

Pricing equity and contingent convertibles with idiosyncratic risk [☆]

Xiaolin Wang^a, Zhaojun Yang^{b,*}

^a*School of Economics, Henan University, Kaifeng 475001, China*

^b*Department of Finance, Southern University of Science and Technology, Shenzhen
518055, China*

Abstract

We consider capital structure including equity, straight bonds (SBs) and contingent convertibles (CoCos). We derive the implied values of equity and CoCos and show that the benefits from issuing CoCos increase dramatically with idiosyncratic risk and risk aversion. The firm value is concave in the CoCos' conversion ratio and optimal conversion ratio increases with risk aversion and idiosyncratic risk. If the claimants are sufficiently risk-averse, the firm should issue less CoCos and equity but more SBs as the idiosyncratic risk rises. If shareholders are sufficiently risk-averse, their risk-shifting incentives disappear. The higher the idiosyncratic risk or the bigger the risk aversion, the higher the leverage.

Keywords: CoCos, Utility-based pricing, Conversion ratio, Risk-shifting, Capital structure

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*Corresponding author. Tel: +86 755 8801 8603.

Email addresses: wofan-1986@163.com (Xiaolin Wang), yangzj@sustc.edu.cn (Zhaojun Yang)

1. Introduction

Motivation. Contingent convertibles (CoCos), also known as contingent convertible bonds, are specific type of corporate securities and the possibility of converting them to equity is contingent on a specified event, that can be different depending on the particular need of an issuer. In the case of that the issuer of CoCos is a bank, the contingent event might, for example, be related to the level of Tier 1 capital falling under a given threshold. In theory, it would be beneficial for any firm to issue CoCos, as argued by [Flannery \(2005\)](#), [Song and Yang \(2016\)](#), [Tan and Yang \(2016\)](#), [Cai et al. \(2019\)](#) and [Yang and Zhao \(2015\)](#) among others. As a matter of fact, since CoCos were first issued by Lloyds Banking Group in November 2009, they have been widely welcomed by banks of many countries in the world. According to the statistics reported by a journalist of the Chinese journal of Moneyweek on December 29, 2014, the banks in China have stepped up issuance: from nothing in 2012 to RMB 358.35 bn yuan of CoCos by December 2014. Most of CoCos issued in China are the so-called write-down CoCo bonds, which are a special type of CoCos with their conversion ratios being zero. Here conversion ratios, also known as ownership stakes, which are the fraction of equity allocated to CoCo holders once CoCos convert into equity.

To the best of our knowledge, all papers in the literature on CoCos assume investors are risk-neutral toward idiosyncratic risk or idiosyncratic risk can be well diversified away. This assumption greatly simplifies the pricing of financial claims. As a starting point of studying CoCos, it is desirable and

essential. Especially, if idiosyncratic risk disappears or is diversified away, we can recover the unique reasonable no-arbitrage price. However, in a real market environment, the idiosyncratic risk faced by a firm is impossible to be completely eliminated. For example, according to [Chen et al. \(2010\)](#), entrepreneurial firms tend to have highly concentrated ownership and lack of diversification is one of the defining characteristics of entrepreneurship. Even though a firm is publicly listed,¹ its equity holders would still take on considerable uninsured idiosyncratic risk. CoCos share much similarity with equity though they are of course different. CoCos indeed belong to a kind of hybrid bonds and are potentially equity. Recently, several banks have announced that they are considering paying managers and executives with such instruments. For example, Credit Suisse is scrapping a scheme that linked bonuses to risky assets after the plan clashed with capital regulations. The 5,500 senior bankers who were offered the scheme in 2012 may now choose between two replacement plans, one of which is CoCo instruments, which are wiped out if the bank's capital falls below a certain level, see [Toshniwal \(2011\)](#). In addition, Barclays has sought approval from the UK regulators for paying its bankers (managing director level and above) with CoCo instruments.² The CoCo instruments in executive compensation are also argued by [Walther and Klein \(2015\)](#), [Kagade and Verma \(2015\)](#) and [Kaal \(2012\)](#) among others. However the executives and managers have undiversified portfolios and naturally such CoCo holders would face significant idiosyncratic risk, see

¹[Cerasi and Daltung \(2000\)](#) point out that bank assets are indivisible and illiquid.

²See Jennifer Hughes and Patrick Jenkins, Barclays Forced to Adapt Cocos Bonus Plan, FINANCIAL TIMES, Feb. 14, 2011.

[Ingersoll \(2002\)](#). It is true that CoCo investors are generally financial institutions who can diversify idiosyncratic risk more effectively. However, the remaining idiosyncratic risk they face would be still significant, in particular if the CoCo issuers are not public-listed companies.

In a word, the pricing of equity or CoCos that overlooks idiosyncratic risk is a shortcoming to be overcome under many situations. In fact, this pricing method would significantly overestimate asset values and usually induces investors to take too big risks. As a result, it would considerably underestimate a firm's leverage and therefore assign the firm a wrong credit score.

To overcome the shortcoming, utility-based models are still the only theoretically defensible way of treating such markets, although there is a widespread practice of using risk-neutral pricing, even when the assets being priced cannot be replicated by trading in other, more primitive assets, as argued by Steven E. Shreve.³ It is true that, to derive pricing kernels of a risk-neutral pricing approach, we also make use of utility functions in solving agents' utility maximization problem. However, this method is very different from our utility-based pricing method we discuss here. In particular, the former leads to a linear pricing schedule but the latter induces a non-linear pricing one.

On the contrary, straight bonds (SBs) would be better priced competitively by diversified lenders, i.e. the idiosyncratic risk resulting from SBs could be well diversified away, as argued by [Chen et al. \(2010\)](#).⁴ For example,

³See page 70 of the book titled *Stochastic Calculus for Finance I* authored by Steven E. Shreve in 2004.

⁴[Chen et al. \(2010\)](#) show further that while nondiversifiable risk does lower the value

credit default swaps, which are liquidly traded assets and depend only on the default risk, can potentially hedge the risk.

Our work. This paper considers the design and pricing of CoCos and optimal capital structure of a firm issuing equity and either SBs or both SBs and CoCos involving idiosyncratic risk. Following [Miao and Wang \(2007\)](#) among others, we assume a firm's cash flow evolves according to an arithmetic Brownian motion. All investors are risk-averse and have access to one risk-free asset and market portfolio to smooth their consumption. We assume SB holders are fully diversified and thus we derive its equilibrium value (market value) according to an equilibrium pricing approach. The cash flow is non-tradable and therefore both CoCos and equity have idiosyncratic risk. Therefore, we derive semi-closed-form solutions of the implied values or consumption utility indifference prices of equity and CoCos by a consumption utility indifference pricing approach under an endogenous bankruptcy triggering level, an exogenously specified conversion ratio. Last, we provide numerical sensitivity analysis by finite difference methods.

Literature review. Following the recent financial crisis, CoCos have been attracting increasing research interests. [Flannery \(2005\)](#) first suggests the idea for CoCos. [Sundaresan and Wang \(2015\)](#) provide the condition that the conversion ratio must satisfy in order for a unique equilibrium to exist and present a design that mitigates the problem of multiple equilibria. [McDonald \(2013\)](#) proposes a form of contingent capital for financial institutions that

of SBs from the perspective of underdiversified investors, the decreased amount of their prices is very small.

converts from debt to equity if two conditions are met: the firm's stock price is at or below a trigger value and the value of a financial index is also at or below a trigger value. [Barucci and Del Viva \(2012a\)](#) study the optimal capital structure of a company issuing perpetual CoCos, equity and SBs with a two-period model. [Barucci and Del Viva \(2012b\)](#) analyze the optimal capital structure of a bank issuing countercyclical CoCos, i.e., the notes that will convert into common shares in a recession.

To take into account idiosyncratic risk, the consumption utility-based indifference pricing method is a good choice. It is a dynamic extension of the static concept of certainty equivalence from economics. Recently, this method has been applied widely, see [Henderson and Hobson \(2002\)](#), [Miao and Wang \(2007\)](#), [Ewald and Yang \(2008\)](#), [Chen et al. \(2010\)](#), [Yang and Yang \(2012\)](#) and [Song, Wang and Yang \(2014\)](#), among others. In particular, [Leung, Sircar and Zariphopoulou \(2008\)](#) and [Liang and Jiang \(2012\)](#) derive the price of a defaultable corporate bond (i.e. SB) also by a utility-based method.

The remainder of the paper proceeds as follows. Section 2 sets up the model and shows the endogenous bankruptcy condition and conversion condition decided by a given conversion threshold of CoCos. Section 3 presents the implied values of CoCos and equity. Section 4 discusses the equilibrium pricing. Section 5 addresses optimal capital structure. Section 6 presents numerical simulations. Section 7 concludes. Proofs of theorems and propositions are relegated to appendices.

2. Model setup

Consider a firm that has invested in a project, of which the total cash flow δ is observable and governed by the following arithmetic Brownian motion:

$$d\delta_t = \mu dt + \rho\sigma dZ_t^1 + \sigma\sqrt{1-\rho^2}dZ_t^2, \quad \delta_0 \text{ given}, \quad (1)$$

where μ is the expected growth rate, σ is the volatility and $Z \equiv (Z^1, Z^2)$ is a two-dimensional standard Brownian motion on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The process Z^1 is independent of Z^2 . We denote by $\mathbb{F} \equiv \{\mathcal{F}_t : t \geq 0\}$ the \mathbb{P} -augmentation of the filtration $\sigma(Z_s; 0 \leq s \leq t)$ generated by process Z . The firm issues equity, SBs and CoCos. For simplicity, both bonds are consol type, meaning they are annuities with infinitive maturity. SBs (CoCos) pay coupon b_1 (b_2), continually in time, until default (conversion). The conversion threshold of CoCos is pre-specified while the default threshold of SBs is endogenously determined by shareholders.

We assume all investors have standard liquid financial opportunities which involves a risky market portfolio and a risk-free asset with interest rate $r > 0$. Let $\{M_t : t \geq 0\}$ denote the value of the market portfolio, which is governed by the following equation:

$$dM_t/M_t = \mu_e dt + \sigma_e dZ_t^1, \quad M_0 \text{ given}, \quad (2)$$

where μ_e and σ_e are the expected return and volatility of the market respectively. Clearly, the parameter ρ in (1) represents the correlation coefficient between the firm's cash flow and the return of the market portfolio. The parameters $\rho\sigma$ and $\sigma\sqrt{1-\rho^2}$ are the systematic and idiosyncratic volatility of the cash flow respectively. Thus, if $|\rho| < 1$, investors face idiosyncratic risk.

