Optimal Investment in Mutually Exclusive Projects and Operating Leverage*

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Abstract

This paper studies investments in exclusive projects with different cost structures. The analysis incorporates the possibility of producing a stochastic revenue stream from two alternative technologies, respectively with stochastic variable cost and fixed cost, and accounts for endogenous operating decisions of project managers. The optimal investment decision is characterized by two boundaries, possibly non-monotone. The effect of operating leverage on managerial policies, investment decisions and values is examined. An application to power generation projects is carried out. The impact of knowledge acquisition, i.e., investments in growth options, is also assessed.

1. Introduction

This paper studies investments in exclusive projects with different cost structures, specifically a variable cost project (VCP) and a fixed cost project (FCP). It derives the optimal investment policy taking account of the endogenous operating decisions for each underlying project. Optimal investment is characterized by two nonlinear and possibly non-monotone boundaries satisfying a system of coupled integral equations. The option to invest in the best of the two projects increases the value of investment and postpones the optimal investment time. The optimal delay can be substantial even if the individual projects are infinitely valuable. Operating leverage emerges as a key determinant of the investment policy. An extension of the model examines the behavior of preliminary investments in knowledge assets for the choice between the alternative projects.

Choices between exclusive alternatives are prevalent in corporate decision-making. Common examples include choices between different production technologies (manufacturing), applications of a given technology (product choice), target firms (mergers and acquisitions), research

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programs (research and development) or advertising strategies (marketing). They are also quite common in individual decision-making, e.g., education choices, employment decisions, entertainment selections, etc.

This paper focuses on situations where the underlying projects exhibit fundamental differences in cost structures, specifically in the mix between fixed and variable costs. An illustrative example is that of utilities who are nowadays faced with non-trivial choices between competing technologies when they invest in new power production units. On the one hand they can build plants using fossil fuels to generate power (e.g., coal- or gas-fired plants). On the other hand they can use Green Energy - hereafter GE - (e.g., wind or solar plants). The distinguishing feature between these alternatives is their respective cost structures. In the former case, there are fixed costs and variable costs. Fixed costs consist of wages and maintenance costs, among others. Variable costs relate to the fuel used as an input in the production process. The price of fuel varies stochastically and is typically positively correlated with the electricity price, hence increases with it. Such a technology has lower operating leverage, depends less on demand and is less risky as a standalone project. In the latter case, the GE input is free and costs are independent of the price of electricity. Such a technology entails fixed costs, is more dependent on demand and has higher operating leverage. Hence, in essence, the choice between the two technologies is a choice between projects with lower and higher operating leverage.

Motivated by this example, we study the general problematic of investing in exclusive alternatives consisting of a VCP and an FCP. The timing of the decision to invest is flexible. Moreover, each of the possible selections involves subsequent timing choices regarding operations. Hence, the firm holds an American-style real option on the maximum of two asset values, which are themselves American contingent claims. Some of the questions arising in this context are as follows. What is the optimal time to invest? Under what conditions is it optimal to invest in the FCP versus the VCP? What is the impact of operating leverage on these decisions? Applied to power generation projects, our model can be used to examine a host of issues. In particular, it can shed light on the relevance of subsidies and on the value of GE. Such issues are important not only for power operators but also for consumers and policy makers.

The model developed considers two underlying sources of uncertainty, i.e., two state variables, one driving revenue, the other costs. The total contribution margin, i.e., the difference between revenue and variable cost, determines the value of the VCP. Revenue is the main factor underlying the value of the FCP. Our first contribution is to solve the problem of the VCP manager in this setting with *two state variables*. In that regard, the value functions associated with the decisions to operate or idle the project are solutions to a system of coupled optimal stopping time problems. We provide an Early Switching Premium (ESP) representation of the solution which decomposes each value function into the value achieved if the state of the project (operating or idling) is maintained forever and a premium, the ESP, which tallies the present value of the gains realized by optimally switching back and forth between operations states. Optimal decisions are then described by a pair of boundaries that depend on the cost factor. The ESP representation of the value functions enables us to show that these boundaries satisfy a system of coupled integral equations of Fredholm type. Although these equations are non-recursive in nature, we are able to design an iterative algorithm to compute the boundaries.

The operating decisions for the FCP are the same as those for the VCP and involve a similar coupled optimal stopping time problem. But the problem is simpler because cash flows depend on revenue alone. The resulting switching boundaries are then constant and the problem can be solved by reducing it to the corresponding free-boundary problem. Optimal boundaries are solutions to a system of algebraic equations.

Our second and main contribution pertains to the investment decision of the firm, namely the decision to undertake the VCP or the FCP. We show that the decision to invest is characterized by two boundaries. When the variable cost factor exceeds the first boundary, then immediate investment in the FCP is optimal. When it falls below the second boundary it is optimal to invest in the VCP. Each of these boundaries depends on the other source of uncertainty, namely revenue. Moreover, both boundaries can fail to be monotone in revenue. In the infinite horizon case, we show that they satisfy a system of coupled integral equations of Fredholm type. In the finite horizon case, the system of coupled integral equations is recursive in nature and a dynamic programming procedure can then be applied.

A project to invest in the best of two production technologies is an American-style dual strike compound max-option. It is a compound option because the value of each underlying project embeds options to shut down or restart production when the relevant state variables reach certain thresholds, hence is inherently nonlinear. It is a dual strike option because the cost of building a VCP differs from the cost of building a FCP. Finally, it is a max-option because of the availability of competing technologies for production. We show that the value of a perpetual project has an Early Investment Premium (EIP) representation. That is, it can be written as the present value of the cumulative local gains realized by optimally investing in the best technology. This premium splits as a premium for investing when the VCP is optimal and one for investing when the FCP is optimal. Each of these components is parametrized by both investment boundaries, reflecting the inherent interdependence between the two decisions. As the boundaries can be computed, the value of the project can be calculated as well.

The optimal investment decision has notable features. First, it is always suboptimal to invest when the values of the two underlying projects coincide and this is true even if both values are extremely large. In this instance, it pays to delay investment even though it would be optimal to invest when a given technology is considered in isolation. Moreover, the premium for postponing investment can be substantial. Second, the optimal behavior when the two project values are close to each other is unusual. There are two cases. In the first case, it is also optimal to delay investment even if one of the projects has significantly more value than the other one. In fact, we identify a cone within which immediate investment is suboptimal. The edges of this cone diverge as the underlying project values increase. This means that the value of waiting can remain positive even when the difference between the underlying projects increases to infinity, i.e., when one project becomes infinitely more valuable than the other one. In the second case, the no-investment region shrinks as the revenue factor becomes large. In this instance the optimal waiting time when both projects have the same net present value becomes asymptotically negligible and the premium for waiting vanishes. We identify and discuss the factors leading to either of the two configurations.

Operating leverage affects decisions and value functions. When the exposure (sensitivity) to the stochastic cost factor increases, i.e., when the VCP has lower operating leverage, the project manager will speed up the decision to idle and postpone the decision to restart operations. Lower operating leverage also decreases the instantaneous benefits from investing in the VCP. This leads to a decrease in the value of the project to invest in the best technology and a substitution of investment in the FCP at the expense of the VCP. When the correlation between revenue and cost increases, shocks to revenue have a more systematic effect on variable cost leading to a decrease in the riskiness of the operating profit of the VCP and its value. This also reduces the value of the project to invest in the best technology and tends to increase the likelihood of investing in the FCP, as opposed to the VCP.

The model is then applied to power generation projects and we conduct a numerical study to gauge the relevance of GE projects for power generation. We focus on the case of a gas-fired plant (GFP) as the technology based on traditional fuels. This reflects the fact that natural gas has become the most efficient source of power production from fossil fuels. A wind plant (WP) is the GE technology considered.[1] Wind power was the second fastest growing segment in that space (behind SPV) and had the second highest share of global electricity production of all renewable energies (behind hydroelectric power) in 2016. A GFP (resp. WP) is an example of a VCP (resp. FCP). We thus consider an operator with a project to build a power plant and who seeks to choose between building a GFP or a WP.

Numerical results underscore the importance of GE for power generation projects. We find that considering an alternative GE-based technology, such as a WP, as a possible choice can lead to a substantial increase in the value of a power project over typical ranges of electricity and gas prices. The value added, i.e., the wind premium, can be large even if the current value of a GFP is significantly larger than that of a WP. It decreases slowly as the gas price decreases and increases when the cost of building a WP decreases. Subsidies are found to play an important role. An increase in subsidies provides incentives to invest earlier in wind power and boosts the wind premium. It also delays or eliminates investments in gas plants.

For an operator of fossil fuel plants, investing in GE power may require a preliminary acquisition of technological expertise and information.[2] Our final contribution is to examine this issue in a two-stage extension of the basic model. In the first stage the firm has three alternatives. It can either invest in the familiar technology, the VCP (i.e., GFP), invest in the unfamiliar technology, the FCP (i.e., the WP), or acquire technological expertise about the FCP and postpone investments. If it chooses the second alternative, it bears an efficiency cost due to imperfect knowledge, hence suboptimal design. If it chooses the last one, it can in a second stage still choose the familiar technology or invest in the FCP instead. The project then becomes a two-stage compound max-option. We show that the first-stage decision, which involves the alternative to acquire technological information, is driven by a triplet of boundaries. Each of these endogenous barriers, depends on the subsequent decisions and their determinants. We examine the properties of these boundaries and, in the context of power generation, study the impact of exogenous parameters of the model, such as operating leverage, on the information acquisition decision and the value of the option to invest in knowledge assets.

The paper relates to several branches of the literature. First, it contributes to the broad literature on growth options, operating leverage and firm value. The relation between operating leverage and risk is examined by Lev (1974). Real options models drawing the link between operating leverage and value premium can be found in Carlson, Fisher and Giammarino (2004),

[1] There are significant differences between technologies available for power production. Valuation of specific technologies and choices between technologies should be carried out on a case-by-case basis.

[2] Energy firms often invest to build real options positions. An example is Statoil's (now Equinor) 2007 acquisition of equity stakes in two wind power technology firms, Sway and ChapDrive. Statoil's objective at the time was to strengthen its position as a potential future producer of wind power. It has since used the technology developed to build the first floating wind farm in the world, the Hywind farm in Scotland, which became operational in 2017. Another example is global energy provider ENGIE who founded ENGIE New Ventures in 2014, a fund dedicated to investments in technology startups, in order to foster internal innovation.

Zhang (2005) and Novy-Marx (2011). In contrast to this literature, our analysis studies the choice between exclusive projects with different leverage characteristics, specifically cost structures (fixed versus variable cost). It highlights, in particular, the role of the variable cost structure for the optimal timing and selection decisions, and the value of the project.

Second, it also relates to the more specialized literature on power generation. Most of the literature in that area focuses on projects involving a given technology.[3] Two exceptions are Decamps, Mariotti and Villeneuve (2006) and Siddiqui and Fleten (2010) who consider a choice between two exclusive alternatives. The first study develops a general method to deal with investments in the best of two technologies with values driven by a common underlying state variable. They show that the optimal investment region can consist of two intervals separated by a continuation interval. The second study adds the possibility of learning about the costs of one of the technologies after investing in it, but before deciding to deploy it. Assuming that the cost of deployment is incurred at the time of investment, they show that the model reduces to a single state variable problem as in Decamps, Mariotti and Villeneuve (2006). They proceed to examine the impact of learning on the optimal investment decision and the project valuation.[4]

The model considered in this paper cannot be reduced to a single state variable. It involves two distinct underlying processes, for revenues and costs, as well as embedded (two-dimensional) optimal stopping time problems for the operating decisions of the underlying projects and the timing and selection decisions of the project manager. We solve this two state variables compound optimal stopping problem in generality, assuming that the underlying processes follow geometric Brownian motions (GBM). In the application to power projects, the first state variable represents the price of electricity and the second one the price of gas. The GBM specification allows the spark spread to take negative values, when the price of gas adjusted by the heat rate exceeds the price of electricity, but also ensures that the price of gas cannot become negative. GBM, as a model of the long run behavior of the electricity price, is standard in the literature on power generation.[5]

Third, the paper is also related to broader literatures on real options and American option pricing. Seminal contributions in the real option area highlight the values of flexibility and of waiting to invest (e.g., Brennan and Schwartz (1985), McDonald and Siegel (1985, 1986); see also Dixit and Pindyck (1994) and references therein). This literature relies on option pricing methods and typically uses partial differential equations to price projects. In contrast, we exploit a characterization of the value function based on the EIP representation, which is rooted in probabilistic/martingale methods (see Kim (1990) and Carr, Jarrow and Myneni (1992) for American options). We also extend the literature on the valuation of multiasset American claims (e.g., Broadie and Detemple (1997)), by considering dual strike compound max-options and designing an implementable algorithm that solves systems of coupled integral equations of Fredholm type for optimal exercise boundaries. The compound structure of our problem

[3]See Deng, Johnson and Sogomonian (1999), Maribu, Galli and Armstrong (2007), Fleten and Nasakkala (2010) for gas-fired plants, and Fleten, Maribu and Wangensteen (2007), Boomsma, Meade and Fleten (2012) for wind plants.

[4]The choice between exclusive alternatives has been studied in other settings as well. See, for instance, Geltner, Riddiough and Stojanovic (1996) for an application to land use and Bakke et al. (2016) for the selection between transmission asset locations. The structure of our problem is more complex as it involves underlying projects with operating decisions, resulting in a dual strike compound max-option.

[5]GBM is also common as a model for the long run behavior of commodity and energy prices. See, e.g., Pindyck (1999) and Schwartz and Smith (2000).

and the presence of uncertain operating leverage modifies some of the properties characterizing optimal exercise decisions for max-options. The advantage of our algorithm compared to, e.g., PDE or Monte-Carlo methods, is the ability to tackle perpetual or long horizon problems, in addition to short horizon problems. This approach can help to address other challenging multi-dimensional stopping problems in real options and American option pricing applications.

Section 2 develops the general model with mutually exclusive VCP and FCP alternatives. Section 3 provides an application in the realm of power generation. Section 4 examines issues pertaining to knowledge acquisition. Conclusions follow. Proofs are in the Appendix.

2. Investments in exclusive projects

2.1. Variable cost project

1. We consider a project generating maximal instantaneous revenue X at cost κY, where Y is a stochastic cost factor (state variable) and $\kappa > 0$ a constant exposure coefficient. Let us assume that the revenue and cost factor processes X and Y follow correlated geometric Brownian motion processes under the risk-neutral measure Q

$$
\begin{aligned}
dX_t &= (r - \delta_X)X_t dt + \sigma_X X_t dW_t \\
dY_t &= (r - \delta_Y)Y_t dt + \sigma_Y Y_t dB_t
\end{aligned}
$$

where $r > 0$ is the interest rate, δ_Z and σ_Z are the (implicit) yield[6] and volatility parameters, respectively, for $Z = \{X, Y\}$. We also denote the correlation between W and B by $\rho \in [-1, 1]$.

We suppose that the project can be operated in two states: 0 - idle and 1 - full operation mode. There is a fixed running cost $k_{i,v} > 0$ for state $i = 0, 1$ with $k_{0,v} < k_{1,v}$, and a fixed switching cost $c_{i,v} \geq 0$ when the manager shifts from state i to state $j \neq i$. We also define the scale factor $\gamma_v \in (0, 1)$, representing the effective operational capacity. The operating profit at time t is $\gamma_v(X_t - \kappa Y_t) - k_{1,v}$. For notational convenience, we absorb the exposure coefficient κ into the initial value of Y, which can be done due to the multiplicative structure of Y. Then the value of the variable cost project once it is started is given by the system of coupled optimal timing problems

$$
V_{0,v}(x, y) = \sup_{\tau \geq 0} \mathsf{E}_{x,y} \left[-\int_0^\tau e^{-rt} k_{0,v} dt + e^{-r\tau}(V_{1,v}(X_\tau, Y_\tau) - c_{0,v}) \right] \tag{2.1}
$$

$$
V_{1,v}(x, y) = \sup_{\tau \geq 0} \mathsf{E}_{x,y} \left[\int_0^\tau e^{-rt}(\gamma_v(X_t - Y_t) - k_{1,v}) dt + e^{-r\tau}(V_{0,v}(X_\tau, Y_\tau) - c_{1,v}) \right] \tag{2.2}
$$

for $x > 0$ and $y > 0$, where the supremum is taken over stopping times of (X, Y) that represent the times of switching from one state to another. The goal now is to find the value functions $(V_{0,v}, V_{1,v})$ and the optimal operating policy. As the problem is two-dimensional, to the best of our knowledge it cannot be solved in a closed form.

