Stock Prices, Changes in Liquidity, and Liquidity Premia

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Abstract

This paper develops a present-value framework that factors in expectations of future market illiquidity. In our framework, an implied liquidity premium is a function of prices, dividends, illiquidity costs, and returns. We find that the liquidity premium for the CRSP market portfolio is significantly priced over short horizons, but its long-horizon evidence is not evident. This finding implies that an illiquidity shock is so transient that even its big variation in the first place could not build up over horizons. At its core, market liquidity risk should be second-order in the long run. We reconcile our findings with some theoretical debate over the importance of the liquidity premium on asset pricing.

JEL Classifications: C12, C32, G12.

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1. Introduction

A perfect market assumes no trade impediments. Standard asset pricing theory (e.g., the Capital Asset Pricing Model) says that all securities are commonly affected by a systematic risk factor. In financial markets, therefore, investors want to be compensated for holding risky securities, which is determined *solely* by the common systematic risk (i.e., market beta).

In reality, however, we encounter various kinds of market imperfections that can bring other sources of premia.[1] For instance, one imperfection is transaction costs. Due to the risk of incurring high costs in liquidating portfolios in the future, investors demand a *liquidity premium* to compensate for potential losses. Therefore, the deep understanding of the liquidity premium is essential for investors to especially forming investment decisions.

The starting point of this article is to acknowledge that it is hard to detect the liquidity premium empirically. The big obstacle of estimating the hard-to-detect *actual* premium motivates us to come up with another way of inference. In this paper, we challenge to identify the *implied* market liquidity premium caused by market liquidity risk,[2] which is *inferred* from a new present-value relationship we propose. With the implied premium in place, our purpose is to examine whether the liquidity premium is a crucial source of price variation.

To the point, we go on to tackle some theoretical debate over the importance of the liquidity premium on asset pricing. On the one side, Amihud and Mendelson (1986) and Lynch and Tan (2011) claim that asset returns carry a substantial size of the liquidity premium. On the other side, Constantinides (1986) and Vayanos (1998) argue that the liquidity premium should be only second-order because investors want to reduce the trading frequency in the presence of transaction costs. To see the debate clearly, suppose that market liquidity risk is so

[1] Vayanos and Wang (2013) provide an extensive review for market imperfections.

[2] Apart from market liquidity risk, investors with leveraged positions (e.g., a hedge fund) can want funding liquidity risk compensation. This funding liquidity risk can interact with the market liquidity risk, which jointly affects required returns (Pedersen, 2015). In this paper, our focus is on market liquidity risk.

second-order that its cumulative effect on prices is *not* significant. When such shocks come up, investors are likely to adjust more myopic demand than intertemporal hedging demand.

This paper finds new results reconciling the aforementioned theoretical debate. It turns out that market liquidity risk just matters in the short run. In the long run, however, investors do *not* seem to mind it seriously since the liquidity risk is *not* expected to last for a long time. The implication is that if you are planning to sell the market portfolio somewhere in the *far* future, in fact, the liquidity risk is *not* relevant to you. Hence, you hardly ask for a market liquidity risk premium because market liquidity risk is basically second-order.

To explain these ideas precisely, Section 2 presents a *new* present-value framework that incorporates illiquidity costs beyond the conventional price–dividend one. We define their linear combination as log liquidity-adjusted price-dividend ratio (hereafter, *pdl* ratio). Motivated by the fact that asset prices should equal expected discounted cash flows, illiquidity costs—referred to as negative dividends (Jones, 2002; Acharya and Pedersen, 2005)—can be another priced factor in a present-value context (henceforth, "illiquidity costs", "negative dividends", and "(il)liquidity" are interchangeably used for convenience).

It is important to note that we leverage *price*-based illiquidity as a mirror image of (positive) dividends, which are *nonstationary*, I(1). This premise is the most challenging because the conventional studies have usually employed *return*-based illiquidity.[3] Simple evidence is that such illiquidity can strongly predict one-period stock returns. Thus, it seems straightforward that it should have a first-order effect on prices, since market illiquidity is pretty persistent enough to last for a long time (i.e., the first-order autocorrelation is high).

No matter which illiquidity concept you use, the conventional literature also overlooks one crucial norm that such illiquidity cannot affect asset prices *independently*. For example,

[3] See Amihud and Mendelson (1986), Amihud (2002), Jones (2002), Pástor and Stambaugh (2003), Acharya and Pedersen (2005), Bekaert et al. (2007), Korajczyk and Sadka (2008), Ben-Rephael et al (2015), and among others.

Amihud (2002), Jones (2002), and Bekaert et al. (2007) assume that market illiquidity is *not* affected by aggregate dividends. Notably, our price–dividend–cost relationship can present the counter example in which market illiquidity must interact with other information sources such as prices and dividends. Going deep, the conventional price–dividend relationship says that return predictability is evident due to the *lack* of dividend growth forecastability (Cochrane, 2008). As such, our price–dividend–cost relationship can address similar questions: How stronger liquidity-premium forecastability is than the others?

For empirical analysis, Section 3 uses the Amihud's (2002) illiquidity measure to estimate the *pdl* ratios (we will elaborate on a full exposition of our proxy choice later). By using annual Center for Research in Security Prices (CRSP) index data from 1926 to 2017, Figure 1 graphs the *pdl* ratios along with the market price–dividend ratios (henceforth, *pd* ratio).

[INSERT FIGURE 1 HERE]

The wedge between the two series offers a great deal of intuition that market illiquidity provides a good estimate of the *trend* in prices. Technically speaking, such negative dividends are nearly a random walk so that illiquidity growth is almost a white noise (or i.i.d.). The rationale behind this is that the *pdl* ratio looks like a trend-adjusted series of the *pd* ratio by our cost proxy. As you will see, such intuition is key in understanding our main results. If market-illiquidity variation deviated away from the trend substantially, prices should embody a large fraction of the long-term liquidity premia in favor of liquidity-premium forecastability.

Such trend implication is analogous to the dividend implication of Cochrane (1994). In short, shocks to dividend growth are so *transient* that they cannot have a *persistent* effect on price variation (Cochrane, 2011). In the similar fashion, shocks to illiquidity growth works. This is possibly because investors are likely reluctant to increase the trading frequency, incurring high costs. More to the point, negative dividends cannot still help to attenuate the *excess price volatility* that is not fully explained by subsequent changes in positive dividends (Shiller, 2014). If market illiquidity was by far first-order, it should take more of the excess volatility recognized as pricing errors *not* justified by subsequent expected cash flows.

In Section 4, we depart from simple predictive regressions and then hypothesize a *constant-expected-liquidity-premium* model. We find that the *pdl* ratios strongly forecast one-period implied liquidity premia inferred from our price–dividend–cost relationship, but *hardly* do the long-term liquidity premia inherent in prices. What this means is that even a big but *less persistent* variation in expected liquidity premia *cannot* lead to a big price change. Thus, market liquidity risk is essentially second-order. Our findings suggest that investors take great care of unexpected illiquidity news in the first place, but do not care much in the long run.

Why do investors react to market illiquidity separately over both horizons? The main answer is due to the transient nature of market illiquidity. To explore this, Section 5 conducts impulse response functions. By design, we formulate a liquidity-premium shock only generated by *one* rise of market illiquidity with *no* information-source changes. As expected, we confirm that prices, dividends, and market illiquidity stay put at the point of the shock onward.

Its transient nature gives us deep understanding of market illiquidity. For example, one conventional norm is that shocks to illiquidity can lead to a rise in expected future illiquidity, which in turn raises expected returns and thus lower current prices. As for the aggregate market at least, the transient illiquidity shock merely lowers prices, *little* relying on the two intermediate channels. Relatedly, the fact that illiquidity growth is *rarely* forecastable so that market illiquidity is nearly a random walk supports our conclusion.

There are two points to discern the extant literature. First, one of the benefits of our framework is that it provides a restrictive structure to identify our price–dividend–cost relationship *uniquely*. With this unique relationship in mind, note that all proportions of our information sources must add up to the whole volatility of the *pdl* ratios in an accounting sense.[4] In contrast, Jones (2002) analyzes the *ad-hoc* interplay between the *pd* ratios and illiquidity, which could induce misleading outcomes, because it does *not* obey the accounting identity.

Second, the explanatory proportions of information sources must be evaluated *relatively* within each information set. As an example, the share of pizza (i.e., the decomposition of prices) depends *entirely* on the number of participants (i.e., the number of information sources). One might doubt that the trend-adjusted illiquidity effect seems quite large (Figure 1). But keep in mind that our whole story must be limited within our information set. If further incorporating another source (e.g., share repurchase), it is *not* surprising that the share of information sources should change *quantitatively* in the relative sense. Nevertheless, we contend that the trend implication of market illiquidity should be intact *qualitatively*, as shown in Figure 1.

Finally, Section 6 concludes the paper. Admittedly, our conclusion relies on which proxies are used. To mitigate the concern, we also use the log quarterly bid-ask spread in Appendix B, but our conclusion is invariant. Indeed, market liquidity risk should be second-order.

2. Theoretical framework

First, we build upon the cross-sectional identity that stock prices and dividends are linked to illiquidity costs. This linkage can be rewritten as a natural log of *net return*, r^*_{t+1}, measured at the end of time $t+1$:

$$r^*_{t+1} = \log\left(\frac{P_{t+1}+D_{t+1}-C_{t+1}}{P_t}\right) = \log\left(1+\frac{D_{t+1}}{P_{t+1}}-\frac{C_{t+1}}{P_{t+1}}\right) + p_{t+1} - p_t, \tag{1}$$

where P_t is the real price of a stock, measured at the end of time period t; D_t is the real dividend during period t; and C_t denotes the illiquidity cost during period t. Here, we assume that C_t is

[4] For example, those of expected returns and dividend growth account for 100% of the volatility of the *pd* ratios.

nonstationary, and use the convention that logs of variables are denoted by lowercase letters, i.e., $p_t \equiv \log(P_t)$.

We then rewrite log net return (1) as

$$r^*_{t+1} = \log(1 + \exp(dp_{t+1}) - \exp(cp_{t+1})) + p_{t+1} - p_t, \tag{2}$$

where $dp_t \equiv d_t - p_t$ is the log dividend-price ratio with sample mean of $\overline{dp}$, and $cp_t \equiv c_t - p_t$ is the log liquidity-price ratio with sample mean of $\overline{cp}$. In log net return (2), the first term on the right-hand-side (RHS) is a nonlinear function of dp_{t+1} and cp_{t+1}. The two ratios will be linearized to derive a new price–dividend–cost relationship.

Second, we apply a first-order Taylor expansion to the nonlinear function above:

$$\log(1 + \exp(dp_{t+1}) - \exp(cp_{t+1})) \approx$$
$$k_l + (\rho_l - 1)p_{t+1} + \rho_l(1/\rho - 1)d_{t+1} - \rho_l(1/\rho - 1/\rho_l)c_{t+1}, \tag{3}$$

where ρ_l and ρ are log-linear discount factors ($\rho_l > \rho$), and k_l is a log-linear coefficient (see Appendix A for more technical details). Substituting (3) into (2) yields the linear difference equation of the log net return:

$$r^*_{t+1} \approx k_l + \rho_l \cdot p_{t+1} + \rho_l(1/\rho - 1)d_{t+1} - \rho_l(1/\rho - 1/\rho_l)c_{t+1} - p_t. \tag{4}$$

Third, we solve difference equation (4) forward with a terminal condition that rules out rational bubbles: $\lim_{j\to\infty}(\rho_l)^j p_{t+1+j} = 0$. We then obtain the following present-value identity without a constant term $k_l/(1-\rho_l)$ in the form of log prices:

$$\begin{aligned} p_t &\approx \sum_{j=0}^{\infty}(\rho_l)^j \left[\rho_l \left[\left(\frac{1}{\rho} - 1\right) d_{t+1+j} - \left(\frac{1}{\rho} - \frac{1}{\rho_l}\right) c_{t+1+j}\right] - r^*_{t+1+j}\right] \\ &\approx \sum_{j=0}^{\infty}(\rho_l)^j \left[\rho_l \left[\left(\frac{1}{\rho} - 1\right) d_{t+1+j} - \left(\frac{1}{\rho} - \frac{1}{\rho_l}\right) c_{t+1+j}\right] - (r_{t+1+j} - lp_{t+1+j})\right], \end{aligned} \tag{5}$$

provided that the state variables on the RHS are observable *ex-post*. In non-stationary present-value form (5), we further impose an ex-post liquidity premium lp_t such that the following accounting identity holds:

$$r_t^* = r_t - lp_t. \tag{6}$$

This new accounting identity means that investors can gain liquidity-adjusted profit r_t^* at the expense of the liquidity premium; large liquidity premium lp_t reduces net profit r_t^*.

