulation are in Table 1.

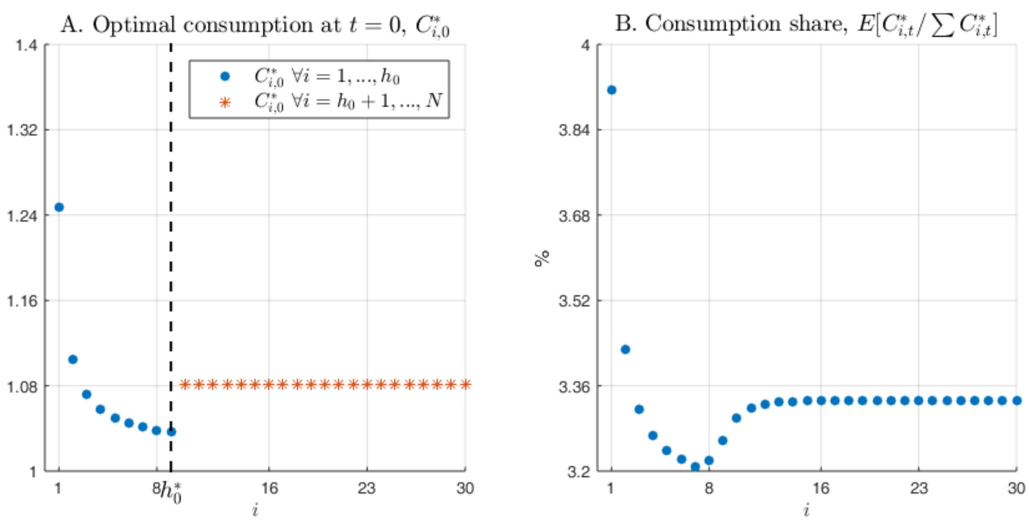

Figure OA.2: **Optimal consumption and consumption share across agents**

Panel A plots the optimal consumption for each agent at time 0 ($t = 0$) in equilibrium. The cut-off stockholder h_0^* is 9th stockholder (dashed vertical line). Therefore, the stockholders range from the first investor to 9th investor and non-stockholders range from 10th to the last (30th). Panel B plots the unconditional consumption share of each agent. To generate this, 1,000 sample paths of economy are simulated. Each path consists of 480 monthly observations (40 years), in total 480,000 months. For both Panel A and B, parameter values for the simulation are in Table 1. Per capital labor income level is normalized to unity.