Once default occurs, SB holders take control and liquidate the firm in an open market. The liquidation value is based on the market value of the firm defined by an equilibrium pricing approach since we assume the buyer is a diversified investment company.

According to the above-mentioned standard liquid financial opportunities, we derive the equilibrium stochastic discount factor, which corresponds to a risk-neutral probability measure \mathbb{Q} , see [Duffie \(2001\)](#) among others. After that, we rewrite the cash flow process δ in (1) as follows:

$$d\delta_t = (\mu - \rho\sigma\eta)dt + \rho\sigma dZ_t^{\mathbb{Q}} + \sigma\sqrt{1 - \rho^2}dZ_t^2, \quad (3)$$

where $\eta = (\mu_e - r)/\sigma_e$ is the Sharpe ratio, $\mu - \rho\sigma\eta$ is the risk-adjusted drift, and $Z^{\mathbb{Q}}$ is a standard Brownian motion under \mathbb{Q} satisfying $dZ_t^{\mathbb{Q}} = dZ_t^1 + \eta dt$.

Thus, the equilibrium value or market value of an unlevered firm is given by:

$$A_0(\delta_t) = \mathbb{E}^{\mathbb{Q}} \left[\int_t^{\infty} e^{-r(s-t)} \delta_s ds | \mathcal{F}_t \right] = \frac{\delta_t}{r} + \frac{\mu - \rho\sigma\eta}{r^2}. \quad (4)$$

We assume the liquidation value of the firm is equal to a fraction $(1 - \alpha)$ of the market value of the unlevered firm after tax, i.e. $(1 - \xi)(1 - \alpha)A_0(\delta_{\tau_1})$, where τ_1 is the default time and ξ is the tax rate. The remaining fraction α is lost due to bankruptcy costs.

We assume CoCos convert into equity once the issuing firm's value gets small enough relative to the value of the outstanding debt. Specifically, the conversion time is given by

$$\tau_2 \equiv \inf \left\{ t \geq 0 : \phi A_0(\delta_t) \leq \frac{b_1}{r} + \frac{b_2}{r} \right\}, \quad (5)$$

where $0 < \phi < 1$ is called conversion leverage. The conversion leverage is

a constant determined by the agreement when it is signed.⁵ Therefore, the conversion threshold, denoted by \bar{x}_2 , is given from (4) and (5) by

$$\bar{x}_2 = \frac{b_1 + b_2}{\phi} - \frac{\mu - \rho\sigma\eta}{r}. \quad (6)$$

Upon conversion of CoCos into equity, we assume the ownership stake (conversion ratio) $0 < \lambda < 1$ is a given constant specified in the agreement when it is signed. Although the ownership stake can take any value between 0 and 1 in essence, we must choose it so large that shareholders will not benefit from such conversion. Otherwise, shareholders might find it optimal to “burn money” to push its cash flow below the conversion threshold.⁶

Throughout the text, borrowing ideas from [Chen et al. \(2010\)](#), we assume there are three groups of investors: inside shareholders, inside CoCo holders and outside SB holders.⁷ The inside equity and CoCos are exposed to idiosyncratic risk and we therefore value them by a utility-based pricing method. On the contrary, SBs are priced by a risk-neutral pricing approach since we assume they can be traded in a competitive market and their idiosyncratic risk can be fully diversified away.

⁵Roughly speaking, once the Tier 1 capital ratio falls below the level $(1 - \phi)$, say 4% if $\phi = 96\%$, conversion is triggered. This rule is similar to [Glasserman and Nouri \(2012\)](#).

⁶ If contractual terms do not dilute the original equity upon conversion, the CoCo issuers would burn money and reduce asset size. By doing so, the issuers can reduce their capital ratio and force the conversion.

⁷It would be more reasonable to assume that investor clienteles in our model include inside/outside shareholders, inside/outside CoCo holders and inside/outside SB holders. All insider investors price their claims by a utility-based method but all outside investors price their claims by a risk-neutral pricing approach. To save space, we leave this problem for future research.

3. Utility-based pricing of equity and CoCos

To fix a utility-based price of an asset, we consider an investor with initial wealth w investing in a standard liquid financial market defined in Section 2. Denote the wealth process of an investor by $W = (W_t)_{t \geq 0}$, his consumption plan by $C = (C_t)_{t \geq 0}$ and his investment strategy by $\theta = (\theta_t)_{t \geq 0}$, where C_t represents the consumption rate selected by the investor at time t and θ_t denote the amount of the wealth allocated to the market portfolio. The dynamics of wealth is given by

$$dW_t = [\theta_t(\mu_e - r) + rW_t - C_t]dt + \theta_t\sigma_e dZ_t^1, \quad t \geq 0, \quad W_0 = w. \quad (7)$$

Definition 3.1. *The investment and consumption strategy (C, θ) is said to be admissible for initial wealth w if (1) the consumption plan C is \mathbb{F} -adapted and takes value on $[0, \infty)$, such that $\int_0^t |C_s|ds < \infty$ (a.s.) for any $t \geq 0$; (2) the investment strategy θ is \mathbb{F} -adapted such that $\int_0^t \sigma_e^2 \theta_s^2 ds < \infty$ (a.s.) for each $t \geq 0$. The set of all admissible strategies is denoted by \mathcal{A} .*

Remark 1. *It is not necessary for the wealth process W given by (7) to remain non-negative at all times. This is because investors are endowed with a project and thus they can borrow from a liquidate financial market guaranteed by their project earnings.*

An investor is characterized by his initial wealth $W_0 = w$ and his preference (utility) $U(\cdot)$. Following many researchers, say [Henderson and Hobson \(2002\)](#), [Miao and Wang \(2007\)](#) and [Song, Wang and Yang \(2014\)](#) among others, we consider the CARA utility, i.e. the exponential utility given by

$$U(c) = -\exp(-\gamma c)/\gamma, \quad c \in \mathbb{R}, \quad (8)$$

where $\gamma > 0$ is the absolute risk aversion parameter.

3.1. Preliminaries

To derive the consumption utility indifference prices (implied values) of equity and CoCos, we first consider the following standard investment and consumption problem.

An investor invests in a risk-free asset and a risky market portfolio to smooth his consumption. The investor chooses an investment and consumption strategy to maximize his expected lifetime time-additive consumption utility

$$V^0(w) = \sup_{(C, \theta) \in \mathcal{A}} \mathbb{E} \left[\int_0^\infty \exp(-\beta t) U(C_t) dt | W_0 = w \right], \quad (9)$$

subject to (7), where $\beta > 0$ is a time-discount rate. Similar to [Merton \(1971\)](#), we easily obtain by dynamic programming that

$$V^0(w) = -\frac{1}{\gamma r} \exp \left(1 - \beta/r - \gamma r \left(w + \frac{\eta^2}{2r^2\gamma} \right) \right). \quad (10)$$

3.2. The implied value of equity

To price a claim, the key is to specify its cash flow. The cash flow of equity has three different expressions during different periods: The first is the residual cash flow $(1-\xi)(\delta_t - b_1 - b_2)$ if $0 \leq t \leq \tau_2$, i.e. the CoCo conversion has not taken place; The second is $(1-\lambda)(1-\xi)(\delta_t - b_1)$ if $\tau_2 \leq t < \tau_1$; The last is 0 if $t \geq \tau_1$, i.e. the firm has been liquidated. So, the claimant's wealth dynamics is given by

$$dW_t = \begin{cases} [\theta_t(\mu_e - r) + rW_t + (1-\xi)(\delta_t - b_1 - b_2) - C_t]dt \\ \quad + \theta_t \sigma_e dZ_t^1, & 0 \leq t < \tau_2; \\ [\theta_t(\mu_e - r) + rW_t + (1-\xi)(1-\lambda)(\delta_t - b_1) - C_t]dt \\ \quad + \theta_t \sigma_e dZ_t^1, & \tau_2 \leq t < \tau_1; \\ (\theta_t(\mu_e - r) + rW_t - C_t)dt + \theta_t \sigma_e dZ_t^1, & t \geq \tau_1. \end{cases} \quad (11)$$

Hence, the claimant should solve the following optimization problem

$$J(w, x) = \sup_{(C, \theta, \tau_1) \in \mathcal{C} \times \Theta \times \mathcal{T}} \mathbb{E} \left[\int_0^\infty \exp(-\beta s) U(C_s) ds \mid W_0 = w, \delta_0 = x \right], \quad (12)$$

subject to (1) and (11) backward by dynamic programming, where \mathcal{T} is the set of all $\{\mathcal{F}_t : t \geq 0\}$ -stopping times taking values on $[0, \infty)$.

Since the controlled system is time-homogeneous, there is a constant default threshold, denoted by \bar{x}_1 , such that the optimal default time is given by $\tau_1 = \inf \{s \geq 0 : \delta_s \leq \bar{x}_1\}$.