Fourth, we take an expectation $E[\cdot|\mathcal{H}_t]$ (or $E_t[\cdot]$) conditional on a new information set $\mathcal{H}_t$ available at time t. This corresponds to the following *ex-ante* (expected) present-value identity:

$$pd_t^l \approx E\left[\sum_{j=0}^{\infty}(\rho_l)^j\left[\beta_1 \Delta d_{t+1+j} - \beta_2 \Delta c_{t+1+j} - r_{t+1+j} + lp_{t+1+j}\right]\middle|\,\mathcal{H}_t\right], \tag{7}$$

where Δ denotes the first difference operator (e.g., $\Delta d_t = d_t - d_{t-1}$), $\beta_1 = \frac{\rho_l(1-\rho)}{\rho(1-\rho_l)} = \frac{1}{1-\bar{C}/\bar{D}} > 1$ because $\rho_l > \rho$, $\beta_2 = \frac{\rho_l - \rho}{\rho(1-\rho_l)} = \frac{\bar{C}/\bar{D}}{1-\bar{C}/\bar{D}} > 0$, and $pd_t^l \equiv p_t - \beta_1 d_t + \beta_2 c_t$ is the log *liquidity-adjusted price–dividend ratio*. Importantly, present-value identity (7) provides a restrictive structure to identify the unique cointegration relationship as below.

PROPOSITION 1: Cointegration restriction

If log prices share a common trend with log dividends and log illiquidity costs, their linear combination must satisfy

$$1 - \beta_1 + \beta_2 = 0,$$

given that expected dividend growth, illiquidity growth, real returns, and liquidity premia are stationary.

Proof: It is straightforward to show Proposition 1 from $\beta_1 = \frac{1}{1-\bar{C}/\bar{D}}$ and $\beta_2 = \frac{\bar{C}/\bar{D}}{1-\bar{C}/\bar{D}}$.

The key premise here is that the illiquidity cost is referred to as a negative dividend (Jones 2002; Acharya and Pedersen 2005). Hence, the total weights of positive and negative dividends can add up to one: $\beta_1 - \beta_2 = 1$. The cointegrating vector between log prices and log total dividends becomes $[1, -(\beta_1 - \beta_2)]' = [1, -1]'$, as in price–dividend ratio $pd_t \equiv p_t - d_t$.

Suppose that Proposition 1 holds with data. Then high prices relative to total dividends (i.e., pd_t^l) signal high expected dividend growth, low expected illiquidity growth, low expected returns, and high expected liquidity premia in the future. At a technical level, the inclusion of negative dividends gives rise to two additional expectations of (a) illiquidity growth and (b) liquidity premia. Notably, these additional sources have an offsetting effect on prices.

To ease of grasp, let us interpret the costs due to *illiquidity* as c_t and inversely the 'gains' due to *liquidity* as $-c_t$. For example, a price rise indicates an *increase* in expected 'liquidity' (i.e., $-c_t - E_t\left[\sum_{j=0}^{\infty}(\rho_l)^j \beta_2 \Delta c_{t+1+j}\right]$). At the same moment, the price rise also signals a *decline* in expected 'liquidity' premia (i.e., $-E_t\left[\sum_{j=0}^{\infty}(\rho_l)^j(-lp_{t+1+j})\right]$), implying lower compensation for bearing market liquidity risk in the future. We will use this conversion with single quotation marks when interpreting our results to avoid confusion (e.g., high liquidity is easier to interpret than low illiquidity).

In sum, our present-value identity provides a unified framework to link the conventional literature (e.g., Campbell and Shiller, 1988; Amihud and Mendelson, 1986). Interested readers are referred to Appendix A.

3. Data and Estimation

3.1. A proxy for stock liquidity

We have one serious obstacle: stock liquidity is hard to define and measure. In fact, the "legitimate" illiquidity costs defined in Section 2 comprise several components (Jones, 2002;

Vayanos and Wang, 2013): the brokerage cost, the bid-ask spread, the price impact, the opportunity cost, and so forth. Practically speaking, it is almost impossible to combine all the components. Although the bid-ask spread is widely used, it alone is not the representative and also does not cover a sufficiently long interval.[5] Indeed, numerous liquidity measures used in the literature are literally proxies, *not* the legitimate illiquidity costs themselves.

To impliment empirical analysis, we want to narrow down the scope of cost c_t to the annual value of Amihud's (2002) measure c_t^* . This proxy has a couple of merits. First, Amihud (2002) shows that c_t^* is an effective measure of the price impact referred to as the concept of Kyle's (1985) lambda. Second, Hasbrouck (2009) shows that c_t^* is *most strongly* correlated with the price impact coefficient of high-frequency Trade and Quote (TAQ) data. Third, Goyenko et al. (2009) show that c_t^* provides a good measure of the high-frequency data, even in the decimals regime.

Concretely, we construct the annual Amihud measure C_t^* from the daily CRSP data from 1926 to 2017. First, we single out ordinary common shares listed on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and the Nasdaq Stock Market (NASDAQ). Moreover, each firm i with more than 200 days of transactions in year t must have the last trading price at the end of the year. Second, we average out the daily Amihud measures for each filtered share in year t in the way of

$$C_t^i \equiv \frac{1}{N_t^i} \sum_{d=1}^{N_t^i} \frac{\left|R_{d,t}^i\right|}{DVOL_{d,t}^i} \times 10^6,$$

where N_t^i is the number of trading days in year t, $|R_{d,t}^i|$ is the absolute daily rate of return on day d in year t, and $DVOL_{d,t}^i$ is dollar volume (i.e., trading volume × price). Note that a high value of c_t^i means high *illiquidity*, which occus when high price variation goes along with low

[5] The U.S. markets transaction data are only available since 1983 (Goyenko et al., 2009).

trading volume. Third, we winsorize all individual cross-section measures in year t over the range of [1%, 99%] to remove outliers.

It is important to note that when aggregating all the cross-section measures into market-wide illiquidity C_t^*, we use the median in year t throughout the paper:

$$C_t^* = \text{median}(C_t^i) \tag{8}$$

for all the filtered firms i. In contrast, most of the conventional literature has tended to use the arithmetic average. We contend that a median reflects a better central tendency for cross-section data than does an average, especially when the data likely suffer from lots of extreme outliers. For ease of grasp, Figure 2 plots the natural logarithms of the cross-section median and average in year t.

[INSERT FIGURE 2 HERE]

Both median and average tend to move in tandem (Figure 2). The apparent downward slope, primarily driven by an increase in dollar volume (denominator), can point to an increase in liquidity due to technological innovations, changes in regulation, and changes in stock market participation (Chordia et al., 2000; Hasbrouck and Seppi, 2001; Jones, 2002; Chordia et al., 2008; Lettau and Nieuwerburgh, 2008). Some spikes are associated with market turmoil, for example, in the recent financial crisis.

Due to the increase in dollar volume over past decades, both series appear to be nonstationary.[6] A large volume of the literature (e.g., Amihud and Mendelson, 1986; Chordia et al., 2001; Jones 2002; Acharya and Pedersen, 2005; among other cross-sectional studies) tends to use either the proportional transaction cost proxies or the first difference in a *return* measure context. Doing so even manifests nonstationary nature inherent in the liquidity data.

[6] The first-order autocorrelation of the average is about 0.95, while that of the median is 1.01.

Hence, we want to keep up the intrinsic feature in a *price* measure context to accord with our cost definition (Section 2).

Next, let us turn to the difference between the two series. Since the early 1990s, their gap has been widening in the way that the means are larger than the medians. We give great attention to the period when prices move substantially relative to dividends. The gap can widen because highly illiquid outliers of small size firms increase in number. In other words, small- and medium-cap firms since then likely have a massive influence on market illiquidity. This finding is related to that of Ben-Rephael et al. (2015), who show that liquidity is significantly priced among NASDAQ stocks in the recent period.

Such a trend-like pattern also has supportable evidence. Acharya and Pedersen (2005) document that the Amihud measure itself is *not* stationary. Hasbrouck (2009) also reveals that the distributions of the TAQ and CRSP/Gibbs measures do *not* seem to be stationary. Ben-Rephael et al. (2015) illustrate several liquidity measures including the Amihud proxy, which also exhibit the observed trend pattern.

3.2. Liquidity-adjusted price–dividend ratios

Direct calculation of *pdl* ratio pd_t^l through Proposition 1 is a difficult task for the following reasons. First, we have to resort to price impact proxy c_t^* , *not* actual legitimate cost c_t. Second, even though c_t^* is assumed to be the perfect proxy, its unit differs from log price p_t and log dividend d_t, because c_t^* is measured in percent per dollar.

To handle this difficulty, we assume that liquidity measure C_t^* is proportional to illiquidity cost C_t: $C_t^* = A \cdot C_t$ as in Bekaert et al. (2007). The logarithm permits $c_t^* = \log C_t^*$ to divide into c_t and proportional constant $a = \log A$: constant a has nothing to do with any stochastic behavior. We then put a restriction on $\hat{\beta}_2 = \hat{\beta}_1 - 1$ through Proposition 1 and estimate $pd_t^l = p_t - \hat{\beta}_1 d_t + \hat{\beta}_2 c_t^*$ through OLS in the Engle-Granger way of

$$p_t = \hat{\beta}_0 + \hat{\beta}_1 d_t - \hat{\beta}_2 c_t^* + \epsilon_t,$$

where $\hat{\beta}_0$ is an intercept; $\hat{\beta}_1$ and $\hat{\beta}_2$ are estimates of β_1 and β_2, respectively; and ϵ_t denotes the cointegrating-equation error. Thus, pd_t^l can be expressed as $\hat{\beta}_0 + \epsilon_t$. In the rest of the paper, we use a hat notation to distinguish such prior estimation from our main analysis in Sections 4 and 5.

To implement this precisely, we use P_t from the annual CRSP value-weighted index from 1926 to 2017 at the end (December) of year t. We also recover dividend D_t from the CRSP value-weighted returns with and without dividends via

$$D_t = \frac{D_t}{P_t} \times P_t = \left(\frac{R_t}{Rx_t} - 1\right) \times P_t,$$

where R_t denotes CRSP value-weighted gross return with dividends, and Rx_t denotes the CRSP value-weighted return without dividends (see Appendix A.2 of Cochrane (2011) for details). All three series (e.g., P_t, D_t, and C_t^*) are deflated by the Consumer Price Index (CPIIND) in December of year t.[7] Another merit of using the annual frequency is that we can avoid strong seasonality in dividend payments.

With these series in hand, we generate the estimates of the shared trend as

$$p_t = \hat{\beta}_0 + \hat{\beta}_1 \cdot d_t - \hat{\beta}_2 \cdot c_t^* = 3.156 + 1.084 \cdot d_t - 0.084 \cdot c_t^*.^{8} \tag{9}$$

All three estimates are significant at the 5% level under the Newey and West's (1987) *t*-statics. The estimates of $\hat{\beta}_1 = \frac{1}{1-\bar{C}/\bar{D}}$ and $\hat{\beta}_2 = \frac{\bar{C}/\bar{D}}{1-\overline{C/D}}$ (Section 2) suggest that the historical amount of

[7] In fact, Amihud measure c_t^* is not perfectly inflation-adjusted. To attain this, one should apply the daily inflation adjustment to individual instruments' measures, even though CPIIND in CRSP is released on a monthly basis. One can use interpolation to obtain daily CPIINDs with monthly CPIINDs, but even doing so does not eschew data processing. For simplicity of calculation, we ignore the precise construction in the paper.

[8] We notice that the estimation result is quite robust. In particular, dynamic least squares yield $p_t = 3.182 + 1.072 \cdot d_t - 0.088 \cdot c_t^*$, and fully-modified least squares deliver $p_t = 3.208 + 1.055 \cdot d_t - 0.096 \cdot c_t^*$.

negative dividends (contingent on price impact c_t^*) is roughly 7.7% of positive dividends on average: $\bar{C}/\bar{D} = \hat{\beta}_2/\hat{\beta}_1$ during our sample.

