Do Put Warrants Unwind Short-Sale Restrictions? Further Evidence from the Taiwan Stock Exchange

Abstract

This study investigates the relationship between stock short-selling restrictions and bearish equity warrants on the Taiwan Stock Exchange—one of the top warrant trading markets—to clarify the substitutive role of put warrants for underlying stocks subject to short-sale constraints. We show that put warrant transactions increase when short selling is prohibited in the spot market and the substitutive increase in trading also leads to wider bid-ask spreads and higher implied volatility for put warrants. Our findings imply that put warrants give pessimistic investors or speculators opportunities to execute bearish transactions when they face short-sales constraints, but these traders also pay higher transaction costs to resolve prohibited short selling. Moreover, we find that the increased trading activities in put warrants could subsequently affect spot market trading through warrant issuers' required dynamic hedging behaviors, providing further support for the substitution effect documented in our analysis.

Keywords: Short-sales restrictions; Short-Selling ban; Put warrants; Bid-ask spreads; Implied volatility

1. Introduction

When an equity market is subject to short sales constraints, the stock short positions are not conductible and investors must seek alternative assets to hedge, realize their pessimistic viewpoints in trading, and arbitrage their private information predicting a negative market impact. This study examines whether bearish equity-linked securities can serve as substitutes for stock short selling. We are particularly interested in equity put warrants, which have received less attention in prior studies. Though the association between bearish equity derivatives and stock short sales has been examined in the literature, prior studies focus more on equity options and the findings are mixed. For example, Figlewski and Webb (1993) argue that investors with short-sale constraints buy put options as substitutes for selling stocks directly. Danielsen and Sorescu (2001) show that the introduction of options trading can alleviate the inefficiency of underlying stocks caused by short-sale constraints. Mayhew and Mihov (2005) use firm size, short interest, and the number of outstanding shares as proxies for implicit short sales restrictions and examine the impacts on the options market. They find that the trading volume of bearish options does not increase when the stock market is subject to short-sale constraints, indicating that the bearish option does not serve as an alternative short-selling instrument.

In a more recent study, Grundy, Lim, and Verwijmeren (2012) examine whether put options served as substitutes for short sales during the September 2008 short-sale ban in the U.S. stock market. They find that put options volume underwent a significant diminution during the ban period. Hao, Lee, and Piqueira (2013) show that short sales tend to be more effective in reflecting bad news information than put options when traders are informed. Given their higher leverage, trading put options is theoretically more profitable than short selling stocks; the findings of Hao et al. (2013), however, imply that investors still prefer stock short selling and do not consider put options to be substitutes.

This study's research question is, can individual stock warrants—the active equity-linked derivatives particularly in Asia-pacific countries—serve substitutes for short selling in the stock market? Focusing on the Taiwan Stock Exchange (TWSE), one of the leading emerging security markets where equity warrants trading is growing rapidly, we examine the relationship between short selling and bearish warrant trading. Related research (e.g., Blau and Wade, 2013; Chen, Chen and Chou, 2019; Blau and Brough, 2015; DeLisle, Lee and Mauck, 2016; Li, Zhao and Zhong, 2016; Lin and Lu, 2016) largely focuses on put options trading because of the flourishing equity options market in the U.S. By contrast, in Taiwan and many other Asian markets such as Hong Kong and South Korea, the equity warrants market is much more liquid and popular than the equity options market, making put warrants more attractive than options to hedgers and speculators in Asian markets and allowing us to examine the information role of these less-explored bearish equity-linked derivatives.[1, 2]

In addition, focusing on warrants rather than options is more suitable for examining the substitution effect of equity-linked securities. In the U.S., options are traded in the futures exchange while, in Taiwan, equity warrants are traded in the stock exchange. If investors really view equity-linked derivatives as alternative instruments for short selling, the substitutes in the same exchange, e.g., stock warrants, should be considered first to avoid latent frictions in searching cross-market instruments. In this regard, put warrants

[1] Hong Kong, Taiwan, and South Korea are usually the top 3 warrants markets in the Asian region. In 2015, Hong Kong, Taiwan, and South Korea were ranked as the No. 1, No. 5, and No. 6 warrants markets in the world.

