# Illiquidity and Stock Returns – II: Cross-section and Time-series Effects

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**Abstract**
Lou and Shu (*RFS*, 2017) decompose Amihud's (2002) illiquidity measure (*ILLIQ)* proposing that its component, the average of inverse dollar trading volume (*IDVOL)*, is sufficient to explain the pricing of illiquidity. Their decomposition misses a component of *ILLIQ* that is related to illiquidity and we find that it affects stock returns significantly both in the cross-section and in time-series. We show that the *ILLIQ* premium is significantly positive after controlling for mispricing, sentiment, and seasonality. In addition, the aggregate market *ILLIQ* outperforms market *IDVOL* in estimating the effect of market illiquidity shocks on realized stock returns.

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## 1. Introduction

In a recent article of the *Review of Financial Studies*, Lou and Shu (2017, henceforth LS) analyze Amihud's (2002) illiquidity measure, *ILLIQ*, the average daily ratio of absolute return to the dollar trading volume and its effect on asset prices. Decomposing *ILLIQ*, LS argue that one of its components denoted *IDVOL*, the average inverse daily dollar volume, is sufficient to explain the *ILLIQ* effect on the cross-section of expected returns. LS also conjecture that the pricing of *ILLIQ* or *IDVOL* does not reflect compensation for illiquidity but it is rather due to mispricing and sentiment, and that its premium is seasonal.

We show that LS's decomposition of *ILLIQ* misses an illiquidity-related component which significantly affects both the cross-section of expected stock returns and the time-series of realized returns in addition to the effects of *IDVOL*. In a "horse race" between *ILLIQ* and *IDVOL* we find that the information in *ILLIQ* is positively priced after controlling for *IDVOL*.[1] Further, the effect of *ILLIQ* on the cross-section of expected return is positive and significant after controlling for mispricing and market sentiment, as well as for seasonality.

*ILLIQ* is a general proxy measure of illiquidity costs which includes *both* the price impact cost and the fixed cost of trading (see Amihud (2002)).[2] Illiquidity, which is multi-dimensional, has been proxied by a number of variables. *ILLIQ* produces *consistent* effects on asset prices both in the cross-section of expected return and in the time-series of realized return and in that it is superior to *IDVOL*. We find that while both measures have a similar effect on expected returns across stocks, *mILLIQ* (aggregate market *ILLIQ*) outperforms *mIDVOL* (aggregate market *IDVOL*) in estimating the time-series effects of market illiquidity shocks on realized returns.

### 1.1. Liquidity and volume

LS find that ln*IDVOL*, natural log of *IDVOL*, has a positive effect on the cross-section of expected returns which is similar to that of ln*ILLIQ*. This is correct. Brennan et al. (1998), Datar et al. (1998), and Chordia et al. (2001) found significant negative pricing of ln*DVOL* (log dollar trading volume) and turnover, and Amihud et al. (2015, p. 357) found that the illiquidity premium on stocks is similar when using *ILLIQ* or *DVOL*.

[1] This is consistent with the evidence in Barardehi et al. (2019) that the absolute return included in *ILLIQ* is important in explaining the cross-section of expected return.
[2] See supporting evidence in Lesmond (2005), Hasbrouck (2009), and Goyenko, Holden and Trzcinka (2009). The last two find that the correlation of *ILLIQ* with the bid-ask spread is similar to that with the price impact.

LS distinguish between volume premium and illiquidity premium. Yet theory suggests that volume is negatively associated with trading costs (Amihud and Mendelson (1986, hereafter AM) and Constantinides (1986))[3] and there is strong empirical support for the negative relation between illiquidity cost and trading volume.[4] Regarding the direction of causality, evidence shows that exogenous liquidity improvements raise trading volume.[5]

## *2.* Decomposing Amihud's Illiquidity Measure

We decompose *ILLIQ* and show that the term missing in LS's analysis presents aspects of illiquidity. *ILLIQ* is the average of $illiq_d$, the daily value of the ratio of absolute daily return $|r_d|$ to dollar trading volume $dvol_d$ in a given period,

$$ILLIQ = (\overline{illiq_d}) = (\overline{|r_d|/dvol_d}), \tag{1}$$

where the superbar indicates the average. The expected value of daily $illiq_d$ is

$$E(illiq_d) = E[|r_d|\cdot(1/dvol_d)] = E(|r_d|)\cdot E(1/dvol_d) + cov(|r_d|, 1/dvol_d)\ . \tag{2}$$

Using the average as an estimator of expected value, we have

$$\ln ILLIQ = \ln[\overline{|r_d|}\cdot\overline{1/dvol_d} + cov(|r_d|, 1/dvol_d)]. \tag{3}$$

LS propose the following decomposition of ln*ILLIQ*,

$$\ln ILLIQ = \ln(\overline{|r_d|}) + \ln(\overline{1/dvol_d})\ , \tag{4}$$

which is accurate only if $cov(|r_d|, 1/dvol_d) = 0$.[6] Denoting LS's illiquidity measure by

$$LSIlliq = \overline{|r_d|}\cdot\overline{1/dvol_d}\ , \tag{5}$$

we have

$$ILLIQ = LSIlliq + cov(|r_d|, 1/dvol_d). \tag{6}$$

We expect that $cov(|r_d|, 1/dvol_d) < 0$ given Karpoff's (1987) finding that $cov(|r_d|, dvol_d) > 0$.

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[3] In AM (Proposition 1), more liquid assets are held in equilibrium by investors who trade them more often, resulting in a positive liquidity-trading volume relation. They support this prediction by empirical evidence. Constantinides's (1986) model predicts lower trading frequency in assets that are more costly to trade.

[4] There is evidence that stocks with higher bid-ask spread have lower trading turnover (Atkins and Dyl (1997)) and that higher illiquidity reduces trading frequency by individual investor (Dias and Ferreira (2004), Naes and Odegaard (2009), Anginer (2010), and Uno and Kamiyama (2010)).

[5] Amihud et al. (1997) find an exogenous increase in stock liquidity generates an increase in trading volume and a decline in illiquidity measured by volatility-to-volume ratio, which is closely related to *ILLIQ*. Similar findings appear in Muscarella and Piwowar (2001) and Kalay et al. (2002). Amihud et al. (1999) find that trading volume increases for stocks whose liquidity improves. Leuz and Verrecchia (2000) find an increase in trading volume and a decline in the bid-ask spreads for firms whose accounting reports became more informative thus reducing asymmetric information and enhancing liquidity.

[6] Similarly, LS's equations (5)-(7) write the mean of the ratio of two random variables as the ratio of the means of these variables. This is accurate only if the variables are uncorrelated, which is not the case there.

We employ a first-order Taylor-series expansion of cov($|r_d|$, $1/dvol_d$) and obtain the following approximation of *ILLIQ* as the sum of *LSIlliq* and a missing term (details are provided in Appendix Analysis A1):

$$ILLIQ \approx LSIlliq - b * CV^2, \qquad (7)$$

where $b$ is the slope coefficient from a regression of $|r_d|$ on $dvol_d$ (and a constant) and $CV$ is the coefficient of variation of $dvol_d$. We expect that $b > 0$ given Karpoff's (1987) findings. Theoretically, $b$ indicates the extent of association between order flow and price movement of the same sign thus being coarsely related to Kyle's (1985) λ. Kim and Verrecchia (1994) theorize that arrival of new information raises illiquidity because of increased diversity of opinion among information processors which increases the positive relation between absolute return and trading volume captured by λ. This suggests a positive association between $b$ and illiquidity for which we provide empirical support below. $CV^2$, which is naturally positive, is known to negatively affect expected return across stocks; see Chordia et al. (2001). Pereira and Zhang (2010) theorize that required return declines in *CV* since higher *CV* provides more opportunities for investors to save on trading costs by timing their trades to high-liquidity periods. Thus, expected returns should be increasing in $-b * CV^2$.[7]

Finally, we define *DIF* as the difference between ln*ILLIQ* and ln*LSIlliq*:

$$DIF = \ln ILLIQ - \ln LSIlliq = \ln(\overline{illiq_d}) - [\ln(\overline{|r_d|}) + \ln(\overline{1/dvol_d})]$$

$$\approx \ln(1 - b * CV^2 / LSIlliq). \qquad (8)$$

*DIF* increases in $-b * CV^2$ and it is expected to have a positive effect on expected return.