There are three reasons why Proposition 1 must hold in estimating cointegrating regression (9). First, doing so ensures that *pdl* ratio pd_t^l is a *sole* representation of present-value identity (7). Second, the sole representation also alleviates a probable concern for log-linear approximation errors.[9] Third, when estimating pd ratio $pd_t = p_t - d_t$ in practice, even log price p_t and log dividend d_t do *not* have the theoretical trend relationship of $[1, -1]$ precisely. What this means is that conducting the direct estimation of pd_t^l does not ensure the cointegration relationship of $p_t - \hat{\beta}_1 d_t + (\hat{\beta}_1 - 1)c_t^*$ where $\hat{\beta}_2 = \hat{\beta}_1 - 1$.

The estimates of $\hat{\beta}_1 = \frac{\hat{\rho}_l(1-\hat{\rho})}{\hat{\rho}(1-\hat{\rho}_l)} = 1.084$ and $\hat{\beta}_1 = \frac{\hat{\rho}_l - \hat{\rho}}{\hat{\rho}(1-\hat{\rho}_l)} = 0.084$ also allow for guessing a discount factor $\hat{\rho}_l$ as the estimate of ρ_l. Specifically, we first calculate $\hat{\rho} = 1/(1 + \exp(\overline{dp})) \approx 0.966$ based on the Campbell and Shiller's present-value identity and then back $\hat{\rho}_l \approx 0.969$ out of the two beta formulas. The fact of $\hat{\rho}_l > \rho$ also grants the validity of present-value identity (7) (Section 2). To sum up, our proxy-dependent expression is

$$pd_t^l \approx E\left[\sum_{j=0}^{\infty}(\hat{\rho}_l)^j\left[\hat{\beta}_1 \Delta d_{t+1+j} - \hat{\beta}_2 \Delta c_{t+1+j}^* - r_{t+1+j} + lp_{t+1+j}\right]\middle|\mathcal{H}_t\right]. \tag{10}$$

Table 1 reports summary statistics for the variables used in the rest of our analysis.

[INSERT TABLE 1 HERE]

The summary statistics show that pd_t^l embodies distinct information from pd_t as also displayed in Figure 1. In Panel A, the standard deviation of pd_t^l (0.269) is less volatile than that of pd_t (0.433). In Panel B, the correlation between pd_t^l and pd_t is 0.621. In Panel C, pd_t^l

[9] If Proposition 1 does not hold with $\hat{\beta}_2 \neq \hat{\beta}_1 - 1$, doing so can create an unidentifiable error ς_t:

$$r_{t+1} \approx \hat{\rho}_l \cdot pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - pd_t^l + lp_{t+1} + \varsigma_{t+1}.$$

gives more stationary evidence than pd_t, although pd_t^l delivers somewhat controversial interpretations for unit root.[10]

4. Empirical analysis

Predictive analysis by present-value logic (10) is another way of representing how much of each information source accounts for *pdl* ratio pd_t^l. The stronger predictability is, the more share each information source has. In contrast, the extant literature usually confines the forecastable relationship between returns and illiquidity exclusively without the present-value structure (e.g., Amihud, 2003; Jones, 2002, Bekaert et al., 2007).

Such forecastable evidence should also be *comprehensively* considered in the accounting sense. As for price–dividend ratios, Cochrane (2008) shows that return predictability is evident due to the *lack* of dividend growth forecastability because return forecastability is a flip side of cash flow forecastability in light of the *conventional* information set. As such, we can address a similar question: How much of illiquidity-growth forecastability by pd_t^l is more predictable than the others?

In the conventional price–dividend relationship, it is worth noting that return forecastability occurs from *cyclical* price fluctuations relative to dividend ones (Cochrane, 1994). From this standpoint, we cast doubt on forecastable evidence of proportional transaction cost proxies (say, price–cost ratios) popularly used in the extant literature: Does the forecastable evidence come from either negative dividends or prices? This unclear question spotlights the strength of our approach that it can distinguish the cause of forecastability.

[10] In particular, pd_t^l appears to be nonstationary in the Phillips-Perron (PP) test but stationary in the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test. However, it is well known that unit root tests such as the PP test lack the power to reject the null hypothesis (Campbell, 2003).

4.1. Simple forecasting regression

We start from four kinds of simple forecasting regressions:

$$\Delta d_{t+1} = a_d + b_d \cdot pd_t^l + \varepsilon_{t+1}^d, \tag{11}$$

$$\Delta c_{t+1}^* = a_{c^*} + b_{c^*} \cdot pd_t^l + \varepsilon_{t+1}^{c^*}, \tag{12}$$

$$r_{t+1} = a_r + b_r \cdot pd_t^l + \varepsilon_{t+1}^r, \tag{13}$$

$$pd_{t+1}^l = a_\phi + \phi \cdot pd_t^l + \varepsilon_{t+1}^{pdl}, \tag{14}$$

where ε_{t+1}^d denotes a dividend shock, $\varepsilon_{t+1}^{c^*}$ denotes an illiquidity shock, ε_{t+1}^r denotes a return shock, and ε_{t+1}^{pdl} denotes a *pdl* shock. The first four rows of Table 2 present estimation results in regressions (11)–(14), respectively. For robustness, the results on *pd* ratio pd_t are reported in the last three rows of Table 2.

[INSERT TABLE 2 HERE]

Overall, short-term forecastable evidence is *not* clear. As mentioned in Stambaugh (1999), it could be argued that our regressor pd_t^l on the RHS is still persistent (Panel C, Table 1), so its inherent near unit root might cause biased estimates (b_x) and *t*-statistics. Simply put, the autocorrelations ϕ of pd_t^l in regression (14) (0.866; 4th row) and pd_t (0.942; 7th row) do not differ from each other at the conventional levels.

Besides, dividend-growth regression coefficient b_d is *not* statistically significant, and R^2 (1.1%) is also quite low (1st row). Even so, the sign of b_d turns out to be negative as opposed to the positive sign by present-value logic (10). However, the forecasting regression of dividend growth Δd_{t+1} on *pd* ratio pd_t (5th row) also delivers the opposite direction as shown in Cochrane (2008). Similarly, log illiquidity growth Δc_{t+1}^* exhibits the opposite sign of $b_{c^*} =$ 0.126 with the R^2 of 0.68% (2nd row).

A key finding is that one-period return coefficient b_r of the *pdl* ratio (3rd row) is *more than* twice as large as that of the *pd* ratio (6th row) in an absolute sense. In particular, our return coefficient $b_r = 0.205$ can be strongly supported by the *t*-statistic of -3.214, despite the small R^2 of 7.77% and the standard deviation 5.5% of the fitted value (i.e., $\sigma(a_r + b_r \cdot pd_t^l)$). In contrast, the return regression on pd_t yields $b_r = -0.085$ with the *t*-statistic of -1.894, the R^2 of 3.49%, and the standard deviation 3.69%. This conclusion is analogous to that of Cochrane (2008).

Why does pd_t^l look like a better return predictor than pd_t? The main answer is that market illiquidity can adjust a price trend (Figure 2). To see this , Figure 3 graphs one-period real return r_{t+1}, the one-period return forecast on pd_t^l (i.e., $a_r + b_r \cdot pd_t^l$; see (13)), and the one-period return forecast on pd_t.

[INSERT FIGURE 3 HERE]

Clearly, the return forecast of pd_t^l better helps to trace the return behavior than does the forecast of pd_t. To highlight the illiquidity effect merely, we also plot the return forecast on the scaled *pd* ratio given by $p_t - \hat{\beta}_1 d_t$, but it cannot outperform the *pdl* forecast.

Does it mean that pd_t^l forecasts more returns than does pd_t? No, because the two ratios underlie different information sets, especially with respect to market illiquidity. To say the least, such illiquidity should carry another information so that it can make the radical difference. For example, illiquidity uncertainty can give rise to the additional liquidity premium not covered by the conventional information set (see Appendix A). From the next section, we will devote special attention to the liquidity premium.

4.2. Short-term liquidity premium

We start off by restating present-value identity (10) in a short period:

$$lp_{t+1} = -\hat{\rho}_l \cdot pd_{t+1}^l - \hat{\beta}_1 \Delta d_{t+1} + \hat{\beta}_2 \Delta c_{t+1}^* + r_{t+1} + pd_t^l. \quad (15)$$

Here, we assume no serious approximation errors forced by $1 - \hat{\beta}_1 + \hat{\beta}_2 = 0$ (Proposition 1) and then link forecasting regressions (11)–(14) to *implied* liquidity premium lp_{t+1} in (15).

Following Cochrane (2008), we regress both sides of (15) on *pdl* ratio pd_t^l. In brief, the LHS of (15) can be expresses as

$$lp_{t+1} = a_{lp} + b_{lp} \cdot pd_t^l + \varepsilon_{t+1}^{lp}, \quad (16)$$

where ε_{t+1}^{lp} denotes a liquidity-premium shock. The RHS respectively yields the following forecast and error identities:

$$b_{lp} = 1 - \hat{\rho}_l \cdot \phi - \hat{\beta}_1 \cdot b_d + \hat{\beta}_2 \cdot b_{c^*} + b_r, \quad (17)$$

$$\varepsilon_{t+1}^{lp} = -\hat{\rho}_l \cdot \varepsilon_{t+1}^{pdl} - \hat{\beta}_1 \cdot \varepsilon_{t+1}^d + \hat{\beta}_2 \cdot \varepsilon_{t+1}^{c^*} + \varepsilon_{t+1}^r. \quad (18)$$

The two identities in (17)–(18) make it clear that the implied liquidity premium is connected to the other four sources of information (i.e., ε^{pdl}, ε^d, ε^{c^*}, ε^r) in the present-value context.

With the regression coefficients in Table 2 in hand, forecast identity (17) produces $b_{lp} = 0.027$, and error identity (18) delivers the 0.4% standard deviation: $\sigma(\varepsilon^{lp}) \approx 0.42\%$. From the '*liquidity*' perspective (Section 2), the converted coefficient (i.e., $-b_{lp} = -0.027$) implies that investors need *lower* compensation for market liquidity risk in response to the observation that high prices relative to total (positive plus negative) dividends today (i.e., a *pdl* rise).

To understand this concretely, suppose that investors observe that prices are low relative to total dividends (i.e., a *pdl* drop) in the event of market turmoil when market liquidity dries up. In this turmoil, investors want a high premium, thereby corresponding to an increase in expected returns: $E_t[r_{t+1}] \approx b_r \cdot pd_t^l = 0.205$, where $pd_t^l = -1$ for example. Additionally, the liquidity dry-up should be compensated for taking the risk of high transaction costs;

therefore, expected net returns should be far higher than expected returns: $E_t[r_{t+1}^*] = E_t[r_{t+1} - lp_{t+1}] \approx (b_r - b_{lp}) \cdot pd_t^l = 0.205 + 0.027 = 0.232$ by accounting identity (6). As a result, prices drop more sharply due to market liquidity risk.

In fact, the unusual signs of b_d and b_{c^*} (Subsection 4.1) might raise attention because they distort the innate attributes of the implied liquidity premium. In Appendix B where we use the quarterly bid-ask spread after 1983, however, we confirm the correct signs of log dividend growth and spread growth. Despite the different time frequencies and proxies, the liquidity-premium coefficient in Appendix B turns out to be positive: $b_{lp} = 0.005$.

Now, we go on to address the empirical question, "Is the liquidity premium of the CRSP market portfolio predictable?" Putting all regression results into perspective, our null hypothesis, the *constant-expected-liquidity-premium model*, has the following form:

$$\mathrm{H}_0\colon b_{lp} = 1 - \hat{\rho}_l \cdot \phi - \hat{\beta}_1 \cdot b_d + \hat{\beta}_2 \cdot b_{c^*} + b_r = 0. \qquad (19)$$

As a result, linear Wald test (19) delivers that $b_{lp} = 0.027$ is substantially far away from the null at the conventional levels; the χ^2-statistic is over 200. This evidence points out that liquidity premium lp_{t+1} is strongly predictable by *pdl* ratio pd_t^l over short horizons. It should thus be priced in a short period, implying that investors *myopically* worry about paying high transaction costs. Hence, current prices should be cheaper due to market liquidity risk.

Does this strong evidence naturally impart a substantive contribution of long-run liquidity premia to prices? In other words, do investors want compensation for taking market liquidity risk in the long run? We do *not* know the concrete answer yet, which essentially hinges on the long-run predictable relationship among prices, dividends, illiquidity, and returns as will be discussed in the next section.