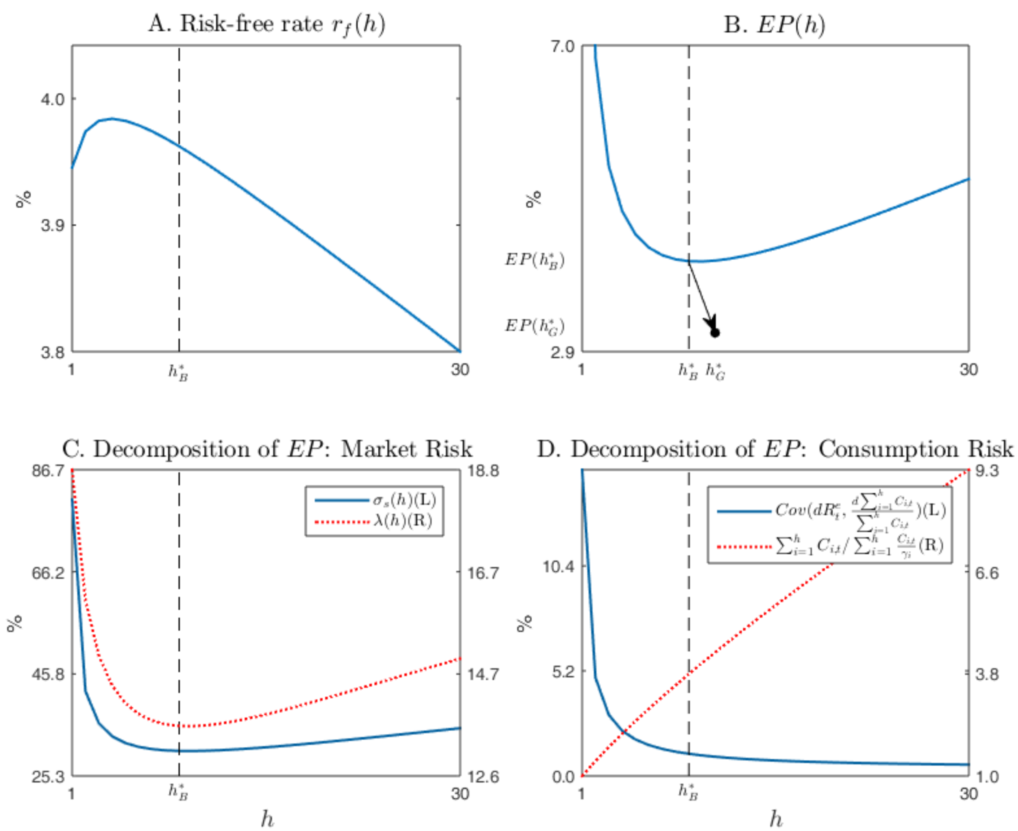

Figure OA.3: **Comparative Statics:** $r_f(h)$, $EP(h)$, $\lambda(h)$, $\sigma_s(h)$, $Cov(h)$, $\Gamma(h)$

This figure plots $r_f, EP, \lambda, \sigma_s$, $Cov(dR_t^e, d\sum_{i=1}^h C_i / \sum_{i=1}^h C_i)$, and $\Gamma \equiv \sum_{i=1}^h C_i / \sum_{i=1}^h \frac{C_i}{\gamma_i}$ as a function of the cut-off stockholder h at the base state. We exogenously include agents to the stock market in a monotonic way from the least risk-averse agent to the most risk-averse agent. That is, the set of stockholders increases as follows: $\{1\}$, $\{1, 2\}$, ..., $\{1, 2, ..., N\}$, as $h = 1, 2, ..., N$. In Panel C, the equity premium is decomposed into the amount of market risk $\sigma_s(h)$ (solid line, left y-axis) and the price of market risk $\lambda(h)$ (dotted line, right y-axis). In Panel D, the equity premium is decomposed into the $Cov(dR_t^e, d\sum_{i=1}^h C_i / \sum_{i=1}^h C_i)$ (solid line, left y-axis), and $\Gamma(h)$ (dotted line, right y-axis). The endogenous cut-off stockholder at the base state h_B^* is 9th stockholder (dashed vertical line). In Panel B, h_G^* denotes the cut-off stockholder at a good state. Parameter values for the simulation are in Table 1.