We estimate the relations between $DIF_{j,s}$ and the other variables, all calculated for each stock $j$ from daily return and volume data over a twelve-month period that ends in month $s$.[8] $ILLIQ_{j,s}$, $|R_{j,s}|$, and $IDVOL_{j,s}$ are, respectively, the average of daily values of $illiq_{j,d,s} = |r_{j,d,s}|/dvol_{j,d,s}$, $|r_{j,d,s}|$, and $1/dvol_{j,d,s}$ ($dvol_{j,d,s}$ is in millions). $DIF_{j,s} = \ln ILLIQ_{j,s} - [\ln|R_{j,s}| + \ln IDVOL_{j,s}]$, $CV_{j,s}$ is the coefficient of variation of $dvol_{j,d,s}$, and $b_{j,s}$ is the slope coefficient from a

[7] Following Harris and Raviv (1993), cov($|r_d|$, $dvol_d$) > 0 may reflect the difference of opinions which by Diether, Malloy, and Scherbina (2002) has negative effect on expected return. This would imply that cov($|r_d|$, $1/dvol_d$) positively affects expected return. We do an indirect test of whether the effect of this term on expected return is related to difference of opinions, which is affected by short-sales constraint. We find that the pricing of $DIF_{j,s}$ is unaffected by whether stocks have high or low (above or below median) institutional holdings which, by Nagel (2005) indicate the ease of short selling.

[8] The twelve-month estimation period follows Amihud (2002). We require return and volume data for at least 200 trading days in that period.

regression of $|r_{j,d,s}|$ on $dvol_{j,d,s}$ (and a constant). The variables are constructed over the years 1955-2016 (744 months).[9] Summary statistics of *ILLIQ*-related variables are in Appendix Table A.1. In each month *s*, we calculate the cross-sectional means, standard deviations, and pairwise correlations and then average them over all 744 months. We find that average $DIF_{j,s}$ is negative since $\text{cov}(|r_{j,d,s}|, 1/dvol_{j,d,s}) \approx - b_{j,s} * CV_{j,s}^2$ is negative (see Equation (7) above), and $DIF_{j,s}$ is negatively correlated with $\ln LSIlliq_{j,s}$ (see Equation (8)). The average monthly cross-stock mean of $CV_{j,s}^2$ is 1.447 and that of $b_{j,s}$ is 0.043. Only 1.1% of the estimated $b_{j,s}$ values are negative, consistent with the theory and empirical evidence that $b_{j,s} > 0$. Both $b_{j,s}$ and $CV_{j,s}^2$ are positively related to illiquidity measured by $LSIlliq_{j,s}$, which does not include these terms. The average monthly cross-stock correlations of $\ln b_{j,s}$ and $\ln CV_{j,s}^2$ with $\ln LSIlliq_{j,s}$ are 0.87 and 0.62, respectively.

Following (8), we estimate monthly cross-stock regressions of $DIF_{j,s}$ on its component variables $\ln CV_{j,s}^2$, $\ln b_{j,s}$, and $\ln LSIlliq_{j,s}$ and find that the average $R^2$ is 0.45 and 0.73 in regressions with and without an intercept, respectively. This implies a high correlation between $DIF_{j,s}$ and a linear combination of its component variables. All three component variables of $DIF_{j,s}$ have highly significant coefficients. When including only $\ln CV_{j,s}^2$ and $\ln b_{j,s}$ (without $\ln LSIlliq_{j,s}$), the average $R^2$ is 0.40 and 0.67 for cross-stock regressions with and without an intercept, respectively, suggesting that $DIF_{j,s}$ reflects illiquidity-related information mainly through $\ln CV_{j,s}^2$ and $\ln b_{j,s}$. (Additional analysis is presented in Appendix Analysis A2.)

## 3. Cross-sectional Analyses

### 3.1. Cross-sectional effects of illiquidity on expected return

We test the cross-sectional effects on expected return of *ILLIQ*, $|R|$, *IDVOL*, and *DIF* by estimating monthly cross-sectional regressions of stock returns on these variables and on commonly used control variables, employing Fama and Macbeth's (1973) method. Similar to LS, we use the natural logarithms of these variables. We estimate the following model:

$$(R_j\text{-}rf)_s = b0_s + b1_s{}'*IL_{j,s-2} + b2_s*Size_{j,s-2} + b3_s*BM_{j,y-1} + b4_s*R12lag_{j,s-2}$$

[9] We include NYSE\AMEX common stocks (codes of 10 or 11) with average price between $5 and $1000 over the twelve-month period. We delete stock-days with negative prices, with trading volume below 100 shares, and with return below -1.0. In calculating $ILLIQ_{j,s}$, $LSIlliq_{j,s}$, $|R_{j,s}|$, and $IDVOL_{j,s}$ for each stock *j* we exclude the day with the highest value of each variable. We censor stocks whose $ILLIQ_{j,s}$, $|R_{j,s}|$, $IDVOL_{j,s}$, $DIF_{j,s}$, or $Size_{j,s}$ (firm's size) are in the extreme 1% in each month *s* to remove potential outliers.

$$+ b5_s{*}R1lag_{j,s-1} + residual_{j,s}\,. \qquad (9)$$

The dependent variable $(R_j\text{-}rf)_s$ is the excess return on stock $j$ in month $s$ and $IL_{j,s}$, a column vector, includes ln$ILLIQ_{j,s}$ and its components ln$LSIlliq_{j,s}$, ln$|R_{j,s}|$, ln$IDVOL_{j,s}$, or $DIF_{j,s}$ calculated over a twelve-month period that ends in month $s$, all lagged by two months as in LS, Amihud et al. (2015), and others. The control variables are *Size*, the market capitalization in logarithm; *BM*, the book-to-market ratio in logarithm;[10] and *R1lag* and *R12lag*, the lagged returns over the previous month and the eleven months from $s$-2 to $s$-12, respectively, to control for the short-term reversal and momentum effects. Table 1 presents the test results of Model (9) for our sample period 1955-2016 of 744 months. The coefficients reflect the premiums in percent.

INSERT TABLE 1

We find that in addition to the coefficient of ln$ILLIQ_{j,s-2}$ being positive and significant, the coefficient of its component $DIF_{j,s-2}$ is positive and significant. It is 1.219 ($t$ = 4.42) when controlling for ln$LSIlliq_{j,s-2}$ whose effect is positive and significant (column (2)) or 0.996 ($t$ = 3.75) when controlling for ln$IDVOL_{j,s-2}$ and ln$|R_{j,s-2}|$ (column (3)). We find that the coefficient of $DIF_{j,s-2}$ is consistently positive and significant when we estimate the model separately over two equal subperiods of 372 months each.[11] Thus, missing *DIF* in the analysis omits valuable information contained in *ILLIQ* that affects expected returns.

We also estimate Model (9) by adding the systematic risks $\beta_{RMrf}$, $\beta_{SMB}$, $\beta_{HML}$, and $\beta_{UMD}$ of the factors of Fama and French (1993) and Carhart (1997), *RMrf*, *SMB*, *HML*, and *UMD*. The results on the significant pricing of *ILLIQ* and its components, including *DIF*, are unchanged (see Appendix Table A.2).

Following our finding in Section 2 that *DIF* is a function of *b*, $CV^2$, and *LSIlliq*, we estimate the model in column (3) replacing $DIF_{j,s-2}$ by $fDIF_{j,s-2}$, the fitted value from a monthly cross-stock regression of $DIF_{j,s-2}$ on ln$CV^2_{j,s-2}$, ln$b_{j,s-2}$, and ln$LSIlliq_{j,s-2}$. We find that the coefficient of $fDIF_{j,s-2}$ is highly significant at 1.261 with $t$ = 3.29 or 1.022 with $t$ = 3.63 when

[10] We use the CRSP and Compustat databases. Book values are from the firm's annual financial report as known at the end of the previous fiscal year and the market value is for December of the year before the year of analysis. We combine the book equity data from Compustat and Ken French's data library, used in Davis et al. (2000). Following Fama and French (1992), we exclude stocks with negative book values.

[11] The coefficients for the first and second subperiods are 0.922 ($t$ = 2.73) and 1.517 ($t$ = 3.47), respectively, in the presence of $LSIlliq_{j,s-2}$.

$fDIF_{j,s\text{-}2}$ is estimated from a cross-stock regression model with or without an intercept, respectively. This is in addition to the positive and significant coefficient of ln$IDVOL_{j,s\text{-}2}$. When $fDIF_{j,s\text{-}2}$ is the fitted value from a cross-stock regression model that includes only ln$CV^2_{j,s-2}$ and ln$b_{j,s\text{-}2}$ (excluding ln$LSIlliq_{j,s\text{-}2}$) with or without intercept, its coefficient is also highly significant being 2.236 ($t$ = 3.55) or 1.026 ($t$ = 3.96), respectively. This indicates that the pricing of $DIF_{j,s\text{-}2}$ is mainly through the two illiquidity-related components ln$CV^2_{j,s-2}$ and ln$b_{j,s\text{-}2}$. (Results for other models of $fDIF_{j,s\text{-}2}$ are in Appendix Analysis A2.)[12] These results suggest that the illiquidity-related information contained in $DIF$ is pertinent for asset pricing.