4.3. Long-term liquidity premium

Following Cochrane (2008), we first divide both sides of short-term forecast identity (17) by $1-\hat{\rho}_l \cdot \phi$ and then rearrange it as

$$\hat{\beta}_1 \cdot b_d^{lr} - \hat{\beta}_2 \cdot b_{c^*}^{lr} - b_r^{lr} + b_{lp}^{lr} = 1, \tag{20}$$

where $b_d^{lr} = b_d/(1-\hat{\rho}_l \cdot \phi)$, $b_{c^*}^{lr} = b_{c^*}/(1-\hat{\rho}_l \cdot \phi)$, $b_r^{lr} = b_r/(1-\hat{\rho}_l \cdot \phi)$, and $b_{lp}^{lr} = b_{lp}/(1-\hat{\rho}_l \cdot \phi)$. In basic, each component of long-term forecast identity (20) stands for the variance fraction of a set of information $[\Delta d_t, \Delta c_t^*, r_t, lp_t]$ on the LHS, so that they add up to the variance of *pdl* ratio pd_t^l (100%) on the RHS. For example, we rewrite the following ex-post identity:

$$pd_t^l = \sum_{j=0}^{\infty}(\hat{\rho}_l)^j\left[\hat{\beta}_1 \Delta d_{t+1+j} - \hat{\beta}_2 \Delta c_{t+1+j}^* - r_{t+1+j} + lp_{t+1+j}\right]. \tag{21}$$

For example, $b_{lp}^{lr} = \frac{\mathrm{Cov}\left(\sum_{j=0}^{\infty}(\hat{\rho}_l)^j lp_{t+1+j},\ pd_t^l\right)}{\mathrm{Var}(pd_t^l)}$ refers to the regression coefficient of long-run liquidity premia $\sum_{j=0}^{\infty}(\hat{\rho}_l)^j\ lp_{t+1+j}$ on pd_t^l in (21). An easy way of computing such long-run estimates is to *infer* them from the results of the simple forecasting regressions (Table 2). Table 3 reports the first three long-run estimates of $\hat{\beta}_1 \cdot b_d^{lr}$, $\hat{\beta}_2 \cdot b_{c^*}^{lr}$, and b_r^{lr} and standard errors calculated from the standard delta method.

[INSERT TABLE 3 HERE]

First, we examine long-run dividend growth forecast $\hat{\beta}_1 \cdot b_d^{lr} = -0.377$. As mentioned before, its sign is unintuitive, which is totally misled by short-term dividend estimate $b_d = -0.056$ (Table 2). The *t*-statistic of $\hat{\beta}_1 \cdot b_d^{lr} = -0.377$ (-0.958) also narrates that the subsequent dividend variation *barely* accounts for the price volatility. In the similar sense, long-run illiquidity growth forecast $\hat{\beta}_2 \cdot b_{c^*}^{lr} = 0.066$ is of little economic significance; the *t*-statistic is 0.168.

Next, we turn to long-run return estimate $b_r^{lr} = -1.276$ with the *t*-statistic of -2.932.[11] This number gives a strong indication for return forecastability, which is analogous to the implication of the *pd* ratios as shown in Cochrane (2008). When using simple excess return forecast b_{er} where $er_{t+1} = a_{er} + b_{er} \cdot pd_t^l + \varepsilon_{t+1}^{er}$, $b_{er}^{lr} = -1.262$ with the *t*-statistic (-2.785) delivers the similar result (last column).[12]

All long-horizon results suggest that market illiquidity (i.e., $\hat{\beta}_2 \cdot b_{c^*}^{lr}$) cannot fully help to attenuate the excess volatility (i.e., b_r^{lr}) inherent in prices because it merely provides a natural price trend. If its variation was so first-order as to deviate away from the trend a lot, the excess volatility referred to as pricing errors should in return be substantially reduced. The interpretation along with the cointegration concept is that when investors observe variation in *pdl* ratios, almost all mean reverting behavior arises from changes in expected returns, *not* from changes in positive and negative dividends; this conclusion does not differ from that of *pd* ratios (Cochrane, 2008).

Now, we move on to address our central question, "Are long-run liquidity premia for the CRSP market portfolio a main source of price variation?" Do investors care about such premia in a long-term perspective? Algebraically, long-term forecast identity (20) leads to $b_{lp}^{lr} = 1 - \left(\hat{\beta}_1 \cdot b_d^{lr} - \hat{\beta}_2 \cdot b_{c^*}^{lr} - b_r^{lr}\right) = 0.167$, implying that about 16.7% of the price volatility comes from changes in expected liquidity premia.

---

[11] Note that all the information sources are not orthogonal. That is why the return estimate over 100% can arise.

[12] To calculate $b_{er}^{lr} = -1.262$, we use the following identity:

$$pd_t^l = E_t\left[\sum\nolimits_{j=0}^{\infty} (\hat{\rho}_l)^j \left\{\hat{\beta}_1 \Delta d_{t+1+j} - \hat{\beta}_2 \Delta c_{t+1+j}^* - er_{t+1+j} - r_{t+1+j}^f + lp_{t+1+j}\right\}\right],$$

where r_t^f is the three-month T bill rate such that $er_t = r_t - r_t^f$.

To assess its economic significance, we hypothesize the constant-expected-liquidity-premium model as

$$\mathrm{H}_0: b_{lp}^{lr} = 1 - \hat{\beta}_1 \cdot b_d^{lr} + \hat{\beta}_2 \cdot b_{c^*}^{lr} + b_r^{lr} = 0. \tag{22}$$

More precisely, we adopt the non-linear Wald test statistic in Eq. (22):

$$\lambda' \left(\partial\lambda/\partial\gamma' \, \theta \, \partial\lambda/\partial\gamma \right)^{-1} \lambda,$$

where $\lambda = 1 - \hat{\beta}_1 \cdot b_d^{lr} + \hat{\beta}_2 \cdot b_{c^*}^{lr} + b_r^{lr}$ is defined as the scalar of deviations imposed by (22); $\gamma = [b_d, b_{c^*}, b_r, \phi]'$ is the vector of the short-term estimates; θ is the estimated variance-covariance matrix in regressions (11)–(14); and $\partial\lambda/\partial\gamma$ is the partial derivative with respect to estimate vector γ. The Wald-test statistic follows a χ^2 distribution with degrees of freedom n equal to the number of observable variables: $n = 4$ in our case. The χ^2-statistic for $\mathrm{H}_0: b_{lp}^{lr} = 0$ is about 2.09, which is close to the null at the conventional levels: the *p*-value is about 72.0%. In sum, the liquidity premium is *not* a main source of price variation, suggesting that investors are not attentive to market illiquidity with great care in forming their *long-term* strategies.

5. Impulse response functions

Why do investors exhibit the different responses to illiquidity in market index returns over both horizons, respectively? To answer this, we go on to investigate how expected liquidity premia move forward through time. We thus display impulse response functions proposed by Cochrane (2011) in the present-value context with reference to a set of error shocks, $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}]$.

5.1. Error standard deviations and correlations

Before jumping into impulse response functions, it is important to scrutinize the empirical relationship between the error shocks $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}]'$. Panel A of Table 4 presents error standard deviations on the diagonal and correlations on the off-diagonal for the error shocks.

[INSERT TABLE 4 HERE]

We find two outstanding observations. First, *pdl* shock ε^{pdl} is strongly correlated with return shock ε^{r}: $\mathrm{Corr}(\varepsilon^{pdl}, \varepsilon^{r}) \approx 50.3\%$. This feature accords well with that of *pd* ratios (Cochrane, 2008); our sample reveals $\mathrm{Corr}(\varepsilon^{pd}, \varepsilon^{r}) \approx 68.4\%$. Of course, ε^{pdl} never comes out independently without dividend shock ε^{d} and illiquidity shock ε^{c^*}: $\mathrm{Corr}(\varepsilon^{pdl}, \varepsilon^{d}) \approx -31.0\%$ and $\mathrm{Corr}\left(\varepsilon^{pdl}, \varepsilon^{c^*}\right) \approx -22.5\%$. Putting all correlation sizes into perspective, we find that return shock ε^{r} is a more dominant factor tied to *pdl* shock ε^{pdl} than the other shocks. Second, liquidity-premium shock ε^{lp} is almost perfectly and positively correlated with *pdl* shock ε^{pdl}. The strong correlation reveals that the *pdl* ratio could be a good proxy for the liquidity premium. For robustness check, we also use the quarterly bid-ask spreads in Appendix B and also find the strong correlation (0.932).

With the second observation in mind, Figure 2 describes that the liquidity premium was high from the last 1990s to the early 2000s, implying that at that time investors took great care of liquidating their portfolios in the *near* future. In the recent global crisis, however, it was relatively low, suggesting that market liquidity risk was *not* a main culprit of the hard time.

5.2 Impulse-response functions

In fact, there are four sources of giving rise to liquidity premia, since by design the implied liquidity premium is inferred from prices, dividends, illiquidity costs, and returns (Section 4).

Among these sources, we draw great attention to an illiquidity-driven shock throughout the analysis. To understand this simply, let us express the gross net-return relationship:

$$R_{t+1}^{*} = R_{t+1} - LP_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t} - \frac{C_{t+1}}{P_t}. \tag{23}$$

As can be seen in (23), in principle, a rise in market illiquidity ($C_{t+1} \uparrow$) with no other changes is a primary driver of raising liquidity premia ($LP_{t+1} \uparrow$). When market illiquidity is *not* a source of price variation at all, it is important to note that our price–dividend–cost relationship must shrink to the conventional price–dividend one.[13]

Our focal point is to examine whether illiquidity shock ε^{c^*} is so transient that investors could become *less* insensitive to market illiquidity in the long run. To conduct the examination, we design the illiquidity shock with no current changes in *pdl* ratio, dividend, and return:

$$\varepsilon^{pdl} = 0, \qquad \varepsilon^{d} = 0, \qquad \varepsilon^{c^*} = 1, \qquad \varepsilon^{r} = 0, \qquad \varepsilon^{lp} = \hat{\beta}_2. \tag{24}$$

Such a shock identification follows a first-order VAR system:[14]

[13] For example, the variance decomposition of present-value identity (21) yields

$$\mathrm{var}(pd_t^l) = \mathrm{cov}\left[pd_t^l, \sum_{j=0}^{\infty}(\hat{\rho}_l)^j \hat{\beta}_1 \Delta d_{t+1+j}\right] - \mathrm{cov}\left[pd_t^l, \sum_{j=0}^{\infty}(\hat{\rho}_l)^j \hat{\beta}_2 \Delta c_{t+1+j}^{*}\right] - \mathrm{cov}\left[pd_t^l, \sum_{j=0}^{\infty}(\hat{\rho}_l)^j r_{t+1+j}\right] + \mathrm{cov}\left[pd_t^l, \sum_{j=0}^{\infty}(\hat{\rho}_l)^j lp_{t+1+j}\right].$$

When market illiquidity is *not* a priced factor (see Appendix A), it should shrink to

$$\mathrm{var}(pd_t) = \mathrm{cov}\left[pd_t, \sum_{j=0}^{\infty}(\rho)^j \Delta d_{t+1+j}\right] - \mathrm{cov}\left[pd_t, \sum_{j=0}^{\infty}(\rho)^j r_{t+1+j}\right].$$

[14] One can add lags into a higher-order VAR system, but we want to keep the parsimonious VAR in the interest of brevity. We admit that such a shock definition might be oversimplification, but it also has a merit of facilitating the analysis simply.

$$
\begin{bmatrix} pd_{t+1}^{l} \\ \Delta d_{t+1} \\ \Delta c_{t+1}^{*} \\ r_{t+1} \\ lp_{t+1} \end{bmatrix} = \begin{bmatrix} \phi \\ b_d \\ b_{c^*} \\ b_r \\ 1 - \hat{\rho}_l \cdot \phi - \hat{\beta}_1 \cdot b_d + \hat{\beta}_2 \cdot b_{c^*} + b_r \end{bmatrix} \cdot pd_t^l + \begin{bmatrix} \varepsilon_{t+1}^{pdl} \\ \varepsilon_{t+1}^{d} \\ \varepsilon_{t+1}^{c^*} \\ \varepsilon_{t+1}^{r} \\ -\hat{\rho}_l \cdot \varepsilon_{t+1}^{pdl} - \hat{\beta}_1 \cdot \varepsilon_{t+1}^{d} + \hat{\beta}_2 \cdot \varepsilon_{t+1}^{c^*} + \varepsilon_{t+1}^{r} \end{bmatrix}. \tag{25}
$$

We also express a cumulative response of prices as $p_t = \sum_{j=1}^{t} \Delta p_j$ and represent the others as $\hat{\beta}_1 d_t = \sum_{j=1}^{t} \hat{\beta}_1 \Delta d_j$, and $\hat{\beta}_2 c_t^* = \sum_{j=1}^{t} \hat{\beta}_2 \Delta c_j^*$ in the spirit of present-value identity (21).[15] Figure 4 plots the impulse response functions in (24).