[2] Li and Zhang (2011) find that derivative warrants typically have higher prices than otherwise identical options in Hong Kong, and the price difference reflects the liquidity premium of derivative warrants over options. Newly issued derivative warrants are much more liquid than options with similar terms.

should be more convenient short-selling substitutes than put options.

Furthermore, the mechanism of the Taiwanese stock market is more suitable than that in the U.S. for examining the influence of bearish equity derivatives under stock short-sale restrictions. In the U.S., stocks are usually traded without short-selling constraints; therefore, restrictions are only implemented as bans for extreme market statuses and their influence can be observed and examined for specific periods. For instance, Grundy et al. (2012) analyze short-sale constraints during the ban period caused by the 2008 financial crisis. On the contrary, prior to 2013, the selected stocks on the TWSE could be sold short directly when the rest of listing companies were regularly subject to short-sale restrictions. By taking advantage of these characteristics, our study can compare short-selling-restricted stocks and short-selling-unrestricted stocks for all available trading days over a relatively long period rather than merely focusing on a specific period of time, thereby generating more insights.

As for studying the extraordinary effects resulting from specific events, e.g., the global financial crisis, analyses of Taiwanese samples also have an advantage over those from other countries. During the 2008 financial crisis period, many countries adopted short-selling bans with different ban settings (see, for example, Beber and Pagano, 2013). However, among major stock markets with active stock options/warrants trading, Taiwan was the only market to launch a short-selling ban for all stocks and the duration of the ban was much reasonable.[3] By comparison, U.S. regulators banned short sales only for specific financial stocks from September 19 through October 8, 2008. In Asian markets, South Korea banned all stocks beginning in early October 2008, but the prohibition lasted until June 1, 2009, which is longer than eight months. Similarly, Japan banned all stocks

[3] The ban started from September 22, 2008 to the end of 2008.

beginning on October 30, 2008, and the ban remained in place even longer, lasting until after mid-2009.[4]

The short-selling ban implemented on the TWSE during the financial crisis period is an ideal experiment for testing the substitution effect of bearish equity-linked securities for two reasons. First, if a short-sale ban is not for all stocks, investors can still find other unrestricted and closely-related companies to sell short instead of purchasing put options or warrants directly. Only comprehensive short-sale bans on all stocks in a market can force investors to seek alternative instruments for equity short selling. Second, if a ban is long-lasting, rather than simply remaining in the derivatives market for their short-selling substitutes in the local market, investors will likely consider moving their transactions to other stock markets without short-sale constraints. When investors expect that a short-selling ban is temporary, they are more likely to stay in the same market and look for trading substitutes that concur with the constraints. Conversely, if investors do not expect the trading ban to terminate soon, they will be compelled to leave for other unrestricted stock markets, which would make examining their substitutive trading behaviors difficult.

Another factor that distinguishes our research from prior studies is that in addition to the substitution effect between put warrants and stock short sales, we test the impact of stock short-sale restrictions on bid-ask spreads and the implied volatility of put warrants—aspects that previous studies have not explored. If put warrants do serve as substitutes for short selling, we expect that increased warrants volume will lead to wider bid-ask spreads and higher volatility. The spread of an asset is generally interpreted as an indicator of the uncertainty and information asymmetry that market makers face. As the

[4] Hong Kong has very active equity warrants market, but it did not launch any short-selling ban in the stock trading for the 2008 financial crisis.

order flow of warrants increases, the market makers who face greater information asymmetry should widen the bid-ask spread. Furthermore, asset returns fluctuate in response to emerging information embedded in the warrants trading order flow. If put warrants written on short-sale restricted stocks have higher trading volume, the implied stock volatility should also be higher. In addition, a short sales ban could make stock prices upwardly biased.

