Value-at-Risk of Equity Index and Index Futures Returns based on Empirical Tail Distribution

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Abstract

The characteristics of tail distribution of three equity indices and the corresponding index futures are examined. The method of L-moments is used to identify the tail distribution of extreme returns. The maximum and minimum returns of equity indexes generally follow generalized extreme value (GEV) distribution (Jenkinson 1955). However, the tail distributions of maximum and minimum returns are diversified in the futures market, implying investors' responses to good and bad information in futures markets are different from the spot markets.

Our evidence shows that tail distributions switch between GEV distribution and generalized logistic (GL) distribution over time. The extreme returns with high L-kurtosis follow GL distribution, but the others follow GEV or generalized normal distribution. The annual value at risk of the equity and futures returns are also discovered. The results show that average risks measured by VaR are larger in the left tails, and the futures generally have less risk.

Key Words: L-moments, Value at risk, tail distribution
JEL: G15, G32

Tail Distribution and Value-at-Risk of Equity Index and Index Futures Returns

1. Introduction

Lately risk management has become a critical issue for financial institutes because of an increase in uncertainty due to persistent financial crises and frequent huge losses to market participants. The Basel Committee on Banking Supervision (BCBS) triggered the development of the value-at-risk (VaR) measurement in 1996, allowing banks to measure market risk using their own internal models. Several measures have been applied to evaluate market risks, and VaR is the one widely used in the financial institutions. The main reason of the popularity of VaR is its simplicity of implementation and intuitional concepts. In the literature, VaR approaches can be classified into three categories according to the assumption of return distribution. The first approach is parametric method which assumes asset returns follow a specific distribution, such as normal or student t distribution (Mittnik & Paolella 2000; Angelidis *et al.* 2004; Bams *et al.* 2005; So & Yu 2006; Agnolucci 2009). The second approach, called historical simulation, uses empirical distribution instead of distributional assumptions to forecast market risk. Historical simulation is a common approach for measuring VaR in this field. Its convenient calculation has led itself becoming the most popular method in banking sector. However, it is criticized for ignoring the issue of serial correlation and heteroscedasticity of volatility. Barone-Adesi *et al.* (1999) introduces the filtered historical simulation applying ARMA or GARCH model to purge the noisy information such as serial correlation within the raw data. Then, an independent and

identical density residual sequence is obtained which can be applied to the conventional HS approach. However, Pritsker (2006) comments the VaR patterns of FHS respond the changes in conditional volatility sluggishly, and respond asymmetrically to large price moves, i.e. risk estimates increase after large losses, but not after large gains. Both of the two methods are not appropriate in practice because of the lack of realistic market characteristics.

Recently, a large number of research focuses on the tail distribution of asset returns by applying extreme value theory, called semi-parameter approach (Longin 1996; Bali 2003; Bingham *et al.* 2003; Gençay & Selçuk 2004; Allen *et al.* 2013; Ergen 2015). The main merit of this method is to fit the tail distribution rather than the entire distribution of asset returns, which significantly increases the accuracy of parameter estimation. Obviously, the distribution of asset returns is the core of risk measurement and VaR modelling. However, there is no conclusive distribution to asset return even Mandelbrot (1963) states that financial returns do not follow normal distribution. To financial institutions and investors, there is a need to identify an appropriate distribution in VaR modelling in both left and right tails.

Most previous research stress on the distribution of spot price changes and its characteristics, especially for equity index (Longin 1996, 2000; Jondeau & Rockinger 2003; Tolikas & Gettinby 2009). Surprisingly, less attention has been put on the identification of the distribution of futures price changes[1] even they play a critical role in the financial market. As known equity index

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[1] According to the basic concept of futures price, the price change of futures contract could be simplified as $\Delta F = \Delta S e^{r \cdot \Delta T}$, where r is risk-free rate and ΔS is stock price change. Thus, the distribution of the movement of futures price is unclear if the distribution of the movement of stock price as mentioned by Mandelbrot (1963)

futures contracts are the principle instruments used to hedge the investors' risks of spot position. Further, the distribution of futures returns is critical in risk management. Basu and Miffre (2013) investigate risk premium of prices for 27 commodity futures, and concluded that commodity futures risk premiums depend on considerations relating to both speculators' hedging pressure and inventory level. They imply the importance of hedging futures risks. Thus, it is critical and essential to find out the distribution of futures return since futures market participants might need to hedge their futures positions.

To the tail distribution of financial returns, some theoretical tail distributions have been proposed. Jenkinson (1955) presents a generalized extreme value (GEV) distribution covering three extreme value distributions: Fréchet, Weibull, and Gumbel distributions. Pickands (1975) shows that the generalized Pareto (GP) distribution is the extreme distribution for the values exceeding a certain threshold. Both GEV and GP distributions have been modelled to measure market risk of various financial returns based on the concept of value at risk (VaR). For example, Ho *et al.* (2000), Longin (1996, 2000), Bali (2003), Byström (2004), and Krehbiel and Adkins (2005) apply GEV distribution to fit the tail distribution of various asset extreme returns, and Neftci (2000), Gençay *et al.* (2003), McNeil (2005), Bao *et al.* (2006), Maghyereh and Al-Zoubi (2006), and Marimoutou *et al.* (2009) use GP distribution to measure market risk of financial asset. From the previous research, it is obvious that the tail distribution would be either GEV or GP distribution. However, some other empirical results are not identical to the two theoretical distributions.

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Gettinby et al. (2004, 2006) apply the method of L-moments to fit the tail distributions of US, UK, and Japan's equity index return. Their results show that the minima and maxima of the equity index returns follow a Generalized Logistic (GLO) distribution rather than other distributions. Tolikas and Gettinby (2009) also show the consistent results in Singapore equity index. However, less research examines the tail distribution of the corresponding futures index returns[2]. In fact, there is very little information about the distribution of futures returns and the risk of futures returns. The main purpose of this paper is to fill up the gap in the literature, identifying the distribution of equity indices and their corresponding futures price changes. Further, the risk patterns of the three pairs of equity index and corresponding futures index are shown based on the proper distributions as well. This paper offers investors and speculators as a critical reference that how risky the futures market is, which has not been discussed comprehensively in related literature.

This paper seeks to characterize the distribution of extreme returns for three equity indices of US, UK, Japan and their corresponding futures prices. This paper makes three important contributions to the literature. First, this paper shows the application of L-moments in estimating tail distribution of equity indices and futures indices, which is found in the previous research rarely. Furthermore, the extreme returns are more likely generalized extreme value distribution or generalized logistic distribution. Actually, the outcomes exhibit the tail distribution changes

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[2] Less research test the tail distribution of futures returns. Cotter (2001) confirms that futures returns are not normal distribution and a difference between upper and lower tails. Hall *et al.* (1989) suggests that the changes of futures prices follow the mixtures of normal distributions with different variances.

across years. Second, this paper also offers some solid evidence that the investors could use futures contracts to manage their positions in stock market since futures indices on average have less risks. However, in Japanese market, futures contracts are riskier than the stock positions. Thus, it seems not appropriate using futures contracts hedge their spot positions. Third, the paper points out that equity and futures indices generally have longer left tails, implying that it is more risks in the left tails.

The paper is organized as follows. Section 2 shows the research method including the identification method of tail distribution and the method of VaR. Section 3 explains the data used in this paper. The research results and analysis are reported in Section 4. Section 5 offers the conclusion.

2.   Methodology

2.1   Extreme returns selection

The selection of extreme returns[3] plays a critical role in fitting tail distribution. There are two main extreme selection methods: block maxima (BM) and peaks-over-threshold (POT). BM method selects extreme return from a fixed block of observations. For example, one extreme return is picked within every five daily returns. In contrast to BM, POT selects the extreme return based on a given threshold. Specifically, the values above the threshold are seen extreme returns. Ferreira and de Haan (2015) suggests BM approach is easier to apply since it has lower

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[3] The return is calculated based on natural log difference between two consecutive trading days.

asymptotic variance and independent observations. Therefore, BM approach is applied in selecting extreme returns in this paper. Another issue is the number of extremes. Using too many extreme returns to fit the tail distribution may not describe the tail distribution well. However, less extreme returns would reduce accuracy of tail distribution fitting. In this paper, weekly extreme returns (an extreme return is selected from every five trading days), including lowest and highest daily returns, are used to fit tail distributions of equity index and its futures price movements. Similar approaches of extreme selection are also used in Longin (1996), Jondeau and Rockinger (2003), Bali (2003), Krehbiel and Adkins (2005), and Bao et al. (2006).

2.2   Distributions fitting and VaR

Generally, there are some different methods to characterize the distribution of financial returns. In the field of risk management, fitting tail distribution with extreme returns improves the accuracy of market risk forecasting, compared with fitting whole distribution with full sample such as Harris and Küçüközmen (2001). Several methods of tail distribution fitting have been widely applied in the literature, for example maximum likelihood estimator (MLE) and method of moments (MOM). The former describes the distribution in large sample well but it assumes the return follows a specific distribution, which is not practical. In risk management, tail distribution fitting needs less observations than fitting the whole distribution. However, MOM could not guarantee that the popular four moments exist.