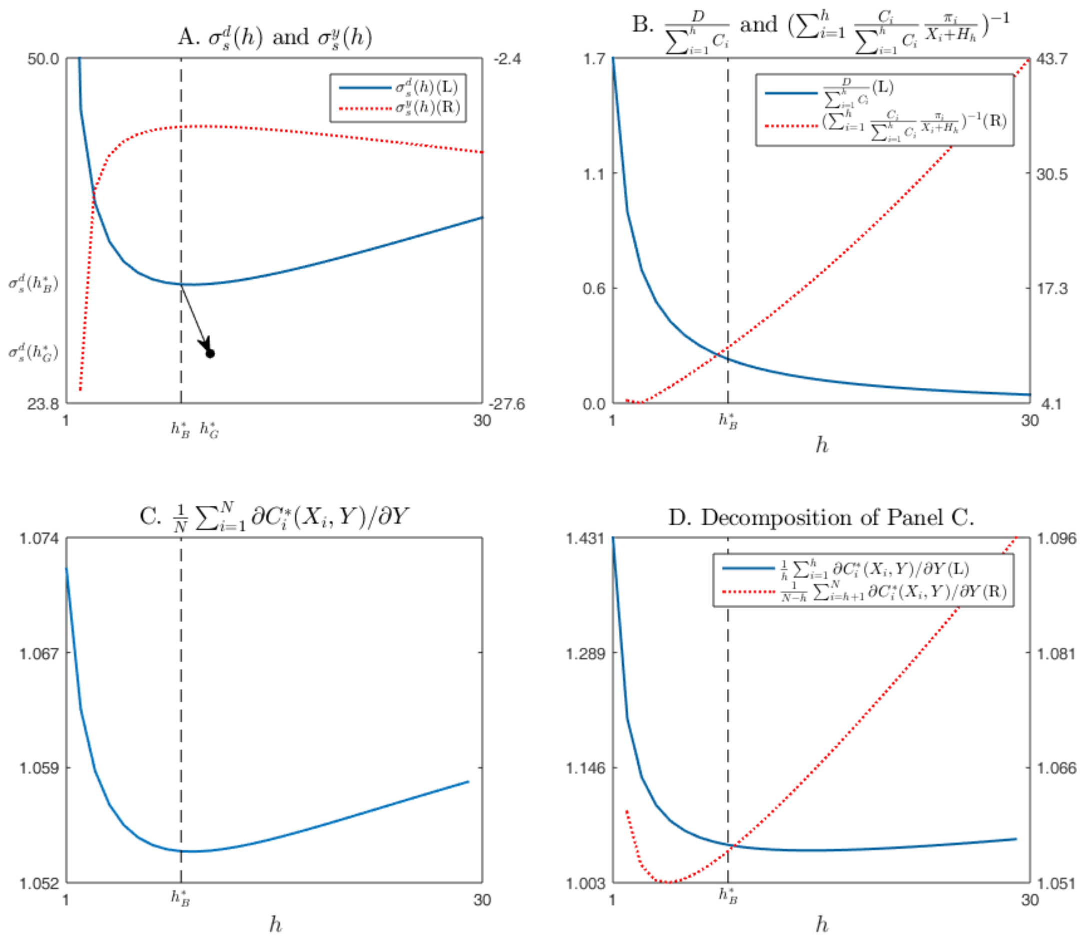

Figure OA.4: **Comparative Statics: Analysis on the stock volatility**

Panel A is σ_s^d (solid line, left y-axis) and σ_s^y (dotted line, right y-axis). Panel B is $\frac{D}{\sum_{i=1}^h C_i}$ (solid line, left y-axis), and $(\sum_{i=1}^h \frac{C_i}{\sum_{i=1}^h C_i}\frac{\pi_i}{X_i+H_h})^{-1}$ (dotted line, right y-axis). Panel C is the average of marginal consumption with respect to labor $\frac{1}{N}\sum_{i=1}^N \partial C_i^*(X_i,Y)/\partial Y$. Panel D is the stockholders' (solid line, left y-axis) and non-stockholders' (dotted line, right y-axis) average of marginal consumption with respect to labor income, respectively, as a function of the cut-off stockholder h at base state. We exogenously include agents to the stock market in a monotonic way from the least risk-averse agent to the most risk-averse agent. That is, the set of stockholders increases as follows: $\{1\}$, $\{1,2\}$, ..., $\{1,2,...,N\}$, as $h = 1,2,...,N$. The endogenous cut-off stockholder at the base state h_B^* is 9th stockholder (dashed vertical line). In Panel B, h_G^* denotes the cut-off stockholder at a good state. Parameter values for the simulation are in Table 1.