[6] The price of a non-storable commodity, such as electricity, can diverge from risk-neutral valuation leading to the emergence of an implicit yield.

First, let us consider the optimal operating policy when the cost factor $Y \equiv 0$ permanently. The problem becomes one-dimensional. The method of solution is well known in this case (see, e.g., Dixit and Pindyck (1994)). Starting from the state 0, the manager should switch and generate revenue when X becomes sufficiently high, i.e., $X \geq b_{0,v}$ for some constant $b_{0,v} > 0$ to be determined. Similarly, when in the state 1, the project should be switched to the idle state when X falls below some threshold $b_{1,v} > 0$. It is clear that the threshold $b_{0,v}$ is above $b_{1,v}$ and also that $b_{1,v} < (k_{1,v} - k_{0,v})/\gamma_v < b_{0,v}$. The details of the solution to this one-dimensional problem can be found in Appendix A.

2. Now we turn to the general case when $Y_0 = y > 0$. As in the previous case, there are two boundaries $b_{0,v}(y)$ and $b_{1,v}(y)$ such that the project is optimally switched from state i to state $j \neq i$ when X hits $b_{i,v}(Y)$. However, now these boundaries are not constants but functions of Y. It is clear that the limits $b_{i,v}(0+)$ were already found in the previous paragraph as the problem becomes one-dimensional due to the absorbing nature of the point $y = 0$. Also one can show that both curves are increasing in $y > 0$. Using hedging arguments again, we have that the value functions $V_{0,v}$ and $V_{1,v}$ solve the corresponding free boundary PDE problems

$$\mathbb{L}_{X,Y} V_{0,v}(x,y) - rV_{0,v}(x,y) = k_{0,v} \quad \text{for } x < b_{0,v}(y) \tag{2.3}$$

$$V_{0,v}(x,y) = V_{1,v}(x,y) - c_{0,v} \quad \text{for } x \geq b_{0,v}(y) \tag{2.4}$$

$$\mathbb{L}_{X,Y} V_{1,v}(x,y) - rV_{1,v}(x,y) = -\gamma_v(x-y) + k_{1,v} \quad \text{for } x > b_{1,v}(y) \tag{2.5}$$

$$V_{1,v}(x,y) = V_{0,v}(x,y) - c_{1,v} \quad \text{for } x \leq b_{1,v}(y) \tag{2.6}$$

where $\mathbb{L}_{X,Y}$ is the joint infinitesimal operator

$$\mathbb{L}_{X,Y} f = (r - \delta_X) x f_x + (r - \delta_Y) y f_y + \tfrac{1}{2}\sigma_X^2 x^2 f_{xx} + \tfrac{1}{2}\sigma_Y^2 y^2 f_{yy} + \rho \sigma_X \sigma_Y xy f_{xy}$$

for $f \in C^{2,2}$ on $(0,\infty)^2$. We also have that smooth and value matching conditions are satisfied at the boundaries $b_{0,v}(y)$ and $b_{1,v}(y)$.

In contrast to the previous case with $Y = 0$, PDEs (2.3) and (2.5) cannot be solved in closed form.[7] Hence we will have to rely on numerical methods. To the best of our knowledge the representation formulas below are new and offer an approach to tackle this type of problems numerically. In most of the literature on switching problems or coupled timing problems, the case of a single stochastic state variable was considered (e.g., Brennan and Schwartz (1985)). In multi-dimensional settings, finite difference methods were mainly employed. However, these methods are not very convenient for our purposes as we will need to use the value functions $V_{0,v}$ and $V_{1,v}$ further in the following sections. The representation formulas presented next also have economic meaning as discussed below.

Theorem 2.1. *The value functions satisfy the early switching premium (ESP) formulas*

$$V_{0,v}(x,y) = -k_{0,v}/r + \pi_0(x,y;b_{0,v}) \tag{2.7}$$

$$V_{1,v}(x,y) = \gamma_v x/\delta_X - \gamma_v y/\delta_Y - k_{1,v}/r + \pi_1(x,y;b_{1,v}) \tag{2.8}$$

[7] Operating costs and switching costs preclude the existence of a closed form solution. In the absence of fixed costs the option to switch becomes an exchange option (see Margrabe (1978)).

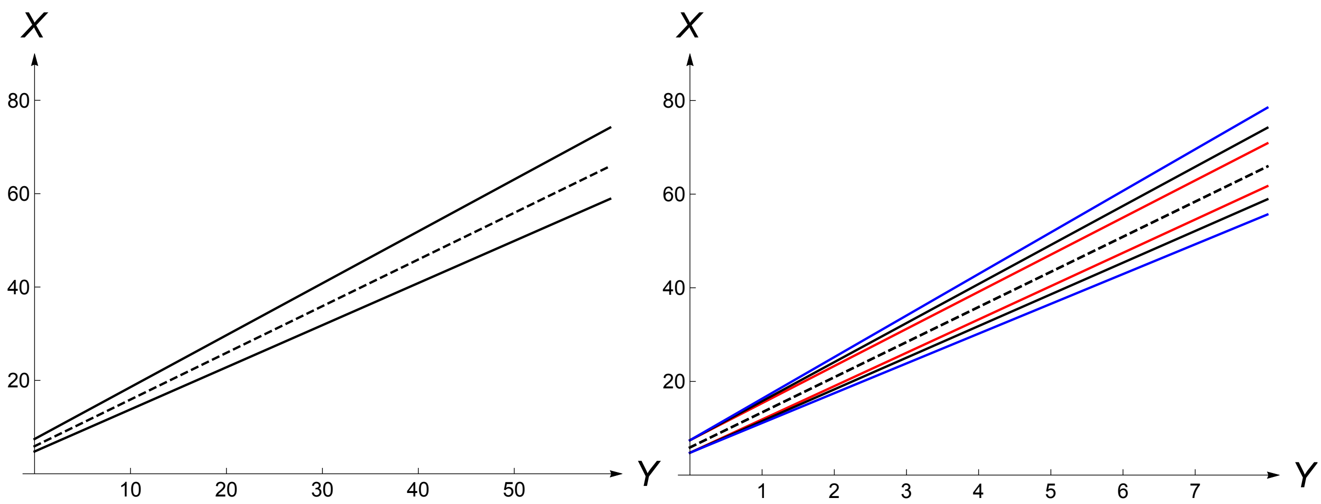

Figure 1: The left panel shows the optimal operating boundaries $b_{0,v}$ (upper solid) and $b_{1,v}$ (lower solid) for the VCP when $\rho = 0.3$. The right panel displays the boundaries for different levels of the correlation parameter: $\rho = 0$ (blue), $\rho = 0.3$ (black), $\rho = 0.6$ (red). The black dashed curve corresponds to $x = y + (k_{1,v} - k_{0,v})/\gamma_v$, where the cash flows for each state have the same value. In the state 0, it is optimal to switch to the state 1 when $X \geq b_{0,v}(Y)$. In the state 1, it is optimal to switch to the state 0 when $X \leq b_{1,v}(Y)$.

for $x, y > 0$ *, where* $\pi_i(x, y; b_{0,v}, b_{1,v})$ *,* $i = 0, 1$*, represent the ESPs defined as,*

$$\pi_0(x, y; b_{0,v}) = \mathsf{E}_{x,y}\left[\int_0^\infty e^{-rt}(\gamma_v(X_t - Y_t) - k_{1,v} + k_{0,v} - rc_{0,v})I(X_t \geq b_{0,v}(Y_t))dt\right]$$
$$\pi_1(x, y; b_{1,v}) = \mathsf{E}_{x,y}\left[\int_0^\infty e^{-rt}(-\gamma_v(X_t - Y_t) + k_{1,v} - k_{0,v} - rc_{1,v})I(X_t \leq b_{1,v}(Y_t))dt\right].$$

The pair of switching boundaries $(b_{0,v}, b_{1,v})$ *satisfy the system of coupled integral equations*

$$\gamma_v b_{0,v}(y)/\delta_X - \gamma_v y/\delta_Y - k_{1,v}/r + \pi_1(b_{0,v}(y), y; b_1) - c_{0,v} = -k_{0,v}/r + \pi_0(b_{0,v}(y), y; b_{0,v}) \tag{2.9}$$
$$\gamma_v b_{1,v}(y)/\delta_X - \gamma_v y/\delta_Y - k_{1,v}/r + \pi_1(b_{1,v}(y), y; b_{1,v}) = -k_{0,v}/r + \pi_0(b_{1,v}(y), y; b_{0,v}) - c_{1,v} \tag{2.10}$$

for $y > 0$ *.*

The early switching premium formulas (2.7)-(2.8) in Theorem 2.1 have a clear interpretation. The value $V_{0,v}(x, y)$ of the project, in state 0, can be decomposed as the present value of cash flows under indefinite operation in the current state, i.e., $-k_{0,v}/r$, plus the present value of the local gains from switching to the state 1. When the manager switches, the instantaneous benefit is the cash flow $(\gamma_v X_t - \gamma_v Y_t - k_{1,v})dt$ and she stops paying the cost $k_{0,v}\, dt$ but at the same time she forgoes the interest, $rc_{0,v}\, dt$ on the switching cost $c_{0,v}$. One can explain the formula (2.8) for the value function $V_{1,v}(x, y)$ in a similar manner. Obviously, the formulas (2.7)-(2.8) do not provide a full characterization of the problem as they depend on unknown optimal switching boundaries. To proceed, one must first solve the system of coupled integral equations (of Fredholm type) (2.9)-(2.10) for the boundaries $b_{0,v}(y)$ and $b_{1,v}(y)$.

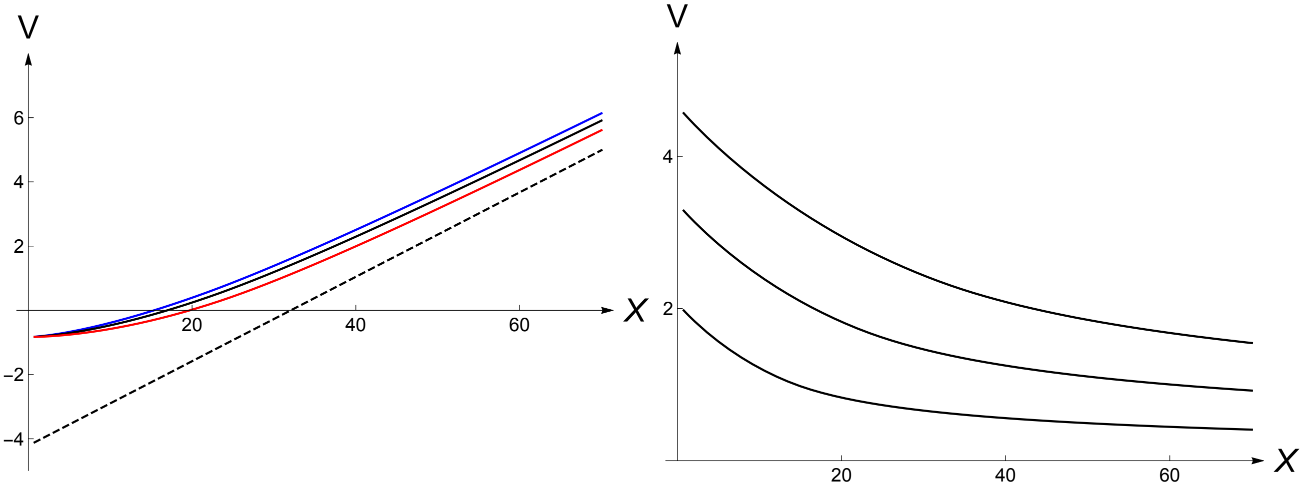

Figure 2: The left panel plots the value function $V_{1,v}(x,y)$ of the VCP versus the value of perpetual operation $V_{cont,v}(x,y)$ (black dashed) as functions of x for given $y>0$ and different levels of the correlation parameter: $\rho=0$ (blue), $\rho=0.3$ (black), $\rho=0.6$ (red). The right panel plots the value of operational flexibility $V_{1,v}(x,y)-V_{cont,v}(x,y)$ as a function of x for three different levels of $y>0$.

3. The Appendix B develops a numerical procedure to solve the system (2.9)-(2.10). This procedure is iterative in nature and produces the pair of boundaries $b_{0,v}(y)$ and $b_{1,v}(y)$ efficiently (typically 2-4 iterations are enough for acceptable accuracy). The resulting boundaries can then be plugged back in (2.7)-(2.8) to find the value of the project in both states. Illustrations are provided in Figures 1 and 2.

Remark 2.2. *The value of the project under perpetual operation is an affine function of the state variables*

$$V_{cont,v}(x,y)=\mathsf{E}_{x,y}\left[\int_0^\infty e^{-rt}(\gamma_v(X_t-Y_t)-k_{1,v})dt\right]=\gamma_v\left(\frac{x}{\delta_X}-\frac{y}{\delta_Y}\right)-\frac{k_{1,v}}{r}$$

for $x,y>0$. The value of operational flexibility, i.e., of the option to switch to the idle state, is the difference $V_{1,v}(x,y)-V_{cont,v}(x,y)=\pi_1(x,y;b_{1,v})$. See Figure 2 for illustration.

The effect of the correlation coefficient ρ on the optimal operating policy is illustrated in Figure 1. As ρ increases, the instantaneous variance of the operating profit decreases. This has two effects on the options to switch. The direct effect is the standard impact of risk on the value of a derivative with convex payoff. A decrease in riskiness reduces value, hence provides incentives for early action. The indirect effect is the associated decrease in the project values $V_{0,v}$ and $V_{1,v}$. This reduction implies that the payoff from switching also decreases, which raises the value of waiting and mitigates the risk effect. The ESP representation in Theorem 2.1, which expresses the switching premium entirely in terms of the fundamentals of the project, shows that the risk effect is the driving force. Hence, the upper boundary $b_{0,v}(Y)$ decreases, the lower boundary $b_{1,v}(Y)$ increases and the cone between the boundaries shrinks, as shown in the plot.

2.2. Fixed cost project

We now turn to the alternative project that also produces revenues driven by the process X. However, the cost factor is a constant in this case and the factor process Y is not relevant. The profit generated by this FCP is an affine function

$$\gamma_f(X_t + s) - k_{1,f} \tag{2.11}$$

of the revenue factor X. The term s stands for an incremental constant revenue stream, e.g., a government subsidy as in the case of green energy projects. The subsidy is a premium on top of the random revenue factor, and is assumed to be constant $s > 0$. The running cost of operations is also constant $k_{1,f} > 0$. It consists of wages and fixed maintenance costs.

The production model assumes that the project can operate in two states: 0 - idle and 1 - operating at capacity level $\gamma_f \in (0, 1]$. Perpetual operation is optimal if $\gamma_f s - k_{1,f}$ exceeds the cost of maintenance $k_{0,f}$ when the project sits idle. We consider the general case and solve the optimal switching problem for the FCP. In contrast to the previous section, the problem here is one-dimensional with single state variable X and thus can be easily solved. In fact, we already provided the solution in our treatment of the VCP when the cost factor Y is permanently zero. Drawing on that result, the value of the FCP (in states 0 and 1, respectively) is given by

$$V_{0,f}(x) = -k_{0,f}/r + A_{0,f}x^p \quad \text{for } x < b_{0,f} \tag{2.12}$$

$$V_{1,f}(x) = \gamma_f(x/\delta_X + s/r) - k_{1,f}/r + A_{1,f}x^q \quad \text{for} \quad x > b_{1,f} \tag{2.13}$$

where $p > 1$ and $q < 0$ are roots to associated quadratic equation, constants $A_{0,f}$, $A_{1,f}$, $b_{0,f}$ and $b_{1,f}$ can be computed numerically as in Appendix A for the VCP case ($Y = 0$). If we start from state 0, we switch to state 1 when $x \geq b_{0,f}$, pay the cost $c_{0,f}$ and obtain the value $V_{1,f}(x)$ of the project in state 1. Thus, for $x \geq b_{0,f}$, we define $V_{0,f}(x) := V_{1,f}(x) - c_{0,f}$ where $V_{1,f}(x)$ is as in (2.13). Similarly, we switch from state 1 to state 0 when $x \leq b_{1,f}$. For $x \leq b_{1,f}$, we define $V_{1,f}(x) := V_{0,f}(x) - c_{1,f}$ where $V_{0,f}(x)$ is as in (2.12). The first two terms in (2.13) give us the value of the project under perpetual operation

$$\mathsf{E}_x \left[\int_0^\infty e^{-rt}(\gamma_f(X_t + s) - k_{1,f})dt \right] = \gamma_f(x/\delta_X + s/r) - k_{1,f}/r \tag{2.14}$$

for $x > 0$. The last term in the right-hand side of (2.13) is the value of the option to switch to the idle state.