In this paper, an extension of method of moment, called L-moments proposed by Hosking

(1990), is applied to fit tail distribution and estimate parameters. Moment-based estimator is appropriate for small sample, and L-moments method characterizes the distribution well even sometimes the conventional moments do not exist. Generally, L-moments could be expressed as a linear combination of probability weighted moments (PWM [4]) proposed by Greenwood *et al.* (1979). Hosking *et al.* (1985) conducted an estimation of generalized extreme value distribution and stated that PWM estimator is fast, straightforward and it also has small biases in small sample estimation compared with MLE. Further, Hosking (1990) and Delicado and Goria (2008) examined the performance of parameter estimation of maximum likelihood, method of moments, and L-moments, and they recommend L-moments method for small sample sizes and also concluded that L-moments method has more efficiencies than MLE. As known, tail distribution fitting of financial returns is highly associated with VaR modelling. The market risk might be over- or under-estimated if the tail distribution cannot be estimated appropriately. Previous research mainly applied MLE, which is more suitable in large sample. However, market risk theoretically happening in the tail area of return distribution is the case of small sample. Thus, L-moments is applied to fit tail-distribution of financial return. According to Hosking (1990), the generalization of L-moments is set as

$$\lambda_r = r^{-1} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} EX_{r-k:r} \tag{1}$$

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[4] Hosking (1990) and Sankarasubramanian and Srinivasan (1999) explained that L-moments are linear function of probability weighted (PWMs).

where the expectation of an order statistic could be expressed as

$$EX_{r-k:r} = \frac{r!}{(r-k-1)!k!} \cdot \int_0^1 x \cdot [F(x)]^{r-k-1} \cdot [1-F(x)]^k \, dF(x).$$

(2)

Equation (1) can be rearranged to obtain first four L-moments as:

$$\lambda_1 = EX = \int_0^1 x \cdot F(x) dF(x)$$

$$\lambda_2 = \frac{1}{2} E(X_{2:2} - X_{1:2}) = \int_0^1 x \cdot [2F(x) - 1] dF(x)$$

$$\lambda_3 = \frac{1}{3} E(X_{3:3} - 2X_{3:3} + X_{1:3}) = \int_0^1 x \cdot [6(F(x))^2 - 6F(x) + 1] dF(x)$$

$$\lambda_4 = \frac{1}{4} E(X_{4:4} - 3X_{3:4} + 3X_{2:4} - X_{1:4}) = \int_0^1 x \cdot [20(F(x))^3 - 30(F(x))^2 + 12F(x) - 1] dF(x)$$

(3)

After obtaining the four L-moments, L-skewness and L-kurtosis can be calculated by the

formulas (4).

$$L-skewness = \frac{\lambda_3}{\lambda_2}$$

$$L-kurtosis = \frac{\lambda_4}{\lambda_2}$$

(4)

If the sample L-statistics shown in Equations (4) are graphically consistent with the theoretical

L-statistics of a contain distribution, then we suggest the tail distribution follows the distribution.

Theoretically, Fisher and Tippett (1928) suggested the tail distribution should be one of Fréchet,

Weibull and Gumbel distribution, and Jenkinson (1955) introduced a generalized extreme value

distribution embracing the three extreme value distributions. However, some empirical studies

obtained diversified conclusions. Gettinby et al. (2004, 2006) investigated the tail distribution of

equity indices in US, UK, and Japan with a long range period. They suggested that GLO

distribution well fitting with daily data in the sample period, even outperforming than GEV

distribution. Recent empirical research confirms this contention. Tolikas (2008) and Tolikas and Gettinby (2009) also presented that GLO distribution fits the tail distribution better than other distribution to other financial assets such as bond, commodities, futures, and stocks. We follow Tolikas and Gettinby (2009) approach to examine the tail distribution of index and futures return based on several distributions such as GEV, GLO, and GPD. In addition, normal distribution, exponential distribution, and Gumbel distribution are also graphically examined.

As the definition of value at risk (VaR), it summarizes an overall loss of a portfolio happening in next trading period with a given probability. Briefly, quantile-based VaR model needs a specific distribution to calculate market risk. As mentioned in the introduction, three approaches with different assumptions to the return distribution are widely applied to measure market risks. In this paper, the best-fitted tail distribution will be applied in VaR model. As known that market changes overtime (Engle 1982; Bollerslev 1986), and the tail distribution changes accordingly. The advantage of using best fitting tail distribution based on BM method to calculate VaR is that it offers the accuracy in the estimation of the tail density of financial returns since the parameter estimation would not be driven by the returns from the body distribution. Besides, the change of market conditions is also considered. Similar approach has been applied in some previous empirical research such as Longin (1996), Lauridsen (2000), and Bali (2003). However, they did not identify the tail distribution based on the market conditions.

3. Data

In this paper, three equity index prices and their corresponding futures nearby prices collected

from DataStream are applied to examine their tail distributions and to assess the value at risk

based on the appropriate tail distributions. The three equity indices are S&P 500 from the United

States, FTSE 100 from United Kingdom, and Nikkei 225 from Japan. Sample periods of both

equity index and futures index of S&P 500 and FTSE 100 are from 2nd January 1985 to 30th

December 2015. Constrained by the data availability, sample period of Nikkei 225 index is from

and futures index is from 2nd January 1985 to 30th December 2015. Nikkei 225 index futures is

from 6th September 1988 to 30th December 2015. After adjusting for holidays, both total

observations of S&P 500 index and index futures are 7,816. Nikkei 225 equity index (index

futures) has 7,816 (7,625) observations. There are 7,830 observations in both FTSE 100 index

and its index futures. The descriptive statistics of the indices are shown in Table 1, indicating that

all of the return sequences are stationary but far away from the normal distribution. On average,

S&P 500 index and its futures returns are more volatile than Nikkei 225 and FTSE 100.

The Q-Q plots of the six sequences are presented in Figure 1. Briefly, Figure 1 indicates that

the extreme value effect exists in all the equity and futures indices returns. From the figures,

negative extreme returns are more than positive ones. In other words, the left tail would be longer

than the right tail, i.e. risk is larger in left tails.

Table 1. Descriptive statistics

	Max. (Min.)	Mean (Std.)	Skewness	Kurtosis	ADF test	JB test	Obs.
S&P500 index	0.1096 (-0.2290)	0.0003 (0.0115)	-1.2734	27.8940	-67.0866	0.0000	7,816
S&P500 futures	0.1775 (-0.3370)	0.0003 (0.0125)	-2.3931	81.2458	-60.8122	0.0000	7,816
Nikkei225 index	0.1323 (-0.1614)	0.0001 (0.0146)	-0.2999	7.6654	-65.1116	0.0000	7,625
Nikkei225 futures	0.1881 (-0.1400)	-0.0001 (0.0154)	-0.0913	8.3646	-60.7330	0.0000	6,718
FTSE100 index	0.0938 (-0.1303)	0.0002 (0.0109)	-0.4958	9.8988	-65.8875	0.0000	7,830
FTSE100 futures	0.0958 (-0.1675)	0.0002 (0.0116)	-0.5902	11.1888	-66.6109	0.0000	7,830

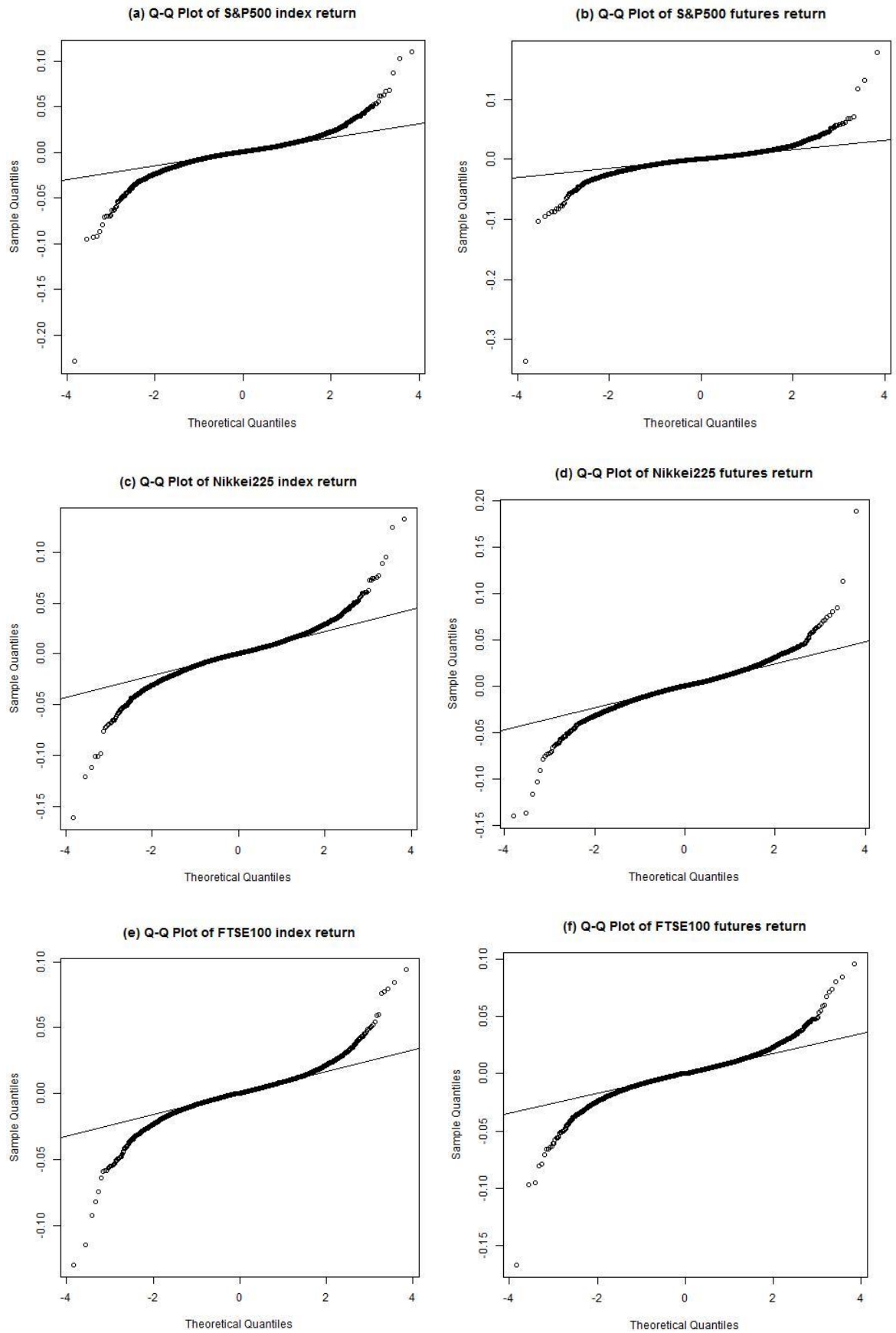Note: The mean values are significant at 1% level.