Miller (1977) posits that asset prices tend to be overvalued when short sale bans prevent pessimistic investors from entering a market. Chang, Cheng, and Yu (2007) show that the overvaluation of stock prices is related to short sale restrictions in the market. If investors move to the put warrants market for substitutive trading, the implied prices of the assets underlying the put warrants should be lower than their spot prices, resulting in higher warrants prices and implied volatilities. Lin and Lu (2015) make a similar argument, contending that the transfer of trading from the stock market to the options market reflects informed traders' opinions and can be observed in option implied volatility. The first study to show that the predictive power of option implied volatilities on stock returns more than doubles around analyst-related events, Lin and Lu (2015) show that a significant proportion of options predictability in stock returns comes from informed options traders' information about upcoming events. The studies described above provide theoretical and empirical support for our conjectures.

In sum, we conduct empirical analysis in three steps. First, we identify our sample companies based on their short-sale constraint status. In each trading day within the sample period, stocks are classified either as "restricted" or "unrestricted" for short selling activities. Next, we examine whether being a short-selling-constrained stock will cause an increase/decrease in the trading volume, bid-ask spreads, and implied volatility of put warrants. Meanwhile, we test the influence of a comprehensive short-selling ban during the 2008 financial crisis in Taiwan. Finally, we perform two-stage regressions to deal with possible endogeneity between put warrants trading volume, bid-ask spreads, and implied volatility. We also conduct robustness tests to validate our findings.

Our key findings in this study are as follows. First, we show that equity short sales and put warrants purchases substitute for each other. A would-be short seller on the TWSE will make substitute trades by purchasing put warrants, meaning restrictions on stock short selling increase the trading volume for corresponding put warrants. Second, we find that the cost of trading warrants, measured by bid-ask spreads, tends to be wider in stocks with short-sale constraints. Also, the implied volatility of put warrants is higher for underlying stocks with short-sell restrictions. Both findings suggest that increases in put warrants trading raise the implicit trading costs of put warrants and make the warrants more expensive as well. Third, we show that during the ban period—when short selling was prohibited for all stocks on the Taiwan stock market—the aggregated trading volume of put warrants declined but *pro forma* unrestricted stocks had consistently higher put warrants trading volumes than restricted stocks.[5]

Finally, by taking advantage of a unique dealers' transaction dataset on the Taiwan Stock Exchange, we find that, under the stock short-sale restriction, market makers' hedging demands for put warrants positively increased dealers' trading in the spot market. This finding indicates that there is indirect selling pressure on restricted stocks via dynamic hedging of dealers' exposures on put warrants. The transfer of increased trading pressure in puts can be attributed to the exemption for market makers who engage in

[5] To be clear, we use the term "*pro forma* unrestricted stocks" to refer to stocks that were restriction-free before the ban period. Although all stocks, including *pro forma* unrestricted stocks, were not allowed to be sold short during the ban period, we use "*pro forma* unrestricted stocks" to differentiate these stocks from stocks that were subject to short-selling constraints (restricted stocks) before the ban period.

making markets for covered warrants. Moreover, Chung, Liu, and Tsai (2014) argue that warrant issuers in Taiwan must conduct delta-neutral strategies to hedge their risks on warrants short positions. This regulation also potentially explains why we find a positive relationship between warrants trading volume and dealers' stock position given the presence of short-selling restrictions.

Contrary to our main findings, both of Battalio and Schultz (2011) and Grundy et al. (2012) find a complementary relationship between bearish equity derivatives and stock short sales. Our reported results, however, can still be compatible with these studies. The conclusions of Battalio and Schultz (2011) and Grundy et al. (2012) are based on put options trading in the U.S., while ours are based on put warrants trading in an Asian market. The differences indicate that different types of bearish equity derivatives do not play the same roles in response to prohibitions on short selling activities. Furthermore, previous findings claim bearish equity options are complementary to stock short positions, but these examinations focus on the influence of the temporary ban initiated by the 2008 financial crisis. Without questioning the validity of previous findings, we believe the reported complementary effects should be more cautiously interpreted for specific periods of time and not necessarily viewed as common impacts for the market during the entire period. The decreased put options volumes for stocks with short-selling constraints may, to some extent, represent the conservatism of investors around the crisis period but not fully relate to substitution or complementary concerns.