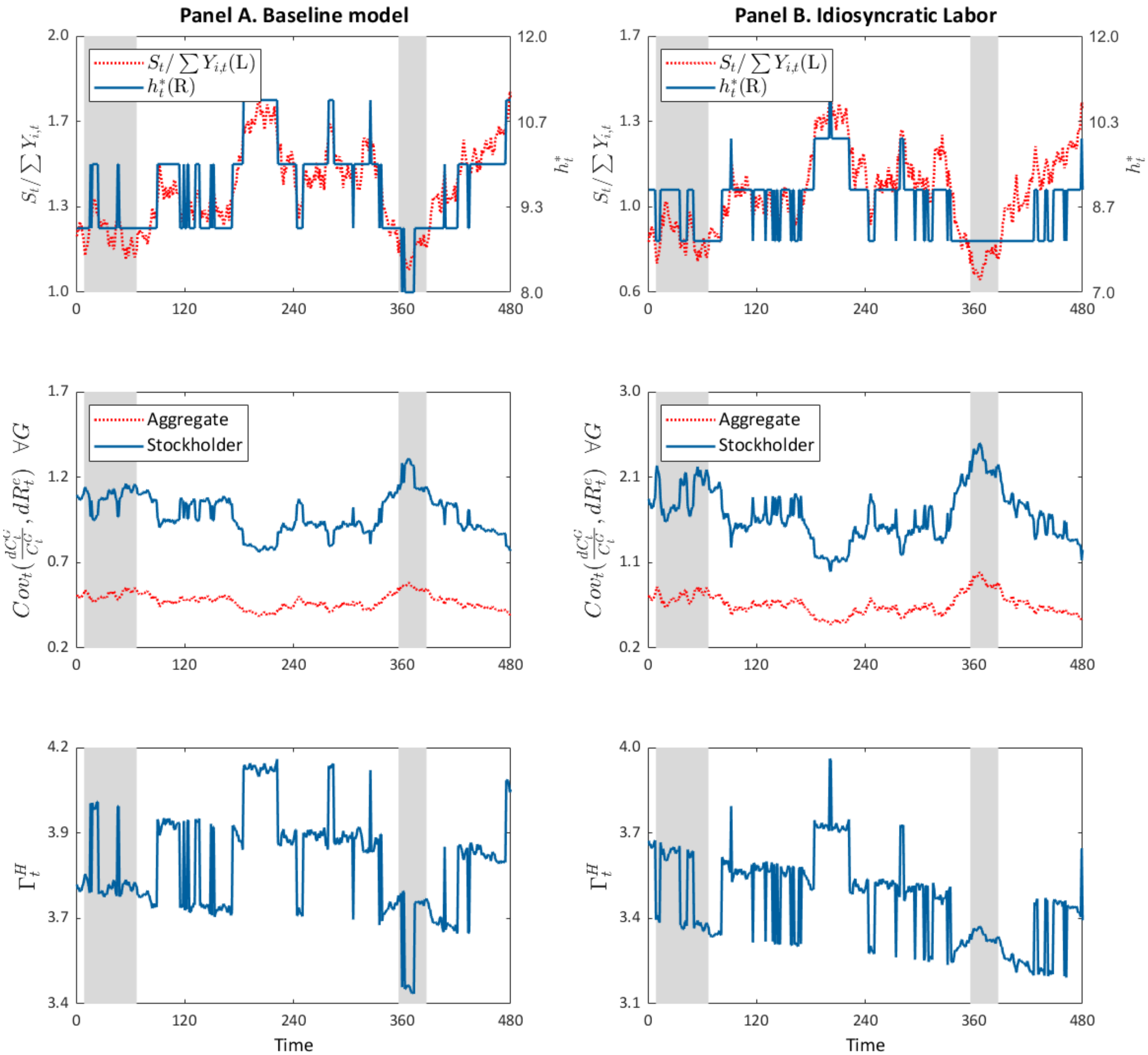

Figure OA.5: **Idiosyncratic labor income setup**

This figure illustrates one sample path of time-varying market participation (right y-axis) and the state variable: the stock market wealth-aggregate labor ratio ($\frac{S_t}{\sum Y_{i,t}}$) (left y-axis) in the top figure, the covariance of stock returns with consumption growth $Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e)$ $\forall G = A, H$ for aggregate consumption and stockholders' consumption in the middle figure, and the price of risk (stockholders' average risk aversion) in the bottom figure. The left panel is the result for the baseline model described in the main body section. The right panel is the result for the idiosyncratic labor income case. The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ based on simulated data. This figure is based on the parameters in Table 1 with $\rho_d = 0.43$, $\rho_y = 0.7$.