Remark 2. *Generally speaking, a default threshold would depend on a corresponding wealth level, even though it is independent of time for a time-homogeneous system. However, it is well-known that the CARA utility we assume here induces the implied value of an asset and default threshold independent of wealth level. This is actually why a lot of papers in the literature make the CARA utility assumption instead of CRRA utility though the latter is more reasonable from an economic viewpoint.*

The problem (12) can be solved backward. First, after the firm goes bankrupt, it is a standard investment consumption problem defined by (9) and (7), and so the value function is given by (10). Second, after CoCos are converted into equity but before the firm goes bankrupt, he should solve the following optimization problem

$$J^1(w, x) = \sup_{(C, \theta, \tau_1) \in \mathcal{C} \times \Theta \times \mathcal{T}} \mathbb{E} \left[\int_0^\infty \exp(-\beta s) U(C_s) ds \mid W_0 = w, \delta_0 = x \right], \quad (13)$$

subject to (1) and

$$dW_t = \begin{cases} [\theta_t(\mu_e - r) + rW_t + (1 - \xi)(1 - \lambda)(\delta_t - b_1) - C_t]dt \\ \quad + \theta_t \sigma_e dZ_t^1, & 0 \leq t < \tau_1; \\ (\theta_t(\mu_e - r) + rW_t - C_t)dt + \theta_t \sigma_e dZ_t^1, & t \geq \tau_1. \end{cases} \quad (14)$$

The Hamilton-Jacobi-Bellman (HJB) equation has the form

$$\begin{aligned} \sup_{c \geq 0, \theta} \{ & (rw + (1 - \xi)(1 - \lambda)(x - b_1) - c)J_w^1 + U(c) + \theta(\mu_e - r)J_w^1 \\ & + \theta \sigma_e \sigma \rho J_{wx}^1 + \frac{\theta^2}{2} \sigma_e^2 J_{ww}^1 \} + \mu J_x^1 + \frac{\sigma^2}{2} J_{xx}^1 - \beta J^1 = 0, \end{aligned} \quad (15)$$

with the following boundary conditions, see e.g. [Krylov \(1980\)](#):

$$J^1(w, \bar{x}_1) = V^0(w), \quad J_x^1(w, \bar{x}_1) = 0 \quad \text{and} \quad \lim_{x \rightarrow +\infty} J^1(w, x) = J^2(w, x), \quad (16)$$

where and throughout the text, the subscript of a function (J^1 here) represents the differentiation with respect to that variable and $J^2(w, x)$ is defined by solving the following optimization problem

$$J^2(w, x) = \sup_{(C, \theta) \in \mathcal{C} \times \Theta} \mathbb{E} \left[\int_0^\infty \exp(-\beta s) U(C_s) ds \mid W_0 = w, \delta_0 = x \right], \quad (17)$$

subject to (1) and

$$dW_t = [\theta_t(\mu_e - r) + rW_t + (1 - \xi)(1 - \lambda)(\delta_t - b_1) - C_t]dt + \theta_t \sigma_e dZ_t^1, \quad t \geq 0. \quad (18)$$

This is because if the current cash flow level $x \rightarrow +\infty$, the bankruptcy will not happen.

We now solve (15) with boundary conditions (16). At first, the optimal consumption and portfolio rules are evidently given by

$$U'(c) = J_w^1(w, x) \quad \text{and} \quad \theta = \frac{-J_w^1}{J_{ww}^1} \frac{\eta}{\sigma_e} + \frac{-J_{wx}^1}{J_{ww}^1} \frac{\sigma \rho}{\sigma_e}. \quad (19)$$

The first equation says that at the optimal solution, the marginal utility of current consumption is equal to the marginal utility of wealth increased if we consume less at present.

Therefore, if the current cash flow level is x , we define the consumption utility indifference price, i.e. the subjective value or implied value, denoted by $E^1(x, b_1)$,⁸ of equity as the solution of the following equation:

$$J^1(w, x) = V^0(w + E^1(x, b_1)). \quad (20)$$

It follows from the last equation of (16), that

$$\lim_{x \rightarrow \infty} V^0(w + E^1(x, b_1)) = J^2(w, x). \quad (21)$$

Noting that similar to Miao and Wang (2007), we are able to get an explicit solution to the optimization problem defined by (17) and (18). We therefore obtain following proposition by substituting (10) and (20) into (15), (16) and (19):

Proposition 3.1. *If CoCos have converted into equity but default has not occurred, the implied value of equity owned by original shareholders satisfies the following ordinary differential equation (ODE):*

$$\begin{aligned} & (1 - \xi)(1 - \lambda)(x - b_1) + (\mu - \rho\sigma\eta)E_x^1 + \frac{\sigma^2}{2}E_{xx}^1 \\ & - \frac{\sigma^2}{2}(1 - \rho^2)r\gamma(E_x^1)^2 = rE^1(x, b_1), \end{aligned} \quad (22)$$

⁸On account of that the controlled system is time-homogeneous and thanks to the assumption of CARA utility, the implied value of equity is a function of the current cash flow level x . In particular, it is independent of the current wealth level w . This conclusion can be proved by a “guess-and-verify” method as used by Miao and Wang (2007).

subject to the value-matching and smooth-pasting conditions⁹

$$E^1(\bar{x}_1, b_1) = 0 \quad \text{and} \quad E_x^1(\bar{x}_1, b_1) = 0 \quad (23)$$

and

$$\begin{aligned} \lim_{x \rightarrow +\infty} E^1(x, b_1) &= (1 - \xi)(1 - \lambda) \left[\frac{x - b_1}{r} + \frac{1}{r^2} (\mu - \rho\sigma\eta \right. \\ &\quad \left. - \frac{1}{2}\sigma^2(1 - \rho^2)(1 - \xi)\gamma(1 - \lambda)) \right]. \end{aligned} \quad (24)$$

The optimal consumption rate is given by

$$\begin{cases} c_t^* = \frac{\beta - r}{\gamma r} + r[W_t + \frac{\eta^2}{2r^2\gamma} + E^1(\delta_t, b_1)], & \tau_2 \leq t < \tau_1; \\ c_t^* = \frac{\beta - r}{\gamma r} + r(W_t + \frac{\eta^2}{2r^2\gamma}), & t \geq \tau_1. \end{cases} \quad (25)$$

And the optimal portfolio rule is given by

$$\begin{cases} \theta_t^* = \frac{\eta}{\gamma\sigma_e} \frac{1}{r} - \frac{\rho\sigma}{\sigma_e} E_x^1, & \tau_2 \leq t < \tau_1; \\ \theta_t^* = \frac{\eta}{\gamma\sigma_e} \frac{1}{r}, & t \geq \tau_1. \end{cases} \quad (26)$$

Last, if CoCos have not converted yet, in the same way, the HJB equation has the form

$$\begin{aligned} \sup_{c \geq 0, \theta} \{ & (rw + (1 - \xi)(x - b_1 - b_2) - c)J_w + U(c) + \theta(\mu_e - r)J_w \\ & + \theta\sigma_e\sigma\rho J_{wx} + \frac{\theta^2}{2}\sigma_e^2 J_{ww} \} + \mu J_x + \frac{\sigma^2}{2} J_{xx} - \beta J = 0, \end{aligned} \quad (27)$$

with the following boundary conditions:

$$J(w, \bar{x}_2) = J^1(w, \bar{x}_2) \quad \text{and} \quad \lim_{x \rightarrow +\infty} J(w, x) = J^3(w, x), \quad (28)$$

where \bar{x}_2 represents the conversion threshold, and $J^3(w, x)$ is given by an explicit solution of the following simpler optimization problem:

$$J^3(w, x) \equiv \sup_{(C, \theta) \in \mathcal{C} \times \Theta} \mathbb{E} \left[\int_0^\infty \exp(-\beta s) U(C_s) ds \mid W_0 = w, \delta_0 = x \right],$$

⁹It is a priori not obvious that the value-matching and smooth-pasting conditions are equivalent to the solution of the constrained maximization problem of shareholders. For this problem, please refer to the discussions by [Song, Wang and Yang \(2014\)](#).

subject to (1) and

$$dW_t = [\theta_t(\mu_e - r) + rW_t + (1 - \xi)(\delta_t - b_1 - b_2) - C_t]dt + \theta_t\sigma_e dZ_t^1, \quad t \geq 0.$$

At first, thanks to (27), the optimal consumption rate and portfolio rule are respectively given by

$$U'(c) = J_w(w, x) \quad \text{and} \quad \theta = \frac{-J_w}{J_{ww}} \frac{\eta}{\sigma_e} + \frac{-J_{wx}}{J_{ww}} \frac{\sigma\rho}{\sigma_e}. \quad (29)$$

Under this situation, the implied value, denoted by $E(x, b_1, b_2)$, of equity is defined by

$$J(w, x) = V^0(w + E(x, b_1, b_2)), \quad (30)$$

and we therefore have

$$\lim_{x \rightarrow +\infty} V^0(w + E(x, b_1, b_2)) = J^3(w, x). \quad (31)$$

Similar to the derivation of Proposition 3.1, we obtain the following theorem:

Theorem 3.2. *If CoCos have not converted into equity, the implied value $E(x, b_1, b_2)$ of equity is a solution of the following ODE:*

$$\begin{aligned} (1 - \xi)(x - b_1 - b_2) + (\mu - \rho\sigma\eta)E_x + \frac{\sigma^2}{2}E_{xx} \\ - \frac{\sigma^2}{2}(1 - \rho^2)r\gamma(E_x)^2 = rE, \end{aligned} \quad (32)$$

subject to the boundary conditions:

$$E(\bar{x}_2, b_1, b_2) = E^1(\bar{x}_2, b_1) \quad (33)$$

and

$$\begin{aligned} \lim_{x \rightarrow +\infty} E(x, b_1, b_2) = & (1 - \xi)\left[\frac{x - b_1 - b_2}{r} + \frac{1}{r^2}(\mu - \rho\sigma\eta) \right. \\ & \left. - \frac{1}{2}\sigma^2(1 - \rho^2)(1 - \xi)\gamma\right], \end{aligned} \quad (34)$$

where \bar{x}_2 is a conversion threshold given by (6), and the function $E^1(\cdot, b_1)$ is given by Proposition 3.1. The optimal consumption rate is given by

$$\begin{cases} c_t^* = \frac{\beta-r}{\gamma r} + r[W_t + \frac{\eta^2}{2r^2\gamma} + E(\delta_t, b_1, b_2)], & 0 \leq t < \tau_2; \\ c_t^* = \frac{\beta-r}{\gamma r} + r[W_t + \frac{\eta^2}{2r^2\gamma} + E^1(\delta_t, b_1)], & \tau_2 \leq t < \tau_1; \\ c_t^* = \frac{\beta-r}{\gamma r} + r(W_t + \frac{\eta^2}{2r^2\gamma}), & t \geq \tau_1. \end{cases} \quad (35)$$

And the corresponding optimal portfolio rule is given by

$$\begin{cases} \theta_t^* = \frac{\eta}{\gamma\sigma_e} \frac{1}{r} - \frac{\rho\sigma}{\sigma_e} E_x, & 0 \leq t < \tau_2; \\ \theta_t^* = \frac{\eta}{\gamma\sigma_e} \frac{1}{r} - \frac{\rho\sigma}{\sigma_e} E_x^1, & \tau_2 \leq t < \tau_1; \\ \theta_t^* = \frac{\eta}{\gamma\sigma_e} \frac{1}{r}, & t \geq \tau_1. \end{cases} \quad (36)$$

We now discuss the implications of this theorem. First, if $\gamma \rightarrow 0$, i.e. the investor is risk-neutral toward idiosyncratic risk, or $|\rho| = 1$, i.e. idiosyncratic risk disappears, the last term of the left-hand side of (32) disappears, and (32) becomes the standard pricing equation under the equilibrium pricing approach. Otherwise, if $\gamma > 0$ and $|\rho| < 1$, due to the fact that the last term of the left-hand side of (32) is strictly negative, for a sufficient large risk-aversion index, (32) says that a larger volatility may lead to a lower value of equity. This result is in sharp contrast to that derived from an equilibrium pricing approach. Intuitively, this is because a larger volatility means a higher risk, and therefore the value of the claim may be less for a risk-averse equity holder. However, if investors are risk-neutral toward idiosyncratic risk, the value of equity may increase with the volatility σ of the cash flow, since shareholders harvest all the cash flow growth but the loss suffered by them is limited thanks to bankruptcy protection.