To the best of our knowledge, ours is the first study to provide new evidence regarding the substitution effect between short sales and bearish equity warrants. The specific features of the TWSE—the longer sample period, the regular short-selling rules, the advantage of trading in the same exchange, and the ideal implementation of the short-selling ban during the global financial crisis period—allow us to perform a more general analysis to reveal the roles of bearish derivatives in serving for equity short sales. In addition, we provide evidence of the transfer of increased trading in puts to dealers' stock positions under short-selling restrictions, a finding that suggests that investors' short positions in derivatives produce indirect impacts on the spot market via market makers' hedging activity for stock warrants. Our findings have important implications for policy makers in Asian capital markets and will benefit the development of this area of research.

The remainder of our study is organized as follows. Section 2 briefly reviews related research; Section 3 describes short-selling restrictions on the TWSE and introduces the warrant market in Taiwan, the sample selection, and the main variables; Section 4 introduces our regression models and the empirical results; and Section 5 concludes the paper.

2. Literature Review

Short-selling activities have been studied for a long time. Some early studies, e.g., Miller (1977), argue that short sale bans prevent pessimistic investors from entering the market and cause price inefficiencies. Moreover, many empirical studies apply the theoretical model introduced by Diamond and Verrecchia (1987), which formulizes the relation between constraints on short selling and asset price adjustment to private information. Most empirical findings concern the information content of short-selling activities. For example, Boehmer, Jones, and Zhang (2008) and Diether, Lee, and Werner (2009) show that short-sellers tend to have valuable information regarding stocks.

While the model developed by Diamond and Verrecchia (1987) indicates that short sellers are informed, Easley, O'Hara, and Srinivas (1998) propose a model hypothesizing that informed traders can establish short positions in the options market when short selling is obstructed by market frictions, e.g., trading costs. In making this argument, Easley et al. (1998) initiated follow-on research in studying the associations between stock short sales and options market trading. For example, Lamont and Thaler (2003) find that put options on subsidiary stocks are much more expensive than call options for equity carve-out stocks, meaning that the high associated short-sale costs around the events lead investors to create synthetic short positions in the options market when they bet against the price movements of these stocks. Both Ofek, Richardson, and Whitelaw (2004) and Evans, Geczy, Musto, and Reed (2009) find that, due to short-sale restrictions, stock market values may be consistently higher than those implied by the options markets, indicating that the options market is more informed and options implicitly serve as alternative instruments for bearish trading strategies. Johnson and So (2012) show that relative options trading volumes have even better prediction abilities for negative stock returns when the short-sale costs of stocks increase. In a recent theoretical prediction study, Atmaz and Basak (2019) show that bid-ask spreads for options, put option implied volatilities, and put-call parity violations are positively related to short-sell cost. The above studies demonstrate that options and short-selling activities might be influenced by each other as well.

Similarly, some studies directly examine whether the introduction of individual equity options can unwind short-sale restrictions. Figlewski and Webb (1993) argue that because of short-sale restrictions, put prices are high relative to call prices when there is a large investor demand for short positions. They also show that short interest increases near options listing dates and suggest that options market makers must hedge the risk of writing puts through short selling. Therefore, if pessimistic investors buy puts or write calls as substitutes for selling short, either to hedge against future price drops or to speculate on potential returns, put prices increase and call prices decrease. Therefore, puts become unusually expensive and the implied volatilities of puts will be higher as well. Recognizing that some previous studies find associations between post-1980 option introductions and negative abnormal returns in underlying stocks, Danielsen and Sorescu (2001) provide evidence that documented abnormal returns around option listing are consistent with the mitigation of short sale constraints resulting from option introductions.