2.3. Investment problem

We consider the problem of a firm with the option to invest in two exclusive alternatives, one project with variable cost, the other with fixed cost. For now we assume that the option is infinitely-lived. We suppose that the projects are built in an idle state and can be immediately run according to the optimal operating policies derived in Sections 2.1 and 2.2. For example, if the operator invests in the VCP when $x \geq b_{0,v}(y)$, then it is optimal to pay the cost $c_{0,v}$ and immediately switch to the operating state with value $V_{1,v}(x, y)$ given in (2.8). On the other hand, one should not invest in a project when it is currently optimal to leave it in an idle state, because the immediate local profit is negative. Hence, we need only consider investing in the VCP when $x \geq b_0(y)$ and the project is optimally switched to state 1 upon paying the fixed cost $c_{0,v}$. Similar arguments can be applied for the FCP. The relevant value of the VCP is $V_v(x, y) = V_{0,v}(x, y)$ given in (2.7) and for the FCP it is $V_{0,f}(x)$ as defined in the paragraph following (2.12).

The option to choose between the two types of projects has payoff,

$$\max\{V_v(x, y) - K_v, \alpha(V_f(x) - K_f)\}$$

where $K_v > 0$ and $K_f > 0$ represent the sunk costs for each project, including the start up costs, and $\alpha > 0$ is a scale factor. The latter can be used to capture various situations of interest. If the firm is capital-constrained, for instance with constraint K_v, the scale factor can be used to equalize the costs of investments in the two alternatives so that $K_v = \alpha K_f$. If the firm is demand-constrained, i.e., there is a fixed market size for the output produced, for instance γ_v, then $\gamma_v = \alpha\gamma_f$ ensures that each of the two projects can service the demand.

The firm can also choose the timing τ of the investment. The value of the project to engage in one of the two alternative projects is therefore given by,

$$V(x, y) = \sup_{0 \leq \tau \leq \infty} \mathsf{E}_{x,y}\left[e^{-r\tau}\max(V_v(X_\tau, Y_\tau) - K_v, \alpha(V_f(X_\tau) - K_f))\right] \tag{2.15}$$

for $x > 0$ and $y > 0$ where the supremum is taken over stopping times $\tau \geq 0$. This is the value of an American option on the maximum of two assets with different strikes, i.e., a dual strike max-option. One of the assets (VCP) depends on two state variables.

The option payoff can be written as

$$G(x, y) = \max(V_v(x, y) - K_v, \alpha(V_f(x) - K_f))$$

for $x > 0$ and $y > 0$. It is clear that the firm should not enter into any project when the payoff is negative, i.e., $G(x, y) \leq 0$.

The two projects have the same net present values (NPVs) when $V_v(x, y) - K_v = \alpha(V_f(x) - K_f))$. Solving for y gives the indifference curve $\widehat{y}(x)$ for $x > 0$. If, for a given x_0, the payoff $\alpha(V_f(x_0) - K_f)$ dominates (is dominated by) $V_v(x_0, y) - K_v$ for all $y > 0$, we define $\widehat{y}(x_0) = 0$ ($\widehat{y}(x_0) = +\infty$). Along this curve, none of the two projects dominates. As $V_v(x, y)$ is decreasing in y, it is clear that for fixed $x > 0$

$$G(x, y) = \begin{cases} G_v(x, y) := V_v(x, y) - K_v & y < \widehat{y}(x) \\ \\ G_f(x) := \alpha\left(V_f(x) - K_f\right) & y > \widehat{y}(x). \end{cases}$$

The curve $\widehat{y}(x)$ can take different forms depending on the scale factor α. There are three possible configurations: $\widehat{y}(x)$ (i) is upward sloping when x goes to $+\infty$, e.g., in case of low values of α; (ii) converges asymptotically to the vertical line in (y, x)-space, e.g., when $\alpha = \gamma_v/\gamma_g$; (iii) is decreasing for large x and eventually becomes zero (note that Y cannot be negative), e.g., when α is high. As the value function $V_v(x, y)$ is not obtained in closed form and has a nonlinear structure, one has to recover the indifference curve numerically by solving an algebraic equation for every $x > 0$. See Figure 3 for illustration. Simplifications occur if the projects are run (state 1) in perpetuity. In this case the value functions have explicit affine structures and the indifference curve is obtained in closed form.

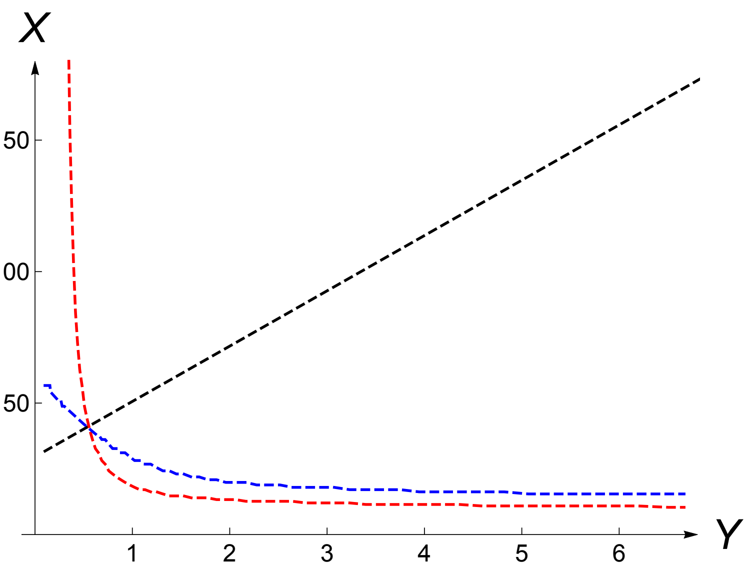

Figure 3: This figure shows different configurations of the indifference curve depending on α : (i) $\widehat{y}$ is upward sloping (black dashed, for low α); (ii) $\widehat{y}$ converges to the vertical line (red dashed, for medium α); and (iii) $\widehat{y}$ is downward sloping and becomes zero for large x (blue dashed, for high α).

2.4. Optimal investment strategy

1. *Investment region.* We now characterize the optimal investment strategy. First, note that immediate investment is optimal when $V(x,y) = G(x,y)$ and waiting is optimal when $V(x,y) > G(x,y)$. The immediate investment region, furthermore, can be split in two parts, one where it is optimal to invest in the VCP, $V(x,y) = G_v(x,y)$, and another where it is optimal to invest in the FCP, $V(x,y) = G_f(x,y)$. As the payoff G is a continuous function, the optimal investment rule is the first time at which either of these two regions is reached.

Let us discuss some obvious properties of the regions. First, when $V_v(x,y) < K_v$ and $V_f(x) < K_f$, it is clear that immediate investment is suboptimal. Second, when $x < b_{0,v}(y)$, one should not enter into the VCP as in this region the project is in the idling state and negative cash flows are immediately collected. Similarly, when $x < b_{0,f}$ immediate investment into the FCP is suboptimal.

When y goes to 0, the project valuation problem reduces to a single state variable problem with payoff

$$\max\left(V_v(x,0) - K_v, \alpha(V_f(x) - K_f)\right)$$

which can be solved in closed form by standard methods, e.g., free-boundary problem with smooth pasting condition. On other hand, when y goes to $+\infty$, the FCP dominates the VCP and the manager faces a one-dimensional problem with the payoff

$$V_f(x) - K_f$$

and the optimal investment boundary $\bar{X}_f$ that can be easily found.

To gain further insights into the structure of the investment region, let us consider the following simple suboptimal strategy: wait until some deterministic time $\tau > 0$ to invest in the project with largest NPV. Exploiting the local time-space formula (see Peskir (2007)) and the optional sampling theorem, we can write the value of this waiting strategy as,

$$
\begin{aligned}
(2.16)\qquad & \mathsf{E}_{x,y}\left[e^{-r\tau}G(X_\tau,Y_\tau)\right] \\
&= G(x,y) + \mathsf{E}_{x,y}\left[\int_0^\tau e^{-rt}H_v(X_t,Y_t)I(Y_t<\widehat{y}(X_t))dt\right] \\
&\quad + \mathsf{E}_{x,y}\left[\int_0^\tau e^{-rt}H_f(X_t)I(Y_t>\widehat{y}(X_t))dt\right] \\
&\quad - \frac{1}{2}\mathsf{E}_{x,y}\left[\int_0^\tau e^{-rt}\frac{\partial G_v}{\partial y}(X_t,Y_t)I(Y_t=\widehat{y}(X_t))dL_t^{\hat{y}}\right]
\end{aligned}
$$

for $(x,y)\in(0,\infty)^2$ where $L^{\hat{y}}$ is the local time process that Y spends at the curve $\widehat{y}(X)$ (this term arises because the payoff G is not smooth along $\widehat{y}$ and one finds its definition, e.g., in Peskir (2007)); H_v and H_f represent the local gains of waiting to invest in the VCP and FCP, respectively, which are defined as,

$$
\begin{aligned}
H_v(x,y) &= \mathbb{L}_{X,Y}G_v(x,y) - rG_v(x,y) = -\gamma_v(x-y) + k_{1,v} + rK_v \\
H_f(x) &= \mathbb{L}_X G_f(x) - rG_f(x) = -\alpha(\gamma_f(x+s) - k_{1,f} - rK_f)
\end{aligned}
$$

for $(x,y)\in(0,\infty)^2$. Hence, the value of waiting until time τ can be decomposed as the immediate investment payoff augmented by the present values of the local gains realized by delaying when (i) the VCP dominates, (ii) the FCP dominate and (iii) the two are equal (on the indifference curve). The local gains in regions (i) and (ii) consist of the instantaneous profits foregone augmented by the investment opportunity cost saved. In region (iii), i.e., on the indifference curve, the local time term has a positive impact on the value of waiting as the payoff G_v is decreasing in y and $dL^{\widehat{y}}\geq 0$.

There are three immediate implications of formula (2.16). First, immediate investment in the VCP is suboptimal when $\gamma_v(x-y) < k_{1,v} + rK_v$ as the local gains H_v of waiting are positive. Second, immediate investment in the FCP is suboptimal when $H_f > 0$, i.e., $\gamma_f(x+s) < k_{1,f} + rK_f$. Third, and most strikingly, investment along the indifference curve is suboptimal even if the current NPVs of the individual projects are arbitrarily large. This follows from the fact that the local time term in (2.16) behaves as $dt/\sqrt{t}$ and dominates dt terms corresponding to H_v and H_f for t sufficiently small. For example, one can choose τ to be deterministic and small enough for any given (x,y) on the indifference curve. This property is robust and holds for any configuration of the curve $\widehat{y}$. We will discuss the intuition behind this result below. The next theorem describes the structure of the immediate investment set.

Theorem 2.3. *There exist two boundaries $b_v:(0,\infty)\to(0,\infty)$ and $b_f:(0,\infty)\to(0,\infty)$ such that immediate investment in the VCP is optimal for $(x,y)\in(0,\infty)^2 : y\leq b_v(x)$ and in the FCP for $(x,y)\in(0,\infty)^2 : y\leq b_v(x)$.*

The optimal investment problem can be easily solved in the extreme cases: when $Y=0$ and $Y=\infty$. The associated problems can be written as follows

$$
(2.17)\qquad \sup_{0\leq\tau\leq\infty}\mathsf{E}_x\left[e^{-r\tau}\max\left(V_v(X_\tau,0)-K_v, V_f(X_\tau)-K_f\right)\right]
$$

$$(2.18) \qquad \sup_{0\le\tau\le\infty} \mathsf{E}_x\left[e^{-r\tau}(V_f(X_\tau)-K_f)\right]$$

for $x>0$. Then $b_f(x)=+\infty$ for $x<\bar{X}_{f,\infty}$, where $\bar{X}_{f,\infty}$ is the optimal threshold for the stand-alone FCP investment project problem (2.18).

The problem (2.17) *can have three types of solutions:*
(i) it is optimal to invest only in the VCP and there is a corresponding threshold $\bar{X}_v$ such that $b_v(x)=0$ for $x\le\bar{X}_v$ (see upper panels in Figure 4);
(ii) it is optimal to invest in the VCP when $X\ge\bar{X}_v$ and in the FCP when $\bar{X}_{f,1}\le X\le\bar{X}_{f,2}$ where $\bar{X}_v$, $\bar{X}_{f,2}$ are optimal thresholds and $\bar{X}_{f,1}=\bar{X}_{f,\infty}$: $b_v(x)=0$ for $x\le\bar{X}_v$ and $b_f(x)=0$ for $x\in[\bar{X}_{f,1},\bar{X}_{f,2}]$ (see lower left panel in Figure 4);
(iii) it is optimal to invest in the VCP when $\bar{X}_{v,1}\le X\le\bar{X}_{v,2}$ and in the FCP when $X\ge\bar{X}_f$ where $\bar{X}_{v,1}$, $\bar{X}_{v,2}$ and $\bar{X}_f$ are optimal thresholds: $b_v(x)=0$ for $x\notin[\bar{X}_{v,1},\bar{X}_{v,2})$ and $b_f(x)=0$ for $x\ge\bar{X}_f$ (see lower right panel in Figure 4).

The first statement of Theorem 2.3 essentially says that the VCP (FCP) investment region is left- (right-) connected in (Y,X)-space. For instance, if immediate investment in the FCP is optimal at (X,Y), then it remains optimal at $(X,Y+\Delta)$ for any $\Delta>0$ (i.e., the region is right-connected). This is intuitively clear, as an increase in the cost factor reduces the payoff from investing in the VCP. We also note that, for convenience, we defined the investment boundaries as functions of X in contrast to Section 2.3 where the switching boundaries were functions of Y.

The second statement describes the investment strategy when Y is close to 0 or goes to $+\infty$. When Y is 0, then the investment problem becomes one-dimensional (see (2.17)). There are three possible regimes described in the statement of theorem. For example, in the first case one obtains the optimal threshold $\bar{X}_v$ and the investment in the VCP must be made only when $X\ge\bar{X}_v$. In other words, $b_v(X)=0$ for $X\le\bar{X}_v$. The other two cases involve disconnected investment regions when $Y=0$.

Now when the cost factor Y goes to ∞, it is clear that the FCP dominates the VCP and the problem reduces to a single dimension as well (see (2.18)). It can then also be solved with a critical threshold $\bar{X}_f$. Therefore, one should not invest at all when Y is large and $X<\bar{X}_f$. In other words, $b_f(x)=+\infty$ for $x<\bar{X}_f$.

The important question is to understand the following: what is the optimal investment policy along the indifference curve $\widehat{y}$ for large values of X? In the case of a single project, it is well known that immediate investment is optimal when the project has sufficiently high NPV. However, when there are several mutually exclusive alternatives, it may well be optimal to wait when two or more projects have the same NPV. This is the case, in particular, if the project values follow geometric Brownian motions (see, e.g., Broadie and Detemple (1997)). It is then optimal to delay investment along the indifference curve even if all projects have infinitely large NPVs. The reason is because the likelihood of an increase in the payoff from waiting is quite substantial (it increases if any of the two project values increases) and this offsets any benefit from immediate investment.