3.3. The implied value of CoCos

The CoCo holder's wealth evolves in the following way:

$$dW_t = \begin{cases} (\theta_t(\mu_e - r) + rW_t + b_2 - C_t)dt + \theta_t\sigma_e dZ_t^1, & 0 \leq t \leq \tau_2; \\ [\theta_t(\mu_e - r) + rW_t + \lambda(1 - \xi)(\delta_t - b_1) - C_t]dt \\ + \theta_t\sigma_e dZ_t^1, & \tau_2 \leq t < \tau_1; \\ (\theta_t(\mu_e - r) + rW_t - C_t)dt + \theta_t\sigma_e dZ_t^1, & t \geq \tau_1. \end{cases} \quad (37)$$

With the conversion threshold and default threshold given by (6) and Proposition 3.1 respectively, its claimant seeks to choose a consumption plan $C \in \mathcal{C}$ and a portfolio rule $\theta \in \Theta$ so as to maximize the expected lifetime utility:

$$G(w, x) = \sup_{(C, \theta) \in \mathcal{C} \times \Theta} \mathbb{E} \left[\int_0^\infty \exp(-\beta s) U(C_s) ds \mid W_0 = w, \delta_0 = x \right], \quad (38)$$

subject to (1) and wealth accumulation equation (37). We solve this backward by dynamic programming as before and obtain the following conclusions:

Proposition 3.3. *If CoCos have converted into 100λ percent of equity and default has not taken place, then the implied value $CB^1(x, b_1)$ of CoCos satisfies the following ODE:*

$$\begin{aligned} \lambda(1 - \xi)(x - b_1) + (\mu - \rho\sigma\eta)CB_x^1 + \frac{\sigma^2}{2}CB_{xx}^1 \\ - \frac{\sigma^2}{2}(1 - \rho^2)r\gamma(CB_x^1)^2 = rCB^1, \end{aligned} \quad (39)$$

subject to the first boundary condition

$$CB^1(\bar{x}_1, b_1) = 0, \quad (40)$$

where \bar{x}_1 is the default threshold given by Proposition 3.1, and the second boundary condition

$$\begin{aligned} \lim_{x \rightarrow +\infty} CB^1(x, b_1) &= \lambda(1 - \xi) \left\{ \frac{x - b_1}{r} + \frac{1}{r^2} [\mu - \rho\sigma\eta \right. \\ &\quad \left. - \frac{1}{2}\sigma^2(1 - \rho^2)\gamma\lambda(1 - \xi)] \right\}. \end{aligned} \quad (41)$$

The optimal consumption rate is given by

$$\begin{cases} c_t^* = \frac{\beta-r}{\gamma r} + r[W_t + \frac{\eta^2}{2r^2\gamma} + CB^1(\delta_t, b_1)], & \tau_2 \leq t < \tau_1; \\ c_t^* = \frac{\beta-r}{\gamma r} + r(W_t + \frac{\eta^2}{2r^2\gamma}), & t \geq \tau_1. \end{cases} \quad (42)$$

And the optimal portfolio rule is given by

$$\begin{cases} \theta_t^* = \frac{\eta}{\gamma\sigma_e} \frac{1}{r} - \frac{\rho\sigma}{\sigma_e} CB_x^1, & \tau_2 \leq t < \tau_1; \\ \theta_t^* = \frac{\eta}{\gamma\sigma_e} \frac{1}{r}, & t \geq \tau_1. \end{cases} \quad (43)$$

Theorem 3.4. *If CoCos have not converted into equity, the implied value $CB(x, b_1, b_2)$ of CoCos satisfies the following ODE:*

$$b_2 + (\mu - \rho\sigma\eta)CB_x + \frac{\sigma^2}{2}CB_{xx} - \frac{\sigma^2}{2}(1 - \rho^2)r\gamma(CB_x)^2 = rCB, \quad (44)$$

subject to the boundary conditions

$$CB(\bar{x}_2, b_1, b_2) = CB^1(\bar{x}_2, b_1), \quad \text{and} \quad \lim_{x \rightarrow +\infty} CB(x, b_1, b_2) = \frac{b_2}{r}, \quad (45)$$

where the conversion threshold \bar{x}_2 is given by (6) and the function $CB^1(\cdot, b_1)$ is given by Proposition 3.3. The optimal consumption rate is given by

$$\begin{cases} c_t^* = \frac{\beta-r}{\gamma r} + r[W_t + \frac{\eta^2}{2r^2\gamma} + CB(\delta_t, b_1, b_2)], & 0 \leq t < \tau_2; \\ c_t^* = \frac{\beta-r}{\gamma r} + r[W_t + \frac{\eta^2}{2r^2\gamma} + CB^1(\delta_t, b_1)], & \tau_2 \leq t < \tau_1; \\ c_t^* = \frac{\beta-r}{\gamma r} + r(W_t + \frac{\eta^2}{2r^2\gamma}), & t \geq \tau_1. \end{cases} \quad (46)$$

And the corresponding optimal portfolio rule is

$$\begin{cases} \theta_t^* = \frac{\eta}{\gamma\sigma_e} \frac{1}{r} - \frac{\rho\sigma}{\sigma_e} CB_x, & 0 \leq t < \tau_2; \\ \theta_t^* = \frac{\eta}{\gamma\sigma_e} \frac{1}{r} - \frac{\rho\sigma}{\sigma_e} CB_x^1, & \tau_2 \leq t < \tau_1; \\ \theta_t^* = \frac{\eta}{\gamma\sigma_e} \frac{1}{r}, & t \geq \tau_1. \end{cases} \quad (47)$$

We discuss the implications of this theorem as follows. First, the last term $\sigma^2(1 - \rho^2)r\gamma(CB_x)^2/2$ on the left-hand side of (44), reflects the effect of the risk attitude of CoCo holders and the idiosyncratic risk volatility on the implied CoCo value. It says clearly that under an incomplete market, the risk aversion γ or the idiosyncratic risk volatility ($\sigma\sqrt{1 - \rho^2}$) reduces the implied value. If γ approaches 0 or idiosyncratic risk is fully diversified, we can derive an explicit implied value, which is just its equilibrium price. In this case, only systematic risk demands risk premium. The second term on the left-hand side of (44), $(\mu - \rho\sigma\eta)CB_x$, presents the risk-adjusted expected growth rate of the cash flow through the systematic volatility of the cash flow and the Sharpe ratio. The risk adjustment can be obtained from the CAPM model. In an incomplete market, idiosyncratic risk is non-diversifiable, and the last term $\sigma^2(1 - \rho^2)r\gamma(CB_x)^2/2$ on the left-hand side of (44) plays an important role on risk adjustment from idiosyncratic risk.

The first equation of (45) is due to the value-matching condition at conversion which says that the implied value of CoCos will not jump due to conversion. And the second equation shows that the implied value of CoCos will converge to the value of default-free bonds with the same coupon if conversion never happens. Equation (46) represents that optimal consumption just consists of a fixed consumption, an excess return of the market portfolio investment and a part of the equivalent risk-free income of the claimant's total wealth including the implied value of the future cash flow. Constant $\frac{\beta-r}{\gamma r}$ is the fixed consumption, which says that the greater the time-discount rate, the more the current consumption. Equation (47) suggests that the hedging demand increases with the absolute values of the correlation coefficient ($|\rho|$)

and the marginal value of CoCos. After default has taken place, the third equalities of (46) and (47) are the standard Merton-style consumption plan and portfolio rule respectively.

Based on the above analysis, we derive the equilibrium prices, which are also the no-arbitrage prices under a complete market, in the following section.

4. Equilibrium prices of corporate securities

Naturally, we assume that the firm gains the SBs' value at the issuance time and the debt holders are competitive. That is, we assume SB investors are well diversified and thus we provide an equilibrium price for SBs, similar to (4). We price SBs by computing the expected sum of their coupons discounted with the risk-free interest rate under the measure \mathbb{Q} , and therefore, we obtain:

Theorem 4.1. *If the current cash flow rate is x and default has not occurred, the equilibrium price of SBs is explicitly given by*

$$B(x, b_1) = \frac{b_1}{r} (1 - e^{\kappa(x - \bar{x}_1)}) + (1 - \alpha)(1 - \xi) \left(\frac{\bar{x}_1}{r} + \frac{\mu - \rho\sigma\eta}{r^2} \right) e^{\kappa(x - \bar{x}_1)}, \quad (48)$$

where $\kappa = -\frac{(\mu - \rho\sigma\eta) + \sqrt{(\mu - \rho\sigma\eta)^2 + 2r\sigma^2}}{\sigma^2} < 0$ and \bar{x}_1 is the default threshold given by Proposition 3.1.

Remark 3. *Theorem 4.1 states that the equilibrium price of SBs is merely the equilibrium price of the coupon payment obtained by its holders before bankruptcy, plus a lump-sum dividend paid at bankruptcy, equalling a fraction $(1 - \alpha)$ of the market value of the unlevered firm.*

In the same way, it is easy to derive the equilibrium prices of equity and CoCos as the corollaries of Theorems 3.2 and 3.4 and Propositions 3.1 and 3.3, respectively. To save space we omit them here.