Figure 1. Q-Q plot of equity index return and index futures return

4. Empirical Results

4.1 Tail distribution of equity index return

Firstly, the tail distributions of the equity and futures return indices are identified based on graphical and numerical approaches. The graphical results using the diagram of L-skewness and L-kurtosis are shown in Figure 2. In the estimating tail distribution, extreme returns in both tails of the three equity indices are selected every five trading days. L-skewness and L-kurtosis are calculated annually based on the extreme returns. In general, the plots spread in the areas of GLO, GEV, and generalized normal (GNO) distribution. In addition, the plots in left tails more concentrate then the ones in right tails. The results are not consistent with Gettinby *et al.* (2004) and Gettinby *et al.* (2006) who suggested that the tails of the three equity indices follow a GLO.

Our graphical results show that both tail distributions could be one of GEV, GLO and GNO distribution[5]. For example, some observations (in red[6]) in Figure 2 clearly follow GLO distribution, but the others seem following GEV or GNO. In the right tails, most of the black plots are difficult to identify which distribution they follow since they are near the cross of GEV and GNO. Here, an emerging question that if the extreme returns follow different distributions across various L-kurtosis needs to answer. To answer this question, the observations are separated

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[5] The densities of the three distributions are shown in the appendix.

[6] In Figure 2, the plots with higher L-kurtosis (larger than 0.18) around generalized logistic distribution are marked in red. Thus, right-tail L-kurtoses of S&P 500 index in 1987, 1989, 1991, 1994, 1997, 1998, 2008, 2011, and 2013-15 are marked in red, and the left-tail ones in 1985, 1987-89, 1991, 1993, 1997-98, 2000, 2003, 2006, and 2008 are red. Right-tail L-kurtoses of Nikkei 225 in 1985-87, 1989, 1995-97, 2000, 2003-05, and 2014 are red, and the left-tail ones in 1987-88, 1991, 1994, 1996, 1998, 2000-02, 2004, 2008, 2010 and 2012 are red. Right-tail L-kurtoses of FTSR 100 index in 1985-87, 1989, 1991-93, 1999, 2005-07, 2009-12, 2014 are marked in red, and the left-tail ones in 1987-89, 1991-93, 1996, 2000, 2002, 2005-2007, 2009.

into two groups: the plots with high L-kurtosis (in red) and the others with low L-kurtosis (in black). Statistically, higher L-kurtosis implies that the extreme return sequence has a fatter shape in the tail area. In other words, the probability of happening extreme events is significant. Graphically, high L-kurtosis observations follow GLO, and the ones with low L-kurtosis follow GEV or GNO. In the left tails in Figure 2, higher L-kurtosis plots tend to be GLO distribution and lower L-kurtosis plots generally follow GEV distribution. It is obvious that fewer minimum returns follow GNO distribution. In the maximum returns (right tails), fewer maximum returns with higher L-kurtosis from S&P 500 follow GLO distribution, but more ones from FTSE 100 and Nikkei 225 index returns follow GLO. The right tails of FTSE 100 and Nikkei 225 index returns, the distributions of the plots with lower L- kurtosis are obscure.

The parameter estimation and numerical test (AD test) results are shown in panel A of Table 2. The estimated scales show that fluctuations in left tails are greater than the ones in right tails, which is consistent with Jondeau and Rockinger (2003). The results imply that most investors in the markets tend to be the risk-averters. About the shape parameter, it dominates tail behaviour of the distribution. The distribution with larger estimated shape parameter theoretically corresponds fatter tail. In addition, the estimated shapes in left (right) tail are positive (negative), and further showing that estimated shapes in left tail are on average larger than the ones of right tail. The estimated scale and shape are consistent with each other, indicating the risk in left tail is larger than the risk in right tail. This results are similar to Jondeau and Rockinger (2003)

focusing on tail index that the probability mass in the right tail increases, as the tail index increases. In the identification of tail distribution, AD test results indicate that GEV distribution cannot be rejected in most tail distributions. Even the results of AD tests are clear, it is obvious that the extreme returns might have diversified distributions, and the results are significantly different from Gettinby et al. (2004) and Gettinby et al. (2006).

The results of AD test are presented in Panel B of Table 2, however there is a discrepancy to Figure 2. General speaking, the observations with high L-kurtosis in left tails, only S&P 500 follows GLO distribution, and the ones in right tails of S&P 500 and Nikkei 225 follow generalized logistic distribution. Some observations with high L-kurtosis, such as left tail of Nikkei 225 and both tails with high L-kurtosis of FTSE 100, are indistinct. The tail distributions are clearer in the part of low L-kurtosis. In the right tails with low L-kurtosis, S&P 500 and Nikkei 225 follow GEV distribution, and FTSE 100 follows GNO distribution. In the left tail with low L-kurtosis, FTSE 100 follows GEV distribution. However, the right tails with low L-kurtosis of S&P 500 and Nikkei 225 cannot be identified. The main attribute of the non-identified phenomenon could be associated with the dispersion of the plots, which might come from national economy conditions.

Table 2. Parameters estimation and distribution tests of the index returns

	Left tail (minima)				Right tail (maxima)			
Panel A	location	scale	shape	AD	location	scale	shape	AD
S&P500 index returns								
GEV	-0.0062	0.0059	0.2217	**0.6828***	0.0076	0.0055	-0.1684	**0.3031***
GLO	-0.0086	0.0044	0.3206	0.0000	0.0098	0.0040	-0.2828	0.0000
GNO	-0.0083	0.0078	0.6727	0.0000	0.0096	0.0070	-0.5901	0.0000
GPA	-0.0005	0.0110	0.0290	0.0000	0.0022	0.0108	0.1182	0.0000
Nikkei225 index returns								
GEV	-0.0089	0.0086	0.1127	**0.2426***	0.0101	0.0078	-0.1039	0.4840
GLO	-0.0124	0.0061	0.2444	0.0143	0.0132	0.0055	-0.2385	**0.5657***
GNO	-0.0121	0.0107	0.5074	0.0000	0.0130	0.0096	-0.4948	0.0000
GPA	-0.0002	0.0180	0.2144	0.0000	0.0021	0.0165	0.2298	0.0000
FTSE100 index returns								
GEV	-0.0063	0.0060	0.1580	**0.6185***	0.0075	0.0055	-0.1263	**0.0927***
GLO	-0.0087	0.0043	0.2756	0.0000	0.0097	0.0039	-0.2537	0.0000
GNO	-0.0085	0.0076	0.5744	0.0000	0.0095	0.0069	-0.5273	0.0000
GPA	-0.0004	0.0119	0.1358	0.0000	0.0019	0.0114	0.1905	0.0000
Panel B	Low L-kurtosis		high L-kurtosis		Low L-kurtosis		high L-kurtosis	
S&P500 index returns								
GEV	**0.7824**		0.0023		0.0414		0.0458	
GLO	0.0015		**0.8945**		0.0112		**0.1027**	
GNO	0.0006		0.0001		0.0004		0.0093	
Nikkei225 index returns								
GEV	**0.8573**		0.0167		0.0178		0.0014	
GLO	0.0109		0.0048		0.0561		**0.3821**	
GNO	0.0087		0.0021		0.0036		0.0000	
FTSE100 index returns								
GEV	0.0387		0.0097		0.0074		0.0048	
GLO	0.0190		0.0103		0.0134		0.0001	
GNO	0.0008		0.0021		**0.4123**		0.0000	