In our setting where the projects' revenues are driven by the same stochastic factor X and the VCP has a stochastic cost component Y, we face a similar situation, i.e., it is always optimal to wait along the indifference curve. The formal explanation was already provided above using the formula (2.16) and a property of the local time process. There are two intuitive

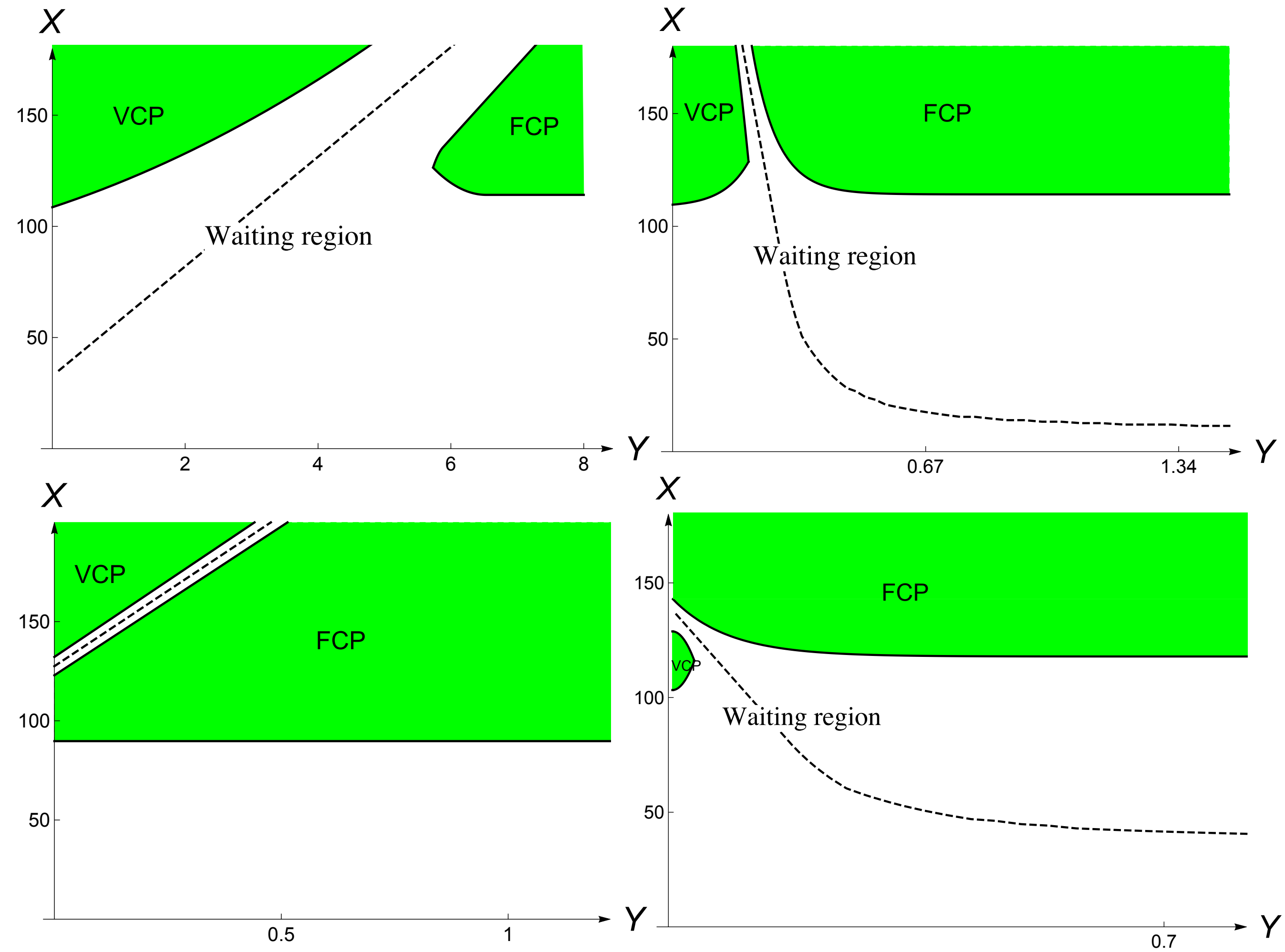

Figure 4: This figure shows the immediate investment region (green color) in the case of 1) capital constraint (upper left panel); 2) demand constraint (upper right panel); 3) low K_f and capital constraint (lower left panel) and 4) high α (lower right panel). Immediate investment in the VCP (FCP) is optimal in the left upper region (in the right upper region). The dashed curve corresponds to $\widehat{y}(x)$, where the two projects have the same net present value. Note the different scale on the horizontal axis.

explanations for this phenomenon. First, recall that the cost factor for the VCP is stochastic and that the decision-maker has the option to choose between the two projects. Consider a point (X, Y) on the indifference curve. If Y decreases, the value of the VCP increases and so does the investment payoff. In contrast, if Y increases, the decision-maker does not lose much as she can proceed with the FCP. Hence, there is a high likelihood of an increase in the payoff if she waits. Second, note that the payoff G can be rewritten in the following way

$$G(x, y) = \alpha(V_f(x) - K_f) + \max(V_v(x, y) - \alpha V_f(x) + \alpha K_f - K_v, 0)$$

for $x, y > 0$. Hence, the option to invest can be decomposed into two parts: an option to invest in the FCP and an exchange option (to give up the FCP and receive the VCP). On the indifference curve, i.e., $y = \widehat{y}(x)$, the immediate payoff of the second part is 0. Now let us consider the strategy of waiting until some deterministic time $t > 0$ and investing at that time. The value of this strategy is given by

$$\mathsf{E}_{x,y}\left[e^{-rt}(\alpha(V_f(X_t) - K_f) + \max(V_v(X_t, Y_t) - \alpha V_f(X_t) + \alpha K_f - K_v, 0))\right].$$

Clearly, as t goes to 0, this value converges to the immediate payoff $G(x, y)$. The first term depreciates at a finite rate as t increases but the second term appreciates at an infinite rate for small t due to the exchange option nature. Hence for any given $x > 0$, we can select $t > 0$ sufficiently small such that the value of this suboptimal strategy dominates immediate investment.

There are, however, major differences in the possible shapes taken by the waiting region around $\widehat{y}$. Illustrations appear in Figure 4. There are two cases. The first case occurs when the indifference curve converges to a vertical line as the cost factor y becomes small (right panels). In this situation, the value of waiting along $\widehat{y}$ becomes very small due to the substantial benefits of early investment for large x. In fact, the optimal waiting time $t > 0$ is close to 0. Hence, both optimal investment boundaries converge asymptotically to the indifference curve. From an economic point of view, this waiting region does not add significant economic value as investment will occur quickly. From a numerical point view, this case is challenging, as it is difficult to capture the exact asymptotic behavior of the boundaries for large x. In fact, the standard dynamic programing approach will not recognize that waiting is optimal at $\widehat{y}$ unless the time step is very small. The second case occurs when the indifference curve is upward sloping (left panels). Here, the variations in y are also important (as x increases, y goes up too) and in fact there is a cone that explodes and in which it is optimal to wait.

As shown in Figure 4, the behavior of the indifference curve, i.e., whether it is upward- or downward-sloping, also influences the shapes of the immediate investment regions. The shape of the VCP is particularly interesting in the downward-sloping case (right panels). If the indifference curve has a vertical asymptote at some $y_0 > 0$, it is optimal to invest in the VCP for all cost factors $y < y_0$ as long as the revenue factor is sufficiently large, i.e., the region is up-connected (upper right panel). If the vertical asymptote corresponds to the vertical axis $y_0 = 0$, the region eventually shrinks to the vertical axis. In this instance, immediate investment in the VCP becomes suboptimal if the revenue factor becomes sufficiently large for all cost factors $y > 0$. Finally, if the indifference curve collides with the vertical axis, the region is bounded and immediate investment in the VCP becomes suboptimal if x is sufficiently large for all $y \geq 0$ (lower right panel). Irrespective of the asymptotic behavior, the boundary of the VCP region can be a non-monotone function of the revenue factor, as displayed in the right panels. The shape of the FCP region is interesting in the upward-sloping case (left-panels). It always extends indefinitely to the right because investing remains optimal when the cost factor increases, i.e., when the NPV of the VCP decreases. But because the region is bounded above by the indifference curve and the surrounding asymptotic waiting cone, it curves back as the revenue factor increases, hence forming a trapezoidal-like shape with open right side (upper left panel). The size of this region depends on model parameters. If the cost K_f of investing in the FCP is sufficiently low, the region extends all the way to the vertical axis (lower left panel). In this instance, if initial factor prices (X, Y) are below the lower edge of the FCP region, the VCP region can never be reached first: investing in the VCP will never be optimal as it is always dominated by the FCP.

2. *Integral equations and EIP.* Theorem 2.4 is the main result of this section. It gives a characterization of the optimal investment strategy and the value of the option to invest in one of the projects.

Theorem 2.4. *Given the optimal investment boundaries* (b_v, b_f) *, the value of the option to invest has the early investment premium (EIP) representation*

$$V(x,y) = \pi(x,y;b_v,b_f) \tag{2.19}$$

for $x,y > 0$ *, where* $\pi(x,y;b_v,b_f)$ *represents the early investment premium (EIP) defined as,*

$$\pi(x,y;b_v,b_f) = \pi_v(x,y;b_v) + \pi_f(x,y;b_f) \tag{2.20}$$

$$\pi_v(x,y;b_v) = -\mathsf{E}_{x,y}\left[\int_0^\infty e^{-rt}H_v(X_t,Y_t)I(Y_t \leq b_v(X_t))dt\right] \tag{2.21}$$

$$\pi_f(x,y;b_f) = -\mathsf{E}_{x,y}\left[\int_0^\infty e^{-rt}H_f(X_t)I(Y_t \geq b_f(X_t))dt\right] \tag{2.22}$$

for $x,y > 0$ *. The optimal investment boundaries solve the pair of coupled integral equations*

$$V_v(x,b_v(x)) - K_v = \pi(x,b_v(x);b_v,b_f) \tag{2.23}$$

$$\alpha(V_f(x) - K_f) = \pi(x,b_f(x);b_v,b_f) \tag{2.24}$$

for $x > 0$ *.*

Theorem 2.4 shows that the value of the option to invest in either of the two projects, hence the best of the two, stems entirely from the EIP (2.19). This premium is the present value of the cumulative instantaneous gains from investing in the VCP when such a choice is optimal and those from investing in the FCP when the latter is optimal. The expression (2.19) is similar to the early exercise premium formula for pricing American-style finite maturity options. However, there is no European-style component due to the perpetual nature of the option to invest.

The theorem also states that the boundaries (b_v, b_f) solve a system (2.23)-(2.24) of coupled integral equations. These equations are derived from the EIP representation of the value of the option held by the investor. As there are two subregions, the premium has two components corresponding respectively to the value of investing when the VCP is optimal ($\pi_v(x,y;b_v)$) and when the FCP is optimal ($\pi_f(x,y;b_f)$). Because each premium component depends on its corresponding boundary, the equations are coupled and cannot be solved separately. Hence, the boundary b_v depends on b_f , and conversely. Also these equations do not have a recursive structure in time or space. The absence of recursivity with respect to time comes from the infinite horizon nature of the problem. The absence of recursivity in space is due to the fact that boundaries are integrated over their respective domains in the premium. As a result the integral equations are not of the typical Volterra type as in standard American option pricing problems.

The effects of model parameters, such as the operating leverage parameters κ and ρ , on the value of waiting and the optimal investment decision are examined in Section 3.6, in the context of the application to power generation technologies.

2.5. Extensions of the model

1. *Finite planning horizon.* In practice, decision-makers do not have an infinite planning horizon (e.g., firms have short-lived shareholders). Here, we describe the case where the investment project is subject to a deadline $T > 0$. Straightforward modifications of the previous results enable us to handle this situation. Theorem 2.3 continues to describe the investment region, except that the boundaries are now functions of time. Thus, there exist two boundaries $b_v : (0,\infty) \times [0,T] \to (0,\infty)$ and $b_f : (0,\infty) \times [0,T] \to (0,\infty)$ such that, at time $t \in [0,T)$, it is optimal to invest in the VCP when $y \leq b_v(x,t)$ or in the FCP when $y \geq b_f(x,t)$. Boundaries are characterized as in Theorem 2.4 where the value of the option to invest, in (2.19), has an additional component $V^e(x,y,t)$ corresponding to the value of a European option to invest in the best of the VCP and FCP at T and the EIP component tallies the gains from early investment up to time T. These modifications also appear in the integral equations (2.23)-(2.24) for the boundaries. As $t \to T$, the boundaries converge to

$$b_v(x,T-) = \min\left(\widehat{y}(x), \underline{y}_v(x), x - (k_{1,v} + rK_v)/\gamma_v\right) \quad \text{for } x \geq 0$$

$$b_f(x,T-) = \begin{cases} \widehat{y}(x) & \text{for } x \geq \max(\underline{x}_f, -s + (k_{1,f} + rK_f)/\gamma_f) \\ +\infty & \text{for } x < \max(\underline{x}_f, -s + (k_{1,f} + rK_f)/\gamma_f) \end{cases}$$

where $\underline{y}_v(x)$ is the curve defined as $V_v(x, \underline{y}_v(x)) = K_v$ for $x > 0$ and $\underline{x}_f$ is the break-even threshold for the FCP $V_f(\underline{x}_f) = K_f$.

The structure of the investment region and the cases which can arise, at any given time t, are the same as displayed in Figure 4. Relative to the infinite horizon case, finite horizon reduces the value of the project to invest in the best project. This means that the boundaries are closer to the indifference curve. For a given planning horizon T, the investment region expands as the horizon approaches (i.e., as $t \to T$). In the limit, the boundary of the VCP region approaches the indifference curve from above (upward-sloping case) or from the left (downward-sloping case) and the boundary of the FCP approaches it from below (upward-sloping case) or from the right (downward-sloping case). Figure 5 illustrates these effects in the capital-constrained case.

2. *The value of innovation.* Suppose that the FCP is based on a new technology, whereas the VCP employs established techniques. The value of the option to invest in the standalone VCP is $V(x,y;\alpha = 0)$, obtained by taking the scaling factor $\alpha = 0$ in (2.19). The value of the option to invest in the best of the VCP and the FCP is $V(x,y;\alpha)$ as given by (2.19). The value of innovation is the difference,

$$\Pi(x,y;\alpha) = V(x,y;\alpha) - V(x,y;\alpha = 0). \tag{2.25}$$

It represents the incremental value generated by the possibility of investing in the new technology as opposed to being limited to the familiar one.

The value of innovation is generally non-negative (in fact strictly positive when the optimal investment region for the FCP is non-empty) because a possible strategy for the option to invest in the best project is to simply ignore the FCP and invest in the VCP. This strategy is suboptimal therefore reduces the value of the option to invest in the best project. It is also clear

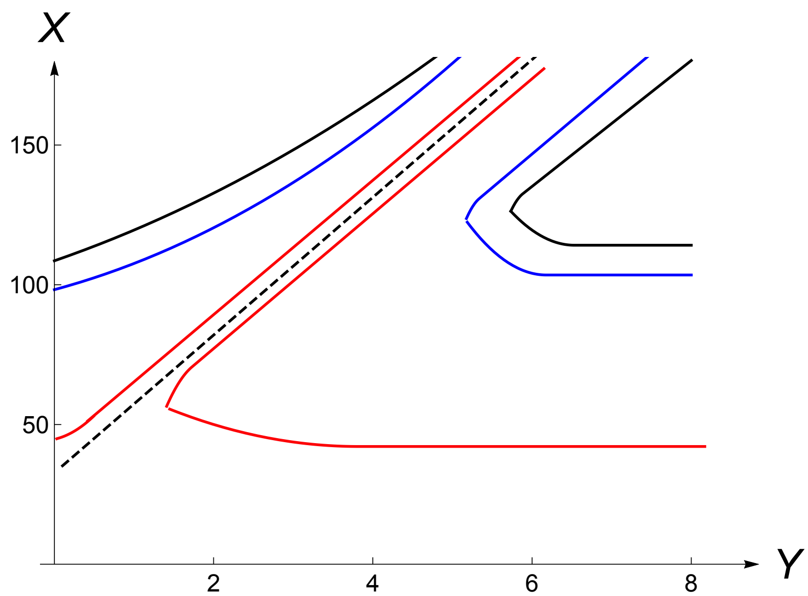

Figure 5: This figure plots the optimal investment boundaries in the finite planning horizon and the capital-constrained case: $T = 1$ year (red line), $T = 10$ years (black line), $T = \infty$ (black solid line). The black dashed line represents the indifference curve.

that the choice of the best time to invest in the standalone VCP reduces the value of innovation. Indeed, investing in the standalone VCP at the optimal stopping time for the project where choices are considered simultaneously is suboptimal for the standalone VCP, therefore magnifies the value of innovation. Hence, the difference in optimal waiting times between the two options has a mitigating effect: it reduces, but does not offset, the gain associated with the availability of the new technology. Finally, it is worth noting that an increase in the subsidy s raises the profit from the FCP, hence the value of innovation.