Some recent studies examine the influence of short-sale constraints by analyzing the short-selling ban in the stock market around the 2008 global crisis period in the U.S or other countries. For example, in studying the 2008 short-sale ban around the world during the financial crisis, Beber and Pagano (2013) find that short-sale bans were harmful to liquidity, but the influence of short selling restrictions on bid-ask spreads was more intense for stocks without listed options than for those with listed options. They suggest that for stocks with listed options, investors can use the options market to gain short exposure during short-sale ban periods.

Also, Battalio and Schultz (2011) find that the 2008 short-sale ban significantly increased the options bid-ask spread for banned stocks. However, when investigating the ratio of option-to-stock volume for U.S. markets during the 2008 short-sale ban period, Battalio and Schultz find no evidence that investors seeking short exposures in banned stocks (financial companies) migrated to the options market. Similarly, Grundy et al. (2012) find a sizeable decrease in option trading volumes for banned stocks relative to unbanned stocks during the 2008 short-sale ban period. They argue that, under the short-sale ban, options trading volume does not experience significant transfer from the stock market to the options market due to the increased options spread. Grundy et al. conclude that, given the constraints on short sales, the complementary effect of an increase in the transaction cost component of the put's price is stronger than the substitutability effect of an increase in the demand for puts given a ban on short sales. In this regard, short-sale bans seem to act as valid restrictions but not driving forces on trading in options.

Although short-selling restrictions limit investors to taking naked short positions in stocks, the spot market may also suffer impacts from derivatives market makers' hedging activity. Some papers (e.g., Ni, Pearson, and Poteshman, 2005; Henderson and Pearson, 2010; Ni, Pearson, Poteshman, and White, 2018) empirically investigate whether and to what extent such derivatives hedging activities impact the spot market. Furthermore, Chung, Liu, and Tsai (2014) examine the impact of covered warrants hedging on the TWSE and find that hedging demand has a significantly pervasive effect on trading volume during the lives of warrants. These papers also motivate us to investigate how derivatives hedging impacts the spot market when naked short selling in stocks is banned.

While the empirical studies summarized above provide various findings regarding the role of bearish equity options and the influence of short-sale bans on stock markets, these studies do not attempt to investigate other equity-linked derivatives, particularly in environments where warrants markets are more active than options markets, e.g., some major Asian economies. We contribute to this strand of literature by directly investigating the effect of equity short selling on the put warrants market in Taiwan, one of the major financial markets in the Asian region, aiming to shed light on a potential complementary/substitutive effect for different bearish equity instruments within the same exchange. For the Taiwan stock market, the features of short-selling restrictions in regular trading days and the ad hoc setting of the short-sale ban for the 2008 financial crisis allow us to achieve a better understanding of the effects of both put warrants and short selling on stocks.

3. Data Description and Methodology

3.1 Short-sale constraints on the TWSE

The Ministry of Finance implemented short sale constraints on the TWSE on September 4, 1998.[6] The restriction prevents investors from short selling stocks when the share prices are running below the closing price of the last trading day or if the closing price of last trading day drops to the lower price limit on the TWSE.[7] The short-selling restriction on the TWSE is similar to the "uptick rule" on the U.S market. The uptick rule requires that investors sell stocks short only when their trading price is higher or not lower than the previous trading price for the same stock.[8] On May 16, 2005, the TWSE unwound the short-sale constraints for 50 blue-chip stocks (TWSE 50 Index constituents); on November 12, 2007, the TWSE further removed the restriction for some major high-tech companies and the constituent stocks of the Mid-Cap 100 Index. On September 23, 2013, the TWSE discontinued the short-selling constraints for most stocks in the market except for when stock prices in the last trading day closed at the lower price limit. Because of the global financial crisis, the TWSE launched a temporary short-selling ban for all stocks in the market from September 22, 2008 to December 31, 2008.

3.2 Warrants market in Taiwan

Stock warrants have been traded on the TWSE since September of 1997, preceding

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[6] The short-selling constraints were launched on the TWSE because of the Asian financial crisis in 1997-1998.

[7] Before May 31, 2015, the upper/lower price limit on the TWSE was +7%/-7% based on the closing price of last trading day. Beginning on June 1, 2015, the price limit was unwound to +10%/-10%.