Note: AD means Anderson-Darling test. The details of AD could be found in Anderson and Darling (1954). The numbers in Panel B are p-values of AD test.
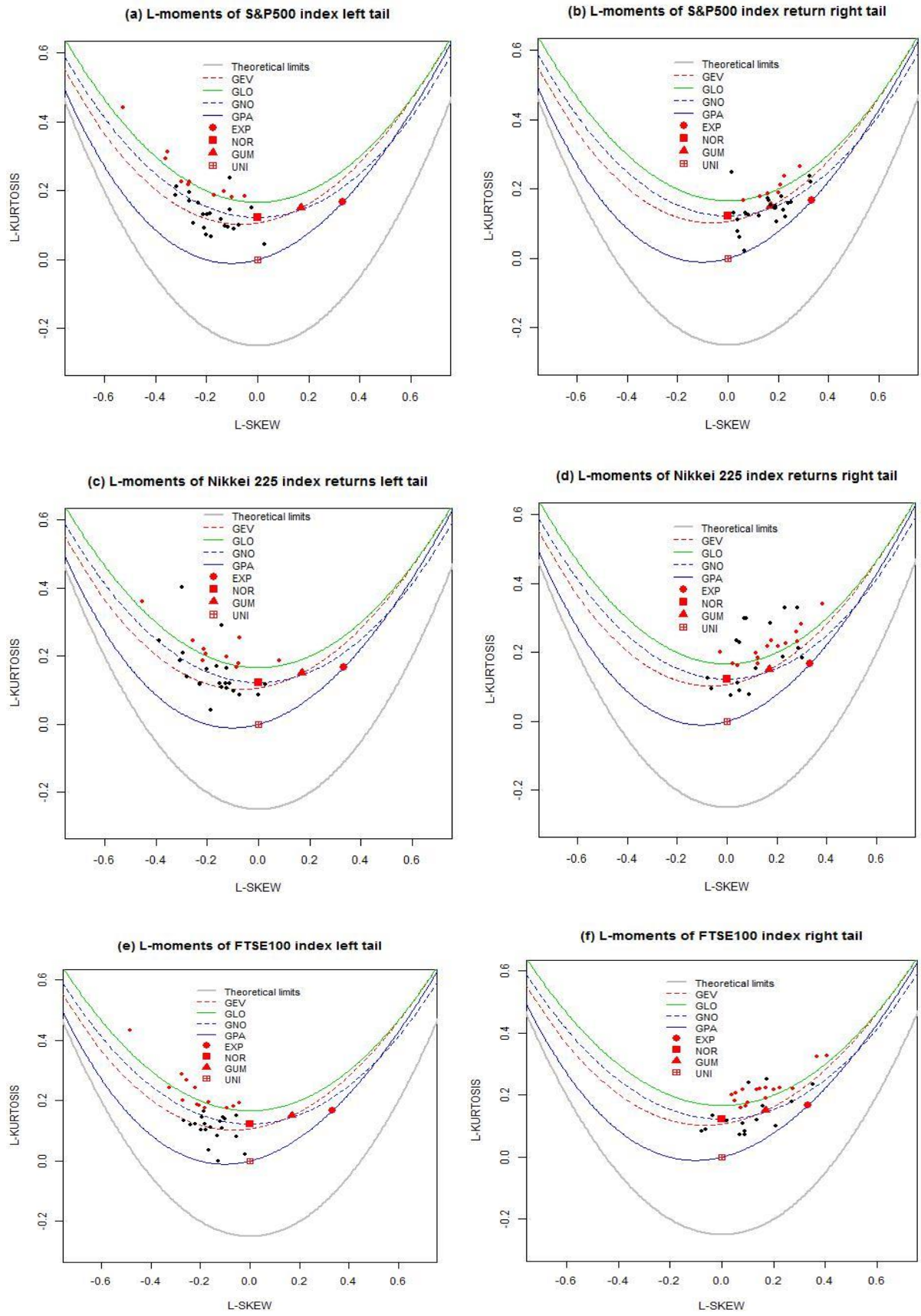
Figure 2. The ratio of L-skewness and L-kurtosis of equity index return

4.2 Tail distribution of index futures returns

The second section characterizes tail distribution of index futures returns, which is less discussed in the related research. The graphical analysis and numerical results are presented in Figure 3 and Table 3. On average, the L-moments scatters of the index futures are not as distinct as the ones of equity indices returns. The plots still spread among GEV, GLO, and GNO distributions. Obviously, high L-kurtosis observations of index futures are less than the ones of equity indices, reflecting the risk of futures is smaller than equity indices. This finding is quite reasonable and consistent with fundamental concepts of investment. In addition, the plots with high L-kurtosis (in red) seem not tend to follow GLO distribution, but the minimum returns (i.e. Figures 3- (a), (c), (e)) with low L-kurtosis are clearly GEV distribution.

The results of estimated parameters are shown in Panel A in Table 3. According to the scales of maximum and minimum returns, left tails have more fluctuations than right tails. This indicates that risks are higher in left tail, and investors buying the index futures contracts take more downside risk. On the other hand, investors could short equity index futures contracts to hedge their portfolio and the right tails have less risk. Another point in Table 3 is the results of AD test shown in panel B, exhibiting that left tails tend to be GEV distribution, especially to S&P 500 and FTSE 100. Even Nikkei 225 follows a GNO distribution, GEV still highly significant (0.7481). In sum, the results of AD test show that no consistent conclusions in the futures tail distributions, which could be found in Figure 3 that the plots are too diversified to identify the tail distribution. The estimated shape parameters are in accord with the scales that left tails are

generally thicker than right tails. The testing results also show that GLO distribution is not the best fit in extreme futures returns, but some evidence show that GEV distribution fits well in some cases.

Further, the observations are divided into high and low L-kurtosis groups, and their AD test results are shown in Panel B in Table 3. The tail distributions of extreme returns with high L-kurtosis (red plots) are quite different from the extreme returns with low L-kurtosis. General speaking, the returns with high L-kurtosis follow GLO distribution although left tail of FTSE 100 future index returns and right tail of Nikkei 225 index futures returns are not. On the other hand, the tail distributions with low L-kurtosis systematically belong GNO distribution even two (right tail of S&P 500 and left tail of Nikkei 225) of them do not follow this distribution. A possible explanation of the difference between the distributions of equity index and futures index returns is that the market participants of the two markets are different. The investors in the futures market are more likely informed and institutional investors (Easley *et al.* 1998; Chen *et al.* 2005). Obviously, different market participants could have diversified behavior, and it may cause different tail distributions.

Table 3. Parameters estimation and distribution tests of the futures returns

	Left tail				Right tail			
Panel A	location	scale	shape	AD	location	scale	shape	AD
S&P500 futures returns								
GEV	-0.0065	0.0065	0.1765	**0.7081***	0.0076	0.0056	-0.2062	0.0000
GLO	-0.0091	0.0047	0.2885	0.0978	0.0099	0.0041	-0.3095	0.0000
GNO	-0.0089	0.0083	0.6024	0.0000	0.0096	0.0072	-0.6483	0.0000
GPA	-0.0001	0.0127	0.1044	0.0000	0.0022	0.0105	0.0547	0.0000
Nikkei225 futures returns								
GEV	-0.0102	0.0090	0.0841	0.7481	0.0109	0.0081	-0.0752	**0.6758***
GLO	-0.0137	0.0062	0.2251	0.1089	0.0141	0.0056	-0.2192	0.6289
GNO	-0.0135	0.0110	0.4664	**0.8614***	0.0139	0.0099	-0.4537	0.3240
GPA	-0.0008	0.0194	0.2649	0.0000	0.0024	0.0177	0.2809	0.0000
FTSE100 futures returns								
GEV	-0.0069	0.0064	0.1478	**0.7789***	0.0081	0.0060	-0.1112	**0.1415***
GLO	-0.0095	0.0046	0.2685	0.0000	0.0105	0.0042	-0.2434	0.0000
GNO	-0.0093	0.0081	0.5592	0.0000	0.0103	0.0074	-0.5053	0.0000
GPA	-0.0005	0.0130	0.1533	0.0000	0.0020	0.0125	0.2170	0.0000
Panel B	Low L-kurtosis		high L-kurtosis		Low L-kurtosis		high L-kurtosis	
S&P500 futures returns								
GEV	0.0637		0.0571		0.0611		0.0875	
GLO	0.0285		**0.6547**		0.0981		**0.7287**	
GNO	**0.3855**		0.0021		0.0058		0.0945	
Nikkei225 futures returns								
GEV	0.0198		0.3012		0.3987		**0.5718**	
GLO	0.0008		**0.3894**		0.0036		0.2846	
GNO	0.0698		0.0657		**0.4652**		0.0687	
FTSE100 futures returns								
GEV	0.4009		**0.5996**		0.2172		0.2385	
GLO	0.0863		0.1892		0.0293		**0.6397**	
GNO	**0.5844**		0.0965		**0.3857**		0.1093	