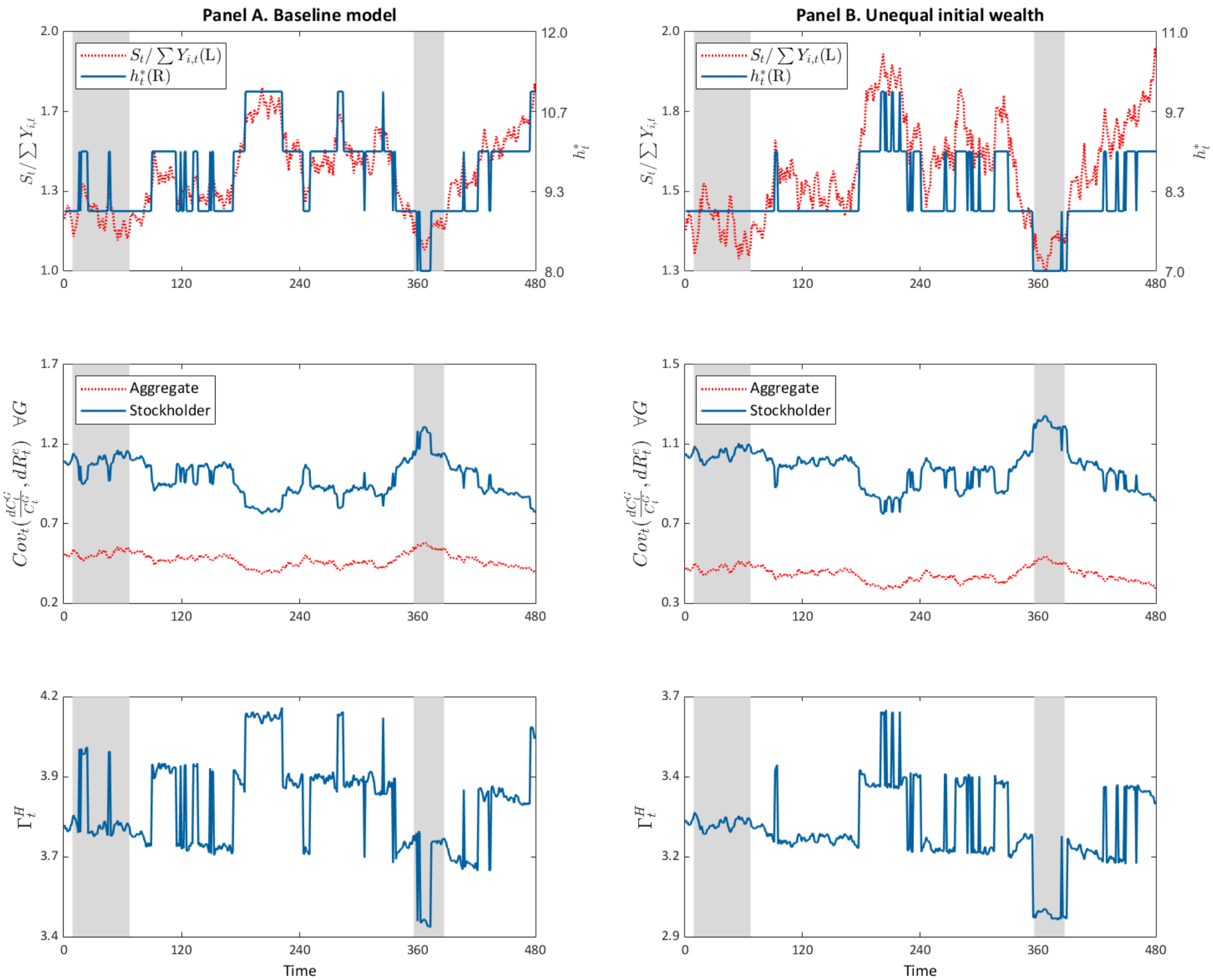

Figure OA.6: **Unequal initial wealth setup**

This figure illustrates one sample path of time-varying market participation (right y-axis) and the state variable: the stock market wealth-aggregate labor ratio ($\frac{S_t}{\sum Y_{i,t}}$) (left y-axis) in the top figure, the covariance of stock returns with consumption growth $Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e)$ $\forall G = A, H$ for aggregate consumption and stockholders' consumption in the middle figure, and the price of risk (stockholders' average risk aversion) in the bottom figure. The left panel is the result for the baseline model described in the main body section. The right panel is the result for the inequality initial wealth endowment case. The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ based on simulated data. This figure is based on the parameters in Table 1.