[INSERT FIGURE 4 HERE]

You find easily that the illiquidity shock is essentially *transient* (Figure 4). Concretely, the illiquidity shock ($\varepsilon^{c^*} = 1$) in (23) leads to a price rise, no dividend move, and a illiquidity drop at the point of the shock spot (time 1; 2nd panel). At this point onward, they stay *there* with no subsequent moves. Clearly, this stay-put evidence highlights the transient nature of market illiquidity. To the point, even a big but *transient* change in illiquidity (strong short-term evidence in Subsection 4.2) can end up with a small price variation (week long-term evidence in Subsection 4.3).

Our results disprove one common *fallacy* that a positive shock to illiquidity corresponds to (i) a rise in expected illiquidity, (ii) a rise in expected returns, and (iii) a drop in prices today, as stated in Acharya and Pedersen (2005). The transient illiquidity shock allows the first two premises (i) and (ii) *hardly* to happen, but *directly* comes to the last outcome (iii). The reason crystalizes again that market illiquidity just provides a natural trend for prices so that its *cyclical*

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[15] we write a change in prices with first-order VAR (25) as

$$
\begin{aligned}
\Delta p_{t+1} &= pd_{t+1}^{l} + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^{*} - pd_t^l \\
&= \underbrace{\left(\phi + \hat{\beta}_1 b_d - \hat{\beta}_2 b_{c^*} - 1\right)}_{= b_p} \cdot pd_t^l + \underbrace{\left(\varepsilon_{t+1}^{pdl} + \hat{\beta}_1 \varepsilon_{t+1}^{d} - \hat{\beta}_2 \varepsilon_{t+1}^{c^*}\right)}_{= \varepsilon_{t+1}^{p}},
\end{aligned}
$$

where $b_p = \phi + \hat{\beta}_1 b_d - \hat{\beta}_2 b_{c^*} - 1$, and $\varepsilon_{t+1}^{p} = \varepsilon_{t+1}^{pdl} + \hat{\beta}_1 \varepsilon_{t+1}^{d} - \hat{\beta}_2 \varepsilon_{t+1}^{c^*}$ denotes a price shock.

variation does *not* affect prices a lot. Another way of interpreting this technically is that it is nearly a random walk so that its cyclical variation could be roughly a white noise (or i.i.d.). Therefore, market liquidity risk should be second-order.

Suppose that market illiquidity has the excess volatility relative to prices and dividends. In this case, the cyclical variation in the *pdl* ratios should arise largely from the excess illiquidity fluctuations in favor of illiquidity-growth predictability. In doing so, we are supposed to reject the constant-expected-liquidity-premium model (Section 4), implying that market liquidity risk should be first-order.

In sum, the whole story above seems similar to that of the *pd* ratios (Cochrane, 2011) despite additional information (e.g., market illiquidity). Evidence is that $b_r^{lr} = \frac{b_r}{1-\hat{\rho}_l\phi} = -1.276$ is compelling in terms of the size and the significance (Subsection 4.3), implying that a small but *persistent* expected return change (week short-term evidence) builds up with horizons toward a huge price variation (strong long-term evidence). At its core, the fact that illiquidity shock ε^{c^*}—additional source of information—should be *transient* is why there is hardly a big difference between the two ratios. If the illiquidity shock was *persistent* as long as return shock ε^r, it should take more of the expected return shares referred to as pricing errors *not* justified by subsequent expected cash flows.

6. Conclusion

This study sheds new light on the long-horizon interplay between market illiquidity and prices. The key finding is that the illiquidity shock is so transient that its cumulative effect does not likely matter for price behavior in the long run. Indeed, investors are sensitive to current unexpected liquidity news, but seem not to consider it with great care in forming long-term portfolio decisions.

An important unanswered question is that improvement in liquidity can also cause a rise in a trading frequency. This causation likely incurs large effective transaction costs and high compensation as a result. Although we cannot present such evidence here, our trend implication suggests that investors should also be reluctant to entail the risk of the effective costs.

One may argue that small- and medium-cap. securities are of greater importance for liquidity pricing than our aggregate example. In this paper, our purpose is to offer an illiquidity pricing channel through present-value logic, *not* to deny the economic significance across asset classes. Even so, our new present-value identity can be applicable to such small- and medium-cap. securities as well. We leave it for future work.

We also open up a new venue of how to gauge the hard-to-detect liquidity premium, which is a function of prices, dividends, costs, and returns. One can attempt different measures of illiquidity costs to distill the implied liquidity premium and then compare which proxy is the essence of price variation. The last thing to remind is that the CRSP dividend series involves all distribution payments to investors, including cash mergers, liquidations, actual dividends, and so forth (Cochrane, 2008). Hence, one can further dissect such an inclusive series to find out which individual component is at its core for price variation. One of the good candidates may be share repurchases and issuances, as shown in Larrain and Yogo (2008). We also leave them for future research.

APPENDIX A: Derivation of log-linear approximation

For an arbitrary nonlinear function $f(x, y)$, it can be approximated around $\bar{x}$ and $\bar{y}$ as

$$f(x,y) \approx f(\bar{x},\bar{y}) + \left.\frac{\partial f}{\partial x}\right|_{x=\bar{x},y=\bar{y}} (x-\bar{x}) + \left.\frac{\partial f}{\partial y}\right|_{x=\bar{x},y=\bar{y}} (y-\bar{y}).$$

Using this first-order Taylor expansion yields Eq. (3):

$$\log(1+\exp(dp_{t+1}) - \exp(cp_{t+1})) = -\log(\rho_l) + \frac{\exp(\overline{dp})}{1+\exp(\overline{dp})-\exp(\overline{cp})}\left(dp_{t+1} - \overline{dp}\right).$$

$$-\frac{\exp(\overline{cp})}{1+\exp(\overline{dp})-\exp(\overline{cp})}(cp_{t+1}-\overline{cp}),$$

where $\rho_l = \frac{1}{1+\exp(\overline{dp})-\exp(\overline{cp})} = \frac{\bar{P}}{\bar{P}+\bar{D}-\bar{C}} > \rho = \frac{1}{1+\exp(\overline{dp})} = \frac{\bar{P}}{\bar{P}+\bar{D}}$, $\frac{\exp(\overline{dp})}{1+\exp(\overline{dp})-\exp(\overline{cp})} = \rho_l(1/\rho-1)$, $\frac{\exp(\overline{cp})}{1+\exp(\overline{dp})-\exp(\overline{cp})} = \rho_l(1/\rho-1/\rho_l)$, $\overline{dp} = \log(1/\rho-1)$, $\overline{cp} = \log(1/\rho-1/\rho_l)$, and

$$k_l = -\log(\rho_l) - \frac{\exp(\overline{dp})}{1+\exp(\overline{dp})-\exp(\overline{cp})}\cdot\overline{dp} + \frac{\exp(\overline{cp})}{1+\exp(\overline{dp})-\exp(\overline{cp})}\cdot\overline{cp}.$$

$$= -\log(\rho_l) - \rho_l(1/\rho-1)\log(1/\rho-1) + \rho_l(1/\rho-1/\rho_l)\log(1/\rho-1/\rho_l).$$

Substituting the Taylor expansion into Eq. (2) delivers the linear difference equation of the log net return:

$$r^*_{t+1} \approx k_l + \rho_l \cdot p_{t+1} + \rho_l(1/\rho-1)d_{t+1} - \rho_l(1/\rho-1/\rho_l)c_{t+1} - p_t.$$

Proposition 1 carries

$$r^*_{t+1} \approx k_l + \rho_l \cdot pd^l_{t+1} + \beta_1 \Delta d_{t+1} + \beta_2 \Delta c_{t+1} - pd^l_t.$$

When illiquidity is *not* a source of price variation, $\rho_l = \rho$ and $k_l = k$, where $k = -\log(\rho) - (1-\rho)\log(1/\rho-1)$ is the log-linear coefficient of log real return (Campbell and Shiller, 1988). In this case, Eq. (4) corresponds to the conventional difference equation of the real return:

$$r_{t+1} \approx k + \rho\cdot p_{t+1} + (1-\rho)d_{t+1} - p_t = k + \rho\cdot pd_{t+1} + \Delta d_{t+1} - pd_t,$$

where $pd_t \equiv p_t - d_t$ is log price–dividend ratio.

Some may argue that our framework requires stationary dp_{t+1} and cp_{t+1} in Eq. (3) to apply the first-order Taylor expansion. If Proposition 1 holds in the real data, we contend that the stationary property for dp_{t+1} and cp_{t+1} is *not* necessary. For example, suppose that $\exp(z)$

is given by $\exp(x) - \exp(y)$. An arbitrary nonlinear function $f(z) = \log(1 + \exp(z))$ can be approximated by the first-order Taylor expansion around $\bar{z}$:

$$f(z) \approx \log(1 + \exp(\bar{z})) + \frac{\partial f}{\partial z}\bigg|_{z=\bar{z}} (z - \bar{z}).$$

When x and y are restricted by $1 - \beta_1 + \beta_2 = 0$ and approximation errors are *not* serious enough to affect our derivation, Eq. (3) can be rewritten with the one state variable:

$$-\log(\rho_l) - \frac{\exp(\overline{dp}) - \exp(\overline{cp})}{1 + \exp(\overline{dp}) - \exp(\overline{cp})} \left(pd_{t+1}^l - \overline{pd^l}\right),$$

where $\frac{\exp(\overline{dp}) - \exp(\overline{cp})}{1 + \exp(\overline{dp}) - \exp(\overline{cp})} = 1 - \rho_l$, $\overline{pd^l} = -\beta_1 \overline{dp} + \beta_2 \overline{cp}$, and $-\log(\rho_l) + (1 - \rho_l)\overline{pd^l} = k_l$. This formula leads to the same representation as Eq. (7).

Eq. (7) represents a unified framework to link the conventional studies as well. First, we allow for negative dividends, which Campbell and Shiller (1988) do not consider. For example, suppose that liquidity is *not* a source of price variation. In this case, $\beta_1 = 1$ and $\beta_2 = 0$ as a result of $\rho_l = \rho$ and $k_l = k$. This assumption corresponds to the conventional Campbell and Shiller's (1988) present-value identity:

$$pd_t \approx \frac{k}{1-\rho} + E\left[\sum_{j=0}^{\infty} \rho^j \left[\Delta d_{t+1+j} - r_{t+1+j}\right] \middle| \Omega_t\right],$$

where Ω_t is the conventional information set generated by the joint history of Δd_t and r_t.

Second, we consider that investment opportunities vary over time, whereas Amihud and Mendelson (1986) do not. Constant variations in expected returns and liquidity premia can allow our present-value identity to be analogous to Amihud and Mendelson's (1986) framework over a holding period. Our framework also supports the conclusion of Amihud and Mendelson (1986), who show that prices decrease with the expected relative bid-ask spreads.

APPENDIX B: Robustness test

For robustness, we use log bid-ask spread s_t displayed in Figure B.1 as another liquidity proxy. To measure price-based illiquidity, we do *not* normalize bid-ask spreads by price levels. Also, it should be noted that the bid-ask spread is available since 1983. Hence, we proceed with the median of all the *quarterly* cross-section bid-ask spreads to enlarge the number of samples until the fourth quarter of 2017. The construction is almost the same as stated in Subsection 3.1 with the exception of *one* condition that each security must have at least 40 trading days over a quarter.

[INSERT FIGURE B.1 HERE]

Proposition 1 allows for estimating *pdl* ratio $pd_t^l = p_t - \hat{\beta}_1 d_t + \hat{\beta}_2 s_t$ displayed in Figure B.2:

$$p_t = 3.291 + 1.060 \cdot d_t - 0.060 \cdot s_t. \tag{B.1}$$

All of them are significant at the conventional levels. These numbers give a sense that the amount of bid-ask spreads is about 5.7% ($= \hat{\beta}_2/\hat{\beta}_1$) of that of dividends over the sample; the autocorrelation of s_t is about one in line with the price-based illiquidity.