A "horse race" between ln$ILLIQ$ and ln$IDVOL$ is problematic given the very high correlation between them across stocks. Following LS, we regress ln$ILLIQ_{j,s}$ cross-sectionally on ln$IDVOL_{j,s}$ (and an intercept) in each month $s$. The residuals from this regression are denoted Rln$ILLIQ_{j,s}$. Table 1, column (4) presents the test results for Model (9) where $IL_{j,s\text{-}2}$ includes Rln$ILLIQ_{j,s\text{-}2}$ and ln$IDVOL_{j,s\text{-}2}$. Following LS (their Table 2B, column (5)) the model includes $IdioVol_{j,s\text{-}2}$, the idiosyncratic volatility calculated as the standard deviation of the daily residuals from a regression of stock returns on Fama-French's (1993) three factors return over the twelve-month estimation period.[13] We find that the coefficient of Rln$ILLIQ_{j,s\text{-}2}$ is 0.736 with $t$ = 4.63 and that of ln$IDVOL_{j,s\text{-}2}$ is 0.099 with $t$ = 3.01.[14] This result indicates that $ILLIQ$ contains priced information on illiquidity which exceeds that in its component $IDVOL$, which is also priced.

We next test the relation between ln$IDVOL_{j,s}$ and Rln$ILLIQ_{j,s}$ and two microstructure measures of illiquidity, Kyle's (1985) $\lambda_{j,s}$ and $Spread_{j,s}$, the dollar quoted spread between the bid and ask prices divided by the spread's midpoint. Data on $\lambda_{j,s}$ are available in the WRDS Intraday Indicator Database for the period 1993-2015[15] and data on $Spread_{j,s}$ are available from CRSP for the period 1993-2016.[16] In regressions of $\lambda_{j,s}$ on ln$IDVOL_{j,s}$ and Rln$ILLIQ_{j,s}$ (and a constant) by

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[12] We also estimate a model as in column (2) of Table 1 replacing $DIF_{j,s\text{-}2}$ by –ln|cov(|$r_{j,d,s\text{-}2}$|, 1/$dvol_{j,d,s\text{-}2}$)| which is based on equation (6). Its coefficient is 0.145 with $t$ = 3.08 and that of ln$LSIlliq_{j,s\text{-}2}$ is 0.248 with $t$ = 3.21.

[13] The results are similar when using ln|$R_{j,s\text{-}2}$| instead of $IdioVol_{j,s\text{-}2}$.

[14] When estimating the model in column (4) for LS's sample period (1964-2012), the coefficients of Rln$ILLIQ_{j,s\text{-}2}$ and ln$IDVOL_{j,s\text{-}2}$ are, respectively, 0.937 ($t$ = 5.58) and 0.083 ($t$ = 2.22). When the model is estimated separately over the two equal subperiods, we find that the coefficient of Rln$ILLIQ_{j,s\text{-}2}$ is 0.948 ($t$ = 4.82) and 0.524 ($t$ = 2.23) in the first and second subperiods, respectively, and the coefficient of ln$IDVOL_{j,s\text{-}2}$ is 0.200 ($t$ = 4.03) and -0.002 ($t$ = -0.04) in the first and second subperiods, respectively.

[15] To be comparable with ln$ILLIQ$ and its component variables, we express $\lambda_{j,s}$ for dollar trading volume in millions.

[16] The variable $Spread_{j,s}$ in CRSP is well populated cross-sectionally from 1993.

the monthly Fama-Macbeth method,[17] we find that the coefficients of ln$IDVOL_{j,s}$ and Rln$ILLIQ_{j,s}$ are 3.722 ($t$ = 17.19) and 6.994 ($t$ = 8.06), respectively. In regressions with $Spread_{j,s}$ as dependent variable, the coefficients of ln$IDVOL_{j,s}$ and Rln$ILLIQ_{j,s}$ are 0.003 ($t$ = 9.94) and 0.008 ($t$ = 8.56), respectively. These results show that *IDVOL* is a proxy measure for illiquidity and that *ILLIQ* contains additional illiquidity-related information that is significantly priced.

LS (Section 4.1) suggest that the illiquidity premium is seasonal, disappearing in January. We test the January effect by regressing the monthly premiums of ln$ILLIQ_{j,s-2}$ and of $DIF_{j,s-2}$ on a constant, a dummy variable *Jan* (= 1 in January; = 0 otherwise), and *RMrf*, the excess market return. For the premium of ln$ILLIQ_{j,s-2}$ from the model in column (1), the intercept is 0.195 ($t$ = 5.59) and the coefficient of *Jan* is -0.126 ($t$ = -0.80),[18] and for the premium of $DIF_{j,s-2}$ from the model in column (2), the intercept is 1.165 ($t$ = 4.20) and the coefficient of *Jan* is 0.112 ($t$ = 0.12). Thus, the illiquidity premium is positive and significant throughout the year.

### 3.2. The illiquidity premium as a function of mispricing, lagged illiquidity, or sentiment

We provide two tests of LS's conjecture that the illiquidity premium "is likely caused by mispricing, not by compensation for illiquidity" (p. 4481). In both tests, we find that the illiquidity premium remains positive and significant after controlling for mispricing.

First, we add to Model (9) $MISP_{j,s-2}$, stock *j*'s average mispricing rank of Stambaugh, Yu, and Yuan (2012) based on 11 anomaly variables. Data are provided by the authors for the period 07/1965-12/2016. The average of the monthly cross-stock correlations of ln$ILLIQ_{j,s}$ and $MISP_{j,s}$ is 0.075, very small. We find that the effect of illiquidity on expected return remains positive and significant in the presence of mispricing, which also affects expected return. The results are presented in the Appendix Table A.3. The coefficients of ln$ILLIQ_{j,s-2}$ and of Rln$ILLIQ_{j,s-2}$ are 0.106 ($t$ = 2.42) and 0.809 ($t$ = 4.58), respectively, and the coefficient of $DIF_{j,s-2}$ in the presence of ln$LSIlliq_{j,s-2}$ is 1.026 ($t$ = 3.89). In a model with $DIF_{j,s-2}$, ln$IDVOL_{j,s-2}$, and ln$|R_{j,s-2}|$, their respective coefficients are 0.832 ($t$ = 3.24), 0.077 ($t$ = 2.02), and -0.176 ($t$ = -0.99). The positive and significant effect of ln$IDVOL_{j,s-2}$ in the presence of $MISP_{j,s-2}$ tests LS's conjecture (p. 4485) that "the volume premium is likely to be attributed to mispricing rather than liquidity premium."

[17] The calculation of the standard errors employs Newey and West's (1986) method with 5 to 6 lags, depending on the availability of $\lambda_{j,s}$ and $Spread_{j,s}$.
[18] When controlling for all four Fama-French-Carhart factors – *RMrf*, *SMB*, *HML*, and *UMD* – the coefficient of *Jan* is 0.010 with $t$ = 0.07 while the intercept is 0.156 with $t$ = 4.31.

Second, we regress the series of the monthly slope coefficients of ln$ILLIQ_{j,s-2}$ from the model in column (1) of Table 1 on the two mispricing factors of Stambaugh and Yuan (2016), $PERF_s$ and $MGMT_s$, which relate to firm's performance and managerial decisions, respectively (and a constant). The model includes $RMrf_s$ as a control. We find that the intercept – the mean illiquidity premium after controlling for the mispricing factors' premiums – is 0.165 with $t$ = 4.32, highly significant, and the coefficients of $PERF_s$ and $MGMT_s$ are 0.016 ($t$ = 1.96) and 0.015 ($t$ = 1.12), respectively.[19] Estimating this regression with the monthly slope coefficient of $DIF_{j,s-2}$ from the model in column (2) as dependent variable, the intercept is 1.150 with $t$ = 3.55, while the coefficients of both $PERF_s$ and $MGMT_s$ are insignificant. The results are qualitatively similar when $RMrf_s$ is excluded from the model.

We thus conclude that the illiquidity premium is positive and significant after controlling for mispricing-related effects.

We revisit our earlier Fama-Macbeth cross-sectional regressions of $\lambda_{j,s}$ and $Spread_{j,s}$ on ln$IDVOL_{j,s}$ and Rln$ILLIQ_{j,s}$ adding $MISP_{j,s}$ to the model. We find that for $\lambda_{j,s}$ as dependent variable, the coefficients of ln$IDVOL_{j,s}$, Rln$ILLIQ_{j,s}$, and $MISP_{j,s}$ are 3.673 ($t$ = 17.44), 6.447 ($t$ = 7.51), and 0.0137 ($t$ = 2.50), respectively, and in regressions with $Spread_{j,s}$ as dependent variable, the coefficients of ln$IDVOL_{j,s}$, Rln$ILLIQ_{j,s}$, and $MISP_{j,s}$ are 0.003 ($t$ = 9.79), -0.00003 ($t$= -2.91), and 0.008 ($t$ = 8.46), respectively. That is, the inclusion of $MISP_{j,s}$ hardly affects the positive strong relation between Rln$ILLIQ$ and ln$IDVOL$ and microstructure measures of illiquidity.