3. *Obsolescence.* Projects, in particular those based on new technologies, can become obsolete. The emergence of alternative technologies, the impact of new regulations or the occurrence of natural phenomena, such as those tied to climate change, are factors that can contribute to such a scenario. Events of this nature affect the option to invest and the value of innovation.

Suppose that the FCP is subject to such risk. Let λ be the arrival rate of an adverse event and assume that its occurrence reduces capacity by a fraction δ, i.e., effective capacity becomes $\gamma_f(1-\delta)$. The case $\delta = 1$ corresponds to complete obsolescence; $\delta \in (0,1)$ is partial obsolescence.

The structure of the solution is modified as follows. The first adjustment pertains to the value of the FCP, hence the associated early investment premium. Suppose that the adverse event has not yet occurred at t. There are two possible states in the future. If the adverse event occurs, the local benefit of investing at v is $\gamma_f(1-\delta)(x+s) - k_{1,f} - rK_f$ Otherwise, it is $\gamma_f(x+s) - k_{1,f} - rK_f$. These events occur with respective probabilities $1 - e^{-\lambda(v-t)}$ and $e^{-\lambda(v-t)}$. The second adjustment pertains to the boundaries which now depend on the occurrence or not of the adverse event. Boundaries are characterized as in Theorem 2.4, but based on the modified EIP.

Obsolescence risk reduces the operating profit of the FCP by the expected cost $(1 - e^{-\lambda(s-t)})\gamma_f \delta(x+s)$. The likelihood of occurrence λ and the capacity cost δ both raise the expected cost, hence weaken the incentives for investing in the FCP. The investment boundaries for the FCP are therefore expected to move out, whereas those for the VCP to move toward the indifference curve. Investments in the VCP substitute for those in the FCP. The value of the option to invest in the best project also decreases, because to payoff from investing in the FCP decreases. As the standalone VCP is not affected, the value of innovation decreases.

3. Application: Investments in power projects

Across industries, firms often face exclusive choices between fixed and variable cost projects. This section focuses on an application to the electric power industry.

3.1. Technology choice for power projects

Nowadays, electric power can be produced using a variety of technologies. Fossil fuel technologies such as coal- and gas-fired plants have traditionally been used to supply base loads in many countries across the globe. New technologies, based on renewable energy sources such as wind and solar photovoltaic, have progressively become more competitive alternatives. We examine the optimal strategy of a power generator who seeks to develop a new plant and can choose between the two types of technologies.

More specifically, we focus on the choice between a gas-fired plant (GFP) and a wind plant (WP). The latest generation of GFP, with carbon-capture technology, is the most efficient among fossil-fuel technologies. With a low heat rate near 7.5 MBtu/MWh it has an efficiency of about 60% . Among technologies based on renewable energy sources, WPs have become very competitive. Taller and more efficient turbines have led to a steady increase in capacity, which reached an average of 37% for 2016 US onshore installations. WPs have the ability to operate $24/7$ which is attractive for utilities. In 2017 , the majority of utility-scale renewable capacity addition came from wind (see EIA, January $10, 2018$).

A GFP is a particular case of a technology with variable costs. It sells electricity at the price X and buys gas at the price Y . The contribution margin is the spark spread $X - \kappa Y$ where $\kappa > 0$ is the heat rate. The operating profit is $\gamma_g(X - \kappa Y) - k_{1,g}$ where γ_g is the capacity factor and $k_{1,g}$ the operating cost. The operator of the plant has the option to idle or operate the plant.[8] In the idling mode, the costs are $k_{0,g} < k_{1,g}$. The optimal operating strategy is described in Section 2.1. The value of a gas-fired plant is denoted by $V_g(x,y)$ and given in Theorem 2.1.

A WP is a special case of a fixed cost technology. It sells electricity at the price X and has operating profit $\gamma_w(X+s) - k_w$ where γ_w is the capacity factor, s the government subsidy and k_w the operating cost. We assume that government subsidies are discontinued after $T_S = 10$ years of operations, which corresponds to current US regulation. Consistent

[8] The model with two states captures the basic operational regimes of a GFP. A more elabotate analysis framework would allow for additional states, for instance to capture differences in restart costs associated with the duration of the idling regime.

with data, we also assume that there is no difference between the running costs of operation and idleness. Perpetual operation is then optimal and the value of the WP is

$$V_w(x) = \gamma_w \left(\frac{x}{\delta_X} + s \frac{1 - e^{-rT_s}}{r} \right) - \frac{k_w}{r} \tag{3.1}$$

for $x > 0$.

The option to choose between the two types of plants has payoff,

$$\max \{ V_g(x, y) - K_g, \alpha(V_w(x) - K_w) \}$$

where $K_g > 0$ and $K_w > 0$ are the costs per MW of capacity for the GFP and the WP, respectively, and $\alpha > 0$ is the scale factor. The case $\alpha = 0$ represents a project to invest solely in the GFP. The power producer can also choose the timing τ of the investment. The value of the investment project is therefore given by,

$$V(x, y; \alpha) = \sup_{0 \le \tau \le \infty} \mathsf{E}_{x,y} \left[e^{-r\tau} \max(V_g(X_\tau, Y_\tau) - K_g, \alpha(V_w(X_\tau) - K_w)) \right]$$

for $x > 0$ and $y > 0$ where the supremum is over all stopping times $\tau \geq 0$. This is the value of an American option on the maximum of two assets with generally different strikes (unless $K_g = \alpha K_w$), i.e., a dual strike max-option. One of the assets, the gas-fired plant, depends on two state variables. Theorem 2.4 shows that $V(x, y; \alpha)$ is the present value of the cumulative instantaneous gains from investing in the WP when such a choice is optimal and those from investing in the GFP when the latter is optimal.[9] , [10]

3.2. Estimation and calibration

Parameter values under the risk-neutral measure are described in Table 1 where the drift $\mu_Z := r - \delta_Z$ for $Z \in \{X, Y\}$. The interest rate is $r = 0.04$. To estimate the volatility parameters of the electricity and gas prices, we collect daily data on Henry Hub Natural Gas Spot Prices from January 2001 to December 2016 and PJM WH Electricity prices for the same time period. As we are interested in the long term trend and would like to remove short term fluctuations, we consider moving averages for the prices with window of 1 year. Finally, for the resulting time series, we estimate $\sigma_X = 0.3$ and $\sigma_Y = 0.25$. The correlation between the two processes is $\rho = 0.3$. Next, to calibrate the drift parameters under the risk-neutral measure, we collect the data for the corresponding forward contracts. We estimate that the drifts $\mu_X = 0$ and $\mu_Y = 0$. For reference, typical values of wholesale electricity prices were in the range $10 - 120$ \$/MWh during the period 2016-2017. Typical values of the gas price

[9] A power producer seeking to build a new plant has the choice between multiple individual technologies as well as multiple hybrid technologies. A project with N alternatives is an American multi-strike compound max-options with N underlying assets, each corresponding to a particular technology. It can be analyzed using the same methods that we employ in the paper, i.e., EIP representation for the project value and integral equations for the relevant boundaries, albeit at the cost of a significant increase in computational difficulty.

[10] Instead of adding capacity, a power producer can also progressively phase out an existing technology and replace it by a new one. A replacement project of this type can be viewed as an American compound spread option. Valuation can be performed using the sequential approach in Section 4. In practice, replacement and expansion projects may be considered simultaneously.

	electricity price $Z = X$	gas price $Z = Y$
Drift μ_Z	0	0
Volatility σ_Z	0.3	0.25

Table 1: Parameter values.

fluctuated between $2.3 - 4.3$ \$/MBtu. Parameter estimates in Table 1 are comparable to estimates reported in Lucia and Schwartz (2002), Maribu, Galli and Armstrong (2007) and Boomsma, Meade and Fleten (2012).

We assume that the capacity factors for the wind and gas plants are, respectively, $\gamma_w = 0.37$ and $\gamma_g = 0.6$, [11] corresponding to average capacity factors in the US. The costs of operations for the gas-fired plant are $k_{0,g} = 3.79$ and $k_{1,g} = 7.33$ \$/MWh.[12] Switching costs are set at $c_{0,g} = 0.004$ M and $c_{1,g} = 0.001$ M (see Fleten, Johansen, Pichler and Ullrich (2016)). These costs capture switches between full capacity operation and idleness for extended periods of time. Costs of restarts increase significantly after a few days due to equipment failure. The running cost for the wind plant is $k_w = 5.33$ \$/MWh[13] and we assume that the subsidy rate is $s = 23$ \$/MWh (see Boomsma, Meade and Fleten (2012)).

The average cost per MW of a wind turbine was about 1.68 M in the U.S. in 2016.[14] For natural gas power plants, typical construction costs vary between 0.7 M and 2.2 M per MW depending on the type of technology deployed. For Combined Cycle plants with Carbon Sequestration, which have a heat rate of about 7.5 MBtu/MWh, the 2016 cost was about 2.15 M.[15] We use these cost estimates, namely 1.68 M and 2.10 M, with a correction for future replacement costs. This gives adjusted investment costs of $K_w = 2.66$ and $K_g = 3.32$. [16]

3.3. Capital-constrained investments

We first consider the case of a firm facing a capital constraint and choosing between two projects with identical investment costs. Letting $K := K_g = \alpha K_w$ ensures that this is the case.

The left panel of Figure 4 displays the structure of the immediate investment region along with the indifference curve $\widehat{y}(x)$ and the rays that are asymptotically tangent to the investment boundaries. The most significant aspect is the emergence of an asymptotic inaction cone (between the two rays) within which it is suboptimal to invest, even if the underlying values become extremely large. The existence of this cone stems from the projects' different exposures

[11] See Tables 6.7.A, 6.7.B in U.S. Energy Information Administration, Electricity Power Monthly, July 2018.

[12] Cost figures are calculated based on estimates reported in U.S. Energy Information Administration, Annual Energy Outlook 2017. We focus on Advanced Natural Gas plants with Combined Cycle and Carbon Sequestration, which have a heat rate of 7.5 M Btu/MWh. For this technology, fixed costs are estimated at 33,210 \$/MWy for 2016, which corresponds to 3.79 \$/MWh. Variable costs are 7.08 \$/MWh. The cost of operations is therefore $k_{1.g} = 3.79 + 50\% \times 7.08 = 7.33$ \$/MWh. The cost if the plant if idle is $k_{0,g} = 3.79$ \$/MWh.

[13] The 2017 report of the U.S. Energy Information Administration estimates the average fixed cost of operations for a wind turbine to be 46,710 \$/MWy, i.e., 5.33 \$/MWh, in 2016.

[14] U.S. Energy Information Administration, Annual Energy Outlook 2017.

[15] U.S. Energy Information Administration, Annual Energy Outlook 2017.

[16] We assume that costs depreciate at 1% per year and that equipment needs to be replaced every 20 years.

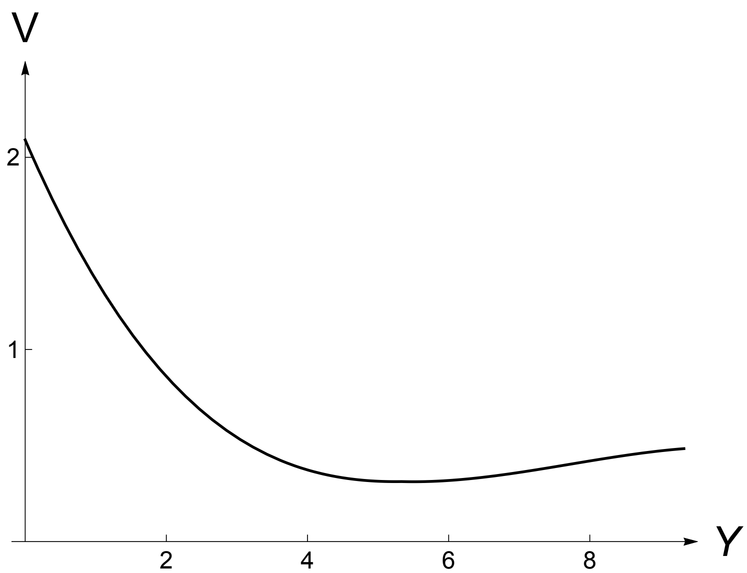

Figure 6: This figure plots the value of waiting to invest along the indifference curve, as a function of the gas price Y , in the capital-constrained case.

to the electricity price, which ensures that both the electricity price X and the gas price Y matter asymptotically. As constant components of revenues and costs become asymptotically irrelevant, the limit continuation region is determined by the relative sizes of the two factors.

3.4. Demand-driven investments

We now consider the case where the firm seeks to meet a given demand for power by choosing between projects that deliver the same effective capacity. Setting the scale factor α so that $\gamma_g = \alpha\gamma_w$ ensures that both technologies deliver the same output.

The right panel of Figure 4 displays the structure of the immediate investment region and the indifference curve $\widehat{y}(x)$ along which the two projects have the same value. Consistent with the results in Section 2 , the boundaries eventually converge to the indifference curve and the continuation region vanishes as the price of electricity becomes very large. Even for moderate electricity prices, the continuation region shrinks to a narrow band. In this region immediate investment remains suboptimal, but the impact on the project value is small. The likelihood of hitting the boundaries in a small time interval is indeed large, implying limited gains for waiting. The reason why the continuation region vanishes asymptotically is because the volatility of the gas price becomes negligible relative to the volatility of the electricity price as we move up the indifference curve. In the limit, the two projects are driven by a common state variable, the electricity price, and the value of waiting converges to zero as in the case of single state variable strategies of the threshold-type are typically optimal.

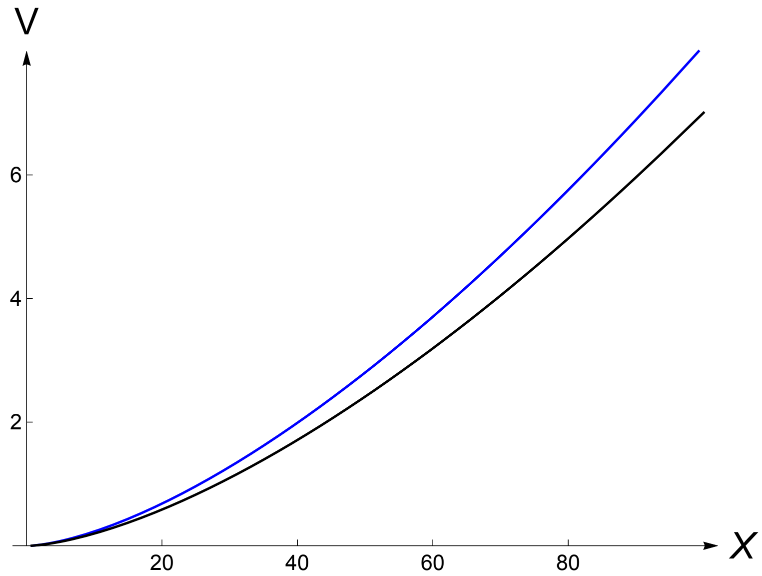

Figure 7: This figure shows that the value of the project $V(X,Y)$ to invest in the case of 1) capital constraint (black line) and 2) demand constraint (blue line) as a function of the electricity price X when the gas price $Y = 2.67$ \$/MBtu.