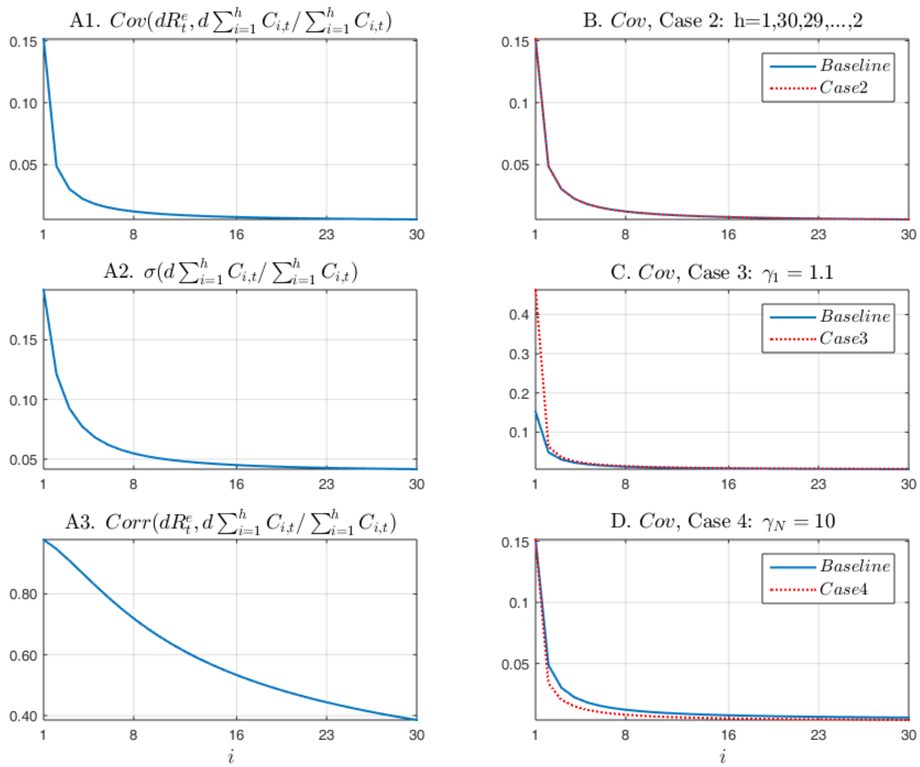

Figure OA.7: **Comparative Statics: Risk-Sharing**

Panel A plots the amount of risk (A1), the stockholders' consumption volatility (A2), and the correlation of stockholders' consumption growth with stock returns (A3) as a function of $h = i$. In this baseline case $\gamma_1 = 1$, $\gamma_N = 50$, and we exogenously include agents to the stock market in a monotonic way from the least risk-averse agent to the most risk-averse agent. That is, the set of stockholders increases as follows: $\{1\}$, $\{1, 2\}$, ..., $\{1, 2, ..., N\}$, as $h = 1, 2, ..., N$. For the Case 2, $\gamma_1 = 1$, $\gamma_N = 50$, the order of inclusion is $h = 1, 30, 29, ..., 2$ (Panel B), For the Case 3, $\gamma_1 = 1.1$, $\gamma_N = 50$, with the ascending order of inclusion (Panel C). For the Case 4, $\gamma_1 = 1$, $\gamma_N = 10$, with the ascending order of inclusion (Panel D). This figure is based on the parameters in Table 1.

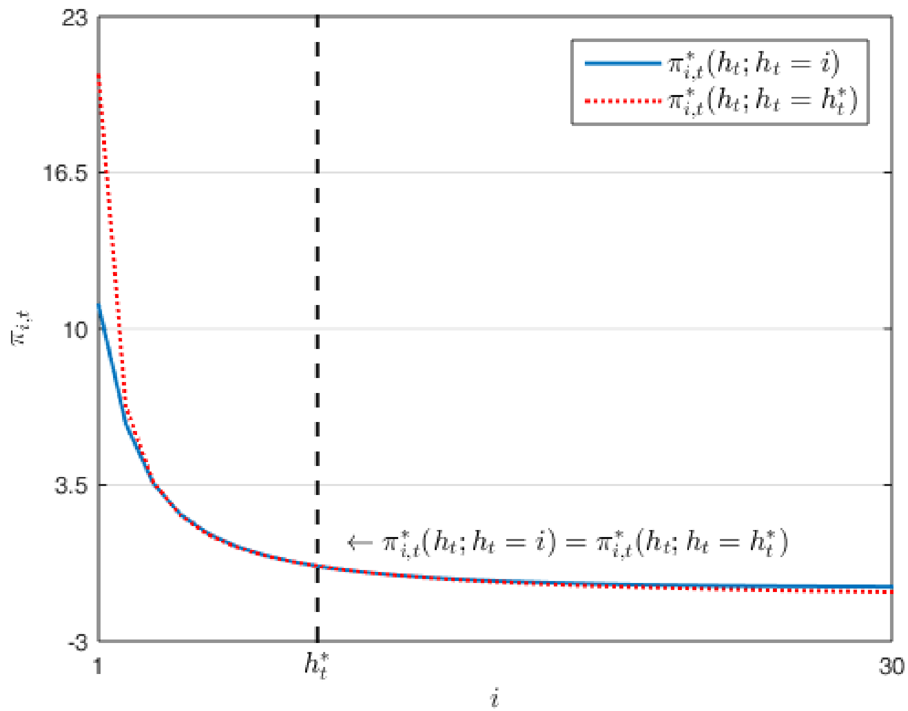

Figure OA.8: **Optimal stock holdings across stockholders**

This figure plots the cross-sectional variation of the optimal stock holdings at time 0 ($t = 0$) for $i = 1, ..., 30$. The optimal stock holding depends on the cut-off stockholder h_t. The solid line is the optimal stock holdings of each stockholder i when each one believes that she is the cut-off stockholder ($h_t = i$). The dotted line is the optimal stock holdings of each stockholder when all stockholders fix the cut-off stockholder ($h_t = h_t^*$). h_t^* is 9th stockholder. This figure is based on the parameters in Table 1.