Next we test LS's finding that the volume-based illiquidity premium is a declining function of lagged market illiquidity. They conclude (p. 4508): "This result does not support the liquidity explanation of the volume premium." LS estimate the following model (their model (12)) with the Fama-French (1993) and Carhart (1997) (FFC) factors as controls.

$$R_t = \alpha + b*Illiq_{t-1} + c*MKT_t + d*SMB_t + e*HML_t + f*MOM_t + u_t. \qquad (10)$$

$R_t$ is the monthly return on a "long-short" portfolio of illiquid-minus-liquid stocks based on turnover quintiles and $Illiq_t$ is the Pastor and Stambaugh's (2003) illiquidity series multiplied by -1. LS find that $b$ is negative and significant. In our analysis, $R_t$ is the return on an $ILLIQ$-based

[19] The intercept remains positive and highly significant at 0.120 with $t$ = 3.01 when including in the regression the four factors of Fama-French-Carhart as controls. Doing the regression without $RMrf_t$, the intercept is 0.101 with $t$ = 2.07 while the slope coefficients of both $PERF_s$ and $MGMT_s$ are insignificant.

long-short portfolio of illiquid-minus-liquid stocks[20] and $Illiq_t$ is $mILLIQ_t$, the market *ILLIQ* the logarithm of average $ILLIQ_{j,t}$ across stocks (see details in Section 4). We find that $b = 0.005$ with $t = 0.16$, insignificant. This does not support LS's suggestion.[21]

We next examine LS's finding (their model (13)) that the positive illiquidity premium is driven by lagged investors' sentiment. We regress $R_t$ on $SENT_{t-1}$ (and a constant) using Baker and Wurgler's (2006) sentiment index available since 7/1965 and find that the slope coefficient[22] is 0.006 with $t = 0.05$, insignificant, and the intercept is 0.515 with $t = 4.21$. Controlling for the FFC factors, the coefficient of $SENT_{t-1}$ is 0.153 with $t = 2.01$ and the intercept – the risk-adjusted illiquidity premium – is 0.353 with $t = 4.42$. By this estimate, the illiquidity premium would be zero if $SENT_{t-1}$ is 2.3 standard deviations below its mean, an event whose probability is 0.011 (under normality; the standard deviation of $SENT_t$ is 1.0). The effect of $SENT_{t-1}$ becomes insignificant over time. Splitting the sample into two subperiods, we find that in the first subperiod, the intercept is 0.527 ($t = 4.38$) and the coefficient of $SENT_{t-1}$ is 0.198 ($t = 2.25$), while in the second subperiod, the intercept is 0.243 ($t = 2.06$) and the coefficient of $SENT_{t-1}$ is 0.096 ($t = 0.57$), insignificant.

## 4. Time-series Analyses: The Effects of Illiquidity Shocks on Aggregate Stock Returns

*ILLIQ* is proposed by Amihud (2002) as an illiquidity measure that produces consistent effects on stock returns in *both* the cross-section and the time-series. Across stocks, *ILLIQ* *positively* predicts expected return and in time-series its market-wide shocks *negatively* affects (contemporaneous) realized returns. An increase in market *ILLIQ*, which is highly persistent, is expected to remain high for a while. This raises expected return and induces a contemporaneous decline in stock prices for given cash flows. The effect of market *ILLIQ* shocks on realized

[20] We follow the methodology in Amihud et al. (2015) and Amihud and Noh (2019). We sort stocks in each month $t$ into three portfolios by volatility (standard deviation of daily returns) due to the positive illiquidity-volatility relation (Stoll, 1978) and then sort stocks by *ILLIQ* within each volatility tercile into five portfolios. *ILLIQ* and volatility are calculated over twelve months up to month $t$. Value-weighted average returns are calculated for each portfolio in month $t + 2$ (skipping one month after the portfolio formation). Then we compute the difference between the average returns on the three highest-*ILLIQ* and those on the three lowest-*ILLIQ* quintile portfolios.

[21] We also follow LS in using Pastor and Stambaugh's (2003) illiquidity measure and their practice of using two-month lag of illiquidity in their cross-sectional analysis. Using $Illiq_{t-2}$ in Model (10), we find that its coefficient is 0.45 with $t = 0.38$, insignificant.

[22] The results are similar after adjusting for finite-sample bias using Amihud and Hurvich's (2004) methodology.

returns is more negative for the less liquid and smaller stocks.[23] There is evidence that causality runs from illiquidity changes to asset prices. Amihud, Mendelson, and Lauterbach (1997) find a price increase for stocks that were moved to a liquidity-increasing trading mechanism,[24] Amihud, Mendelson, and Uno (1999) find a rise in prices of stocks whose liquidity increased due to facilitation of trading, and Kelly and Ljungqvist (2012) find that stock prices declined following exogenous termination of analysts' coverage which raised stock illiquidity.

We calculate the aggregate monthly series of *ILLIQ*-based variables as follows. For each stock $j$ and month $t$, we calculate the values of $ILLIQ_{j,t}$ and of its components $IDVOL_{j,t}$ and $|R_{j,t}|$ and then calculate month-$t$ cross-stock value-weighted average.[25] The resulting series, transformed into logarithm, are denoted $Y_t = mILLIQ_t$, $mIDVOL_t$, and $m|R_t|$, respectively. Shocks in each of these series are calculated by an AR(2) model over a rolling window of 60 months that ends in month $n$.[26] The shock in month $n$+1 denoted $dY_{n+1}$ is the difference between the actual value of the series and its predicted value using the slope coefficients estimated over the preceding 60 months. Thus, our method is forward-looking. The series $dmDIF_t$ is the difference $dmILLIQ_t – (dm|R_t| + dmIDVOL_t)$. The series $dY_{n+1}$ are calculated for the period 1955-2016, 744 months. The variables are in percent. Summary statistics for the series are presented in Appendix Table A.4. In a regression of $dmILLIQ_t$ on $dmIDVOL_t$ (and a constant), $R^2 = 0.48$ meaning that $dmIDVOL_t$ explains only half of the time-series variation in $dmILLIQ_t$. And in a regression of $dmILLIQ_t$ on $dm|R_t|$ and $dmIDVOL_t$, $R^2 = 0.79$ implying that a fifth of the information in $dmILLIQ_t$ is not included in the two component series.

We test the effects on realized stock returns of shocks to market illiquidity, $dmILLIQ_t$, and of its component by estimating the following time-series regression model, where $dY_t$ is a column vector that includes a subset of the variables $dmILLIQ_t$, $dmIDVOL_t$, $dm|R_t|$, and $dmDIF_t$:

$$RMrf_t = a + b'*dY_t + residual_t, \qquad (11)$$

---

[23] The effect of shocks to market illiquidity on realized returns is similar to the effect of shocks to market risk in French et al. (1987). For empirical support on the negative relation between market illiquidity shocks and realized returns on stocks and bonds, see a review in Amihud et al. (2013) and recent evidence in Harris and Amato (2019). Karolyi et al. (2012) use market illiquidity shocks in analyzing liquidity commonality.

[24] Similar results are found by Muscarella and Piwowar (2001), Kalay, Wei, and Wohl (2002), and Jain (2005).

[25] The weights are the market capitalizations at the end of the preceding month. The same stock filters used in the cross-section analysis are employed. Excluded are stock-months with less than 15 days of valid return and volume data and those with values at the top 1% of $ILLIQ_{j,t}$, $|R_{j,t}|$, or $IDVOL_{j,t}$.

[26] The model is $Y_t$ = a0 + a1*$Y_{t-1}$ + a2*$Y_{t-2}$ + residual$_t$. For $Y_t = mILLIQ_t$ or $mIDVOL_t$, the model includes a third term a3*$T_t$ where $T_t$ is the serial number of the observation, to account for a time trend in these series.

INSERT TABLE 2

Our findings in Table 2 are as follows:

(i) The coefficient of either $dmILLIQ_t$ or $dmIDVOL_t$ is negative and significant (columns (1) or (2), respectively). The coefficient of $dmILLIQ_t$ is twice as negative as that of $dmIDVOL_t$ and the respective $R^2$ values are 0.23 and 0.05 implying that $dmILLIQ_t$ provides a better fit.

(ii) In a “horse race” between $dmILLIQ_t$ and $dmIDVOL_t$ where both are in the model (column (3)), the coefficient of $dmILLIQ_t$ is *negative* and significant, consistent with theory and with the positive cross-sectional effect of $ILLIQ_{j,s}$ on expected return, whereas that of $dmIDVOL_t$ is *positive* and significant, inconsistent with theory given the positive cross-stock effect of $IDVOL_{j,s}$ on expected return. Testing the model in column (3) for two equal subperiods, 1955-1985 and 1986-2016, we find that the coefficient of $dmILLIQ_t$ is consistently negative and significant in both subperiods, while that of $dmIDVOL_t$ is positive. In the second subperiod, even in a model with $dmIDVOL_t$ alone (as in column (2)), its coefficient is -0.008 with $t$ = -0.49, insignificant.