Note: AD means Anderson-Darling test. The details of AD could be found in Anderson and Darling (1954). The numbers in Panel B are p-values of AD test.
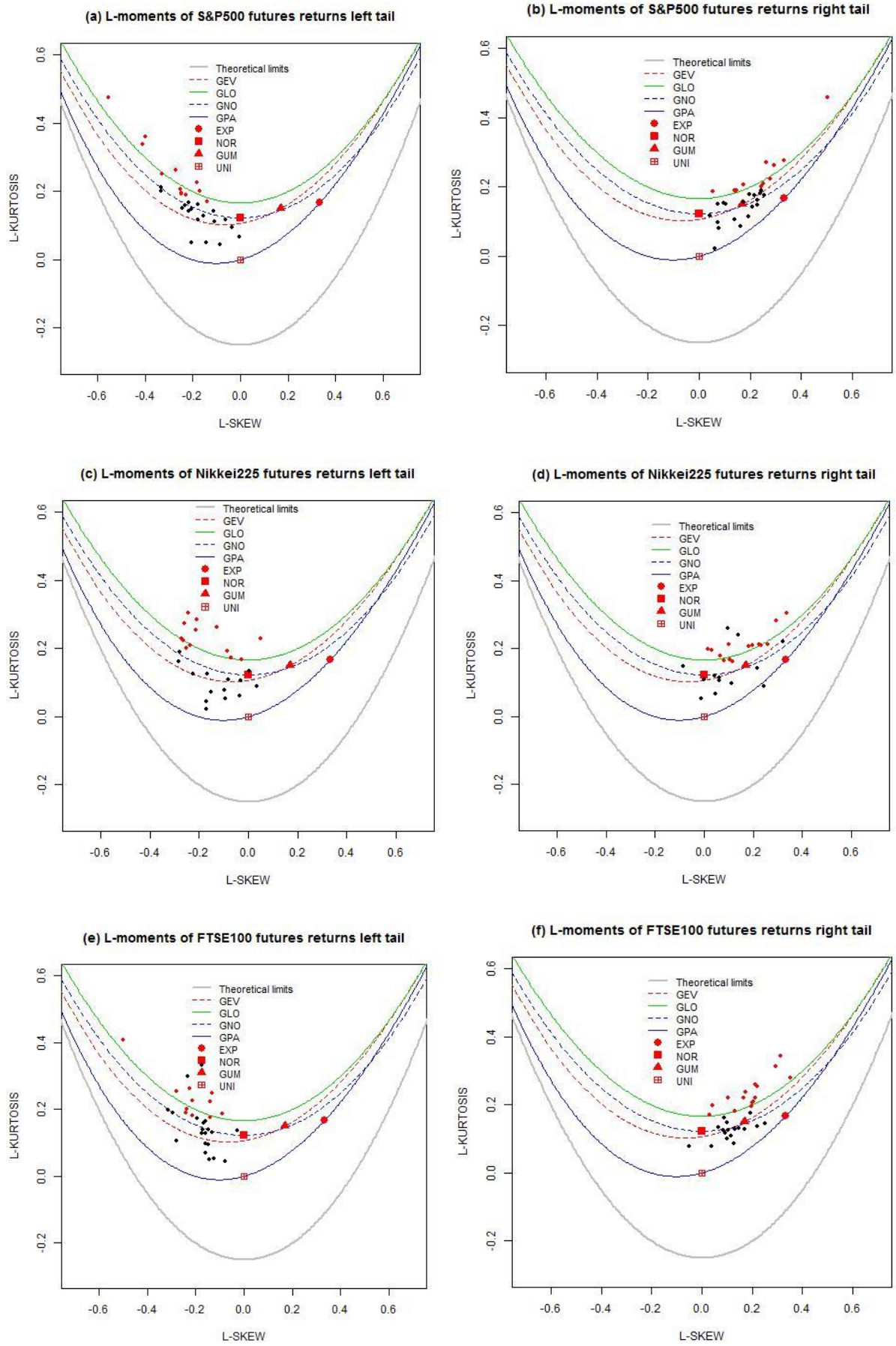
Figure 3. The ratio of L-skewness and L-kurtosis of index futures return

4.3 The patterns of L-moments and the tail distributions in financial crisis periods

It is known that skewness and kurtosis dominate the thickness and height of the return distribution, which play critical role in risk management. The patterns of L-skewness (tau 3) and L-kurtosis (tau 4) of the equity indices and the corresponding futures returns are shown in Figures 4 and 5. The evidence shows that there are some differences between equity index and equity index futures. In the case of equity indices, on average, the L-skewness in left tail are more stable than the ones in the right tails. The patterns of equity index L-skewness (in blue) of maximum returns (right tails) in Figure 4 show that there are three main peaks around 1987, 1997, and 2009 corresponding to three critical financial crises. Similar patterns are also found in the minimum returns (left tails), especially the historical low in 1987. The large L-skewness in right tails and the small L-skewness in left tails in the crisis period indicate that the tail distributions are driven more far away from the center of the distribution. In other words, the three equity indices have longer tail in the period of financial crises.

The mean of L-skewness in Table 4 shows that the absolute values of L-skewness in left tail are larger than the ones in right tails. Specifically, the equity indices have longer left tail and riskier than the right tail, and the left tails are stable compared with the right tails. The significant differences between minimum and maximum returns show investors' asymmetric reactions to the good and bad news. In the part of L-kurtosis (in red), the L-kurtosis are smaller and stable in left tails than the ones in right tails. Only the effects of financial crisis happened in 1987 are reflected in the both left and right patterns of L-kurtosis. Surprisingly, little effect of the crises

happened in 1997 and in 2008 could be found in the L-kurtosis patterns. Possible explanation is that L-kurtosis describes the shape of body distribution rather than the tail distribution.

Figure 5 describes L-skewness and L-kurtosis of the three equity index futures. According to the value of L-skewness, the left tails are generally longer than right tails, which is consistent with the ones of equity indices. In addition, L-skewness in left tails are more stable, except of Nikkei 225 index futures. Contrary to L-skewness, L-kurtosis in the right tails are larger and stable than the ones in left tails. Comparing the results of equity indices and futures indices in Figure 4, 5, and Table 4, the tail distributions of the equity indices and the index futures returns change overtime, and it implies tail distributions are not stable across market and time periods. Further, the equity indices tend to have longer left tails, implying they are riskier than the index futures. The results also support some solid evidence that the investors could use futures contracts to hedge or manage the market risks of their spot positions.

Market participants may be more interested of identifying the tail distributions of index and index futures returns in the years of financial crises happened in 1987, 1997, and 2007 to 2009. The results of AD tests displayed in Table 5 show that the left tails of the equity indices in the crisis periods tend to be GEV distribution, especially in 1987. However, the maximum returns present GLO distribution. For example, maximum returns of FTSE 100 in 1997 and 2007 to 2009 tend to follow GLO distribution. In contrast, the tail distributions of index futures returns are quite diversified. Around 1987, only the left tails of S&P 500 and FTSE 100 are identified. In 1997, both maximum and minimum returns of Nikkei 225 follow GEV distribution, and both

minimum returns of FTSE 100 and S&P 500 index futures tend to be GNO distribution. Briefly, maximum returns of Nikkei 225 and FTSE 100 index futures in 2007 to 2009 tend to be generalized logistic distribution. However, it is not consistent with each other in the case of minimum returns in 2007 to 2009.

From section 4.1 to 4.3, tail distributions of the equity indices and the corresponding futures indices are examined from various perspectives. However, the results are not entirely consistent with previous research. For example, Longin (1996) clarified the asymptotic distribution of stock return as a Fréchet distribution based on a single parameter. Harris and Küçüközmen (2001) examined the distributions of equity index returns of UK and US, and they argued that the skewed generalized t distribution fits better than other distributions. They focused on the whole distribution of equity index return rather than tail distribution. Furthermore, Gettinby et al. (2004), Gettinby et al. (2006), Tolikas (2008), Tolikas and Gettinby (2009) strongly stated that tail distributions of UK, US, Japan, and Singapore equity index returns are GLO distribution. However, it is known that the stock market changes by time and thus the tail distribution of equity index returns seem not likely staying in a specific distribution[7]. On the other hand, investors' asymmetric responses to good and bad news in the stock market might lead the differences between left- and right-tails. Another reason causing the results in this paper different from previous research may attribute to the sample period.