[INSERT FIGURE B.2 HERE]

Figure B.2 also shows clearly that the bid-ask spread provides a natural price trend. For this reason, we notice that subsequent qualitative implications are almost similar. Therefore, we do not repeat the details to conserve space and instead devote attention to a couple of primary points.

Our present-value identity has the form:

$$pd_t^l \approx E\left[\sum_{j=0}^{\infty} (\hat{\rho}_l)^j \left[\hat{\beta}_1 \Delta d_{t+1+j} - \hat{\beta}_2 \Delta s_{t+1+j} - r_{t+1+j} + lp_{t+1+j}\right] \middle| \mathcal{H}_t\right]. \tag{B-2}$$

We set $\hat{\rho}_l$ to 0.995. To obtain this number, we first compute the annual value of $\hat{\rho} = 1/(1 + \exp(\overline{dp})) = 0.977$ from 1983 to 2017 and then transform it into the quarterly value of $\hat{\rho} = \sqrt[4]{0.977} = 0.994$. With the resulting $\hat{\rho} = 0.994$ in effect, we can back the value of $\hat{\rho}_l$ out of the beta formulas in Proposition 1.

[INSERT TABLE B.1 HERE]

Table B.1 calculates the results of simple predictive regressions. Likewise, the short-term forecastable evidence is not reliable because the autocorrelation of pd_t^l is about 0.91 close to unit root. Noteworthy are the signs of $b_d = 0.044$ and $b_s = -0.076$, which show the expected signs as shown in Eq. (B-2). The forecast identity brings us to compute liquidity premium coefficient b_{lp} as

$$b_{lp} = 1 - \hat{\rho}_l \cdot \phi - \hat{\beta}_1 \cdot b_d + \hat{\beta}_2 \cdot b_s + b_r = 0.005.$$

The linear Wald test for $\mathrm{H}_0\colon b_{lp} = 0$ says that $b_{lp} = 0.005$ is far away from the null at the conventional levels, confirming that the liquidity premium should be price on a short-term basis. Next, we move on to the long-run forecasting estimates reported in Table B.2, all of which are calculated based on the short-run coefficients.

[INSERT TABLE B.2 HERE]

One thing to note is that dividends present a dominant share to explain the price volatility: $\hat{\beta}_1 \cdot b_d^{lr} = 47.2\%$. This share is even larger than $b_r = -42.9\%$ and $b_{er} = -42.7\%$ in an absolute sense. Given that the CRSP dividends contain all payments to shareholders, the popularity of share repurchases since 1983 might cause such a result; the enactment of Rule 10b-18 in November 1982 has permitted firms to buy back a predetermined fraction of their shares under strict supervision. Nevertheless, the shares of $b_r = -42.9\%$ and $b_{er} = -42.7\%$

are evidence in favor of return predictability. With b_r in place, we calculate the long-term liquidity premium estimate as $b_{lp}^{lr} = 1 - \hat{\beta}_1 \cdot b_d^{lr} + \hat{\beta}_2 \cdot b_s^{lr} + b_r^{lr} = 0.052$. The non-linear Wald test for $\mathrm{H}_0: b_{lp}^{lr} = 0$ shows that its time variability is *not* economically significant; the *p*-value is about 92.0%. Likewise, the resulting liquidity premium is *not* significantly priced in the long run because it is just associated with the price trend, *not* with its cyclical fluctuations.

[INSERT TABLE B.3 HERE]

Let us see the error correlations and standard deviations reported in Table B.3 and compare them with those of Table 4. The common feature is that liquidity-premium shock ε^{lp} exhibits a strong correlation with *pdl* shock ε^{pdl} (0.932). Conversely, we discover in Table B.3 that *pdl* shock ε^{pdl} and dividend shock ε^{d} are strongly and negatively correlated (-0.800), indicating that the *pdl* shock can be a large fraction of expected cash flow news since 1983, whereas Table 4 illustrates that a *pdl* move is almost expected return news. Even so, it is still intact that market liquidity risk is essentially second-order.

[INSERT FIGURE B.3 HERE]

To see this, illiquidity shock ε^{s} is designed to be *solely* accompanied by liquidity-premium shock ε^{lp}:

$$\varepsilon^{pdl} = 0, \qquad \varepsilon^{d} = , \qquad \varepsilon^{s} = 1, \qquad \varepsilon^{r} = 0, \qquad \varepsilon^{lp} = \hat{\beta}_2 = 0.06. \tag{B-3}$$

In Figure B.3, you confirm stay-put moves shown in Figure 4.

APPENDIX C: Over-identifying restriction on a GMM

In Appendix C, all endogenous variables are redefined as deviations from their sample means. This change is justifiable because constant terms do not affect any stochastic behavior. Doing so has another advantage of increasing one degree of freedom (see Hypothesis C).

Our empirical question here is, how much expected net return r^* accounts for expected return r? To conduct the investigation, unlike the previous ex-post identity of $r_t^* = r_t - lp_t$, we replace net return $r_t^* = r_t - lp_t$ with actual real return r_t by $r_t^* = \omega \cdot r_t$, where ω is the proportion of the net return to the real return. This proportion can be estimated through tests of over-identifying restriction on a GMM.

HYPOTHESIS C: $\mathbf{H_0}$: $\boldsymbol{E[v_{t+1}|\mathcal{H}_t] = E\left[v_{t+1} \cdot I_t^{\mathcal{H}}\right] = 0}$

We have the following forward-looking moment equation by $r_t^* = \omega \cdot r_t$:

$$pd_t^l = \hat{\rho}_l \cdot pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - \omega \cdot r_{t+1} + v_{t+1}.$$

By redefining the variables discussed above, we do not need to estimate the intercept in the forward moment equation above. Residual v_{t+1} should be orthogonal to instrumental variables $I_t^{\mathcal{H}}$ in information set $\mathcal{H}_t = [pd_t^l, \Delta d_t, \Delta c_t^*, r_t]'$:

$$E\left[\left[pd_t^l - \left\{\hat{\rho}_l \cdot pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - \omega \cdot r_{t+1}\right\}\right] \cdot I_t^{\mathcal{H}}\right] = 0.$$

Table C.1 presents the results of over-identifying restriction on a GMM. Note that the number of the instrumental variables (simply, rank) can be up to five due to $\mathcal{H}_t = [pd_t^l, \Delta d_t, \Delta c_t^*, r_t]'$ including a constant. To conserve space, we report three simple cases (a) just-identification with rank 1, $\mathcal{H}_t = [pd_t^l]'$, (b) over-identification with rank 2, $\mathcal{H}_t = [pd_t^l]'$ and a constant, and (c) over-identification with full rank 5.

[INSERT TABLE C.1 HERE]

As a result, the proportion ω spans from about 1.118 (rank 5) to 1.131 (rank 1), all of which are statistically different from one. This finding gives another evidence in favor of liquidity premium predictability over short horizons; otherwise, ω should not be far from one. The number ω such that $E_t[r^*_{t+1}] = \omega E_t[r_{t+1}]$ also suggests that the one-period expected return hovers around 88% (=1/1.131) – 89% (=1/1.118) of the one-period net return from 1926 to 2017 for the market portfolio. The remaining 10-11% thus come from the variability of the liquidity premium to fill the void in constituting the net return.

The GMM results can pertain to our main findings in Subsection 4.2. The fact that $b_r = -0.205$ and $b_{lp} = 0.027$ can lead roughly to $b_{r^*} \approx b_r - b_{lp} = -0.232$ by accounting identity (6). With these numbers in place, the resulting proportion ω is about 88% ($= 0.205/0.232$), implying that about 12% ($= 1 - \omega$) of the net return is accounted for by the liquidity premium.

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Table 1. Summary statistics

Panel A: Means and standard deviations from 1927 to 2017

	Δd_t	Δc_t^*	r_t	er_t	pd_t	pd_t^l
Mean	0.054	−0.081	0.065	0.057	3.365	3.156
Standard deviation	0.143	0.410	0.197	0.199	0.433	0.269

Panel B: Contemporaneous correlations from 1927 to 2017

	Δd_t	Δc_t^*	r_t	er_t	pd_t	pd_t^l
Δd_t	1.000					
Δc_t^*	**−0.344**	1.000				
r_t	**0.652**	**−0.613**	1.000			
er_t	**0.649**	**−0.671**	**0.980**	1.000		
pd_t	−0.115	**−0.214**	0.054	0.038	1.000	
pd_t^l	**−0.246**	−0.041	0.002	−0.013	**0.621**	1.000

Panel C: Unit root tests from 1927 to 2017

	Δd_t	Δc_t^*	r_t	er_t	pd_t	pd_t^l
PP	**−11.314**	**−5.925**	**−9.261**	**−8.852**	−1.593	−2.503
KPSS	0.055	0.313	0.056	0.069	**0.869**	0.067

NOTE: Δd_t is log real dividend growth, Δc_t^* is log illiquidity growth contingent on Amihud (2002) price impact proxy c_t^* (see Figure 2), r_t is log real return, er_t is log excess return, pd_t is the conventional price–dividend ratio, and $pd_t^l = p_t - \hat{\beta}_1 d_t + \hat{\beta}_2 c_t^*$ is the liquidity-adjusted price–dividend ratio, where $\hat{\beta}_1 = 1.084$ and $\hat{\beta}_2 = 0.084$ in Eq. (9). The PP test has the null that a variable has a unit root, and the KPSS test has the null that a variable is stationary. We allow for an intercept and select an optimal lag in each test equation, based on the SC. We report adjusted *t*-statistics in the PP test and Lagrange Multiplier (LM) statistics in the KPSS test, respectively. Significant statistics at the 5% level are highlighted in bold face.

Table 2. Forecasting regressions

Regression	b_x or ϕ (S.E.)	t-statistic	R^2 (%)	$\sigma(bx)$ (%)
$\Delta d_{t+1} = a_d + b_d \cdot pd_t^l + \varepsilon_{t+1}^d$	−0.056 (0.062)	−0.899	1.10	1.50
$\Delta c_{t+1}^* = a_{c^*} + b_{c^*} \cdot pd_t^l + \varepsilon_{t+1}^{c^*}$	0.126 (0.123)	1.023	0.68	3.39
$r_{t+1} = a_r + b_r \cdot pd_t^l + \varepsilon_{t+1}^r$	−0.205 (0.064)	−3.214	7.77	5.50
$pd_{t+1}^l = a_\phi + \phi \cdot pd_t^l + \varepsilon_{t+1}^{pdl}$	0.866 (0.056)	15.394	74.73	23.25
$\Delta d_{t+1} = a_d + b_d \cdot pd_t + \varepsilon_{t+1}^d$	−0.027 (0.038)	−0.724	0.69	1.19
$r_{t+1} = a_r + b_r \cdot pd_t + \varepsilon_{t+1}^r$	−0.085 (0.045)	−1.894	3.49	3.69
$pd_{t+1} = a_\phi + \phi \cdot pd_t + \varepsilon_{t+1}^{pd}$	0.942 (0.039)	24.106	88.31	40.69

NOTE: We report slope estimate b_x or ϕ with respect to each forecasting regression (1st column) for state variable x and its standard errors in parentheses. The t-statistic estimated from a GMM method is corrected for heteroscedasticity.

Table 3. Long-run implications

	$\hat{\beta}_1 \cdot b_d^{lr}$	$\hat{\beta}_2 \cdot b_{c^*}^{lr}$	b_r^{lr}	b_{er}^{lr}
Long-run estimate	−0.377	0.066	−1.276	−1.262
(S.E.)	(0.394)	(0.968)	(0.435)	(0.453)
t-statistic	−0.958	0.168	−2.932	−2.785

NOTE: We report the long-run slope coefficient calculated as $b_x^{lr} = b_x/(1 - \hat{\rho}_l \phi)$ in long-run forecast identity (20). Here, b_x and ϕ is the one-period regression coefficient in Table 2, and $\hat{\rho}_l$ is set to 0.969. We use $\hat{\beta}_1 = 1.084$ and $\hat{\beta}_2 = 0.084$ in Eq. (9). The *t*-statistic is calculated through the standard delta method.