(iii) The coefficient of $dmDIF_t$ is *negative* and significant controlling for $dmIDVOL_t$ and $dm|R_t|$ (column (4)).[27] The negative coefficient of $dmDIF_t$ is consistent with the positive cross-sectional effect of $DIF_{j,s}$ on expected return in the presence of $\ln IDVOL_{j,s}$ and $\ln|R_{j,s}|$. Intuitively, since $mDIF_t$ is highly persistent (its serial correlation is 0.87), a rise in $mDIF_t$ implies higher future values of $mDIF_{t+1}$ and lower average values of $CV_{t+1}$, which is undesirable by investors by Pereira and Zhang’s (2010) theory. Thus, a positive shock in $mDIF_t$ lowers contemporaneous market prices and generates lower realized returns, which is what we find.

In Panel B we estimate Model (11) with $SMB_t$ as dependent variable, including $RMrf_t$ as a control variable given its correlation with market illiquidity shock. We find that the coefficient of $dmILLIQ_t$ is negative and significant while that of $dmIDVOL_t$ is positive and insignificant when both are included in the model (column (5)). In the model in column (6) with all components of $mILLIQ_t$ – $dmDIF_t$, $dmIDVOL_t$, and $dm|R_t|$ – all coefficients are negative and significant.

Testing LS’s suggestion that the illiquidity effect is a January phenomenon, we add to Model (11) for $dY_t = dmILLIQ_t$ two variables, $Jan_t$ and $dmILLIQ_t*Jan_t$, where $Jan_t = 1$ in January (0 otherwise). We find (in Appendix Table A.5) that the coefficient of $dmILLIQ_t$ is -0.120 with $t$

[27] The coefficient of $dmDIF_t$ is similar when $dmDIF_t$ is calculated as the prediction errors from an AR(2) model of $mDIF_t = mILLIQ_t – [m|R_t| + mIDVOL_t]$.

= -11.57 while the coefficient of $dmILLIQ_t$*$Jan_t$ is insignificant. With $SMB_t$ as dependent variable, the coefficient of $dmILLIQ_t$ is -0.051 with $t$ = -7.68 and that of the interaction term is again insignificant. We thus conclude that the negative and significant effect of $dmILLIQ_t$ on aggregate stock returns persists in *both* January and non-January months.

Next, we estimate the effects on stock returns of $dmILLIQ_t$ and $dmIDVOL_t$ in the presence of shocks to $m\lambda_t$. This series is a monthly equally-weighted average (in logarithm) of Kyle's (1985) λ, a price impact measure estimated from intraday trades and quotes data, which positively affects expected return (Brennan and Subrahmanyam (1996) and Huh (2014)).[28] Data on $m\lambda_t$ for 1/1983-12/2009 is kindly provided by Sahn-Wook Huh. We calculate the illiquidity shocks series $dm\lambda_t$ as we do for $dmILLIQ_t$ and $dmIDVOL_t$. The results are in Table 2, Panels C and D. We find that the coefficient of $dm\lambda_t$ is negative and highly significant (column (7)), as expected and it becomes insignificant when including $dmILLIQ_t$ (column (8)) whose coefficient is negative and highly significant. Yet, when both $dm\lambda_t$ and $dmIDVOL_t$ are included in the model (column (9)), the coefficient of $dm\lambda_t$ is negative and significant while that of $dmIDVOL_t$ is insignificant. The insignificant effect of $dmIDVOL_t$ in the presence of $dm\lambda_t$, whose effect is consistent with the theory on the effect of illiquidity shocks on returns, means that it is not the volume component alone in $mILLIQ_t$ that generates its effect on stock returns. When all components of $dmILLIQ_t$ are included in the model (column (10)), their coefficients are all negative and significant in the presence of $dm\lambda_t$. The results are similar when the dependent variable is $SMB_t$. The significant effect of $dmILLIQ_t$ in the presence of $dm\lambda_t$ may attest to *ILLIQ* being a broader measure of illiquidity than market price impact alone.

Finally, we test LS's suggestion that the illiquidity effect reflects investors' sentiment using $dSENT_t$, the monthly change in Baker and Wurgler's (2006) sentiment index. The correlation between $dSENT_t$ and $dmILLIQ_t$ is -0.092, very low. Adding $dSENT_t$ to Model (11) for $dY_t = dmILLIQ_t$ we find that the coefficient of $dmILLIQ_t$ is -0.120 with $t$ = -10.99 and that of $dSENT_t$ is -2.245 with $t$ = -1.95. With $SMB_t$ as dependent variable and $RMrf_t$ included as a control variable, the respective coefficients are -0.055 ($t$ = -7.36) and 0.370 ($t$ = 0.45), insignificant. Thus, the effect of market illiquidity shocks on realized returns remains negative and highly significant after controlling for the effect of sentiment changes.

[28] We thank an anonymous referee for suggesting this test.

In Appendix Table A.6, we present additional results. First, in Panel A, we examine months with *opposite* signs of $\Delta mILLIQ_t$ and $\Delta mIDVOL_t$, the first difference in the respective market illiquidity series, indicating opposite reading of changes in market illiquidity. We find that in these months, the signs of changes of four benchmark measures of illiquidity[29] are consistent with the sign of $\Delta mILLIQ_t$ but are opposite to the sign of $\Delta mIDVOL_t$ (rows (4) to (7)).[30] Moreover, we find that in these months, the market price reaction is negatively related to $\Delta mILLIQ_t$, as expected of the effect of illiquidity shocks on stock returns, and positively related to $\Delta mIDVOL_t$ in rows (2) and (3), which is contrary to expectations. Second, in Panel B, we find that the stock return correlation with $\Delta mILLIQ_t$ is twice more negative than it is with $\Delta mIDVOL_t$ in rows (2) and (3). We also find that in rows (4) to (7), the correlations of $\Delta mILLIQ_t$ with the four benchmark measures of illiquidity are far greater than those of $\Delta mIDVOL_t$. In summary, these results suggest that $\Delta mILLIQ_t$ is the better measure of illiquidity changes.

Another finding in Appendix Figure A.1 is that during two major illiquidity crises – those of October 19, 1987 when stock price sharply fell and illiquidity increased and October, 2008 following Lehman Brothers' bankruptcy, market $ILLIQ_t$ has risen sharply, consistent with other benchmark measures of illiquidity, while market $IDVOL_t$ remained practically unchanged.

## 5. Concluding Remarks

We compare the performance of Amihud's (2002) illiquidity measure *ILLIQ* to the performance of its component *IDVOL*, the average inverse dollar trading volume, which LS propose is a sufficient alternative based on their decomposition of *ILLIQ*. We show that LS's decomposition misses an illiquidity-related component of *ILLIQ* that is priced in both the cross-section of expected return and in the time-series of realized aggregate stock returns. We also show that *ILLIQ* is significantly priced after controlling for mispricing, sentiment, and

[29] These benchmark measures are: (i) $\Delta m\lambda_t$, the first difference in $m\lambda_t$, the logarithm of monthly cross-stock average of Kyle's (1985) λ from Huh (2014); (ii) $\Delta mQSP_t$, the first difference in $mQSP_t$, the value-weighted average (in logarithm) of the quoted relative bid-ask spreads for NYSE\AMEX stocks using CRSP (since 1993); (iii) $\Delta mESP_t$, the first difference in $mESP_t$, the logarithm of the average effective relative bid-ask spread calculated by Abdi and Ronaldo (2017) for NYSE stocks; (iv) $iPSIlliq_t$, the innovations in the market illiquidity series of Pastor and Stambaugh (2003) (multiplied by -1) available for the period 08/1962 to 12/2016.

[30] This is consistent with Pastor and Stambaugh (2003, p. 657) who point out the problem in using volume to depict market liquidity: "While measures of trading activity such as volume and turnover seem useful in explaining cross-sectional differences in liquidity, they do not appear to capture time variation in liquidity. Although liquid markets are typically associated with high levels of trading activity, it is often the case that volume is high when liquidity is low."

seasonality. Further, the effects of shocks in the time-series of market *ILLIQ* on aggregate stock returns are consistent with theory and with the effect of *ILLIQ* in the cross-stock analysis, while the effects of *IDVOL* and shocks in market *IDVOL* do not always exhibit such consistency in cross-section and time-series analyses.