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[7] Many studies focus on the characteristics of return distribution (for example, return volatility) changes over time (Bollerslev 1986; Schwert 1989; Engle 2002; Justiniano & Primiceri 2008). As known, if one of the four moments changes over time, then it implies the distributions of assets returns change over time as well.
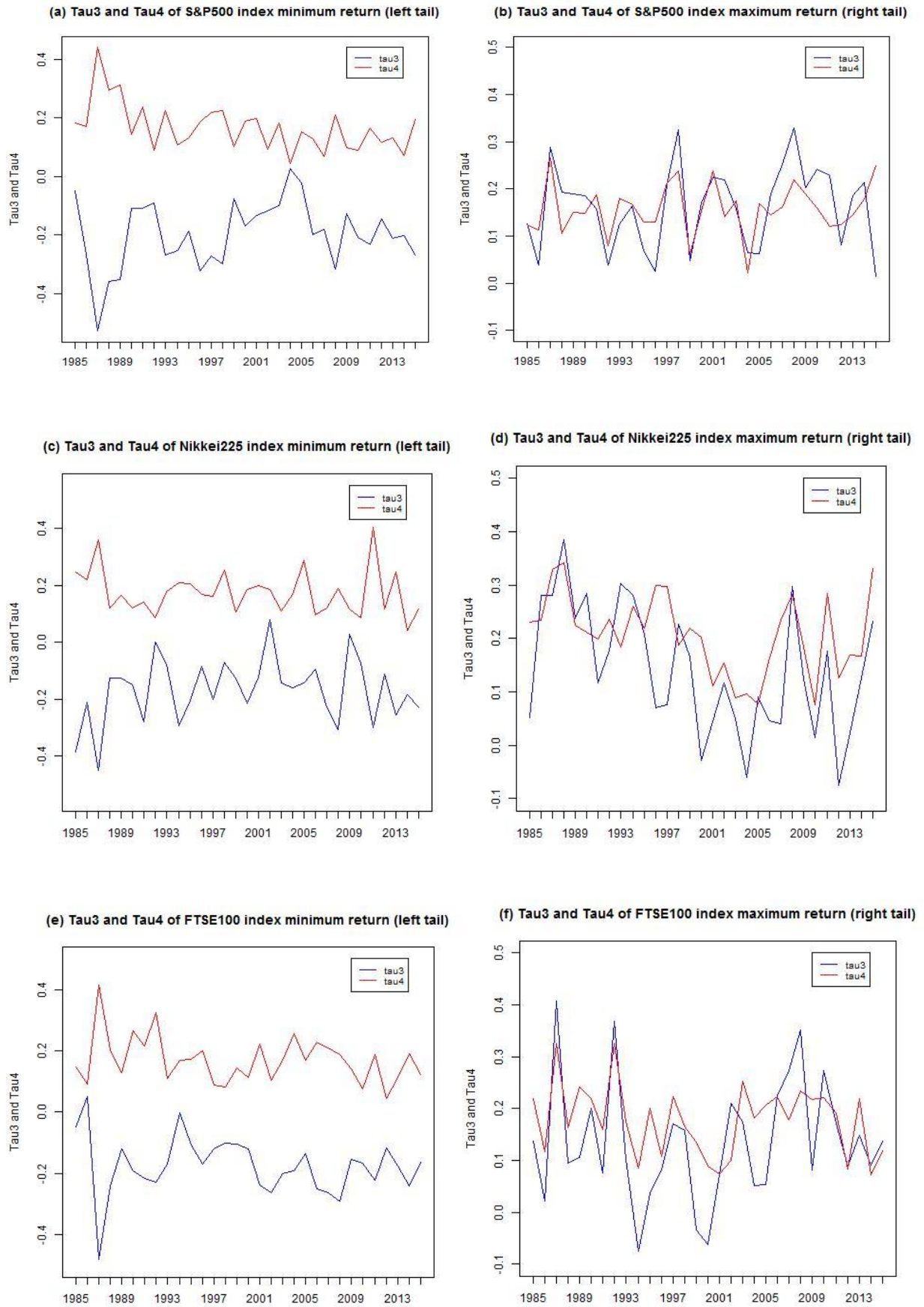
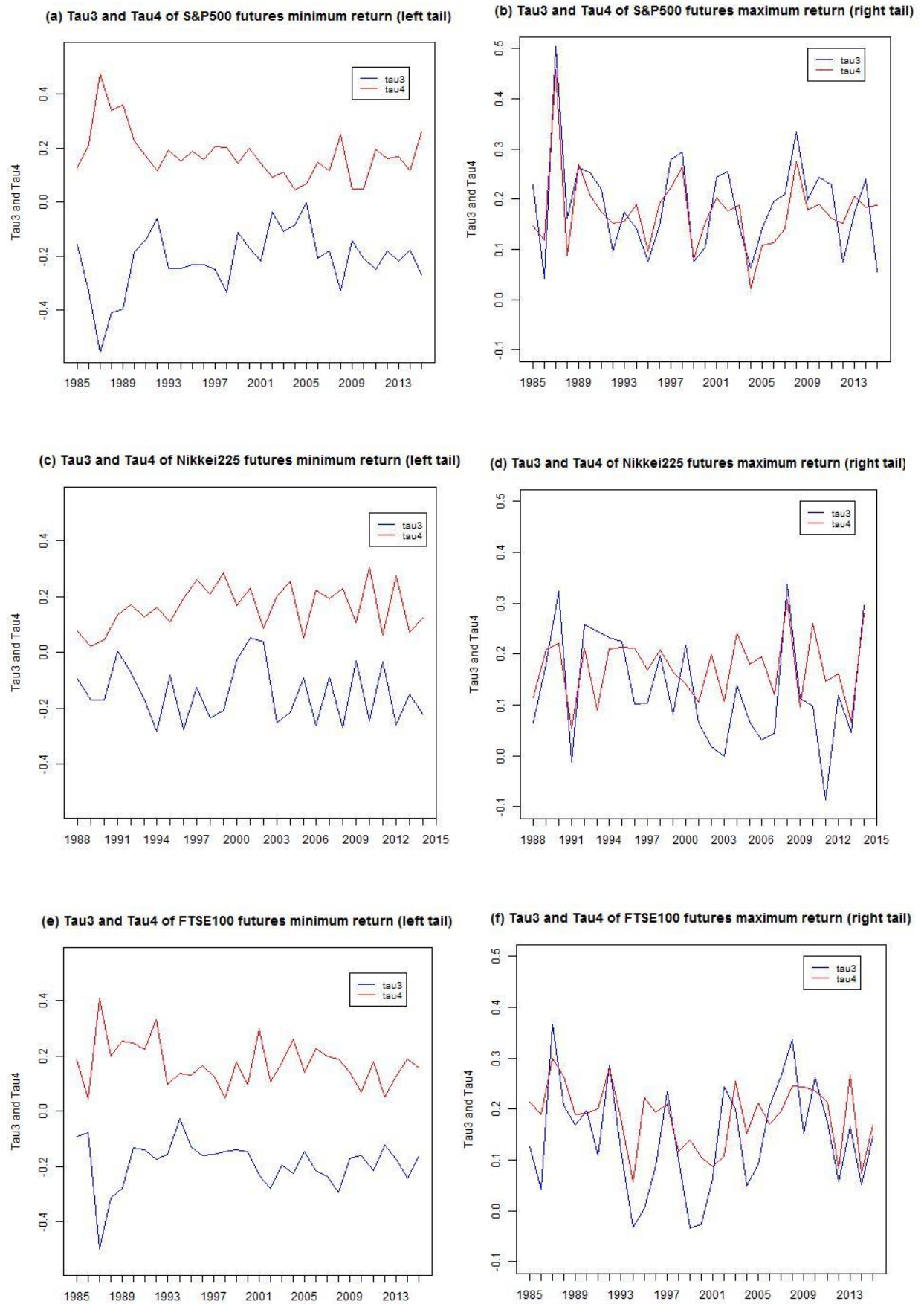Figure 4. The ratio of L-skewness and L-kurtosis of equity index return

Figure 5. The ratio of L-skewness and L-kurtosis of index futures return

Table 4. Mean and scale of L-skewness and L-kurtosis of indices and futures indices returns

	Equity index		Equity index futures	
	Left tail	Right tail	Left tail	Right tail
L-skewness				
S&P 500	-0.2184	0.1681	-0.2022	0.1862
	(0.1191)	(0.0891)	(0.0863)	(0.0699)
Nikkei 225	-0.2053	0.1535	-0.1656	0.1531
	(0.0948)	(0.1082)	(0.0975)	(0.1168)
FTSE 100	-0.1771	0.1356	-0.1674	0.1446
	(0.0951)	(0.1038)	(0.0577)	(0.0882)
L-kurtosis				
S&P 500	0.1807	0.1645	0.1561	0.1601
	(0.0892)	(0.0583)	(0.0635)	(0.0523)
Nikkei 225	0.1695	0.1801	0.1436	0.1716
	(0.0891)	(0.0661)	(0.0880)	(0.0772)
FTSE 100	0.1565	0.1753	0.1493	0.1663
	(0.0843)	(0.0804)	(0.0666)	(0.0669)

Table 5. Identifications of tail distributions in the periods of financial crises

	Equity index		Equity index futures	
	Left tail	Right tail	Left tail	Right tail
1987				
S&P 500				
GEV	**0.8413**	0.7769	0.8014	0.0000
GLO	0.6769	**0.9157**	**0.8450**	0.0000
GNO	0.0000	0.6366	0.0000	0.0000
Nikkei 225				
GEV	**0.9563**	0.0128	-	-
GLO	0.9319	0.0000	-	-
GNO	0.7059	0.0000	-	-
FTSE 100				
GEV	**0.3007**	0.0102	**0.8013**	0.0103
GLO	0.0000	0.0000	0.0000	0.0113
GNO	0.0000	0.0000	0.0000	0.0308
1997				
S&P 500				
GEV	0.9913	0.9398	0.9588	0.9277
GLO	**0.9963**	**0.9918**	0.8875	**0.9708**
GNO	0.8035	0.8677	**0.9612**	0.5954
Nikkei 225				
GEV	0.8639	**0.9739**	**0.8484**	**0.9901**
GLO	**0.8979**	0.9466	0.8466	0.9726
GNO	0.8601	0.9468	0.7739	0.9564
FTSE 100				
GEV	0.9375	0.9498	0.7974	**0.9885**
GLO	0.7343	**0.9939**	0.5354	0.9764
GNO	**0.9533**	0.9235	**0.8473**	0.9863
2007-2009				
S&P 500				
GEV	0.7588	**0.5769**	0.8104	**0.3138**
GLO	0.6049	0.0000	0.7130	0.0000
GNO	**0.8351**	0.0000	**0.8420**	0.0000
Nikkei 225				
GEV	**0.8584**	0.7971	0.9185	0.8792
GLO	0.7953	**0.9697**	**0.9290**	**0.9896**
GNO	0.8373	0.4961	0.8558	0.0000
FTSE 100				
GEV	**0.9722**	0.9234	**0.9482**	0.6052
GLO	0.8339	**0.9960**	0.8552	**0.8732**
GNO	0.9625	0.0000	0.8104	0.0000

Note: The numbers in this table are the p-values of AD tests.