Table 4. Error standard deviations and correlations

	ε^d	ε^{c^*}	ε^r	ε^{pdl}	ε^{lp}
ε^d	14.2	−33.8	65.2	−31.0	−31.0
ε^{c^*}	−33.8	40.8	−61.6	−22.5	−22.5
ε^r	65.2	−61.6	18.9	50.3	50.3
ε^{pdl}	−31.0	−22.5	50.3	13.5	100
ε^{lp}	−31.0	−22.5	50.3	100	0.4

NOTE: Each number stands for error standard deviations on the diagonal (%) and correlation on the off-diagonal (%). The error shocks are dividend shock ε^d, illiquidity shock ε^{c^*} subject to Amihud's (2002) illiquidity proxy in Eq. (8), return shock ε^r, *pdl* shock ε^{pdl}, and implied liquidity-premium shock ε^{lp} by error identity (18).

Table B. 1. Forecasting regressions and error standard deviations

Regression	b_x or ϕ (S.E.)	t-statistic	R^2 (%)	$\sigma(bx)$ (%)
$\Delta d_{t+1} = a_d + b_d \cdot pd_t^l + \varepsilon_{t+1}^d$	0.044 (0.030)	1.448	1.45	1.4
$\Delta s_{t+1} = a_s + b_s \cdot pd_t^l + \varepsilon_{t+1}^s$	−0.076 (0.030)	−2.546	5.75	2.5
$r_{t+1} = a_r + b_r \cdot pd_t^l + \varepsilon_{t+1}^r$	−0.042 (0.024)	−1.791	2.81	1.4
$pd_{t+1}^l = a_\phi + \phi \cdot pd_t^l + \varepsilon_{t+1}^{pdl}$	0.906 (0.035)	26.019	83.71	29.5

NOTE: We report slope estimate b_x or ϕ with respect to each forecasting regression (1st column) for state variable x and its standard errors in parentheses. The t-statistic estimated from a GMM method is corrected for heteroscedasticity.

Table B. 2. Long-run implications

	$\hat{\beta}_1 \cdot b_d^{lr}$	$\hat{\beta}_2 \cdot b_s^{lr}$	b_r^{lr}	b_{er}^{lr}
Long-run estimate	0.472	−0.047	−0.429	−0.427
(S.E.)	(0.223)	(0.209)	(0.209)	(0.204)
t-statistic	2.120	−0.209	−2.056	−2.091

NOTE: We report the long-run slope coefficient calculated as $b_x^{lr} = b_x/(1 - \hat{\rho}_l \phi)$ in long-run forecast identity (20). Here, b_x and ϕ is the one-period regression coefficient in Table B. 1, and $\hat{\rho}_l$ is set to 0.995. We use $\hat{\beta}_1 = 1.060$ and $\hat{\beta}_2 = 0.060$ in Eq. (B-1). The *t*-statistic is calculated through the standard delta method.

Table B. 3. Error standard deviations and correlations

	ε^d	ε^s	ε^r	ε^{pdl}	ε^{lp}
ε^d	11.8	11.0	24.9	−80.0	−62.5
ε^s	11.0	10.0	−9.2	−11.5	−39.8
ε^r	24.9	−9.2	8.1	37.9	56.4
ε^{pdl}	−80.0	−11.5	37.9	13.0	93.2
ε^{lp}	−62.5	−39.8	56.4	93.2	0.1

NOTE: Each number stands for error standard deviations on the diagonal (%) and correlation on the off-diagonal (%). The error shocks are dividend shock ε^d, illiquidity shock ε^s subject to bid-ask spread proxy in Eq. (B-1), return shock ε^r, *pdl* shock ε^{pdl}, and implied liquidity-premium shock ε^{lp} by error identity: $\varepsilon^{lp} = -\hat{\rho}_l \cdot \varepsilon^{pdl} - \hat{\beta}_1 \cdot \varepsilon^d + \hat{\beta}_2 \cdot \varepsilon^s + \varepsilon^r$, where $\hat{\rho}_l = 0.995$, $\hat{\beta}_1 = 1.060$, and $\hat{\beta}_2 = 0.060$.

Table C. 1. GMM test

$I_t^{\mathcal{H}}$ (rank, identification)	Implications of GMM	Significance level for J test
pd_t^l (1, just)	$pd_t^l = \hat{\rho}_l \cdot pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - 1.131 \cdot r_{t+1} + \nu_{t+1}$ (0.049)	–
pd_t^l and constant (2, over)	$pd_t^l = \hat{\rho}_l \cdot pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - 1.126 \cdot r_{t+1} + \nu_{t+1}$ (0.043)	0.830
$[pd_t^l, \Delta d, \Delta c_t^*, r_t]'$ and constant (5, over)	$pd_t^l = \hat{\rho}_l \cdot pd_{t+1}^l + \hat{\beta}_1 \Delta d_{t+1} - \hat{\beta}_2 \Delta c_{t+1}^* - 1.118 \cdot r_{t+1} + \nu_{t+1}$ (0.034)	0.583

NOTE: The table reports the results of over-identifying restrictions on the GMM through continuous updating. Here, $I_t^{\mathcal{H}}$ denotes the instrumental variable, and the rank is the number of the instrumental variables. The last column presents *p*-values for H_0: $E\left[v_{t+1} \cdot I_t^{\mathcal{H}}\right] = 0$ in terms of the *J*-statistics.

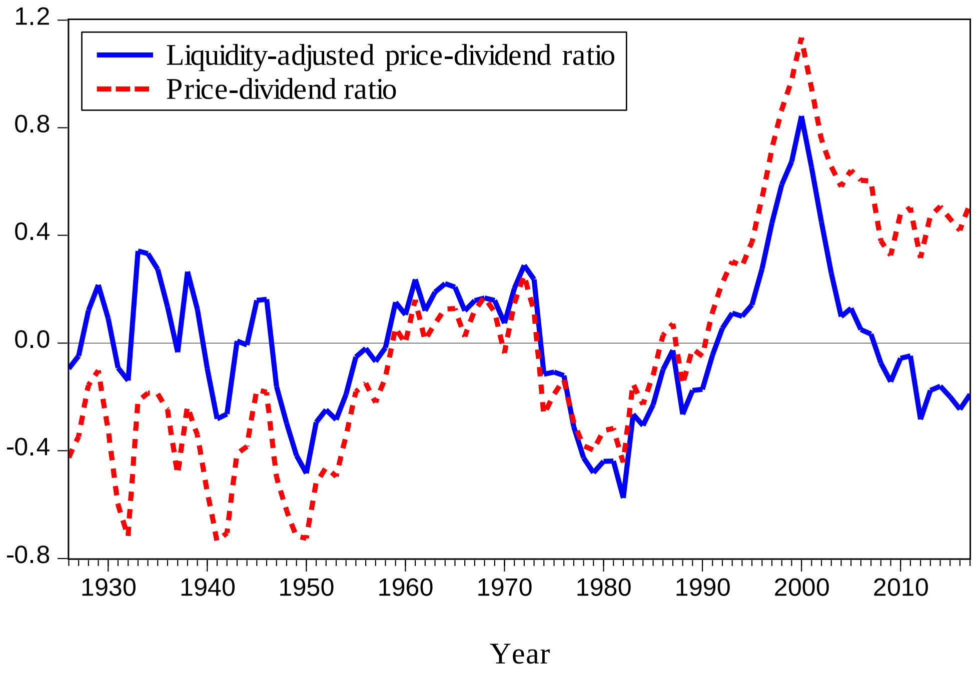

Figure 1. Price–dividend and liquidity-adjusted price–dividend ratios. The two ratios are plotted from 1926 to 2017. They are redefined as deviations from their sample means.

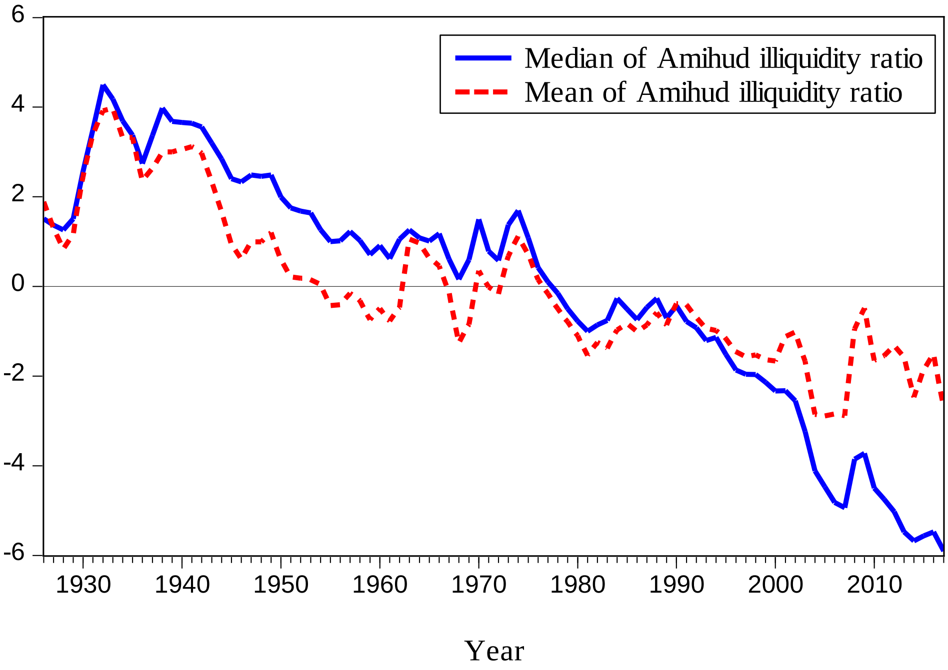

Figure 2. Log Amihud (2002) illiquidity ratio c_t^*. See Subsection 3.1 for calculation. In particular, c_t^* is deflated the CPI provided by CRSP at the end of month in year t. We also redefine c_t^* as deviations from its sample mean from 1926 to 2017.

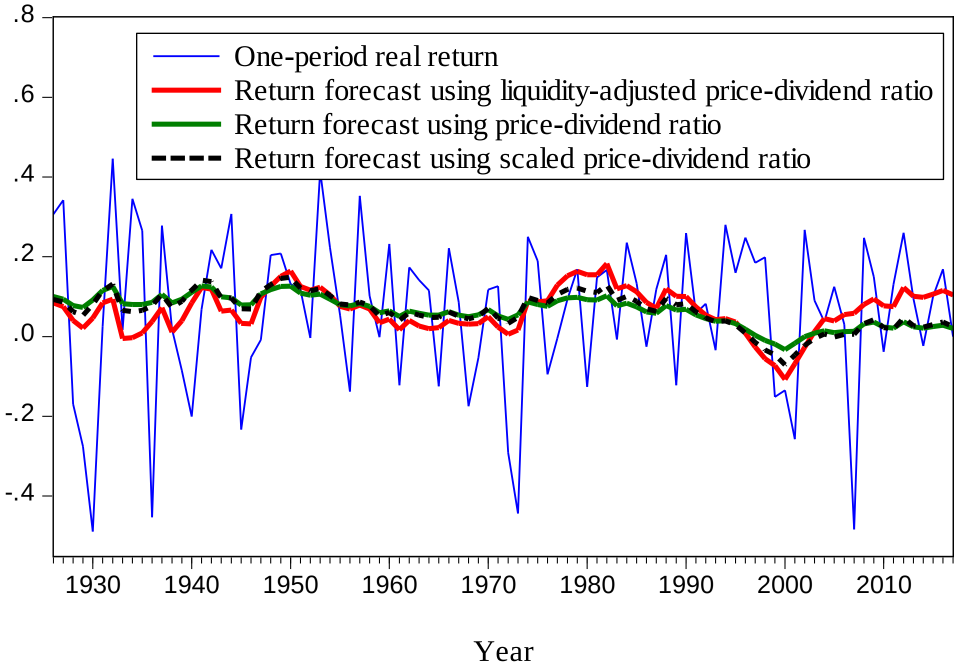

Figure 3. Forecast and one-period ex-post returns. The figure plots one-period real return r_{t+1} and the fitted regression values of simple forecasting regressions in Subsection 4.1 on *pdl* ratio $pd_t^l = p_t - \hat{\beta}_1 d_t + \hat{\beta}_2\, c_t^*$, *pd* ratio $pd_t = p_t - d_t$, and scaled *pd* ratio $p_t - \hat{\beta}_1 d_t$: i.e., $a_x + b_x \times$ regressor.