The key question is whether illiquidity, which is costly and undesirable, is priced regardless of which proxy measure is used. Naturally, no single measure completely encompasses all aspects of illiquidity.[31] While this study provides evidence on the pricing of illiquidity and its components in the cross-section and time-series of stock returns, there is a need for a unified and comprehensive modeling of the pricing of illiquidity and its components in dynamic equilibrium from the following three angles: (1) the cross-sectional effect on expected return of the level of illiquidity, (2) the time-series effect on realized return of market illiquidity, and (3) the pricing of exposure to market illiquidity shocks using illiquidity risk factor, which applies (2). Such modeling is called for given the proliferation of research on the pricing of illiquidity *both* as a stock-specific characteristic and – using the time-series of market illiquidity – as a source of systematic risk.

[31] Harris and Amato (2019) find significant pricing power of low-frequency illiquidity measures employing alternative simple ratios constructed from volatility and volume. This calls for a principal component approach that would integrate low-frequency illiquidity measures.

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**Table 1. The Effect of Illiquidity Variables on Expected Stock Return: Fama-Macbeth Cross-sectional Regressions.**

This table presents the averages of slope coefficients from monthly Fama-Macbeth cross-sectional regressions of the following model:

$$(R_j - rf)_s = b0_s + b1_s{}'{*}IL_{j,s-2} + b2_s{*}Size_{j,s-2} + b3_s{*}BM_{j,y-1} + b4_s{*}R12lag_{j,s-2} + b5_s{*}R1lag_{j,s-1} + residual_{j,s.} \quad (9)$$

$(R_j - rf)_s$ is the month-$s$ return of stock $j$ in the excess of the risk-free rate. $IL_{j,s}$ is a column vector of *ILLIQ*-related variables. $ILLIQ_{j,s}$ is the average of daily values of $illiq_{j,d,s} = |r_{j,d,s}|/dvol_{j,d,s}$, where $r_{j,d,s}$ and $dvol_{j,d,s}$ are, respectively, the daily return and dollar trading volume (in millions) of stock $j$ on day $d$, calculated over twelve months that end in month $s$. $|R_{j,s}|$ and $IDVOL_{j,s}$ are the averages of $|r_{j,d,s}|$ and $1/dvol_{j,d,s}$, respectively, over the same twelve months. $LSIlliq_{j,s} = |R_{j,s}| * IDVOL_{j,s}$. $DIF_{j,s} = \ln ILLIQ_{j,s} - \ln LSIlliq_{j,s} = \ln ILLIQ_{j,s} - [\ln|R_{j,s}| + \ln IDVOL_{j,s}]$ and "ln" indicates natural logarithm. $\mathrm{Rln}ILLIQ_{j,s}$ is the residual from month-$s$ cross-stock regression of $\ln ILLIQ_{j,s}$ on $\ln IDVOL_{j,s}$ (and a constant). $IdioVol_{j,s-2}$ is the standard deviation of the residuals from a regression of daily returns on the daily values of the Fama-French three factors estimated over twelve months that end in month $s$. The sample period is 1/1955-12/2016, 744 months. The control variables are $Size_{j,s-2}$, the market capitalization in logarithm; $BM_{j,y-1}$, the book-to-market ratio in logarithm for the end of the previous calendar year; $R1lag_{j,s-1}$ and $R12lag_{j,s-2}$, the lagged returns over the previous one month and the preceding eleven months (months $s$-2 to $s$-12), respectively. The slope coefficients are in percent and the $t$-statistics are in parentheses.

| Explanatory variables | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $\ln ILLIQ_{j,s-2}$ | 0.102 (2.45) | | | |
| $\ln LSIlliq_{j,s-2}$ | | 0.111 (2.64) | | |
| $\ln\|R_{j,s-2}\|$ | | | -0.317 (-2.10) | |
| $\ln IDVOL_{j,s-2}$ | | | 0.093 (2.64) | 0.099 (3.01) |
| $\boldsymbol{DIF_{j,s-2}}$ | | **1.219 (4.42)** | **0.996 (3.75)** | |
| $\mathbf{Rln}\boldsymbol{ILLIQ_{j,s-2}}$ | | | | **0.736 (4.63)** |
| $IdioVol_{j,s-2}$ | | | | -0.663 (-8.82) |
| Control variables: $Size_{j,s-2}$, $BM_{j,y-1}$, $R12lag_{j,s-2}$, $R1lag_{j,s-1}$ | | | | |
| *Average Adjusted* $R^2$ | 5.62% | 5.95% | 7.48% | 7.52% |

## Table 2. The Effect of Shocks in Market Illiquidity Series on Realized Stock Returns

This table presents the monthly time-series regressions of realized stock returns on $dmILLIQ_t$, $dm|R_t|$, $dmIDVOL_t$, and $dmDIF_t$ for the period 1/1955-12/2016. The first three are the shocks to the time-series $mILLIQ_t$, $m|R_t|$, and $mIDVOL_t$, the (logarithm of) monthly market averages of, respectively, $illiq_{j,d,t} = |r_{j,d,t}|/dvol_{j,d,t}$, $|r_{j,d,t}|$, and $1/dvol_{j,d,t}$, where $r_{j,d,t}$ and $dvol_{j,d,t}$ are the daily return and daily dollar volume of stock *j* on day *d* of month *t*. These variables are first averaged for each stock over the days of each month and then averaged across stocks in each month to produce the market series. Some filters apply. The average across stocks is value-weighted using market capitalization at the end of the preceding month. This produces the market series $mILLIQ_t$, $m|R_t|$, and $mIDVOL_t$. In addition, $m\lambda_t$ is the logarithm of monthly cross-stock equally-weighted average of Kyle's (1985) λ estimated from intraday trades and quotes. The series is provided by Huh (2014) for the period 1/1983-12/2009. The shocks in each of these series indicated by the prefix "*d*" are calculated by estimating an AR(2) model over a rolling window of 60 months ending in month *n* (the models for $mILLIQ_t$, $mIDVOL_t$, and $m\lambda_t$ also include a time trend) and setting the shock in month *n*+1 as the difference between the actual value of the series and its predicted value, using the estimated slope coefficients from the preceding 60-month window. We define $dmDIF_t = dmILLIQ_t - (dm|R_t| + dmIDVOL_t)$.
The dependent variables are $RMrf_t$ in Panels A and C, the market excess return over the risk-free rate, and $SMB_t$ in Panels B and D, the return on the portfolio of small-minus-big stocks, respectively. The regressions include intercepts (not reported). The slope coefficients are in percent. The *t*-statistics (in parentheses) employ robust estimation of standard errors (White (1980)).

| | Panel A: $RMrf_t$ | | | | Panel B: $SMB_t$ | | Panel C: $RMrf_t$ | | | | Panel D: $SMB_t$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| $dmILLIQ_t$ | -0.122<br>(-12.35) | | -0.156<br>(-9.57) | | -0.063<br>(-6.66) | | | -0.084<br>(-4.22) | | | -0.057<br>(-4.76) | |
| $dmIDVOL_t$ | | -0.061<br>(-5.48) | 0.051<br>(3.21) | -0.103<br>(-10.94) | 0.014<br>(1.61) | -0.047<br>(-6.83) | | | -0.013<br>(-0.69) | -0.055<br>(-2.97) | | -0.017<br>(-1.29) |
| $dm\|R_t\|$ | | | | -0.155<br>(-9.72) | | -0.063<br>(-6.70) | | | | -0.129<br>(-4.99) | | |
| $\boldsymbol{dmDIF_t}$ | | | | **-0.143**<br>**(-6.32)** | | **-0.051**<br>**(-3.86)** | | | | **-0.103**<br>**(-3.21)** | | |
| $dm\lambda_t$ | | | | | | | -0.843<br>(-3.23) | -1.790<br>(-1.39) | -4.709<br>(-3.12) | -0.548<br>(-0.42) | -0.127<br>(-0.12) | -1.717<br>(-1.73) |
| $RMrf_t$ | | | | | 0.079<br>(2.47) | 0.078<br>(2.42) | | | | | 0.058<br>(1.11) | 0.124<br>(2.54) |
| *Adjusted* $R^2$ | 0.23 | 0.05 | 0.25 | 0.25 | 0.15 | 0.15 | 0.04 | 0.14 | 0.04 | 0.18 | 0.11 | 0.05 |

## APPENDIX

### Appendix Analysis A1: The derivation of *DIF* in Equation (8)

We derive an approximation of cov($|r_d|$, $1/dvol_d$) that gives rise to Equation (7). For random variable $Y$, the first-order Taylor-series expansion of $\frac{1}{Y}$ around its mean gives

$$\frac{1}{Y} \approx \frac{1}{E[Y]} - \frac{1}{(E[Y])^2}(Y - E[Y]).$$

Then its covariance with random variable $X$ is given by

$$Cov(X, \frac{1}{Y}) \approx -\frac{Cov(X,Y)}{(E[Y])^2}.$$

Now let $X = |r_d|$ and $Y = dvol_d$, the absolute value of return and dollar trading volume on day $d$ in a given period, respectively. We then have

$$Cov\left(|r_d|, \frac{1}{dvol_d}\right) \approx -\frac{cov(|r_d|, dvol_d)}{\overline{DVOL}_d^2} = -\frac{cov(|r_d|, dvol_d)}{var(dvol_d)} * \frac{var(dvol_d)}{\overline{DVOL}_d^2} = -b * CV^2,$$

where $b = \frac{cov(|r_d|, dvol_d)}{var(dvol_d)}$ is the slope coefficient from a regression of $|r_d|$ on $dvol_d$ and a constant and $CV$ is the coefficient of variation of $dvol_d$.