4.4 Value at risk of equity index and futures returns

The market changes over time, the tail distribution and the risks of the equity indices and the corresponding futures indices might be changed every year accordingly. This section goes further to test the annual tail distributions of equity indices and futures indices, and the best-fitting tail distribution will be applied to measure value at risk of the equity indices and futures indices. The extreme returns of the equity index and futures index returns in each year are collected to fit the annul tail distributions. The results are exhibited in Table 6, showing the tail distributions may actually change according to the market conditions. Even the results in Table 6 seem obscure; however, some critical information are revealed from the results. Generally, the minimum (maximum) returns of S&P 500 index more likely follow GNO (GEV) distribution, but FTSE 100 index has a consistent distribution (GLO) in both left and right tails. The minimum returns of Nikkei 225 index are obscure, and obviously, the maximum returns strongly follow GLO distribution.

In the equity index futures column, most of equity index futures returns tend to follow GLO distribution. Some extreme returns would be more likely GNO or GEV distribution such as the minimum returns of S&P 500 and FTSE 100 index futures. Particularly, the maximum futures returns of FTSE 100 significantly follow GLO distribution. Generally, the results of index futures are partially consistent with Gettinby *et al.* (2006), especially in the tails of Nikkei 225 index futures.

As looking at the period of different crises, some interesting information could be found. The

index tail distributions of minimum returns in 1987 tend to be GEV distribution, and the

maximum returns could be seen as a GLO distribution (S&P 500 and FTSE 100). However, both

tail distributions of index futures return are diversified. In the period of crisis in 1997, the tail

distributions of index returns are as similar as their corresponding index futures returns. For

example, left (right) tail of S&P 500 returns is GEV (GLO) distribution, and the same

distributions could be found in S&P 500 index futures returns. Both of Nikkei 225 index and

index futures returns are GEV distribution. To FTSE 100 index and its index futures, the

minimum returns of FTSE 100 and its minimum futures returns are GNO distribution, and the

maximum returns of FTSE 100 index and FTSE 100 index futures are GLO distribution. The

results imply that the investors' behaviours in stock and futures markets are consistent with each

other. However, tail distributions of the three-pair indices around 2007 to 2009 are obscure and

difficult to explain. This phenomenon could be attributed the complicated natures and causes of

the financial crisis around 2007 to 2009.

As pointed out by previous research, the distribution of asset returns is the core of the

calculation of risks. After we obtain the tail distributions, they would be applied to measure value

at risk of the financial indices. Figure 6.[8] shows the patterns of value at risk with 1% probability

of the indices and index futures returns. Obviously, using best fitting tail distribution to measure

market risks of the indices and the corresponding index futures returns really captures some

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[8] The estimated VaRs are based on the best-fitting tail distributions, and the estimation of parameters converge at the setting criteria, 10^{-5}. Thus, the peaks and feet in Figure 6 reflect the real market conditions.
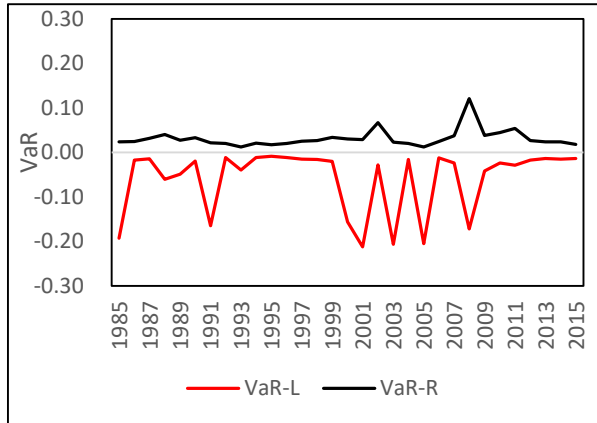
fluctuations caused by financial crises. On average, the risks of index futures are systematically smaller than equity index returns. It implies that it is reasonable using futures contracts to hedge investors' spot positions. Further, the risks in left tails (in red) of equity index returns are significantly larger than the ones in the right tails (in blue), indicating that the risks of long positions are greater. Unexpectedly, the Japanese equity market as well as its futures market has the largest market risks among the three pairs of financial returns. In addition, equity index in Japan has more risks than futures market, which means that it is not appropriate to use futures contracts to manage the risks from stock market. In the case of British market, equity and futures market have less impacts from the financial crisis around 2008. In sum, the best-fitting tail distribution could be used to measure market risks of stocks and futures returns. Furthermore, this method also appropriately reflects the market conditions.

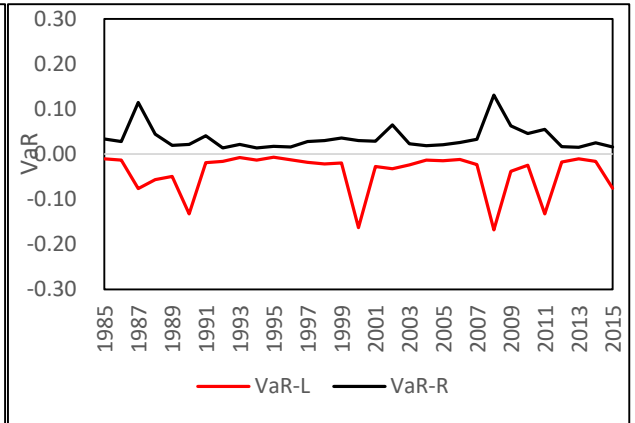Table 6. Annual tail distributions of equity indices and index futures

	Equity index						Equity index futures					
	S&P 500		**Nikkei 225**		**FTSE 100**		**S&P 500**		**Nikkei 225**		**FTSE 100**	
	Left	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left	Right
1985	GLO	GEV	GEV	GLO	GNO	GLO	GEV	GEV			GLO	GLO
1986	GNO	GEV	GLO	GEV	GEV	GNO	GEV	GEV			GEV	GLO
1987	GEV	GLO	GEV	GEV	GEV	GLO	GLO	GEV			GEV	GNO
1988	GLO	GNO	GEV	GLO	GLO	GLO	GLO	GNO	GEV	GNO	GEV	GLO
1989	GLO	GEV	GLO	GLO	GNO	GLO	GLO	GLO	GNO	GEV	GLO	GLO
1990	GEV	GNO	GEV	GEV	GLO	GLO	GLO	GLO	GNO	GEV	GLO	GLO
1991	GLO	GLO	GNO	GLO	GLO	GLO	GNO	GEV	GLO	GEV	GLO	GLO
1992	GEV	GEV	GEV	GLO	GEV	GEV	GNO	GLO	GLO	GLO	GEV	GEV
1993	GLO	GLO	GLO	GEV	GEV	GLO	GEV	GEV	GEV	GNO	GNO	GLO
1994	GNO	GEV	GEV	GLO	GLO	GNO	GNO	GLO	GNO	GLO	GNO	GEV
1995	GNO	GNO	GLO	GLO	GLO	GLO	GEV	GNO	GEV	GLO	GEV	GLO
1996	GNO	GNO	GLO	GLO	GLO	GNO	GEV	GLO	GEV	GLO	GNO	GLO
1997	GEV	GLO	GEV	GEV	GNO	GLO	GEV	GLO	GEV	GEV	GNO	GLO
1998	GEV	GLO	GLO	GEV	GEV	GLO	GNO	GLO	GLO	GLO	GNO	GNO
1999	GEV	GEV	GEV	GLO	GLO	GNO	GEV	GEV	GLO	GLO	GLO	GNO
2000	GLO	GLO	GNO	GLO	GNO	GEV	GLO	GLO	GLO	GNO	GNO	GEV
2001	GLO	GLO	GLO	GNO	GLO	GEV	GNO	GLO	GEV	GEV	GLO	GEV
2002	GEV	GEV	GLO	GLO	GNO	GNO	GEV	GNO	GNO	GLO	GNO	GNO
2003	GLO	GLO	GNO	GEV	GEV	GLO	GNO	GLO	GLO	GEV	GEV	GLO
2004	GEV	GEV	GNO	GNO	GLO	GLO	GEV	GEV	GLO	GLO	GLO	GLO
2005	GLO	GLO	GLO	GEV	GLO	GLO	GEV	GNO	GEV	GLO	GEV	GLO
2006	GNO	GNO	GEV	GLO	GLO	GLO	GNO	GNO	GLO	GLO	GLO	GEV
2007	GNO	GEV	GNO	GLO	GLO	GEV	GNO	GNO	GLO	GEV	GLO	GLO
2008	GLO	GEV	GEV	GLO	GEV	GEV	GLO	GEV	GLO	GLO	GNO	GLO
2009	GNO	GLO	GNO	GLO	GNO	GLO	GNO	GEV	GEV	GEV	GNO	GLO
2010	GNO	GNO	GEV	GEV	GNO	GEV	GNO	GEV	GLO	GLO	GNO	GLO
2011	GNO	GNO	GLO	GLO	GEV	GLO	GLO	GEV	GEV	GLO	GEV	GLO
2012	GNO	GEV	GNO	GEV	GEV	GEV	GNO	GLO	GLO	GLO	GEV	GEV
2013	GNO	GNO	GLO	GLO	GNO	GLO	GEV	GLO	GNO	GEV	GNO	GLO
2014	GNO	GEV	GNO	GLO	GLO	GEV	GNO	GNO	GNO	GEV	GEV	GEV
2015	GEV	GLO	GNO	GLO	GNO	GEV	GLO	GLO	GNO	GEV	GEV	GLO
GNO	**12**	8	9	2	9	5	**12**	7	7	3	11	4
GEV	9	**12**	11	10	9	9	11	12	9	10	11	7
GLO	10	11	11	**19**	**13**	**17**	8	12	**12**	**15**	9	**20**
$\overline{\text{VaR}}$	0.0692	0.0310	0.0913	0.0343	0.0472	0.0279	0.0411	0.0350	0.0988	0.0385	0.0478	0.0259
Scale	0.1047	0.0203	0.1127	0.0189	0.0427	0.0211	0.0462	0.0272	0.1021	0.0196	0.0484	0.0132
Max.	0.5053	0.1209	0.4102	0.0956	0.2108	0.1185	0.1684	0.1307	0.4055	0.0877	0.1689	0.0652
Min.	0.0093	0.0118	0.0071	0.0095	0.0098	0.0098	0.0071	0.0133	0.0087	0.0137	0.0092	0.0036