[8] The SEC adopted a Rule 10a-1, also known as "tick test". The rule mandates that a short sale can only occur at a price above the most recently traded price (plus tick) or at the most recently traded price if that price exceeds the last different price (zero-plus tick).

the introduction of futures trading in Taiwan. According to reported TWSE statistics, Taiwan's warrant market was the second largest in Asia and the fifth largest among global markets by the end of 2015. All equity warrants on the TWSE are issued by securities firms. Warrants on the same underlying stocks can be issued more than once by the same issuer and different issuers. The security firm has to apply for the approval of the Financial Supervisory Commission (FSC) to issue new warrants. The issuer is allowed to keep a portion of the whole issuance as inventory to make the market for warrant transactions. Subject to the regulations, the security firms that issue warrants are responsible for acting as dealers or market makers for warrant trading. Since stock warrants are traded on the TWSE, the trading mechanisms for warrants and the underlying stocks are identical. Therefore, warrants are essentially traded like common stocks.

To further facilitate the diversification of financial instruments and provide investors a hedging platform, put warrants were introduced in July 2003 on the TWSE, while the tradable index products—ETFs—were also launched around the same time. The call and put warrants on ETFs were launched in July 2004. By the end of 2015, there were 10,542 warrants on the TWSE, but only 825 warrants had ETFs as their underlying assets. More than 90% of the warrants on the TWSE are individual stock warrants.

3.3 The sample selection

Given that the short-selling constraints were initially removed for some stocks on the TWSE on May 16, 2005, our sample period starts from this date and extends to December 31, 2015; thus, it covers a nearly 11-year period—much longer than other studies in the literature. The main dataset used in this study contains all put warrants traded on the TWSE during this period. We obtain the data on put warrants from the Taiwan Economic Journal (TEJ) database, which provides data about warrants' characteristics (e.g., announcement date, listing date, strike price, time to maturity, exercise ratio), the number of shares for warrants issued, daily trading prices, and daily trading volume. Furthermore, we collect spot market data for underlying stocks from the same database, including daily closing prices, high and low prices, daily stock trading volume, and stock index prices.

We set up some criteria to exclude the observations of illiquid warrants or those that have very spare trading volumes: (1) the number of trading shares is less than 10 in more than half of all available trading days in a month; (2) the time to maturity is greater than 365 days or less than 30 days; (3) the bid-ask spread is greater than 0.5; and (4) the implied volatility is unavailable.[9]

3.4 Put warrant trading volume, bid-ask spreads, and implied volatility

Our analysis is based on daily observations obtained from the TWSE. We examine the daily trading volume of each warrant. In untabulated results, we also consider dollar volume numbers to measure warrants market trading activities; the results are qualitatively similar to our reported findings. To examine the implicit trading costs of warrants, we estimate put warrant bid-ask spreads. Due to the unavailability of intra-day bid-ask spreads data for warrants for our study, we adopt the approach developed by Corwin and Schultz (2012) to estimate bid-ask spreads. This approach requires only the daily high and low prices of two consecutive trading days to estimate daily bid-ask spreads. Corwin and Schultz show that the bid-ask spreads estimated using their approach perform as well as actual daily bid-ask spreads. The basic formula we use to estimate the warrant bid-ask spreads is listed below:

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[9] Lin and Lu (2016) apply similar filters to mitigate the influence of illiquid options and eliminate possible recording errors.

$$[Ln\left(H_t^o / L_t^o\right)]^2 = \left[Ln\left(\frac{H_t^A\left(1+S/2\right)}{L_t^A\left(1-S/2\right)}\right)\right]^2 \tag{1}$$

S(%) is the bid-ask spread of a warrant. The equation assumes that the daily high prices are buyer-initiated trades that increase by half of the spread, and the daily low prices are seller-initiated trades that decrease by half of the spread. Then $H_t^o(L_t^o)$ is the observed high (low) price on day *t*, and $H_t^A(L_t^A)$ is the actual high (low) price on day *t*.[10] Appendix A describes all the details of this estimation. Finally, to estimate implied volatility of warrants, we apply the well-known Black-Scholes pricing formula to back out the implied volatilities for European put warrants and estimate the implied volatilities of American put warrants using a binomial tree model.