From Equation (6), we now have the following approximation in Equation (7):

$$ILLIQ \approx LSIlliq - b * CV^2.$$

Since LS carry out their analysis in logarithmic term, we have

$$\ln ILLIQ \approx \ln[(LSIlliq - b * CV^2) * LSIlliq / LSIlliq],$$

and the omitted term – the difference between ln*ILLIQ* and ln*LSIlliq* – is

$$DIF = \ln ILLIQ - \ln LSIlliq \approx \ln(1 - b * CV^2 / LSIlliq),$$

which is Equation (8).

**Appendix Analysis A2: Tests of the relationship between *DIF* and its components**

In Appendix Analysis A1, we have the first-order approximation of cov($|r_d|$, $1/dvol_d$) using Taylor-series expansion that leads to, for stock $j$ in month $s$,

$$DIF_{j,s} = \ln ILLIQ_{j,s} - \ln LSIlliq_{j,s} \approx \ln(1 - b_{j,s} * CV_{j,s}^2 / LSIlliq_{j,s}) .$$

There can be more information in higher-order terms not included in the approximation $\ln(1 - b_{j,s} * CV_{j,s}^2 / LSIlliq_{j,s})$ that is pertinent to asset pricing. In addition, empirically, the approximation term is estimated with error. We thus carry out a more detailed analysis as follows. Define the residual term that includes higher-order terms:

$$Resid_{j,s} = DIF_{j,s} - \ln(1 - b_{j,s} * CV_{j,s}^2 / LSIlliq_{j,s}).$$

We first run monthly cross-stock regressions of $Resid_{j,s}$ on $\ln b_{j,s}$, $\ln CV_{j,s}^2$, and $\ln LSIlliq_{j,s}$ (and an intercept) and find that average $R^2$ is 0.42 and that the coefficients of the three component variables are highly significant with respective coefficients of 0.060 ($t$ = 32.37), 0.045 ($t$ = 17.05) and -0.084 ($t$ = -47.49).[32] The highly significant coefficients indicate that $DIF_{j,s}$ includes material information in higher-order terms not captured by the approximation $\ln(1 - b_{j,s} * CV_{j,s}^2 / LSIlliq_{j,s})$ alone. We denote $fResid_{j,s}$ the fitted value of $Resid_{j,s}$ from its monthly cross-stock regressions on $\ln b_{j,s}$, $\ln CV_{j,s}^2$, and $\ln LSIlliq_{j,s}$ (with an intercept).

We then estimate Model (9) with $IL_{j,s-2}$ including $\ln(1 - b_{j,s-2} * CV_{j,s-2}^2 / LSIlliq_{j,s-2})$, $fResid_{j,s-2}$, and the two components of $LSIlliq_{j,s-2}$, $\ln|R_{j,s-2}|$ and $\ln IDVOL_{j,s-2}$ as in column (3) of Table 1. We find that the coefficients of *ILLIQ*-related variables included in $IL_{j,s-2}$ are as follows:

| | |
|---|---|
| $\ln(1 - b_{j,s-2} * CV_{j,s-2}^2 / LSIlliq_{j,s-2})$: | 0.823 with $t$ = 2.80 |
| $fResid_{j,s-2}$ : | 1.934 with $t$ = 2.81 |
| $\ln\lvert R_{j,s-2}\rvert$ : | -0.335 with $t$ = -2.13 |
| $\ln IDVOL_{j,s-2}$ : | 0.129 with $t$ = 3.55. |

[32] The calculation of the standard errors employs Newey and West's (1986) method with 7 lags.

This result shows the positive and significant pricing of the two components of $DIF_{j,s-2}$: the approximation term $\ln(1 - b_{j,s-2} * CV_{j,s-2}^2 / LSIlliq_{j,s-2})$ and $fResid_{j,s-2}$, a function of the three component variables that captures residual higher-order terms.

In another test, we estimate an unconstrained cross-stock regression model of $DIF_{j,s}$ as a function of $\ln(1 - b_{j,s} * CV_{j,s}^2 / LSIlliq_{j,s})$, $\ln b_{j,s}$, $\ln CV_{j,s}^2$, and $\ln LSIlliq_{j,s}$. In monthly cross-stock regressions, the average $R^2$ is 0.85 and the coefficients of the four component variables are 0.790 ($t$ = 34.74), 0.068 ($t$ = 30.35), 0.037 ($t$ = 16.89), and -0.087 ($t$ = -27.09), respectively. Notably, the coefficient of the approximation term $\ln(1 - b_{j,s-2} * CV_{j,s-2}^2 / LSIlliq_{j,s-2})$ is the largest and most significant. When the model is estimated with an intercept, average $R^2$ is 0.45 and the coefficients of all four component variables are also highly significant.

We then estimate Model (9) with $IL_{j,s-2}$ including $fDIF_{j,s-2}$, the fitted value of $DIF_{j,s-2}$ from monthly cross-stock regressions of $DIF_{j,s}$ on the four component variables above. We employ the model in column (3) of Table 1 that includes the two components of $LSIlliq_{j,s-2}$, $\ln|R_{j,s-2}|$ and $\ln IDVOL_{j,s-2}$. We find that the coefficient of $fDIF_{j,s-2}$ is highly significant at 0.983 with $t$ = 3.34, in addition to the coefficient of $\ln IDVOL_{j,s-2}$ being positive and significant. When using $fDIF_{j,s-2}$ from a cross-stock regression model that includes an intercept, its coefficient is 1.281 with $t$ = 3.26.

In Sum, these results indicate the significant pricing of the illiquidity-related information included in $DIF_{j,s}$, measured by a function of $b_{j,s}$, $CV_{j,s}^2$, and $LSIlliq_{j,s}$.

**Table A.1. Summary Statistics of the *ILLIQ*-related Variables**

For each stock $j$, we calculate the averages of the daily values of $illiq_{j,d,s} = |r_{j,d,s}|/dvol_{j,d,s}$, $|r_{j,d,s}|$, and $1/dvol_{j,d,s}$, where $r_{j,d,s}$ and $dvol_{j,d,s}$ are, respectively, the daily return and dollar trading volume on day $d$. The averages of these values for each stock over the preceding twelve months that end in month $s$ are $ILLIQ_{j,s}$, $|R_{j,s}|$, and $IDVOL_{j,s}$, respectively. We also define $LSIlliq_{j,s} = |R_{j,s}| * IDVOL_{j,s}$. The prefix "ln" indicates natural logarithm. $DIF_{j,s} = \ln ILLIQ_{j,s} - \ln LSIlliq_{j,s} = \ln ILLIQ_{j,s} - [\ln|R_{j,s}| + \ln IDVOL_{j,s}]$. The table presents the time-series averages of the monthly cross-sectional statistics of the variables over the sample period of 1955 to 2016, 744 months.

| Variables | Mean | Standard Deviation | Pairwise Correlations | | | |
|---|---|---|---|---|---|---|
| | | | $\ln ILLIQ_{j,s}$ | $\ln LSIlliq_{j,s}$ | $\ln\|R_{j,s}\|$ | $\ln IDVOL_{j,s}$ |
| $\ln ILLIQ_{j,s}$ | -3.34 | 1.98 | 1.0 | | | |
| $\ln LSIlliq_{j,s}$ | -3.17 | 2.02 | 0.99 | | | |
| $\ln\|R_{j,s}\|$ | -4.13 | 0.34 | 0.45 | 0.45 | | |
| $\ln IDVOL_{j,s}$ | 0.97 | 1.89 | 0.99 | 0.99 | 0.30 | |
| $DIF_{j,s}$ | -0.17 | 0.09 | -0.37 | -0.41 | -0.24 | -0.38 |

**Table A.2. The Effect of *ILLIQ* and Its Components on Expected Return, controlling for Systematic Risks:**

This table is similar to Table 1 except that we add to Model (9) the systematic risks, factor loadings under Fama and French (1993) and Carhart (1997). The systematic risks, $\beta_{RMrf}$, $\beta_{SMB}$, $\beta_{HML}$, and $\beta_{UMD}$ of the respective factors *RMrf*, *SMB*, *HML* and *UMD*, are estimated over a rolling window of past 60 months up to month *s*-2 and added to the explanatory variables in Model (9).