3.5. Green energy valuation and welfare

The value of green energy is the value of innovations based on GE technologies. In the context of this section, it is the incremental value generated by the possibility of investing in the alternative wind technology as opposed to being limited to the gas-fired technology. This wind premium is given by $\Pi(x, y; \alpha)$ in (2.25) where $V(x, y; \alpha)$ is the value of the project to invest in the best of the two technologies and $V(x, y; \alpha = 0)$ the value of the project to build a GFP.

The wind premium measures the welfare gained by considering the option to invest in the best technology instead of being limited to the GFP. To see this recall that the Marshalian surplus is the sum of consumer surplus and producer surplus. The consumer surplus for an extra unit of capacity, in our model with exogenous prices, is null. This is true for each of the two projects because the additional quantity of electricity produced has no price impact, hence the incremental area under the demand curve is null.[17] The producer surplus for an extra unit of capacity is given by the profit, i.e., marginal revenue minus marginal cost, for each project under consideration.[18] Under these circumstances, the present value of the producer surplus is the present value of the Marshalian surplus and it corresponds to the value of the project under consideration. The welfare gain is the incremental present value of the Marshalian surplus realized by considering the project to invest in the best technology as opposed to the project to invest in the GFP. This gain is then the difference in the project values, which corresponds

[17]For a small price change $\varepsilon \to 0$, the incremental consumer surplus at time t for an additional unit of installed GFP capacity is $\int_{X-\varepsilon}^{X} \gamma_g dv \to 0$. The same argument applies for an additional unit of WP capacity.

[18]For a small price change $\varepsilon \to 0$, the incremental producer surplus at time t for an additional unit of installed GFP capacity is $\gamma_g(X - \varepsilon) - \gamma_g \kappa Y - k_{1,g} - \int_{X-\varepsilon}^{X} S(v)dv \to \gamma_g(X - \kappa Y) - \gamma_g - k_{1,g}$ where $S(v)$ is the original supply curve. A similar argument applies for an additional unit of WP capacity.

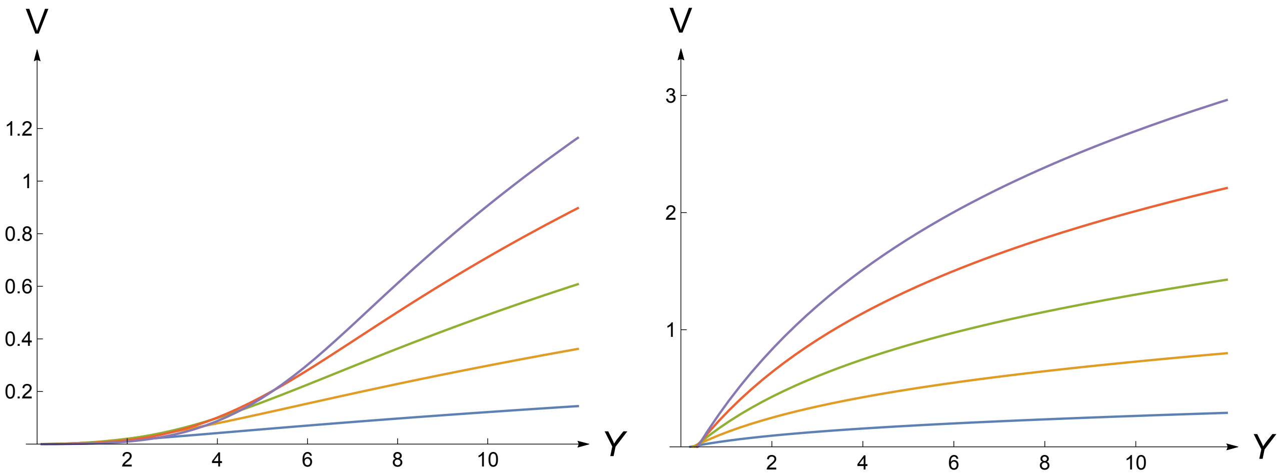

Figure 8: This figure shows the value of wind premium $\Pi(x,y;\alpha) = V(x,y;\alpha) - V(x,y;\alpha=0)$ in the case of 1) capital constraint (left panel) and 2) demand constraint (right panel) as a function of the gas price Y for different electricity prices X : 20\$/MWh (blue), 40\$/MWh (orange), 60\$/MWh (green), 80\$/MWh (red), 100\$/MWh (purple). Note the different scale on the vertical axis.

to the wind premium.

The wind premium, or welfare gain, depends on all the parameters of the model. It inherits the properties of the value of innovation described at the end of Section 2. In particular, it is increasing in the subsidy rate s . Subsidizing wind power is therefore welfare enhancing.

Figure 7 displays $V(x,y;\alpha)$ for the capital- and demand-constrained cases. Figure 8 illustrates the incremental welfare benefits associated with a potential investment in the wind farm. It plots the wind premium $\Pi(x,y;\alpha)$ for constant values of the electricity price. The left (right) panel shows the capital-constrained (demand-constrained) case. This premium can be substantial. It increases as the price of gas increases and converges to an upper bound as the price of gas becomes large. The premium is larger in the demand-constrained case due to the greater efficiency of the WP.

3.6. Operating income and leverage

The GFP is equivalent to a long position in the revenue stream (partially) hedged by a short position in the operating cost. The variable part of the operating cost depends on the heat rate κ and the correlation coefficient ρ between the electricity price and the gas price. Variations in these coefficients modify the effectiveness of the hedge, hence affect the performance of the GFP and the value of the project to invest in the best technology.

An increase in the heat rate amounts to a decrease in the efficiency of the GFP as more gas is required to produce a MWh of electricity. The variable cost thus increases and both the total contribution margin and the operating leverage decrease. The increase in κ therefore reduces the payoff from an investment in the GFP, causing the gas boundary to shift left (decrease). It also shifts the wind boundary to the left due to the coupling of optimal decisions. Investments in the gas fired technology are postponed whereas those in the wind technology are sped up. The left panel of Figure 9 illustrates these effects.

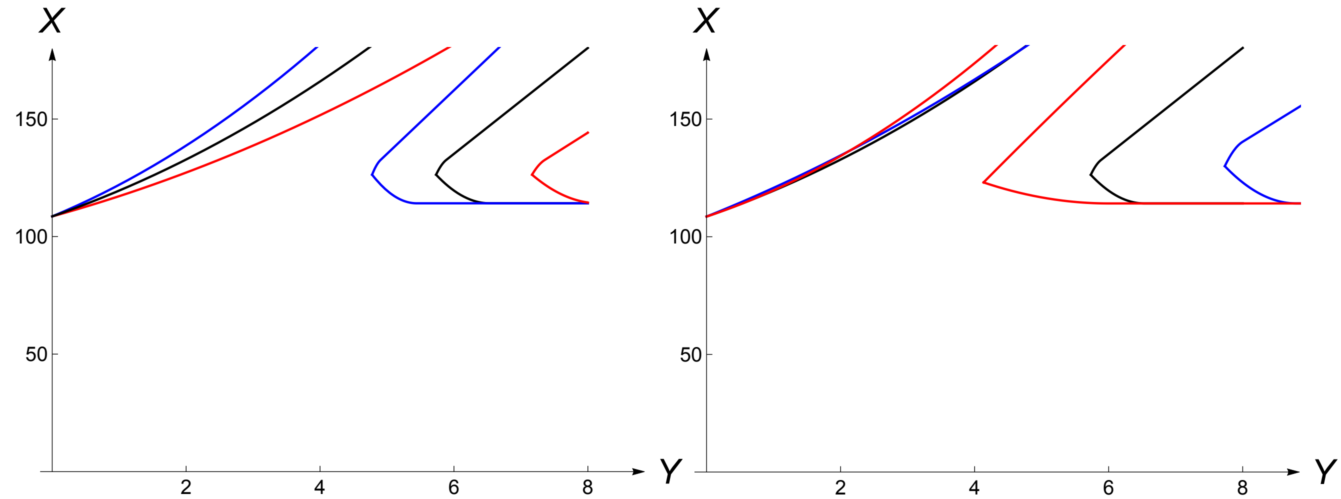

Figure 9: This figure shows effect of the heat rate κ and the correlation parameter ρ on the optimal investment boundaries in the case of capital constraint. Left panel: $\kappa = 6$ MBTu/MWh (red line), $\kappa = 7.5$ MBTu/MWh (black line), $\kappa = 9$ MBTu/MWh (blue line). Right panel: $\rho = 0$ (blue line), $\rho = 0.3$ (black line), $\rho = 0.6$ (red line).

The effects of an increase in the correlation between the electricity and gas prices are similar to those above, as displayed in the right panel of Figure 9, albeit less transparent. When the correlation coefficient increases, the value of the GFP decreases (see Section 2.1 and Figure 2). This leads to a direct reduction in the payoff from investing in that technology. The increase in correlation has a second effect because it reduces the riskiness of the GFP. By convexity, this reduces the value of the option to invest in the best of the two technologies. The combined effect is a decrease in the value of the investment project along with an unambiguous leftward shift in the wind boundary. The direct effect in the GFP value induces a leftward shift in the gas boundary, which is partly mitigated by the reduction in the overall value of the project.

3.7. Subsidies and investment costs

The left panel of Figure 10 shows the impact of the subsidy s in the capital-constrained case. As s increases the payoff from investing in the WP increases and so does the incentive to invest in that technology. The wind boundary therefore shifts left and the immediate wind-investment region expands. At the same time, because optimal decisions are coupled, the gas boundary also shifts left and the immediate gas-investment region contracts. Investments in the WP therefore substitute for investments in the GFP, both directly and indirectly. The indirect effect is due to the fact that it is optimal to wait longer before committing to build a GFP.

This plot also shows that the impact on the wind boundary is substantial. Moreover, it illustrates the limits of regulations based on subsidies in the capital-constrained case. As subsidies go to infinity, the upper part of the wind boundary converges to the lower edge of the no-investment cone. The cone itself is not affected. Hence, regulation based on a fixed feed-in tariff is unable to reduce the size of the waiting region beyond the asymptotic cone.

As a result of technological advances, the cost of wind plants has substantially decreased during the last decade. For a given cost constraint K, such a reduction amounts to an increase

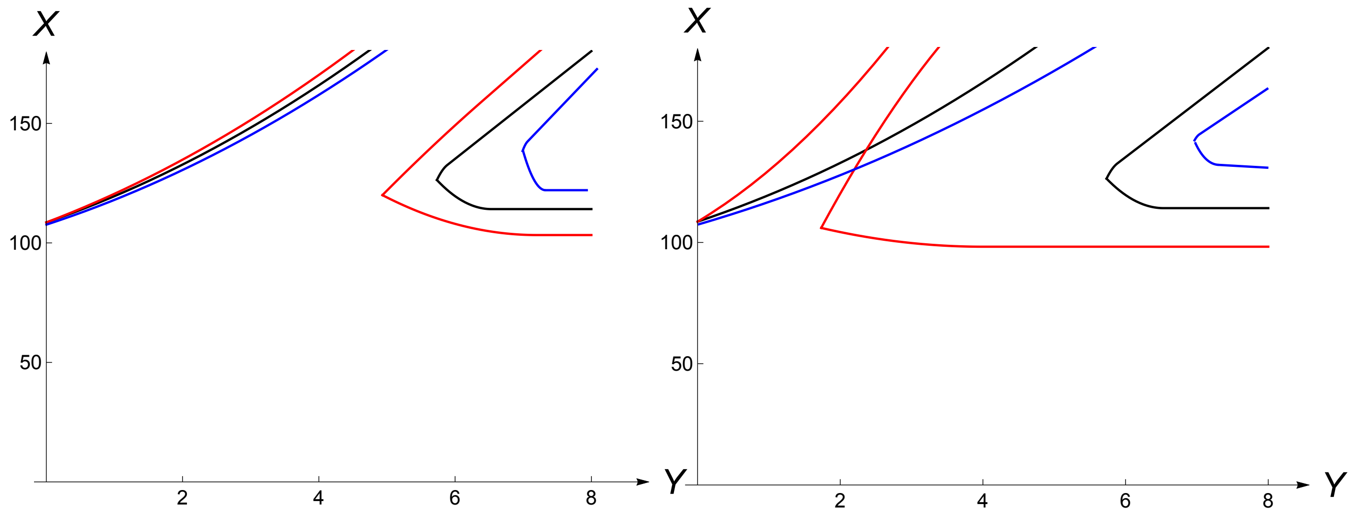

Figure 10: This figure shows effect of the subsidy s and the investment cost K_w of FCP on the optimal investment boundaries in the case of capital constraint. Left panel: $s = 15$ \$/MWh (blue line), $s = 23$ \$/MWh (black line), $s = 30$ \$/MWh (red line). Right panel: $K_w = 1.5$ (red line), $K_w = 1.68$ (black line), $K_w = 1.75$ (blue line).

in the scale of an investment in wind power. The right panel of Figure 10 shows the impact of a decrease in K_w. As cost decreases, the payoff from an investment in the WP increases, thereby increasing the value of the timing option. The wind boundary therefore shifts left (decreases) and the immediate wind-investment region expands. At the same time, because optimal decisions are coupled, the gas boundary also shifts in the leftward direction (decreases) and the immediate gas-investment region contracts. Investments in the WP therefore substitute for investments in GFP, both directly and indirectly. The indirect effect is due to the fact that it is optimal to wait longer before committing to build a GFP.

4. Investing in growth options: knowledge acquisition

This section examines the decision to invest in knowledge-building assets. Section 4.1 presents the example of a leading utility that invested in the option to build renewable power. General results are in Section 4.2, for the choice between a VCP and an FCP. The analysis is then specialized to the power generation context in Section 4.3.

4.1. Xcel Energy: an example

Xcel Energy is a utility holding company based in Minnesota, resulting from the merger of three predecessors, Public Service Company of Colorado, Southwestern Public Service Company and Northern States Prower. Its activities focus on the production, transmission and distribution of power and the production, transport and distribution of natural gas. The company operates in Western and Midwestern states and services over 3.5 million customers with electricity and over 2 million with natural gas (see Xcel Energy 2017-Corporate Responsibility Report).

In 2007, most of Xcel's power generation capacity came from about 60 coal and natural gas plants, 20 hydroelectric plants, 2 nuclear plants and 1 wind plant. Total dependable capacity reached 15,699 MW. The wind plant, located in Ponnequin-Weld County, CO, was Colorado's first wind farm. It consisted of 44 wind turbines build over the period 1999-2001 with total capacity of 25 MW. Xcel owned 37 of the 44 turbines. In addition to the power generated by its own plants, Xcel also purchased from other operators under long term supply contracts. At the end of 2007, it was the nation's leading utility for wind power purchases, with about 2,500 MW under contract.

December 2007 marked a turning point in Xcel's green energy strategy. On Christmas day, an extreme weather event hit Northeastern Colorado, bringing a record snowfall along with high winds that delivered three quarters of Xcel's generating capacity in an hour and nearly overloaded the grid. The event underscored the variability of wind and the unreliability of the day-ahead forecasts based on public information that were used at the time to predict load and available capacity. In the aftermath, Xcel partnered with the National Center for Atmospheric Research (NCAR) to develop reliable wind forecasts and with the National Renewable Energy Laboratory (NREL) to develop a reliable model for the relation between turbine output and wind speed. As part of the project, it installed a network of weather recording stations located at turbine height to provide accurate measurements of wind power. The project led to the development of high-resolution models that update every 15 minutes and produce wind forecasts up to 168 hours ahead. The density of the data collection grid permits the observation of terrain effects within a single wind farm and the making of individual predictions for each turbine (see Mahoney et al (2012) for an overview of the system).[19] In conjunction with the wind forecasting model, the firm developed an automatic dispatch model, i.e., an information technology infrastructure that automatically manages load and generation. This system optimizes the resources necessary for rapid ramp-up in case of an adverse wind event. Xcel estimates that the project led to a 17-38 percent improvement in its operations and savings of 22 M from 2009-2012 (Montgomery (2013)). Savings through the end of 2016 were estimated at 60 M for an investment of about 3.8 M (Baskin (2016)).