5. Optimal Capital Structure

In practice, there are seldom firms who take pure equity financing and conversely a mix of debt and equity is common. A general explanation for this phenomenon is that debt financing can reduce the tax burden on firms. This explanation is reasonable, but not complete. In fact, as seen in our numerical analysis, another reason is more important in a risk-averse world. Namely, in contrast to pure equity financing, a mix of equity, SBs and CoCos can produce considerable diversification benefits in addition to tax shields.

In essence, all diversification benefits are generated from the nonlinear qualities of the utility-based asset prices combining with idiosyncratic risk. This is because the utility-based asset prices are derived from a nonlinear pricing schedule, which refers to any pricing structure where the total charges payable by purchasers are not proportional to the quantity of their obtained assets. In particular, as stated at Proposition 1.8 in the book of [Carmona \(2009\)](#), the indifference price (implied value) of a claim is a nondecreasing and convex function of its payoff. It means that the implied value of a claim is less than the sum of all the implied values of its parts. On the contrary, the diversification benefits disappear once the prices are derived from a risk-neutral pricing method since the latter corresponds to a linear pricing schedule.

Naturally, we want to know what the best combination of equity, SBs and

CoCos, i.e. optimal capital structure, should be to maximize the total firm's value. Motivated by [Duffie \(2001\)](#),¹⁰ we assume the original owner of the firm will sell the firm to the equity and CoCo holders at the consumption utility indifference prices defined in this paper respectively, and to the diversified SB investors at the equilibrium price. In other words, the firm values its equity at the implied value $E(x, b_1, b_2)$ and its CoCos at $CB(x, b_1, b_2)$. Diversified lenders price debt in competitive capital markets at $B(x, b_1)$, which does not contain the idiosyncratic risk premium.

Doing so, we must specify the ownership stake of CoCos at the same time. However, to the best of our knowledge, no papers in the literature consider the optimal ownership stake. This is because all papers in the literature assume investors are risk-neutral toward idiosyncratic risk and therefore, the selection of the conversion ratio is independent of the total firm value. However, if investors are risk-averse, the situation is fundamentally different, i.e. the total firm value is closely related with the ownership stake. For this reason, optimal capital structure problem is to maximize the sum of the three prices of equity, SBs and CoCos over all admissible ownership stake λ and coupons b_1 and b_2 .

More specifically, given the exogenous conversion threshold and endogenous bankruptcy conditions, the original owner divides the total cash flow of the firm into three parts that are uniquely determined by the ownership stake λ and coupons b_1 and b_2 . That is, to derive an optimal capital structure

¹⁰See the second paragraph from the bottom on Page 260 of the book titled *Dynamic Asset Pricing Theory* authored by Darrell Duffie in 2001.

for a firm, we need only to solve the following optimization problem:

$$\max_{b_1, b_2, \lambda} \{E(x, b_1, b_2) + CB(x, b_1, b_2) + B(x, b_1)\}, \quad (49)$$

where the function $E(\cdot, b_1, b_2)$, $CB(\cdot, b_1, b_2)$ and $B(\cdot, b_1)$ are given by Theorems 3.2, 3.4 and 4.1 respectively.

Thanks to [Chen et al. \(2010\)](#), we may also interpret the target function in (49) as the total value that one needs to pay to acquire the firm by buying out the shareholders, CoCo holders and SB holders.

There are seldom explicit solutions of optimal capital structure including CoCos even based on an equilibrium pricing approach, let alone the utility-based pricing we discuss here. Naturally, an explicit solution to the optimization problem defined by (49) is not available and thus, we provide numerical simulations in the next section.

6. Comparative statics and numerical simulations

In this section, we perform numerical simulations and focus on parameter regions where conversion and default will not happen immediately. Following [Koziol and Lawrenz \(2012\)](#) and [Glasserman and Nouri \(2012\)](#) among others, unless otherwise stated, the baseline parameter values are selected as follows: $r = 0.05$, $\delta_0 = x = 5$, $\phi = 0.96$, $b_1 = 4$, $b_2 = 0.8$, $\sigma = 0.6$, $\mu = 0.18$, $\eta = 0.35$, $\gamma = 1$, $\rho = 0.7$, $\xi = 0.2$, $\alpha = 0.4$ and the ownership stake $\lambda = 0.4$.¹¹

¹¹ By contrast, [Koziol and Lawrenz \(2012\)](#) suppose a firm's cash flow is driven by a geometric Brownian motion and their conversion time is the first time of the cash flow hitting the boundary $\phi(d + c)$, where $\phi > 0$ is a constant, d is the aggregate deposit payment and c is CoCos' coupon rate. Our conversion rule is similar to [Glasserman and Nouri \(2012\)](#).

6.1. The effects of risk aversion and cash flow volatility

Table 1 states that the implied value of equity decreases quickly but that of CoCos decreases slowly as investors get more risk-averse. The difference results from the fact that equity has more cash flow risk than CoCos. As predicted by (6), conversion threshold \bar{x}_2 keeps unchanged. The SBs' price is generally independent of risk aversion. However, as explained in Table 1, a bigger risk aversion induces a higher default threshold \bar{x}_1 and naturally a bit less SBs' price.

Table 1: Impacts of **risk aversion** (γ) on default threshold \bar{x}_1 , conversion threshold \bar{x}_2 , equilibrium price B of SBs and the implied values E , CB of equity and CoCos respectively.

γ	$\gamma \rightarrow 0$	0.01	0.1	1	1.5	2	2.5
\bar{x}_1	1.74	1.74	1.78	2.05	2.16	2.26	2.33
\bar{x}_2	4.34	4.34	4.34	4.34	4.34	4.34	4.34
B	72.60	72.60	72.49	71.50	71.07	70.69	70.37
E	19.98	19.80	18.99	13.40	11.53	10.15	9.09
CB	10.98	10.95	10.81	9.57	8.99	8.47	8.02

Table 2 depicts the same results with Table 1 with regard to cash flow volatility (σ) instead of risk aversion. It displays that the implied values of equity and CoCos and the equilibrium price of SBs decrease as the volatility grows. The former is obvious and the latter holds since SB holders need more systematic risk premium for a larger volatility. In contrast to standard pricing method, there is a U-shaped relation between the default threshold and volatility. It turns out that a higher cash flow volatility has two opposite

effects on the endogenous default threshold \bar{x}_1 : One decreases the default threshold due to the liquidation option effect and the other increases the threshold since investors are risk-averse.

Table 2: Impacts of **cash flow volatility** (σ)

σ	0.5	0.55	0.6	0.65	0.7
\bar{x}_1	2.02	2.05	2.05	2.04	2.03
\bar{x}_2	3.85	4.10	4.34	4.59	4.83
B	76.64	74.26	71.50	68.52	65.42
E	16.52	14.79	13.40	12.21	11.17
CB	12.14	10.73	9.55	8.66	8.04

6.2. Risk-taking incentives

To study the risk-taking incentives of shareholders of a firm issuing CoCos, we assume the firm has an option to increase cash flow volatility σ by choosing different technologies. We check whether CoCos will enhance risk-prevention incentive to shareholders or not. To this end, we compute the implied values of equity while cash flow volatility changes from $\sigma = 0.1$ to $\sigma = 0.9$ for several different risk aversion parameters, ownership stakes and correlations. The numerical results are reported in Figures 1~6.

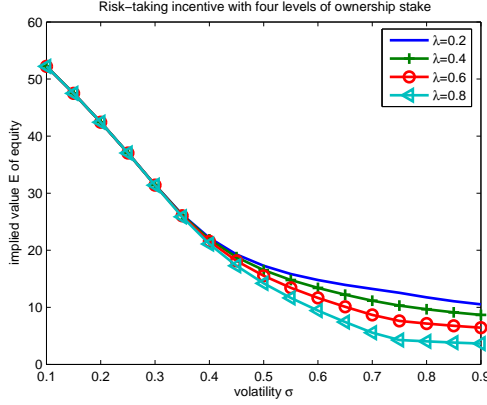


Figure 1: This figure shows the impact of **volatility** (σ) on the implied value of equity with four levels of ownership stake (risk aversion $\gamma = 1$ and correlation coefficient $\rho = 0.7$).

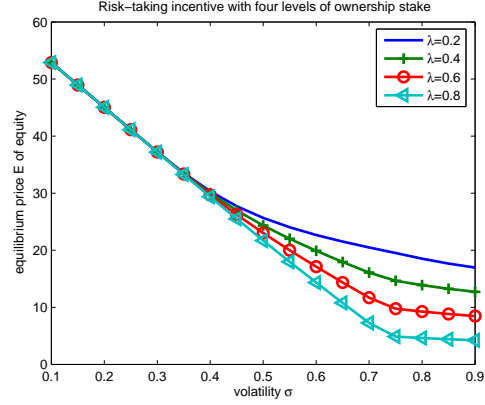


Figure 2: This figure shows the impact of **volatility** (σ) on the equilibrium price of equity with four levels of the ownership stake (risk aversion $\gamma \rightarrow 0$ and correlation coefficient $\rho = 0.7$).

Figures 1 and 2 depict the impact of volatility σ on the implied value of equity with four ownership stakes λ when risk aversion $\gamma = 1$ and $\gamma \rightarrow 0$ respectively. They show that if the volatility is small enough, the ownership stake almost has no impact on the implied value of equity. Namely, if the issuing firm's profits keep steady growth, the ownership stake has little effect on the interests of equity holders. This conclusion is counter-intuitive at first sight. It turns out that a smaller volatility leads to a lower possibility of conversion and naturally the impact of ownership stake on the implied value of equity is very limited. Figures 3 and 4 state that there are risk-taking incentives for an investor who is risk-neutral toward idiosyncratic risk but there are not for a risk-averse investor, even his risk-aversion index is small, say $\gamma = 0.5$. In general, the higher the ownership stake or the larger the risk

aversion index, the weaker the incentives. The intuition behind this result is that if shareholders are risk-averse, they demand an extra idiosyncratic risk premium, which definitely decreases the implied value of equity. This effect makes shareholders prefer projects with a lower idiosyncratic volatility. Figure 5 shows further that the implied value of equity decreases faster with cash flow volatility for a more risk-averse investor. In other words, there are fewer risk-taking incentives for a more risk-averse shareholder. All the results explain that in a risk-averse world, CoCos can prevent shareholders from investing in high-risk projects and indirectly increase the financial safety of the issuing firms.