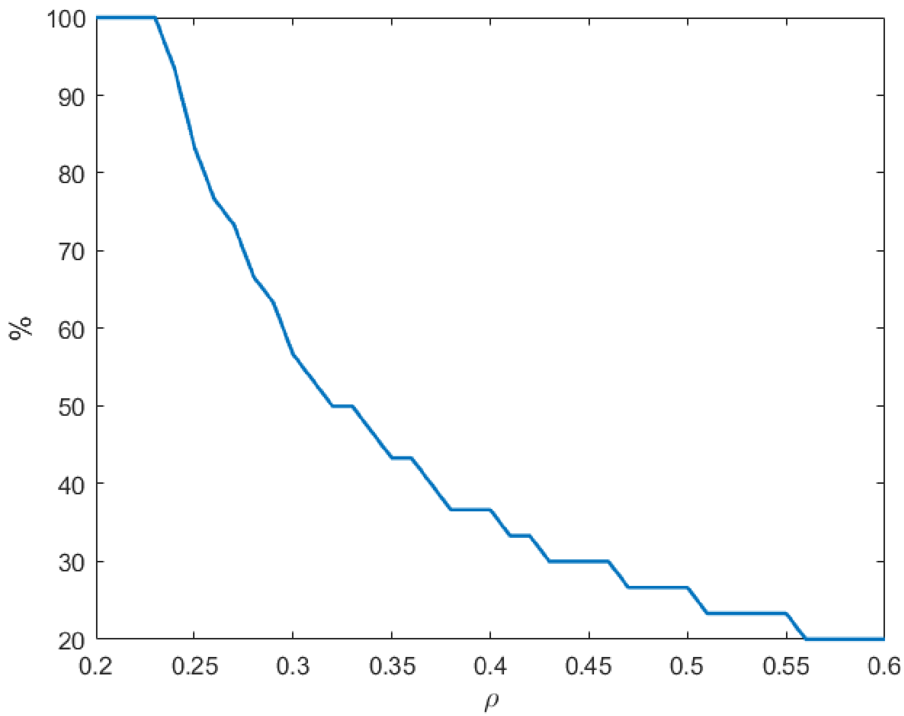

Figure OA.9: **Market participation rate with ρ**
This figure plots market participation rate with different values of correlation between dividend growth and non-financial income growth at $t = 0$. Other parameter values are reported in Table 1.

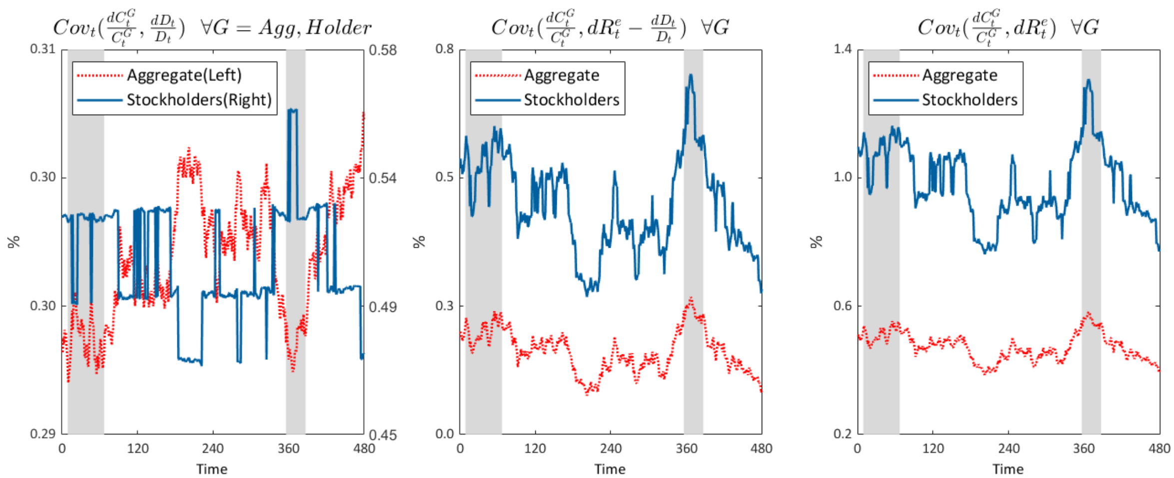

Figure OA.10: **Return decomposition for amount of consumption risk**
This figure displays one sample path of the covariance of consumption growth with cash flow component of returns $Cov_t(\frac{dC_t^G}{C_t^G}, \frac{dD_t}{D_t})$ (left), discount rate component of returns $Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e - \frac{dD_t}{D_t})$ (middle), and total returns $Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e)$ (right) for aggregate consumption ($G = A$) and stockholders' consumption ($G = H$), respectively. The dotted line is for aggregate consumption and the solid line is for stockholders' consumption. Parameter values for the simulation are in Table 1. The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ based on simulated data. Parameter values for the simulation are in Table 1.