This acquisition of knowledge assets paved the way for subsequent investments in wind plants. By 2013 Xcel owned and operated 3 WP. By 2015, this number grew to 5, for a total capacity of 850 MW. It also added further wind generation capacity by entering long term supply contracts. Since that time, Xcel has continued an ambitious expansion in wind power using a multi-pronged approach. In 2017, it introduced a program to retire aging coal pants and replace them with renewable power. It also announced the development of 12 new wind farms across 7 states and a goal of exceeding 10,000 MW of wind capacity in its system by 2022. At that time, it anticipates producing 48 percent of its power supply from renewable sources.

4.2. Two-stage model

As the example above illustrates, a firm specializing in a certain type of projects with variable costs, may not have the expertise required to deploy an alternative technology with different cost structure and operate it in the most efficient way. It may also be unable to precisely assess the value of such an alternative using its existing human capital resources. In order to improve its decision-making process, the firm can then invest in knowledge-building assets prior to the

[19] See also https://ral.ucar.edu/solutions/products/empirical-wind-energy-conversion-algorithm

actual selection of the best project. This section develops a stylized model with an information acquisition (IA), i.e., knowledge-building, stage.

The decision-making process has now two stages. In the first stage, the firm has three alternatives. It can invest in a VCP using a technology it is familiar with, invest in the FCP with unfamiliar technology bearing an efficiency cost or invest in the acquisition of information about the alternative technology. If it decides to acquire information, it can in a second stage either invest in the VCP or in the FCP. In this setting, both the decision to acquire information and the timing of the acquisition are endogenous. Decisions regarding the choice between the three types of projects and the timing of that choice are also endogenous.

The model for the VCP is as described in Section 2. The net present value of investing in it is $V_v(x, y) - K_v$. The model for the FCP is a generalization of the previous one, allowing for an endogenous efficiency choice tied to information acquisition. First, we assume that the theoretical capacity factor γ_f is unknown ex-ante, given by a random variable with prior distribution function $F(\gamma)$ and support $[0, 1]$. Second, if the project is not optimally designed for the theoretical capacity factor γ_f , there is an efficiency loss ϕ resulting in an effective capacity

$$\gamma_e = \gamma_f(1 - \phi) \tag{4.1}$$

where ϕ is assumed to be constant. Under full information, $\phi = 0$ so that effective capacity equals theoretical capacity. In this instance the FCP model corresponds to the model in Section 2. Under incomplete information, sub-optimal design results in an efficiency loss $\phi \in (0, 1]$.

Technological information permitting the optimization of capacity utilization can be acquired at a cost K_I . Once the information is acquired, the theoretical capacity is revealed and equal to γ_f . Optimal project design implies that the effective capacity is then also equal to γ_f . The net present value of the FCP conditional on complete information about γ_f is $V_f(x; \gamma_f) - K_f$, with $V_f(x; \gamma_f)$ as in Section 2. If the firm chooses to forgo IA, it operates under incomplete information. It can still invest in the FCP, but also bears the efficiency cost due to suboptimal design. The net present value of the FCP conditional on imperfect information is $V_f^i(x; \phi) - K_f$ where,

$$V_f^i(x; \phi) = \int_0^1 V_f(x; \gamma(1 - \phi))dF(\gamma) \tag{4.2}$$

for $x > 0$ and $\phi \in (0, 1]$.

The stage two decision of the firm has been solved in Section 2 with the option value denoted by $V(x, y; \gamma_f)$ for a known value of γ_f . In the first stage, the firm has three alternatives and solves,

$$V_{\mathrm{I}}(x, y) = \sup_{0 \le \tau \le \infty} \mathsf{E}_{x,y} \left[e^{-r\tau} \max(V_v(X_\tau, Y_\tau) - K_v, V_f^i(X_\tau; \phi) - K_f, V_{\mathrm{II}}(X_\tau, Y_\tau) - K_I) \right] \tag{4.3}$$

where $V_f^i(x; \phi)$ is defined above in (4.2) and the value of the project in stage II is given by

$$V_{\mathrm{II}}(x, y) = \mathsf{E}_F[V(x, y; \gamma_f(\omega))] = \int_0^1 V(x, y; \gamma)dF(\gamma) \tag{4.4}$$

for $x, y > 0$.

In this section we focus on the first stage problem (4.3), i.e., whether or when to acquire information.

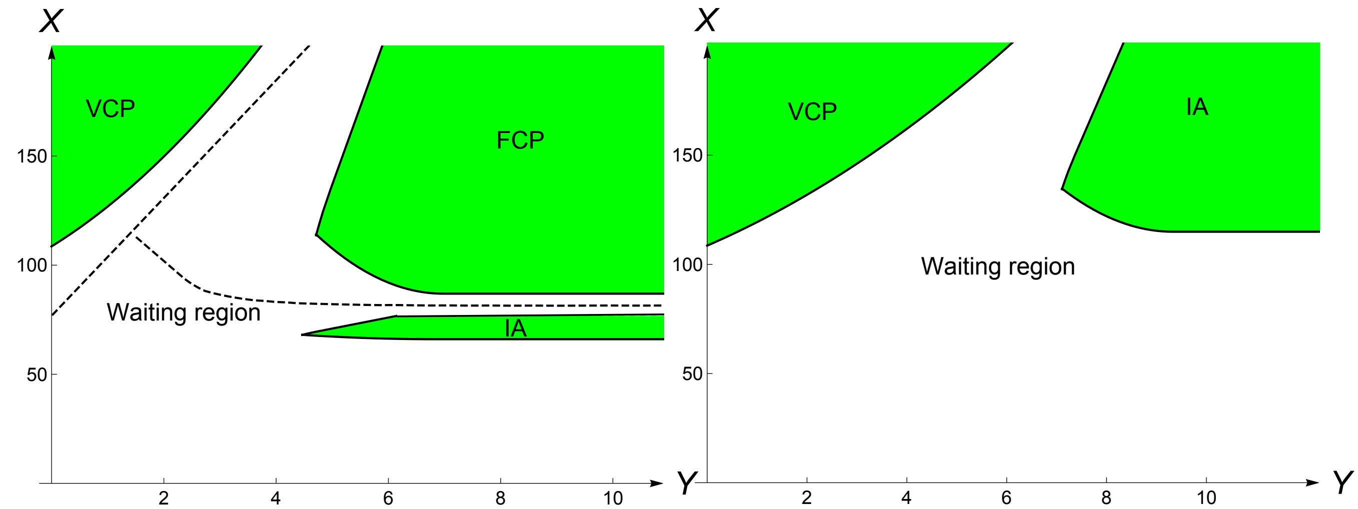

Figure 11: This figure shows the immediate investment region (green color) in the case of 1) $\phi = 0$ and low K_I (left panel); 2) high ϕ (right panel). Dashed lines represent indifference curves.

Theorem 4.1. *There exist three boundaries* $b_{v,I} : (0,\infty) \to (0,\infty)$ *,* $b_{f,I} : (0,\infty) \to (0,\infty)$ *and* $b^i_{f,I} : (0,\infty) \to (0,\infty)$ *such that it is optimal to invest in VCP when* $y \leq b_{v,I}(x)$ *, to acquire information when* $y \geq b_{f,I}(x)$ *and to invest in the FCP by forgoing information acquisition when* $y \geq b^i_{f,I}(x)$ *.*

When Y *goes to infinity, the optimal timing decision can be inferred from the one-dimensional (state variable) problem*

$$\sup_{0 \leq \tau \leq \infty} \mathsf{E}_x \left[e^{-r\tau} \max(V^i_f(X_\tau;\phi) - K_f, V_{II}(X_\tau,\infty) - K_I) \right] \tag{4.5}$$

for $x > 0$ *.*

Theorem 4.1 shows that the immediate investment region splits into three subregions (see the left panel of Figure 11 for illustration). In the upper subregion, where $y \leq b_{v,I}(x)$, investment in the VCP is optimal. In the intermediate subregion, where $y \geq b^i_{f,I}(x)$, direct investment in the FCP is optimal, In the lower subregion, where $y \geq b_{f,I}(x)$, investment in IA is optimal. The subregions are separated by two indifference curves along which waiting dominates. The first indifference curve is the set of points along which the NPVs of investments in the VCP and in the inefficient FCP are the same. The second one is the set of points where the NPVs of investments in the inefficient FCP and IA are the same. Along these curves, waiting is optimal even if the the values of the underlying stand-alone projects become extremely large. The intuition for this property is the same as in Section 2. When the efficiency cost of direct investment increases, the intermediate subregion shrinks. It eventually vanishes when ϕ becomes sufficiently large (see the right panel of Figure 11).

4.3. Valuation of knowledge assets

Theorem 4.2 is the main result of this section. It gives a characterization of the optimal investment boundaries for the first stage.

Theorem 4.2. *The optimal investment boundaries* $(b_{v,I}, b^i_{f,I}, b_{f,I})$ *solve the system of three integral equations,*

$$V_v(x, b_{v,I}(x)) - K_v = \pi_I(x, b_{v,I}(x); b_{v,I}, b^i_{f,I}, b_{f,I}) \tag{4.6}$$

$$V^i_f(x; \phi) - K_f = \pi_I(x, b^i_{f,I}(x); b_{v,I}, b^i_{f,I}, b_{f,I}) \tag{4.7}$$

$$V_{II}(x, b_{f,I}(x)) - K_I = \pi_I(x, b_{f,I}(x); b_{v,I}, b^i_{f,I}, b_{f,I}) \tag{4.8}$$

for $x > 0$ *and where* $\pi_I(x, y; b_{v,I}, b^i_{f,I}, b_{f,I})$ *represents the early investment premium (EIP) defined as,*

$$\pi_I(x, y; b_{v,I}, b^i_{f,I}, b_{f,I}) = \pi_{v,I}(x, y; b_{v,I}) + \pi^i_{f,I}(x, y; b^i_{f,I}) + \pi_{f,I}(x, y; b_{f,I}) \tag{4.9}$$

$$\pi_{v,I}(x, y; b_{v,I}) = -\mathsf{E}_{x,y}\left[\int_0^\infty e^{-rt} H_v(X_t, Y_t) I(Y_t \le b_{v,I}(X_t)) dt\right] \tag{4.10}$$

$$\pi^i_{f,I}(x, y; b^i_{f,I}) = -\mathsf{E}_{x,y}\left[\int_0^\infty e^{-rt} H^i_{f,I}(X_t, Y_t) I(Y_t \ge b^i_{f,I}(X_t)) dt\right] \tag{4.11}$$

$$\pi_{f,I}(x, y; b_{f,I}) = -\mathsf{E}_{x,y}\left[\int_0^\infty e^{-rt} H_{f,I}(X_t, Y_t) I(Y_t \ge b_{f,I}(X_t)) dt\right] \tag{4.12}$$

where

$$-H_v(x, y) = \gamma_v (x - \kappa y) - k_{1,v} - rK_v \tag{4.13}$$

$$-H^i_{f,I}(x, y) = -\int_0^1 H_f(x; \gamma(1-\phi))\, dF(\gamma) \tag{4.14}$$

$$-H_{f,I}(x, y) = -\int_0^1 H_f(x; \gamma)\, I(y \ge b_f(x; \gamma)) dF(\gamma) - rK_I \tag{4.15}$$

$$-H_f(x; \gamma) = \alpha(\gamma(x + s) - k_{1,f} - rK_f) \tag{4.16}$$

for $x, y > 0$ *. The value of the two-stage project is given by*

$$V_I(x, y) = \pi_I(x, y; b_{v,I}, b^i_{f,I}, b_{f,I}) \tag{4.17}$$

for $x, y > 0$ *.*

Theorem 4.2 shows that the boundaries for stage I also satisfy a system of integral equations. These equations follow from the EIP representation of the value of the two-stage project, given in (4.17). There are now three premium components, respectively associated with the local net gains from investing in the VCP, those from investing in technological information acquisition and those from investing directly in the FCP. For the VCP investment, the structure of the local net gains is similar to that in the benchmark model of Section 3. The trigger boundary $b_{v,I}(X)$ is nevertheless different from the VCP boundary $b_v(X; \gamma_f)$ for stage II, which also corresponds to the VCP boundary in the benchmark model. For the information acquisition investment, the structure of the local net gains is different. It is especially interesting to note that benefits are only collected when information acquisition is optimal and it is optimal to immediately invest in the FCP for some possible values of γ_f , i.e., when $Y > \max(b_{f,I}(X), b_f(X; \gamma_f))$ for some γ_f .

Hence, there is no value creation if waiting to invest is ex-post optimal for all possible γ_f , i.e., after information is acquired. This surprising result follows because the second stage project does not produce benefits until it is implemented: prior to investment its value grows at the risk free rate, therefore has a null dividend yield. The opportunity cost rK_I , in contrast, is always incurred starting immediately after information is acquired. Finally, for the direct investment in the FCP, the local net gain is the expected value of the gain under full information reduced by the efficiency loss associated with suboptimal design.

It is also interesting to note the respective positions of the boundaries for stages I and II. Figure 12 displays a situation with 3 possible values of the FCP capacity factor, $\gamma_f = 0.34, 0.37$ and 0.4 , each with equal probability $1/3$. The case displayed corresponds to parameter values such that direct investment in the FCP is suboptimal in stage 1. Observe that the information boundary (solid curve) is above the second stage wind boundary corresponding to the highest capacity factor $\gamma_f = 0.4$ (lowest dashed curve). This is a structural result that will always hold. It reflects the fact that the local benefits of investing in the FCP are null (i.e., waiting to invest is optimal) if the revenues are below the lowest possible ex-post FCP boundary. For similar reasons, the first stage VCP boundary is always below the highest possible second stage VCP boundary. Indeed, if it were higher, it would be optimal to invest in the VCP in the first stage irrespective of the FCP capacity factor. Information would then be irrelevant. As long as information provides ex-post benefits for some value of γ_f , this cannot happen. As for the FCP boundary for direct investment, it is also greater than the second stage FCP boundary for the highest capacity factor $\gamma_f = 0.4$. This is due to the uncertainty about theoretical capacity and the efficiency loss for suboptimal design.

Figure 12 shows that the first stage information boundary can exceed the second stage FCP boundary corresponding to the lowest capacity factor $\gamma_f = 0.34$. This seemingly surprising situation can occur when the marginal contribution of the VCP is sufficiently low. In such an event the value of the VCP tends to zero and the project reduces to a choice between two alternatives, an efficient FCP with an information acquisition stage and an investment stage, or an inefficient FCP under incomplete information (in the plot, the second alternative is suboptimal). Because information is costly, it can then pay to wait before acquiring information even if the cost factor exceeds the highest possible second stage FCP boundary.

4.4. Application: power growth options

In the context of power projects, the effects of government subsidies are especially relevant. As the subsidy size increases, the local gains from information acquisition and wind power investment increase, leading to an increase in the overall value of the project. This raises the value of information, hence the first stage payoff from information acquisition, and shifts the information boundary to the left. Due to the interactions between the three boundaries, it also shifts the first stage gas boundary and the first stage wind boundary for direct investment to the left, hence optimally delaying potential first stage investments in gas power and speeding up potential first stage direct investments in wind power. The right panel of Figure 13 illustrates these effects for parameter values leading to suboptimal direct investment in the first stage.