In particular, our results here are in contrast to the standard results on CoCos, say those derived by [Koziol and Lawrenz \(2012\)](#) among others, which argue that CoCos induce the risk-shifting incentives. This difference results from our assumption that investors are risk-averse. For this reason, a high-risk project might have much less value than that in a risk-neutral world. Another reason is that [Koziol and Lawrenz \(2012\)](#) do not take into account the change of the risk-adjusted drift parameter while investing in a project with a higher risk. Doing so, they actually assume that the expected return of the investment project increases with its risk at the same time. As a result, investors in this case would have more risk-shifting incentives.

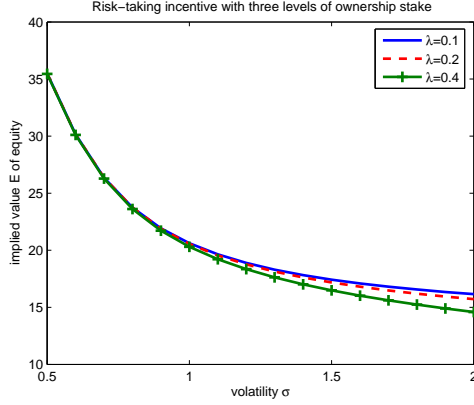


Figure 3: This figure shows the impact of **volatility** (σ) on the implied value of equity with three levels of the ownership stake (risk aversion $\gamma = 0.5$ and correlation coefficient $\rho = 0.2$).

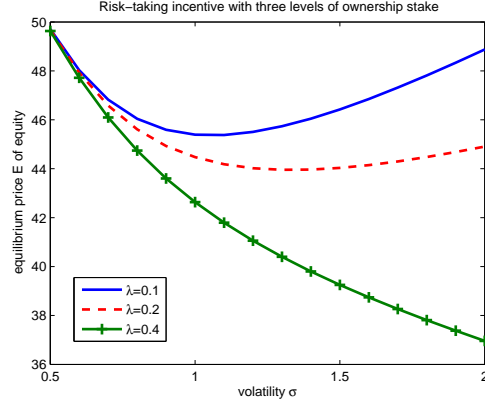


Figure 4: This figure shows the impact of **volatility** (σ) on the equilibrium price of equity with three levels of the ownership stake (risk aversion $\gamma \rightarrow 0$ and correlation coefficient $\rho = 0.2$).

Figure 6 turns to the impact of cash flow volatility σ and correlation (ρ) on the implied value of equity. In general, the same story described in Figure 5 happens again. However, if the cash flow is strongly negatively correlated with the market portfolio,¹² the implied value of equity will conversely increase with cash flow volatility (σ). This is true since a negatively correlated project is just like an insurance product and can decrease the total systematic risk faced by shareholders.

¹²Generally speaking, as a matter of fact, the correlation is positive.

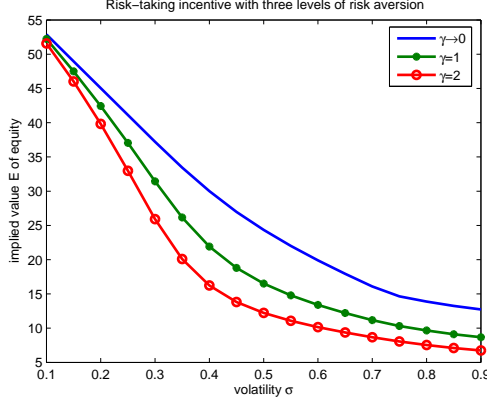


Figure 5: The impact of **volatility** (σ) on the implied value of equity under different risk aversion parameters.

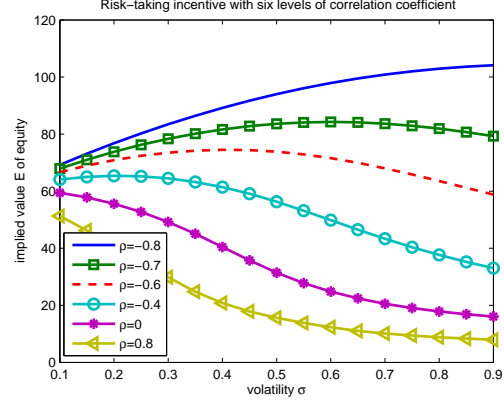


Figure 6: This figure shows the impact of **volatility** (σ) on the implied value of equity under different correlations.

6.3. Analysis on capital structure

To compare our model with a classical capital structure, in this and next subsections, we maximize the total firm value over the coupon rates b_1 and b_2 for a previously given ownership stake $\lambda = 0.4$. We will discuss optimal ownership stake as well in Subsection 6.5.

We fix optimal capital structure for the given ownership stake $\lambda = 0.4$ with and without CoCos respectively. First, we ignore taxes by letting $\xi = 0$ to measure diversification benefits. Second, we take into account tax benefits by letting $\xi = 0.2$. Finally, we compare the conclusions in a risk-averse world with those in the risk-neutral one.

Tables 3~6 present simulation results. Panel A of Table 3 shows optimal capital structure without tax, i.e. $\xi = 0$. In a classical theory, it is unfavorable to take debt financing since it only increases bankruptcy costs. This is incorrect in our model as seen from Panel A of Table 3. The reason is that

both CoCos and SBs produce diversification benefits. By contrast, in a risk-neutral world, the diversification benefits disappear and so the SBs' coupon rate is zero while the CoCos' coupon rate can take any value as seen in the panel. It is emphasized that in optimal capital structure even without tax, only if CoCo investors are risk-averse, it is favorable to issue CoCos but CoCos must convert into equity immediately (since $\bar{x}_2 \geq 5.02 > \delta_0 = 5$) as seen in Panel A of Table 3. The reason is that CoCos still provide diversification benefits.

Both Panel A and Panel B of Table 3 state that the firm should issue less CoCos and equity and more SBs if investors are more risk-averse. This is because for a more risk-averse investor, the subjective value (implied value) of the same risk asset is less. Meanwhile, SBs are priced by the equilibrium pricing approach and thus it does not depend on risk aversion. Therefore, the firm should sell less CoCos to a more risk-averse investor and naturally the amount of SBs issued gets more.

Table 3 further shows that the optimal leverage, default threshold and conversion threshold increase with risk aversion but the total firm value reversely decreases with the index.

Panels A~D of Table 5 show the optimal capital structure with different idiosyncratic risk volatility levels ($\varepsilon \equiv \sigma\sqrt{1 - \rho^2}$) while systematic risk keeps unchanged. It says that if investors are risk-averse enough, say $\gamma = 1$, once idiosyncratic risk rises, the firm should sell less CoCos and equity but more SBs. However, if the investors are risk-neutral toward idiosyncratic risk ($\gamma \rightarrow 0$), the opposite holds true. The main reason is that the utility-based price of a risk asset is less than its equilibrium price and the difference increases

with idiosyncratic risk. Hence, the firm would rather allocate more cash flow to SB investors if the investors of equity or CoCos are more risk-averse or idiosyncratic risk goes up. On the contrary, if investors are risk-neutral, then an increased idiosyncratic risk will raise the values of equity and CoCos as well since both of them involve a call option on the firm's asset. Therefore, the firm should sell less SBs and increase the amount of equity and CoCos.

Table 5 presents that conversion threshold and default threshold increase with SBs' coupon rate. A larger bankruptcy loss rate will induce higher expected bankruptcy costs and consequently the firm should decrease the amount of SBs issued. In this way, the possibility of bankruptcy drops and the negative effect of an increased bankruptcy loss rate becomes less.

Tables 3 and 5 show that optimal leverage $(CB + B)/(E + CB + B)$ is always greater than that if CoCos are not issued. It increases with idiosyncratic risk volatility ε and risk aversion γ . However, the opposite holds true in a risk-neutral world from Panel D of Table 5. Hence, the diversification benefits lead to a seemingly counterintuitive prediction: More risk-averse investors prefer a higher leverage. This happens because investors are risk-averse and both equity and CoCos are exposed to idiosyncratic risk while SBs not. It can be further explained as follows.

First, a natural measure of a leverage in our model is the implied leverage, which is defined as the ratio of the debt value, i.e. the sum of the implied value of CoCos and the equilibrium value of SBs, to the total value of the firm, which equals the debt value plus the implied value of equity. Accordingly, the implied (subjective) value of equity (due to nondiversifiable risk) has significant impact on the leverage and ignoring subjective valuation will

substantially underestimate the firm's leverage. In fact, as pointed out in Subsection 6.1, the value of equity decreases quickly and that of CoCos decreases slowly as risk aversion rises. This means that the more risk-averse the investors, the higher the leverage.

Second, diversification motives make the firm issue more debt, which further raises the leverage ratio of the issuing firm. The more risk-averse the investors, the stronger incentive they have to diversify idiosyncratic risk. While numerical results depend on model parameter assumptions, the analysis provides support for our intuition that the firm's need for diversification and the subjective valuation discount for bearing nondiversifiable idiosyncratic risk are key determinants of the implied leverage for a firm, as argued by [Chen et al. \(2010\)](#). Our analysis also explains that the classical approach to fix the leverage of a firm would lead to a wrong tradeoff between equity and debt and its leverage might be significantly underestimated.

Table 3: This table plots the impact of **risk aversion** γ on the optimal capital structure including equity, SBs and CoCos. We consider two business income tax rates ($\xi = 0$ or $\xi = 0.2$) and three risk aversion levels. The case " $\gamma \rightarrow 0$ " corresponds to the equilibrium pricing. In particular, the first line of Panel A represents that the total firm value is independent of the coupon of CoCos. The total firm value is given by $T = E + CB + B$, and the leverage given by $L \equiv (CB + B)/(E + CB + B)$.