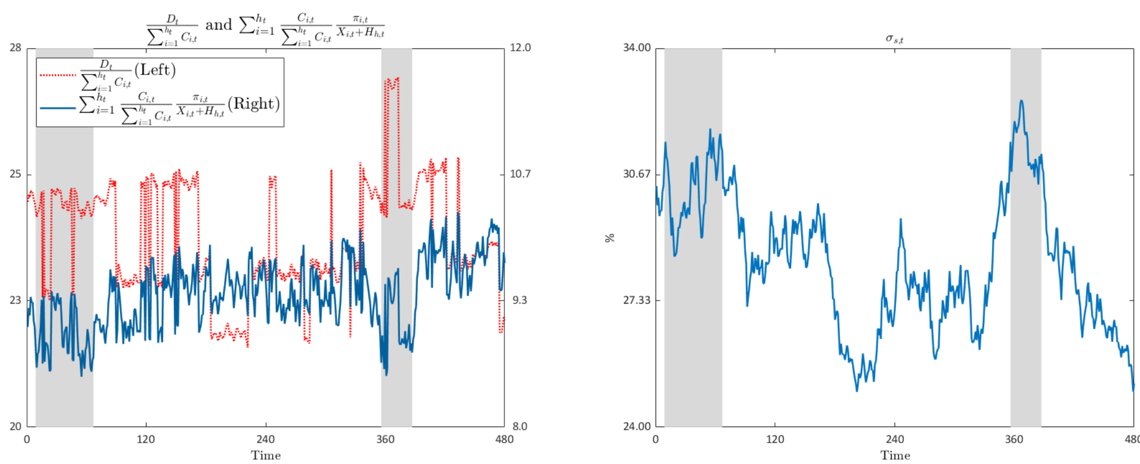

Figure OA.11: **Time-variation in the conditional stock volatility**

The left figure shows the dividend share in the stockholders' consumption (left y-axis) and the risky asset share in total wealth (right y-axis) given $\sigma_{s,t}^d = \sigma_d \frac{D_t}{\sum_{i=1}^{h_t} C_{i,t}} / \sum_{i=1}^{h_t} \frac{C_{i,t}}{\sum_{i=1}^{h_t} C_{i,t}} \frac{\pi_{i,t}}{X_{i,t}+H_{h,t}}$. The right figure is the corresponding conditional stock volatility in this economy. The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ based on simulated data. Parameter values for the simulation are in Table 1.

Table OA.1: Probit regression of stock ownership and risk Risk appetite

Table OA.1 reports the Probit regression of households stock ownership or households' unwillingness to take financial risks on the observable characteristics. The SCF data from 1989, 1992, 1995, 1998, 2001, 2004, 2007, 2010, and 2013. The first dependent variable takes one if a household has positive holding either in stock (hstocks=1) or mutual funds excluding MMMFs (hnmmf=1) otherwise zero. The second dependent variable takes one if a household reports that they have no tolerance for investment risk otherwise zero. The regressors are age of household (age), age squared (age^2), an indicator for race not being white/Caucasian (race=1), the number of kids (kids), an *highschool* indicator for at least 12 but less than 16 years of education for head of household (educ>11 and educ<16), an *college* indicator for 16 or more years of education (educ >16), the log of real total household income before taxes (income), the log of real dollar amount in checking and savings account (log(checking+saving)) (set to zero if checking and savings = 0), and indicator for checking and savings account = 0, an indicator for dividend income (X5709=1), and year dummies. For the second dependent variable, the log of one plus stock and mutual funds holding amount is also included. Robust standard errors are used for Z-statistic and statistical significance at the 10%, 5%, and 1% levels are denoted by *, **, ***, respectively.

Independent Variable	Dependent Variable: Stock ownership	Dependent Variable: Unwillingness to take risk
age	0.004**	-0.009***
age^2	-1.8×10^{-5}	2.8×10^{-4}***
$kids$	-0.030***	0.040***
$1_{i\in highschool}$	0.281***	-0.250***
$1_{i\in college}$	0.610***	-0.578***
$1_{i\in nonwhite}$	-0.305***	0.209***
$log(income)$	0.234***	-0.189***
$1_{income=0}$	2.591***	-2.332***
$log(chk+saving)$	0.089***	-0.072***
$1_{chk+saving=0}$	0.555***	-0.322***
$1_{Div>0}$	1.393***	-0.324***
$log(1+holding)$	-	-0.049***
1_{1992}	-0.028*	0.024
1_{1995}	0.043***	-0.109***
1_{1998}	0.247***	-0.232***
1_{2001}	0.266***	-0.179***
1_{2004}	0.134***	-0.100***
1_{2007}	-0.009	-0.173***
1_{2010}	-0.143***	-0.015
1_{2013}	-0.245***	-0.082***
1_{2016}	-0.270***	-0.200***
$Cons$	-4.787***	2.631***
Number of Obs.	238,880	238,880
Pseudo R^2	0.414	0.247