Operating leverage effects are displayed in the left panel of Figure 13, for the same configuration of parameters. As shown in Section 3, an increase in the heat rate κ , which amounts to a reduction in the efficiency of the GFP, reduces the value of the GFP. This decreases the

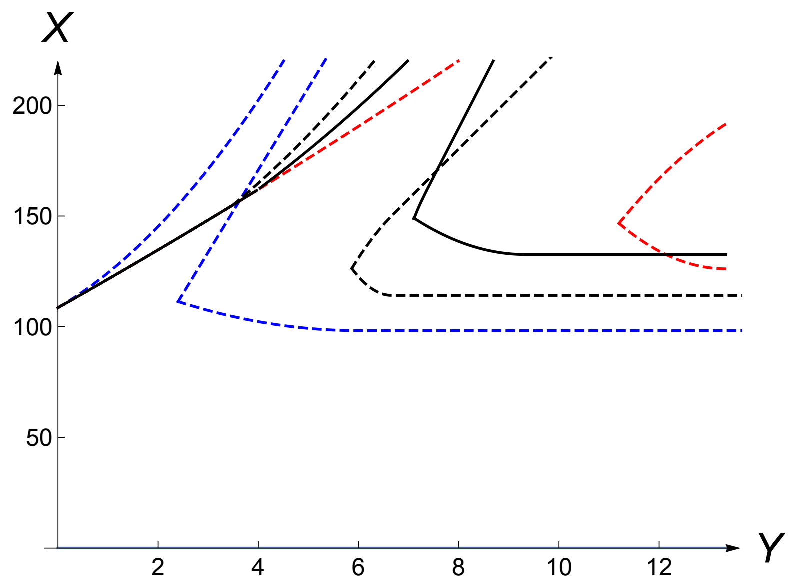

Figure 12: This figure plots the optimal investment boundaries for stage I (solid black line), and for stage II: $\gamma_f = 0.34$ (dashed red), $\gamma_f = 0.37$ (dashed black), $\gamma_f = 0.4$ (dashed blue). Other parameter values are as estimated in Section 3

second stage value function V_{II}, hence the payoff from IA, as well as the first stage payoff from investing in the GFP. The first (second) effect increases (decreases) the value of waiting to invest in information. Overall, the second effect dominates and reduces the value of waiting, thus moving the IA boundary to the left. Similar effects would combine to move the wind boundary for direct investment to the left, if parameter values led to the optimality of such investment in the first stage.

5. Conclusion

In this paper, we examined the optimal investment in the best of two projects with different types of cost structures, variable and fixed. We showed that the immediate investment policy is characterized by a pair of boundaries satisfying a system of coupled integral equations of the Fredholm type. The value of the investment project has an early investment premium representation consisting of two components, each tallying the local gains achieved when investment in a given project is optimal. In this context, we showed that it is always optimal to postpone investment when the NPVs of the two projects are equal. This is true even if these values are arbitrarily large. In fact, for some configurations of parameter values, waiting is optimal as long as the underlying revenue and cost factors lie in a cone. Thus, waiting can be optimal even if the discrepancy between the individual project values becomes infinitely large. Operating leverage emerges as an important factor. When it decreases, the welfare gain from investing in the FCP increases, speeding up investments in the FCP at the expense of those in the VCP.

The model was applied to examine optimal investment decisions in the power generation sector. Power can be produced using various technologies entailing different types of costs. We focused on the choice between a gas-fired plant and a wind plant. The possibility of using

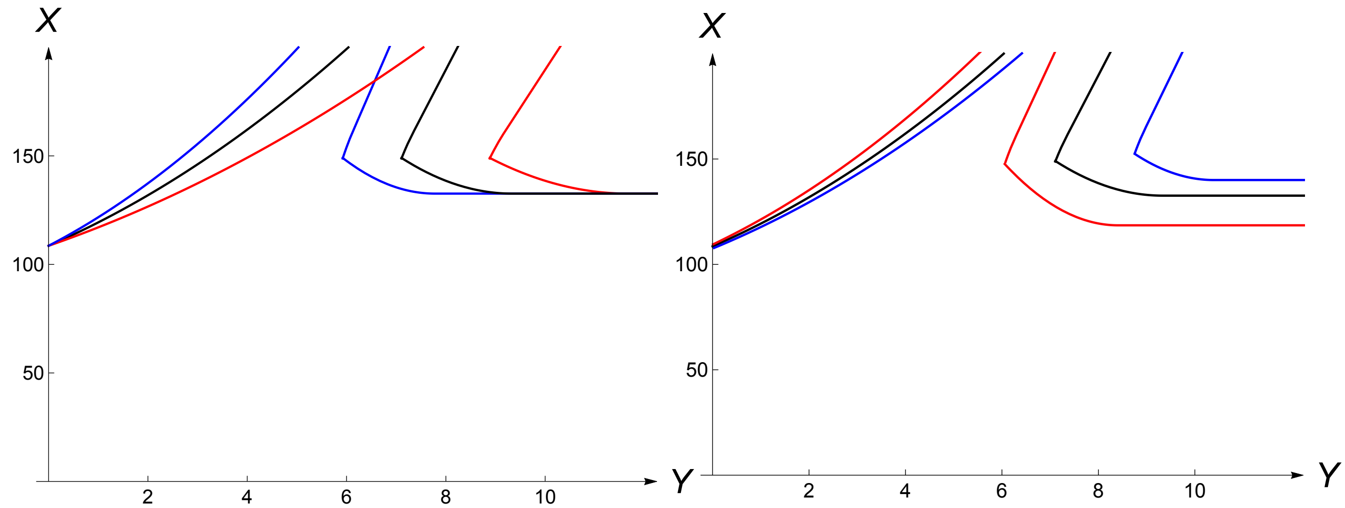

Figure 13: This figure shows effect of the heat rate κ and the subsidy rate s on the optimal investment boundaries of stage I. Left panel: $\kappa = 6$ MBTu/MWh (red line), $\kappa = 7.5$ MBTu/MWh (black line), $\kappa = 9$ MBTu/MWh (blue line). Right panel: $s = 15$ (blue line), $s = 23$ (black line), $s = 30$ (red line). Other parameter values are as estimated in Section 3

wind to produce electricity has a significant impact on the value of a power generation project. The welfare gain (premium) associated with the renewable source is an increasing function of the price of gas. This premium is large for typical values of the underlying prices, and has a significant impact on the value of a power project. It can remain positive even when the price of natural gas decreases and the spark spread increases. This provides an economic rationale as to why renewable power generation, e.g., wind, remains competitive even in the face of sharp declines in the cost certain fossil fuels.

Our analysis provides additional perspective regarding the importance of renewable energy for modern societies. It is clear that renewable energy sources have a major role to play in alleviating the negative consequences, such as global warming and other climate change phenomena, associated with reliance on fossil fuels. This study shows that renewable energy sources provide immediate welfare benefits for agents even if the economics of the problem would suggest that they are dominated by fossil fuels for power generation. The mere availability of power production based on renewable sources implies that it might be optimal to postpone investments and consider green power generation as a viable alternative to fossil fuel power generation.

The paper also provides new insights about investments in growth options, that is knowledge-building investments designed to increase information about components of growth options. In the particular case of power projects, information about proven technologies is more widely available and the cost of acquiring technical expertise to build and optimize the design of a plant has decreased. This is no longer true for advanced technologies which are either experimental or proprietary in nature. Implementation, in these cases, involves a transfer of technology, which is likely to be very costly. Alternatively, it may involve the formation of a partnership with or the outright acquisition of the firm owning the technology. Exploration of these issues might be a fruitful avenue for future research.

Finally, it might be worth mentioning that our model with choices among competing alternatives each with embedded options has further applications beyond power projects. One possible domain of application is human capital investments and pertains to decisions involving alternative career paths. For instance, a choice to invest in an engineering degree or a medical degree involves an initial decision about the timing of the investment and about the path to follow, each of which entails several subsequent decisions about whether to stop with the initial degree or pursue subsequent more advanced degrees at later endogenous times. Another application is in the area of conglomerate formation, where a firm grows by acquiring several entities, each engaged in activities that usually differ from that of the acquiring firm and with its own growth options. In such intertwined structures, the risks associated with any given firm's cash flows have an effect on the timing of the acquisition and the selection among the candidate targets. The analysis developed in the present paper shows that the value functions for such multi-stage timing and selection problems can be written as the present values of cash flows collected over specific future events and provides tools for the computation of the relevant decision boundaries.

Appendix A: Optimal switching problem for VCP when $Y = 0$

This Appendix briefly explains how to solve the switching problem for VCP when $Y = 0$. Using standard hedging arguments, one can show that the value functions $V_{0,v}$ and $V_{1,v}$ satisfy the second-order linear ODEs in the continuation region

$$
\begin{aligned}
&(r-\delta_X)xV_{0,v}'(x) + \frac{\sigma_X^2}{2}x^2V_{0,v}''(x) - rV_{0,v}(x) = k_{0,v} \quad \text{for } x < b_{0,v} \\
&(r-\delta_X)xV_{1,v}'(x) + \frac{\sigma_X^2}{2}x^2V_{1,v}''(x) - rV_{1,v}(x) = -\gamma_v x + k_{1,v} \quad \text{for } x > b_{1,v}.
\end{aligned}
$$

These equations can be solved, respectively, as follows

$$
\begin{aligned}
&V_{0,v}(x) = A_0x^p + B_0x^q - k_{0,v}/r \quad \text{for } x < b_{0,v} \\
&V_{1,v}(x) = A_1x^p + B_1x^q + \gamma_v x/\delta_X - k_{1,v}/r \quad \text{for} \quad x > b_{1,v}
\end{aligned}
$$

where A_0, B_0, A_1, B_1 are unknown constants and p, q are roots of the associated quadratic equation with $p > 1$ and $q < 0$. It is obvious that $V_{0,v}(0+) = -k_{0,v}/r$ and $V_{1,v}(x)$ is growing at a slower rate than linear. Hence, we can deduce that $B_0 = 0$ and $A_1 = 0$. Next, we impose smooth and continuous pasting conditions at the boundaries $b_{i,v}$, $i = 0, 1$. Hence we derive the system of four algebraic equations

$$
\begin{aligned}
&A_0b_{0,v}^p - k_{0,v}/r = B_1b_{0,v}^q + \gamma_v b_{0,v}/\delta_X - k_{1,v}/r - c_{0,v} \\
&pA_0b_{0,v}^{p-1} = qB_1b_{0,v}^{q-1} + \gamma_v/\delta_X \\
&A_0b_{1,v}^p - k_{0,v}/r - c_{1,v} = B_1b_{1,v}^q + \gamma_v b_{1,v}/\delta_X - k_{1,v}/r \\
&pA_0b_{1,v}^{p-1} = qB_1b_{1,v}^{q-1} + \gamma_v/\delta_X
\end{aligned}
$$

with four unknowns $(b_{0,v}, b_{1,v}, A_0, B_1)$. It can be shown that this system has a unique solution that is easy to find by numerical methods.

Appendix B: Numerical method

This Appendix develops a new numerical approach to solve the system (2.9)-(2.10). First, we rewrite it in a more convenient manner. For this, we consider the premium π_1 and write the indicator function $I(X_t \leq b_1(Y_t))$ as $1 - I(X_t \geq b_1(Y_t))$. Then, by integrating over dt on $(0, \infty)$ we obtain that

$$\pi_1(x, y; b_1) = -\gamma_v x/\delta_X + \gamma_v y/\delta_Y + k_{1,v}/r - k_{0,v}/r - c_{1,v} + \tilde{\pi}_1(x, y; b_1)$$

and the alternative characterization $V_1(x, y) = -k_{0,v}/r - c_{1,v} + \tilde{\pi}_1(x, y; b_1)$, where we defined

$$\tilde{\pi}_1(x, y; b_1) = \mathsf{E}_{x,y}\left[\int_0^\infty e^{-rt}(\gamma_v(X_t - Y_t) - k_{1,v} + k_{0,v} + rc_{1,v})I(X_t \geq b_1(Y_t))dt\right]$$

for $x > 0$ and $y > 0$. Now if we insert the new expression of π_1 into (2.9)-(2.10) some of the terms will disappear and the system becomes

$$\tilde{\pi}_1(b_0(y), y; b_1) - \pi_0(b_0(y), y; b_0) = c_{0,v} + c_{1,v} \tag{5.1}$$

$$\tilde{\pi}_1(b_1(y), y; b_1) - \pi_0(b_1(y), y; b_0) = 0 \tag{5.2}$$

for $y > 0$.

Now if we take expectations inside of the integrals, we can derive more explicit representations and rewrite the first equation (5.1) (analogously, we repeat it for the second one)

$$\begin{aligned} c_{0,v} + c_{1,v} = {} & \gamma_v b_0(y) \int_0^\infty e^{-\delta_X t}(\mathsf{P}^X_{b_0(y),y}(X_t \geq b_1(Y_t)) - \mathsf{P}^X_{b_0(y),y}(X_t \geq b_0(Y_t)))dt \\ & - \gamma_v y \int_0^\infty e^{-\delta_Y t}(\mathsf{P}^Y_{b_0(y),y}(X_t \geq b_1(Y_t)) - \mathsf{P}^Y_{b_0(y),y}(X_t \geq b_0(Y_t)))dt \\ & - (k_{1,v} - k_{0,v} - rc_{1,v}) \int_0^\infty e^{-rt}\mathsf{P}_{b_0(y),y}(X_t \geq b_1(Y_t))dt \\ & + (k_{1,v} - k_{0,v} + rc_{0,v}) \int_0^\infty e^{-rt}\mathsf{P}_{b_0(y),y}(X_t \geq b_0(Y_t))dt \end{aligned}$$

for $y > 0$ where we defined new probability measures $d\mathsf{P}^X = e^{-rt}(X_t/X_0)d\mathsf{P}$ and $d\mathsf{P}^Y = e^{-rt}(Y_t/Y_0)d\mathsf{P}$, respectively. Hence, we present the system of equations (5.1)-(5.2) as

$$b_0(y) = \frac{y \cdot I^Y(b_0(y), y; b_0, b_1) + I_1(b_0(y), y; b_1) - I_0(b_0(y), y; b_0) + c_0 + c_1}{I^X(b_0(y), y; b_0, b_1)} \tag{5.3}$$

$$b_1(y) = \frac{y \cdot I^Y(b_1(y), y; b_0, b_1) + I_1(b_1(y), y; b_1) - I_0(b_1(y), y; b_0)}{I^X(b_1(y), y; b_0, b_1)} \tag{5.4}$$

for $y > 0$ where the functions

$$I^X(x, y; b_0, b_1) = \gamma_v \int_0^\infty e^{-\delta_X t}(\mathsf{P}^X_{x,y}(X_t \geq b_1(Y_t)) - \mathsf{P}^X_{x,y}(X_t \geq b_0(Y_t)))dt$$

$$I^Y(x, y; b_0, b_1) = \gamma_v \int_0^\infty e^{-\delta_Y t}(\mathsf{P}^Y_{x,y}(X_t \geq b_1(Y_t)) - \mathsf{P}^Y_{x,y}(X_t \geq b_0(Y_t)))dt$$

$$I_0(x,y;b_0) = (k_{1,v} - k_{0,v} + rc_{0,v}) \int_0^\infty e^{-rt} \mathsf{P}_{x,y}(X_t \geq b_0(Y_t))dt$$
$$I_1(x,y;b_1) = (k_{1,v} - k_{0,v} - rc_{1,v}) \int_0^\infty e^{-rt} \mathsf{P}_{x,y}(X_t \geq b_1(Y_t))dt$$

for $x, y > 0$.

Therefore we can solve the system of integral equations (5.3)-(5.4) numerically as the fixed point problem by induction

$$b_0^{(n)}(y) = \frac{y \cdot I^Y(b_0^{(n-1)}(y), y; b_0^{(n-1)}, b_1^{(n-1)}) + I_1(b_0^{(n-1)}(y), y; b_1^{(n-1)}) - I_0(b_0^{(n-1)}(y), y; b_0^{(n-1)}) + c_0 + c_1}{I^X(b_0^{(n-1)}(y), y; b_0^{(n-1)}, b_1^{(n-1)})} \tag{5.5}$$

$$b_1^{(n)}(y) = \frac{y \cdot I^Y(b_1^{(n-1)}(y), y; b_0^{(n-1)}, b_1^{(n-1)}) + I_1(b_1^{(n-1)}(y), y; b_1^{(n-1)}) - I_0(b_1^{(n-1)}(y), y; b_0^{(n-1)})}{I^X(b_1^{(n-1)}(y), y; b_0^{(n-1)}, b_1^{(n-1)})} \tag{5.6}$$

for $y > 0$, $n \geq 1$ and using, e.g., $b_i^{(0)}(y) \equiv b_i(0+)$, $i = 0, 1$, as the initial curves.

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