| Explanatory variables | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $\ln ILLIQ_{j,s-2}$ | 0.096 (3.16) | | | |
| $\ln LSIlliq_{j,s-2}$ | | 0.108 (3.34) | | |
| $\ln\lvert R_{j,s-2}\rvert$ | | | -0.329 (-2.54) | |
| $\ln IDVOL_{j,s-2}$ | | | 0.096 (3.20) | 0.092 (3.11) |
| **$DIF_{j,s-2}$** | | **0.861 (3.83)** | **0.644 (2.91)** | |
| **$\mathrm{Rln}ILLIQ_{j,s-2}$** | | | | **0.511 (3.79)** |
| $IdioVol_{j,s-2}$ | | | | -0.558 (-7.29) |
| $\beta_{RMrf,j,s-2}$ | 0.015 (0.17) | 0.017 (0.20) | 0.126 (1.86) | 0.088 (1.23) |
| $\beta_{SMB,j,s-2}$ | -0.012 (-0.22) | -0.010 (-0.18) | 0.043 (1.01) | 0.048 (1.09) |
| $\beta_{HML,j,s-2}$ | 0.100 (2.16) | 0.096 (2.08) | 0.083 (2.02) | 0.100 (2.40) |
| $\beta_{UMDj,s-2}$ | -0.069 (-1.08) | -0.070 (-1.12) | -0.065 (-1.11) | -0.083 (-1.40) |
| Control variables: $Size_{j,s-2}$, $BM_{j,y-1}$, $R12lag_{j,s-2}$, $R1lag_{j,s-1}$, | | | | |
| *Average Adjusted* $R^2$ | 7.67% | 7.85% | 8.65% | 9.04% |

**Table A.3. The Effects of Illiquidity and Mispricing on Expected Return**

This table is similar to Table 1 where we estimate Fama-Macbeth monthly cross-sectional regressions of stock returns on stock characteristics with the following adjustments. Columns (1) to (4) include an additional control variable, $MISP_{j,s}$, constructed by Stambaugh, Yu, and Yuan (2012) by combining each stock's rankings on 11 anomaly variables computed at the end of each month *s*. Data for the sample period from 7/1965 to 12/2016 are obtained from the authors' web site. The slope coefficients are in percent and the *t*-statistics are in parentheses.

| Explanatory variables | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| **$MISP_{j,s-2}$** | **-0.017 (-8.73)** | **-0.017 (-8.75)** | **-0.016 (-9.51)** | **-0.015 (-9.19)** |
| ln$ILLIQ_{j,s-2}$ | 0.106 (2.42) | | | |
| ln$LSIlliq_{j,s-2}$ | | 0.114 (2.59) | | |
| ln$\|R_{j,s-2}\|$ | | | -0.176 (-0.99) | |
| ln$IDVOL_{j,s-2}$ | | | 0.077 (2.02) | 0.077 (2.00) |
| **$DIF_{j,s-2}$** | | **1.026 (3.89)** | **0.832 (3.24)** | |
| **Rln$ILLIQ_{j,s-2}$** | | | | **0.809 (4.58)** |
| $IdioVol_{j,s-2}$ | | | | -0.602 (-7.47) |
| Control variables: $Size_{j,s-2}$, $BM_{j,y-1}$, $R12lag_{j,s-2}$, $R1lag_{j,s-1}$ | | | | |
| *Average Adjusted $R^2$* | 5.88% | 6.19% | 7.68% | 7.68% |

**Table A.4. Summary Statistics on Shocks to the Market Illiquidity Series**

The variables are defined in Table 2.

| Variables | Mean (Std. Deviation) (in %) | Pairwise Correlation | | |
|---|---|---|---|---|
| $dmILLIQ_t$ | 0.208 (17.16) | $dmILLIQ_t$ | $dm\|R_t\|$ | $dmIDVOL_t$ |
| $dm\|R_t\|$ | -0.017 (16.73) | 0.437 | | |
| $dmIDVOL_t$ | -0.031 (16.40) | 0.691 | -0.170 | |
| $dmDIF_t$ | 0.256 (10.51) | -0.141 | -0.614 | -0.161 |

**Table A.5. The Effect of Market Illiquidity Shocks on Realized Stock Returns, controlling for the January effect**

The variables are defined in Table 2. $Jan_t$ = 1 in the month of January and zero otherwise. The time-series regressions include intercepts (not reported). The slope coefficients are in percent. The *t*-statistics are presented in parentheses, employing the robust estimation of standard errors by White (1980).

| Explanatory variables | Dependent variable | |
|---|---|---|
| | $RMrf_t$ | $SMB_t$ |
| $dmILLIQ_t$ | -0.120 (-11.57) | -0.051 (-7.68) |
| $Jan_t$ | -0.140 (-0.24) | 1.650 (4.09) |
| $dmILLIQ_t$*$Jan_t$ | -0.029 (-0.94) | -0.001 (-0.05) |
| $RMrf_t$ | | 0.087 (2.80) |
| Adjusted $R^2$ | 0.23 | 0.17 |

**Table A.6. The Effects of Opposite Changes in *mILLIQ* and *mIDVOL***

Panel A presents the means of variables for two subsamples of months in which $\Delta mILLIQ_t$ and $\Delta mIDVOL_t$ have opposite signs where $\Delta$ indicates the first differences in the series that are presented in Table 2. The benchmark market illiquidity series are $m\lambda_t$, the logarithm of monthly cross-stock equally-weighted average of Kyle's (1985) $\lambda$ estimated from intraday transactions and quotes data, reflecting price reaction to order flow, provided by Huh (2014); $mQSP_t$, the logarithm of the value-weighted market average of the quoted relative bid-ask spread, the dollar spread divided by the spread midpoint, using CRSP daily data for NYSE\AMEX stocks; $mESP_t$, the logarithm of the market average of the effective relative bid-ask spread, calculated by Abdi and Ronaldo (2017); $iPSIlliq_t$, the innovations in the market liquidity series of Pastor and Stambaugh (2003) multiplied by -1 to make it an illiquidity series. The sample period is 1/1950-12/2016, 804 months. The series $m\lambda_t$ is available for 1983-2009 (324 months), the series $\Delta mQSP_t$ is available for 1993-2016 (288 months), and the sample period for $iPSIlliq_t$ is 8/1962-12/2016 (653 months). N is the default number of months in each estimation and n is the sample size for a particular variable with a shorter sample period. The numbers in parentheses are *t*-statistics. The numbers in Panel A are in percent. Panel B presents the pair-wise correlations of $\Delta mILLIQ_t$ and $\Delta mIDVOL_t$ with the other variables.

| | | Panel A: Means of variables | | Panel B: Correlations | |
|---|---|---|---|---|---|
| | | (1) | (2) | (3) | (4) |
| | | $\Delta mILLIQ_t > 0$ & $\Delta mIDVOL_t < 0$ | $\Delta mILLIQ_t < 0$ & $\Delta mIDVOL_t > 0$ | …with $\Delta mILLIQ_t$ | … with $\Delta mIDVOL_t$ |
| (1) | N | 75 | 91 | 804 | |
| (2) | $RMrf_t$ | -2.585 (-4.57) | 1.274 (2.94) | -0.500 | -0.235 |
| (3) | $SMB_t$ | -0.903 (-3.02) | 0.281 (1.17) | -0.350 | -0.184 |
| (4) | $\Delta m\lambda_t$ (n = 27, 38) | 5.881 (1.84) | -5. 237 (-1.90) | 0.326 | 0.040 |
| | | | | n = 323 | |
| (5) | $\Delta mQSP_t$ (n = 38, 42) | 4.952 (1.46) | -6.284 (-2.02) | 0.315 | 0.066 |
| | | | | n = 288 | |
| (6) | $\Delta mESP_t$ | 13.387 (7.47) | -9.579 (-6.83) | 0.474 | -0.055 |
| (7) | $iPSIlliq_t$ (n = 57, 71) | 2.149 (2.01) | -1.720 (-3.17) | 0.307 | 0.069 |
| | | | | n = 653 | |

**Figure A.1. Market Illiquidity Series *MILLIQ* and *MIDVOL* during Two Financial Crises**
This figure depicts the time-series behavior of market illiquidity series during each of the stock market crises in 1987 and 2008. We use the value-weighted market average series of $MILLIQ_t$ and $MIDVOL_t$ which are similar to $mILLIQ_t$ and $mIDVOL_t$, respectively, except that we do not take the logarithmic transformation. Similarly, we employ three illiquidity benchmark series: $M\lambda_t$, $MESP_t$, and $MQSP_t$, based on $m\lambda_t$, $mESP_t$, and $mQSP_t$, whose details are provided in Table A.6. The values presented in the plots are relative to their average levels of series in the first half of the year when each crisis occurred. A value above (below) 1 means an increase (decrease) relative to the average level in the first half of the corresponding year.

The left (right) panel presents the monthly series of $MILLIQ_t$ and $MIDVOL_t$ with the benchmark illiquidity series for 1987 (2008) relative to their average levels in the first half of 1987 (2008). The crisis occurred in October, 1987 (2008).