γ	b_1	B	b_2	CB	E	T	$L(\%)$	\bar{x}_2	\bar{x}_1
Panel A:									
$\xi = 0$									
$\gamma \rightarrow 0$	0	0	—	—	—	113.34	—	—	-2.26
$\gamma = 1$	3.55	65.68	≥ 1.90	14.43	19.60	99.71	80.35	≥ 5.02	1.66
$\gamma = 2$	4.20	74.27	≥ 1.25	8.33	10.48	93.08	88.74	≥ 5.02	2.53
Panel B:									
$\xi = 0.2$									
$\gamma \rightarrow 0$	3.35	62.66	1.20	16.40	24.87	103.93	76.07	4.08	1.09
$\gamma = 1$	4.00	71.50	1.10	10.44	12.83	94.77	86.46	4.65	2.05
$\gamma = 2$	4.20	73.02	1.00	7.75	8.89	89.66	90.08	4.76	2.46

Table 4: This table reports the impact of **risk aversion** γ on the optimal capital structure including equity and the straight bond only. The partial aim is to compare the results with those reported in Table 3. The total firm value is given by $T = E+B$, and the leverage given by $L = B/(E+B)$.

γ	b_1	B	E	T	L (%)	\bar{x}_1
Panel A: $\xi = 0$						
$\gamma \rightarrow 0$	0	0	113.34	113.34	0	-2.26
$\gamma = 1$	4.20	74.49	18.97	93.46	79.70	2.47
$\gamma = 2$	4.55	77.20	10.06	87.26	88.48	3.07
Panel B: $\xi = 0.2$						
$\gamma \rightarrow 0$	4.05	73.29	29.19	102.48	71.52	1.79
$\gamma = 1$	4.35	74.92	15.11	90.03	83.22	2.55
$\gamma = 2$	4.50	75.16	9.21	84.36	89.09	2.99

Table 5: This table reports the impact of **idiosyncratic risk volatility** ($\varepsilon \equiv \sigma\sqrt{1 - \rho^2}$) and **bankruptcy loss rate** (α) on the optimal capital structure including equity, SBs and CoCos. We consider two business income tax rates ($\xi = 0$ or $\xi = 0.2$) and three levels of the idiosyncratic risk volatility ($\varepsilon = 0.30$, $\varepsilon = 0.43$ or $\varepsilon = 0.56$). We compare the implied values in Panel B with the equilibrium prices in Panel D. We take the baseline parameter values unless otherwise stated in the table.

ε	b_1	B	b_2	CB	E	T	L (%)	\bar{x}_2	\bar{x}_1
Panel A:									
$\xi = 0$									
$\varepsilon = 0.30$	2.80	54.20	≥ 2.65	21.00	29.96	105.17	71.51	≥ 5.02	1.01
$\varepsilon = 0.43$	3.55	65.68	≥ 1.90	14.43	19.60	99.71	80.35	≥ 5.02	1.66
$\varepsilon = 0.56$	4.00	70.10	≥ 1.30	10.83	14.06	94.99	85.20	≥ 5.02	1.97
Panel B:									
$\varepsilon = 0.30$	3.80	70.78	1.15	12.87	15.47	99.12	84.39	4.50	1.97
$\varepsilon = 0.43$	4.00	71.50	1.10	10.44	12.83	94.77	86.46	4.65	2.05
$\varepsilon = 0.56$	4.25	71.99	1.05	8.30	10.49	90.78	88.45	4.86	2.15
Panel C:									
$\alpha = 0.6$									
$\varepsilon = 0.30$	3.40	63.91	1.40	16.49	18.01	98.41	81.70	4.34	1.57
$\varepsilon = 0.43$	3.60	64.99	1.35	13.45	15.23	93.67	83.74	4.50	1.65
$\varepsilon = 0.56$	3.80	65.22	1.30	11.00	12.98	89.20	85.45	4.65	1.70
Panel D:									
$\gamma \rightarrow 0$									
$\varepsilon = 0.30$	3.45	65.59	1.15	16.10	22.96	104.65	78.06	4.13	1.46
$\varepsilon = 0.43$	3.35	62.66	1.20	16.40	24.87	103.93	76.07	4.08	1.09
$\varepsilon = 0.56$	3.30	60.38	1.25	16.56	26.37	103.31	74.47	4.07	0.73

Table 6: This table reports the impact of **idiosyncratic risk volatility** ε and **loss rate** (α) on the optimal capital structure including equity and SBs only. The partial aim is to compare the results with those reported in Table 5.

ε	b_1	B	E	T	$L(\%)$	\bar{x}_1
Panel A: $\gamma = 1, \xi = 0, \alpha = 0.4$						
$\varepsilon = 0.30$	3.65	68.74	31.24	99.98	68.75	1.97
$\varepsilon = 0.43$	4.20	74.49	18.97	93.46	79.70	2.47
$\varepsilon = 0.56$	4.50	75.22	13.37	88.59	84.91	2.68
Panel B: $\gamma = 1, \xi = 0.2, \alpha = 0.4$						
$\varepsilon = 0.30$	4.25	76.56	18.83	95.39	80.26	2.51
$\varepsilon = 0.43$	4.35	74.92	15.11	90.03	83.22	2.55
$\varepsilon = 0.56$	4.50	73.40	12.06	85.46	85.89	2.59
Panel C: $\gamma = 1, \xi = 0.2, \alpha = 0.6$						
$\varepsilon = 0.30$	3.95	71.51	25.55	94.06	76.03	2.18
$\varepsilon = 0.43$	4.00	69.36	18.79	88.15	78.68	2.19
$\varepsilon = 0.56$	4.10	67.41	15.61	83.02	81.20	2.20
Panel D: $\gamma \rightarrow 0, \xi = 0.2, \alpha = 0.4$						
$\varepsilon = 0.33$	4.15	76.38	26.68	103.06	74.11	2.16
$\varepsilon = 0.42$	4.05	73.29	29.19	102.48	71.52	1.79
$\varepsilon = 0.50$	4.00	70.77	31.27	102.04	69.34	1.43

6.4. A comparison between two capital structures with and without CoCos

By comparing Table 3 with Table 4 and Table 5 with Table 6 respectively, we find that the optimal leverage with CoCos is greater than that without CoCos. The total firm value increases considerably if CoCos are issued and

they can not only decrease bankruptcy risk but also significantly increase the total firm value, even if tax shields are not taken into account.

For example, Tables 3 and 4 indicate that under our baseline parameter values, if risk aversion $\gamma = 1$, the total firm value increases by $(94.77 - 90.03)/90.03 = 5.26\%$; if $\gamma = 2$, the number is $(89.66 - 84.36)/84.36 = 6.28\%$. Generally speaking, the higher the risk aversion, the larger the amount increased. To the extreme, if investors are risk-neutral toward idiosyncratic risk, i.e. $\gamma \rightarrow 0$, the increased amount is very limited as seen in Tables 3~6. Tables 5 and 6 further indicate that the higher the idiosyncratic risk the larger the increased amount of the total firm value if CoCos are issued.

Now we turn to default probability. For this aim, it suffices to compare default thresholds between capital structures with and without CoCos though it is direct to derive explicit default probabilities. Tables 3~6 state that if CoCos are issued, default threshold \bar{x}_1 decreases steadily, no matter whether investors are risk-averse or not.

Furthermore, to compare the risk-taking incentive in the case where CoCos are issued with that where CoCos are not issued, we provide Figures 7 and 8. Figure 7 takes the correlation $\rho = 0.2$ and ownership stake $\lambda = 0.2$ and Figure 8 takes $\rho = 0.2$ and SBs' coupon rate $b_1 = 4 + 0.8 = 4.8$ excluding CoCos. The results explain that there are no obvious evidences on whether CoCos enhance the risk-taking incentive of shareholders or not. As shown in Subsection 6.2, if investors are a bit risk-averse, say $\gamma = 0.5$, Figures 7 and 8 say that shareholders have not risk-taking incentives no matter if CoCos are issued or not. Jensen and Meckling (1976) point out that there is an incentive problem associated with risky debt: After debt is in place,

managers have incentive to take on riskier projects to take advantage of the option-type payoff structure of equity. However, there is little empirical evidence in support of such risk-shifting behaviors. [Chen et al. \(2010\)](#) argue that one possible explanation is that managerial risk aversion can potentially dominate the risk-shifting incentives. Obviously, our conclusions are quite consistent with their explanation.

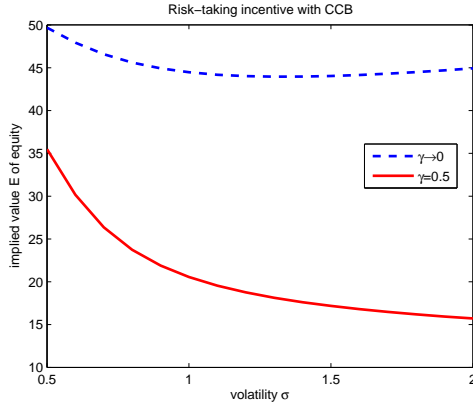


Figure 7: The figure presents the impact of **volatility** (σ) on the implied value of equity with CoCos under different risk aversion parameters ($\rho = 0.2, \lambda = 0.2$).

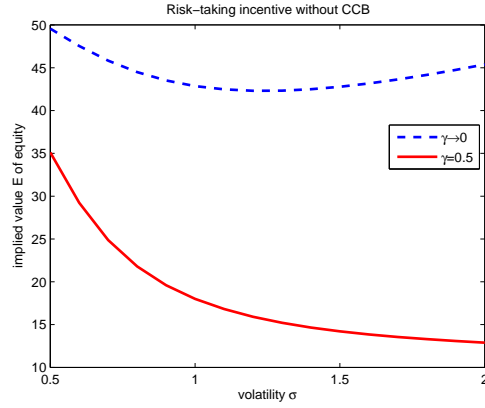


Figure 8: The figure presents the impact of **volatility** (σ) on the implied value of equity without CoCos under different risk aversion parameters ($\rho = 0.2, b_1 = 4.8$).

6.5. Optimal ownership stake and conversion leverage

To design CoCos, we must specify its coupon, conversion threshold and conversion ratio or its ownership stake. As far as we know, no paper in the literature discusses what optimal ownership stake should be. This happens because the problem is unimportant in a risk-neutral world. However, in our model, investors are risk-averse and ownership stake is therefore a key determinant in designing CoCos, as seen in [Figure 9](#) and [Table 7](#).

Figure 9 depicts the effect of ownership stake (λ) on the total firm value under risk aversion $\gamma = 1, 2$ respectively. It indicates that the firm value is a globally concave function of the stake.