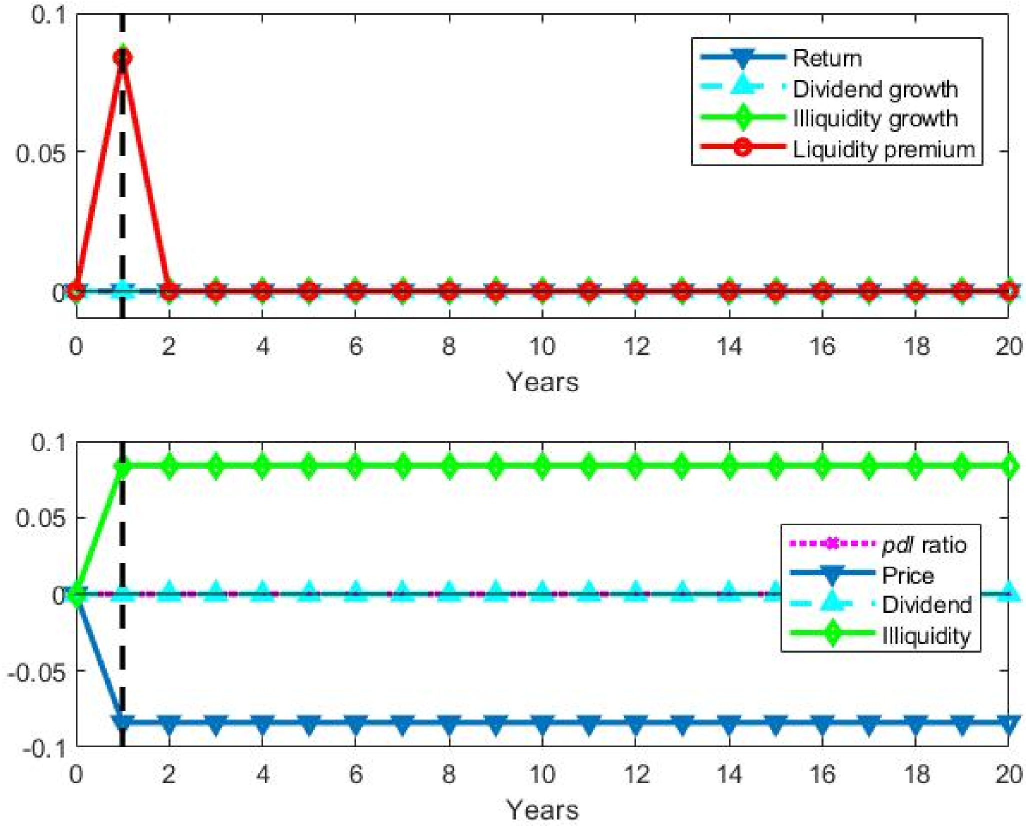

Figure 4. Impulse response to *illiquidity* shock with no moves in *pdl* ratio, dividend, and return. The impulse response functions are plotted based on the first-order VAR (25). The first panel uses the five error shocks, $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}]$, where liquidity-premium shock ε^{lp} is inferred from error identity (18): $\varepsilon^{lp} = -\hat{\rho}_l \cdot \varepsilon^{pdl} - \hat{\beta}_1 \cdot \varepsilon^{d} + \hat{\beta}_2 \cdot \varepsilon^{c^*} + \varepsilon^{r}$, where $\hat{\rho}_l = 0.969$, $\hat{\beta}_1 = 1.084$, and $\hat{\beta}_2 = 0.084$. The second panel further uses the price shock $\varepsilon^{p} = \varepsilon^{pdl} + \hat{\beta}_1 \cdot \varepsilon^{d} - \hat{\beta}_2 \cdot \varepsilon^{c^*}$ in footnote 15. We identify the *illiquidity* shock as $\left[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{c^*}, \varepsilon^{r}, \varepsilon^{lp}\right] = [0,0,1,0,\hat{\beta}_2]$ in Eq. (24). The vertical dashed line represents the starting time of the shock.

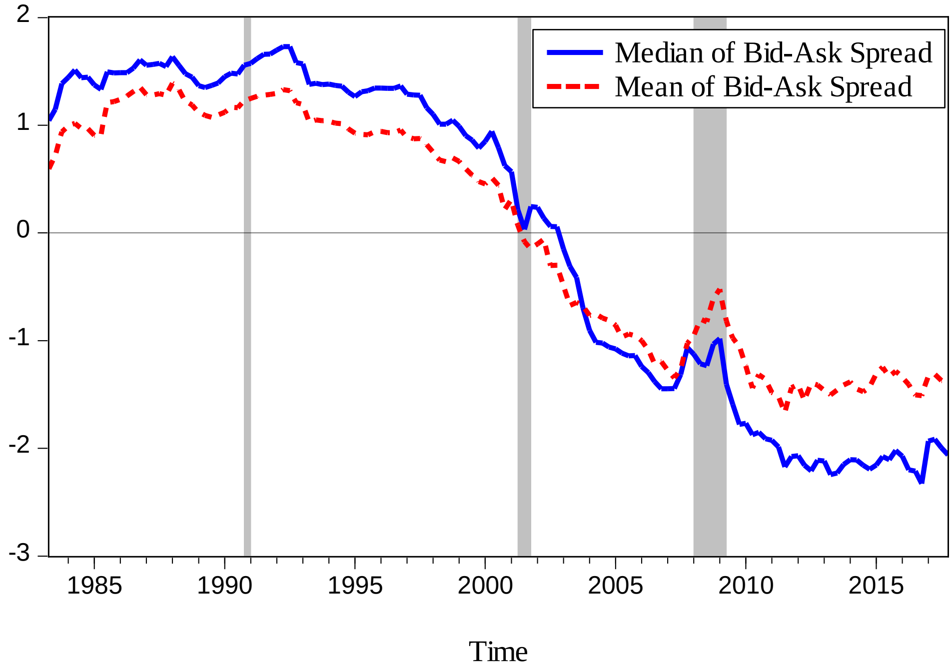

Figure B. 1 Log bid-ask spread s_t. We plot both median and mean of all the cross-sectional spreads from the first quarter of 1983 (1983Q1) to the fourth quarter of 2017 (2017Q4). Both of them are redefined as deviations from its sample means and deflated the CPI provided by CRSP at the end of month in quarter t. Shaded areas represent the NBER recessions for the period following the peak through the trough.

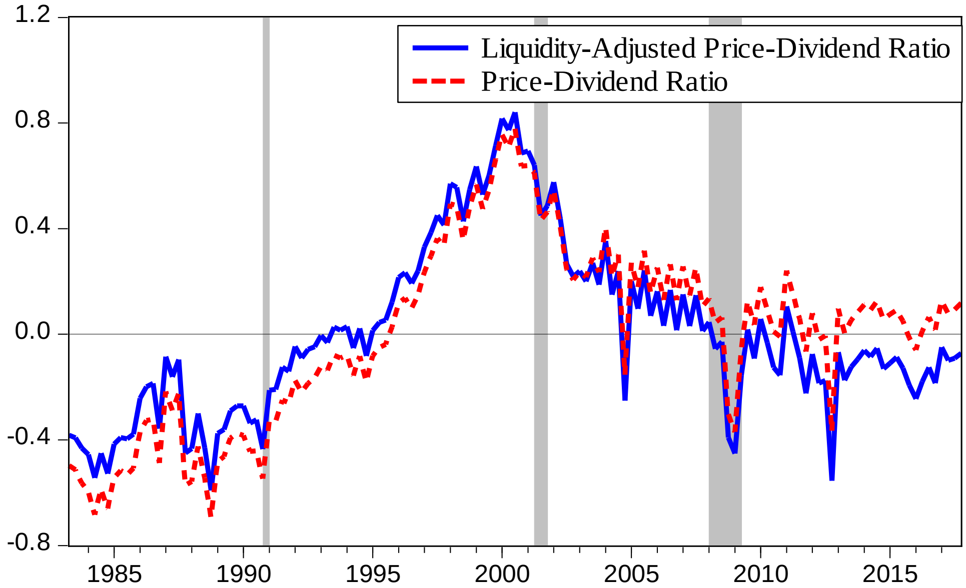

Figure B. 2 Price–dividend and liquidity-adjusted price–dividend ratios. The two ratios are plotted over the period from 1983Q1 to 2017Q4. They are redefined as deviations from their sample means. Shaded areas represent the NBER recessions for the period following the peak through the trough.

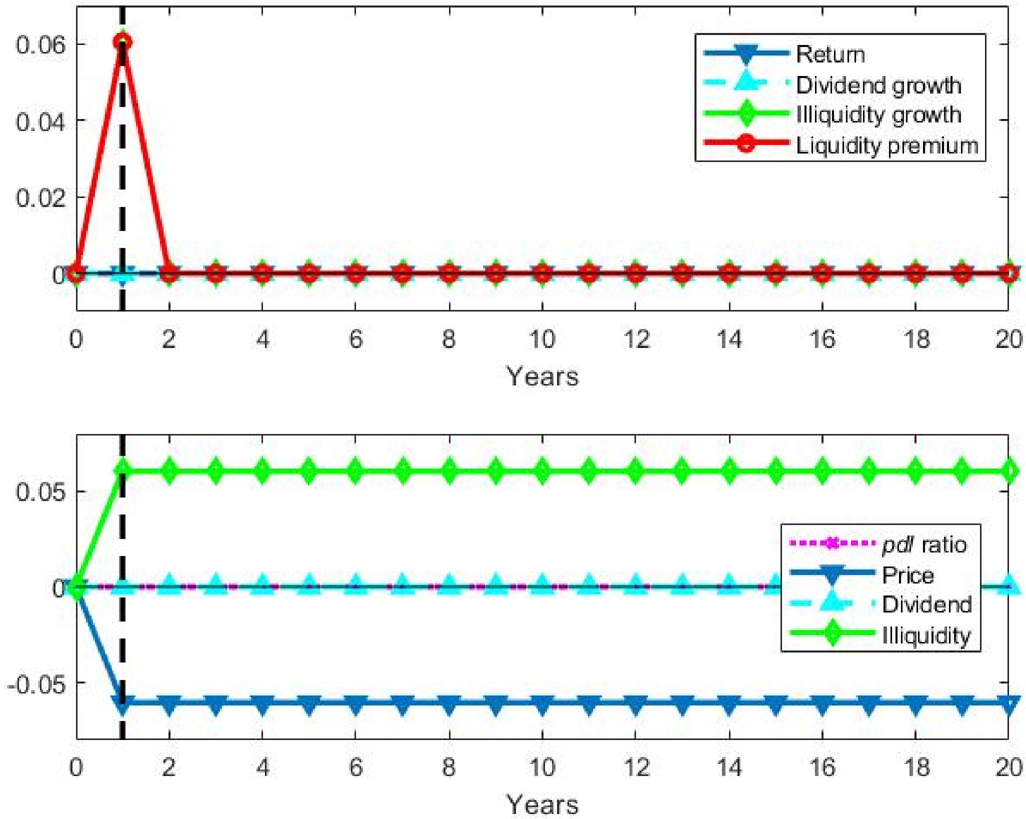

Figure B. 3. Impulse response to *illiquidity* shock with no moves in *pdl* ratio, dividend, and return. The impulse response functions are plotted based on the similar form of the first-order VAR (25). The first panel uses the five error shocks, $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{s}, \varepsilon^{r}, \varepsilon^{lp}]$, where liquidity-premium shock ε^{lp} is inferred from the error identity: $\varepsilon^{lp} = -\hat{\rho}_l \cdot \varepsilon^{pdl} - \hat{\beta}_1 \cdot \varepsilon^{d} + \hat{\beta}_2 \cdot \varepsilon^{s} + \varepsilon^{r}$, where $\hat{\rho}_l = 0.995$, $\hat{\beta}_1 = 1.060$, and $\hat{\beta}_2 = 0.060$ in Eq. (B-1). The second panel further uses the price shock: $\varepsilon^{p} = \varepsilon^{pdl} + \hat{\beta}_1 \cdot \varepsilon^{d} - \hat{\beta}_2 \cdot \varepsilon^{s}$. We identify the *pdl* shock as $[\varepsilon^{pdl}, \varepsilon^{d}, \varepsilon^{s}, \varepsilon^{r}, \varepsilon^{lp}] = [0,0,1,0,\hat{\beta}_2]$. The vertical dashed line represents the starting time of the shock.