Note: In this table, GNO, GEV, and GLO are abbreviation of generalized normal distribution, generalized extreme value distribution, and generalized logistic distribution. $\overline{\text{VaR}}$ is the mean of annual value at risk, and all of the $\overline{\text{VaR}}$ are significantly different from zero. For convenience of understanding, the VaR numbers are shown in positive format. Theoretically, the VaR numbers in left tails should be negative.
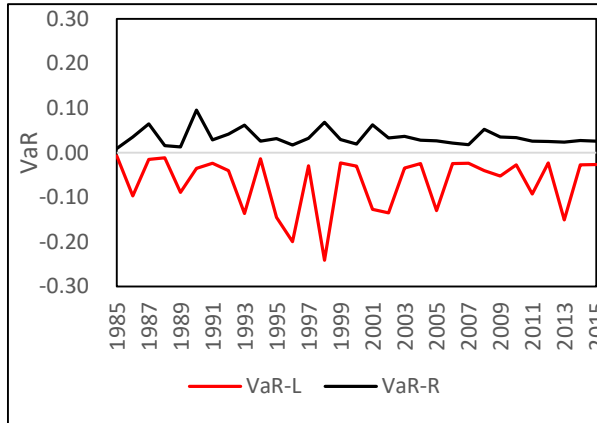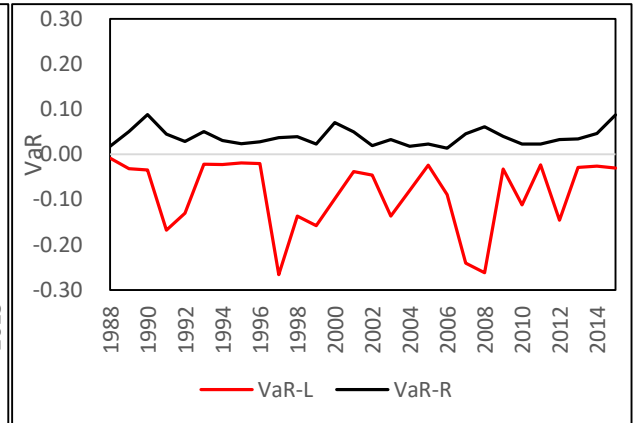
(a) VaRs of S&P 500 index returns
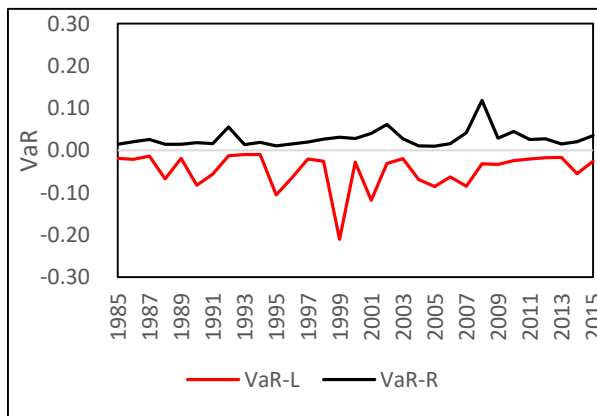
(b) VaR of S&P 500 index futures returns

(c) VaRs of Nikkei 225 index returns

(d) VaRs of Nikkei 225 index futures returns

(e) VaRs of FTSE 100 index returns

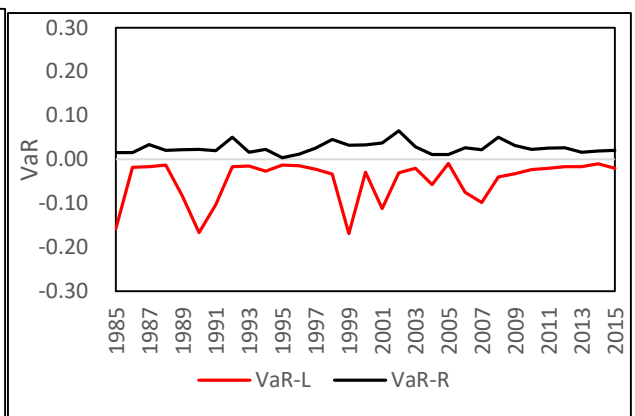(f) VaRs of FTSE 100 index futures returns

Figure 6. The patterns of annual VaRs of equity indices and index futures returns

5. Conclusion

In this study, the characteristics of tail distribution of three major equity indices, as well as the corresponding index futures, have been discovered. It is known that the tail distribution plays a critical role in risk measuring and pricing modelling. In the literature, many attentions have been paid on the tail distribution of equity index. However, less research focused on the tail distribution of index futures. According to Jenkinson (1955), tail distribution would follow GEV distribution. Nerveless, some empirical studies provided the entirely different results. Thus, it is worthy to identify the tail distribution of equity index and equity index futures.

This paper goes further, not only tail distributions of index returns, but also futures returns are also examined. Three major equity indices and the corresponding index futures are examined in this paper. The method of L-moments is applied to estimate the parameters of the tail distributions. Our results are quite different from Gettinby et al. (2004, 2006), Tolikas (2008) and Tolikas and Gettinby (2009), who argued that the tail distribution of equity index is GLO distribution. Obviously, the discrepancies exist in our results but some patterns could be found in both the minimum and maximum returns, especially in the period of financial crises. General speaking, the tail distributions of equity indices switch over GEV distribution and GLO distribution. The extreme returns of index futures tend to follow GLO distribution, particularly the maximum returns. The evidences also show that the minimum returns of both S&P 500 index and it futures are significantly GNO distribution. Among the three regions, the maximum returns between

indices and index futures are quite consistent with each other, implying using futures to hedge investors' spot positions works. The similar pattern could be found in the left tails of the three pairs of sequences.

Furthermore, our results also point out that the tail distributions of minimum and maximum returns are different, which implies the asymmetric effects of investors' behavior. Generally, higher L-kurtosis observations follow GLO distribution, and some lower L-kurtosis ones (S&P 500 and Nikkei 225 index returns) tend to be GEV distribution. In the part of index futures, both minimum and maximum returns with low L-kurtosis have a tendency of GNO distribution, but the majority of minimum and maximum returns with high L-kurtosis follow GLO distribution. In the final section, annual tail distribution is tested, and the overall results of the tails consistently tend to follow GLO distribution.

In the final, some potentials could be extended further. For example, the differences among GEV, GNO, and GLO are still obscure, and the causes of distribution transition could be the further issues in this area. In addition, the transition probability of the distribution could be modelled; however, it is not the core in this paper.

Appendix. Tail distributions and corresponding VaRs

1. Generalized Extreme Value Distribution (GEV)

Probability density function:

$$F(x) = \exp\left\{ -\left[1 + \xi\left(\frac{x-\mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\} \tag{A.1}$$

2. Generalized Logistic Distribution (GLO)

Probability density function:

$$F(x) = \frac{\xi \exp\left(-\frac{x-\mu}{\sigma} \right)}{\sigma\left[1 + \exp\left(-\frac{x-\mu}{\sigma} \right) \right]^{1+\xi}} \tag{A.2}$$

3. Generalized Normal Distribution (GNO)

Probability density function:

$$F(x) = \Phi(y)$$
$$\text{where} \quad y = -\xi^{-1} \log\left[1 - \xi\frac{(x-\mu)}{\sigma} \right] \tag{A.3}$$

And $\Phi(y)$ is the distribution function of the standard normal distribution.

■

From (A1) to (A3), μ is location parameter, σ is scale parameter, and ξ is shape parameter. VaR based on the three distribution could be obtained by inverting the density with a given probability p.

$$\text{VaR}(x; p) = F_x^{-1}(p) \tag{A4}$$

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