Table 7 reports that, if investors are risk-neutral, the total firm value is independent of the ownership stake. By contrast, if investors are risk-averse, optimal ownership stake increases with both risk aversion γ and idiosyncratic risk volatility ε . This phenomenon explains that the firm should allocate more wealth to CoCo holders upon conversion. Similar to Panel B of Table 3, Panel A of Table 7 also states that the firm should issue less CoCos and equity and more SBs if investors are more risk-averse.

Table 7: This table plots the impact of **risk aversion** γ and **idiosyncratic risk level** ε on optimal capital structure including equity, SBs and CoCos. We consider three risk aversion levels and three levels of idiosyncratic risk volatility. We take the baseline parameter values unless otherwise stated.

γ	b_1	B	b_2	CB	E	T	$L(\%)$	λ	\bar{x}_2	\bar{x}_1
Panel A:										
$\varepsilon = 0.43$										
$\gamma \rightarrow 0$	3.35	62.66	1.20	16.40	24.87	103.93	76.07	—	4.08	1.09
$\gamma = 1$	4.10	73.04	1.10	11.75	10.32	95.11	89.15	0.55	4.76	2.09
$\gamma = 2$	4.40	76.02	0.90	8.33	5.99	90.34	93.37	0.65	4.86	2.49
Panel B:										
$\gamma = 1$										
$\varepsilon = 0.3$	3.85	71.52	1.15	13.84	13.69	99.05	86.18	0.5	4.55	1.99
$\varepsilon = 0.43$	4.10	73.04	1.10	11.75	10.32	95.11	89.15	0.55	4.76	2.09
$\varepsilon = 0.50$	4.25	73.86	1.05	10.93	8.48	93.27	90.91	0.6	4.86	2.12

Note: If $\gamma \rightarrow 0$, the total value of the firm is unaffected by the ownership stake λ . And we take the baseline parameter value $\lambda = 0.4$.

Figure 10 plots the effect of conversion leverage and risk aversion on the total firm value. As expected, the total firm value increases monotonically with conversion leverage. This is because the higher the conversion leverage, the more the tax shield while bankruptcy costs are unchanged. However, as a matter of fact, the conversion leverage must be less than a pre-specified level under a suitable financial regulatory environment, say $\phi \leq 0.95$.

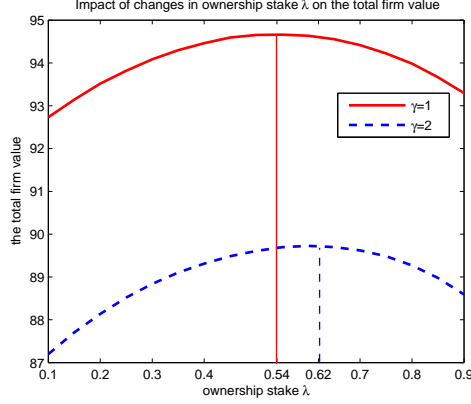


Figure 9: This figure shows the impact of ownership stake (λ) on the total firm value with two levels of risk-aversion index.

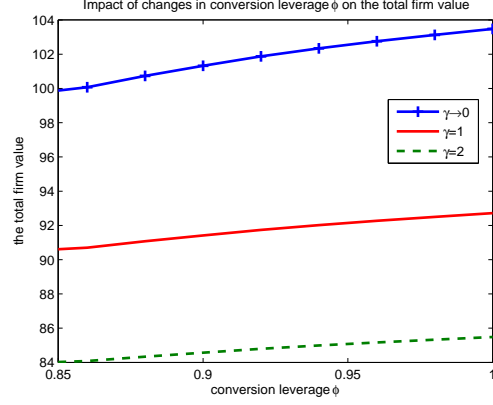


Figure 10: This figure shows the impact of conversion leverage (ϕ) on the total firm value with coupons $b_1 = 2.5, b_2 = 2.3$.

7. Conclusions

After the recent global financial crisis, one of the most common suggestions is to introduce contingent convertible bonds (CoCos) into capital structure of a firm, which is too important to fail. Actually, CoCos can be issued by any firms. To the best of our knowledge, all papers in the literature on CoCos are based on the following assumptions: The market is complete or investors are risk-neutral toward idiosyncratic risk. However, in practice, it is common for a risk-averse investor to invest in an incomplete market. For this reason, we relax the assumption and discuss the pricing of equity and CoCos and optimal capital structure in an incomplete market for a risk-averse investor, based on the consumption utility indifference pricing approach, while straight bonds (SBs) are priced by an equilibrium pricing approach since their idiosyncratic risk can be presumably diversified away.

We derive the explicit equilibrium prices of all corporate securities and semi-closed-form implied values of equity and CoCos based on the consumption utility indifference pricing method under an exogenously given conversion threshold and an endogenous default threshold. Following that, we analyze optimal capital structure by a numerical method.

We perform numerical simulations by finite difference methods and provide a comparison between the capital structures with and without CoCos. The results show that: (i) CoCos can increase the total firm value and the increased value increases dramatically with idiosyncratic risk volatility and the degree of risk aversion of investors; (ii) The total firm value is concave in the ownership stake and there is a unique optimal one, which increases with risk aversion; (iii) If investors are risk-averse enough, the issued amounts of CoCos and equity (SBs) will decrease (increase) with idiosyncratic risk but if investors are risk-neutral, the opposite holds true; (iv) There are risk-taking incentives for investors who are risk-neutral toward idiosyncratic risk but there are not if investors are somewhat risk-averse, no matter if CoCos are issued or not; (v) The higher the idiosyncratic risk or the more risk-averse the investors, the greater the leverage and the leverage including CoCos is always greater than that excluding CoCos.

Appendices

Appendix A Proof of Proposition 3.1

According to Bellman's principle of optimality, $J^1(w, x)$ can be equivalently written as

$$\begin{aligned} & J^1(w, x) \\ &= \sup_{(C, \theta, \tau_1) \in (\mathcal{C}, \Theta, \mathcal{T})} \mathbb{E} \left[\int_0^{\tau_1} e^{-\beta t} U(C_t) dt + e^{-\beta \tau_1} V^0(W_{\tau_1}) | W_0 = w, \delta_0 = x \right], \end{aligned} \quad (\text{A.1})$$

subject to (1) and (14). Therefore, by a standard computation, we obtain (15).

From the first-order condition (19) and exponential utility (8), the optimal consumption rate is evidently given by

$$c = -\frac{1}{\gamma} \ln J_w^1(w, x). \quad (\text{A.2})$$

Substitute (A.2) and the second equation of (19) into (15), we immediately get

$$\begin{aligned} & (rw + (1 - \lambda)(1 - \xi)(x - b_1) + \frac{1}{\gamma} \ln J_w^1 - \frac{1}{\gamma}) J_w^1 + \mu J_w^1 + \frac{1}{2} \eta^2 J_{xx}^1 \\ & - \frac{((\mu_e - r) J_w^1 + \sigma \sigma_e \rho J_{wx}^1)^2}{2 \sigma_e^2 J_{ww}^1} - \beta J^1 = 0. \end{aligned} \quad (\text{A.3})$$

According to (10) and (20), $E^1(x, b_1)$ satisfies

$$J^1(w, x) = -\frac{1}{\gamma r} \exp \left(1 - \beta/r - \gamma r \left(w + E^1(x, b_1) + \frac{\eta^2}{2r^2 \gamma} \right) \right). \quad (\text{A.4})$$

If the current cash flow rate x is large enough, according to (21), $E^1(x, b_1)$ satisfies

$$J^2(w, x) = -\frac{1}{\gamma r} \exp \left(1 - \beta/r - \gamma r \left(w + E^1(x, b_1) + \frac{\eta^2}{2r^2 \gamma} \right) \right). \quad (\text{A.5})$$

Plugging (A.4) and (A.5) back into HJB (A.3) and boundary condition (16) respectively leads to (22) and boundary conditions (23) and (24) of Proposition 3.1. Substituting (A.4) into (A.2) and the second equation of (19) we derive the optimal consumption rate (25) and the portfolio rule (26).

Appendix B Proof of Theorem 3.2

Thanks to Bellman's principle of optimality, the optimization problem (12) can be equivalently written as

$$\begin{aligned} J(w, x) \\ = \sup_{(C, \theta) \in (\mathcal{C}, \Theta)} \mathbb{E} \left[\int_0^{\tau_2} \exp(-\beta t) U(C_t) dt + \exp(-\beta \tau_2) J^1(W_{\tau_2}) | W_0 = w, \delta_0 = x \right], \end{aligned} \quad (\text{B.1})$$

subject to (1) and (11). By a standard computation, we then derive (27). From the first term of the first-order condition (29) and the exponential utility (8), the optimal consumption rate is at once given by

$$c = -\frac{1}{\gamma} \ln J_w(w, x). \quad (\text{B.2})$$

Thus, it follows from (B.2) and the second equation (29) that

$$\begin{aligned} (rw + x - b_1 - b_2 + \frac{1}{\gamma} \ln J_w - \frac{1}{\gamma}) J_w + \mu J_w + \frac{1}{2} \eta^2 J_{xx} \\ - \frac{((\mu e - r) J_w + \sigma \sigma_e \rho J_{wx})^2}{2 \sigma_e^2 J_{ww}} - \beta J = 0. \end{aligned} \quad (\text{B.3})$$

According to (10) and (30), $E(x, b_1, b_2)$ satisfies

$$J(w, x) = -\frac{1}{\gamma r} \exp \left(1 - \beta/r - \gamma r \left(w + E(x, b_1, b_2) + \frac{\eta^2}{2r^2\gamma} \right) \right). \quad (\text{B.4})$$

Therefore, if the current cash flow rate x is large enough, according to (31), $E(x, b_1, b_2)$ satisfies

$$J^3(w, x) = -\frac{1}{\gamma r} \exp \left(1 - \beta/r - \gamma r \left(w + E(x, b_1, b_2) + \frac{\eta^2}{2r^2\gamma} \right) \right). \quad (\text{B.5})$$

Substituting (B.4) and (B.5) into (B.3) and the boundary condition (28) respectively gives (32) and the boundary conditions (33) and (34) in Theorem 3.2. Plugging (B.4) back into (B.2) and the second equation of (29) gives the optimal consumption rate (35) and the portfolio rule (36).

The proofs of Proposition 3.3 and Theorem 3.4 are similar to those of Proposition 3.1 and Theorem 3.2 respectively and so we omit them here.

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