3.5 Description statistics

There are more European puts warrants than American put warrants on the TWSE. Although the first put warrant was issued in 2003, the number of outstanding put warrants was initially low and only started growing substantially in 2008. Our sample reflects these characteristics. Table 1 presents descriptive statistics for the put warrants used in our analysis. Within our sample period, 439 stocks had ever had put warrants outstanding. In total, nearly 17,000 put warrants were traded on the TWSE, given that 12% of put warrants were European contracts. We report the average daily trading volume, the bid-ask spread, and the implied volatility of put warrants by years in the last three columns of Table 1. The overall average daily put warrants trading volume across all years is 234, but the number decreased over time, particularly in relatively recent years, implying that

[10] As noted in Corwin and Schultz (2012), the estimated high-low spread can be negative for some two-day periods. In these cases, they suggest making several adjustments for negative daily values, including setting the negative values to zero, treating them as missing, or possibly setting them as equal to the minimum tick size divided by the closing price. In this study, we apply the third method to avoid bias.

different puts warrants also substituted for each other in trading. The average put warrants bid-ask spread and implied volatility are 0.097 and 0.6395, respectively, and both of them tended to decrease slightly over time, suggesting that the put warrants market became more liquid as the whole market grew.

[Table 1 about here]

Using the underlying stocks associated with our put warrant sample, we classify stocks based on the status of their short-selling restrictions. Table 2 shows the number of "unrestricted" stocks. Within our sample period, each of the TWSE 50 or Med-Cap 100 constituents was free from short-sale constraints. For the rest of other listing stocks, the TWSE did not allow investors to execute short-selling transactions until September 23, 2013 when the short-selling restrictions were basically removed from all stocks on the TWSE with a few exceptions—for example, if the net asset value per share was less than $10 NTD, or when stock prices in last trading day closed at the lower price limit. Given the mechanism, we can specify whether a stock had short-selling constraints for trading on a specific day. We label the stocks subject to short-sell constraints as restricted stocks and the ones without the restrictions as unrestricted stocks.

The first column of Table 2 shows the total number of unrestricted stocks on the TWSE across each year. As the table shows, there were distinct increases in the number of stocks in 2007 and 2013, which were associated with policy deregulations in 2007 and 2013, respectively. For the comparison, the second column reports the numbers of unrestricted stocks with outstanding put warrants. On average, only nearly one-third of unrestricted stocks (281 out of 834) had active put warrants traded on the TWSE. This implies a possible substitution effect: when investors can execute a bearish strategy by short selling stocks directly without constraints, the put warrants trading demand becomes relatively weak.

[Table 2 about here]

4. Regression Models and Empirical Results

4.1 The effect of short-sale restrictions on put warrants trading

To examine the relationship between short sales restriction and put warrants trading, we conduct the following regression:

$$\begin{aligned} Put\ volume_{j,i,t} &= \beta_0 + \beta_1 D_{i,t}^{Unrestricted} + \beta_2 Stock\ volume_{i,t} + \beta_3 Stock\ return_{i,t} \\ &+ \beta_4 Stock\ return_{i,t-1} + \beta_5 Index\ return_t + \beta_6 Index\ return_{t-1} \\ &+ \beta_7 Ln(Size)_{i,t} + \beta_8 (Time\ to\ maturity)^{-1}{}_{j,i,t} + \beta_9 Stock\ spread_{i,t} \\ &+ \varepsilon_{j,i,t} \end{aligned} \tag{2}$$

where $Put\ volume_{j,i,t}$ is the daily trading volume for put warrant *j* written for stock *i* on day *t*. $D_{i,t}^{Unresticted}$ is a dummy variable that equals 1 if stock *i* is not subject to the short-sale restriction on day *t* and equals 0 otherwise. The coefficient of $D_{i,t}^{Unresticted}$ will explain whether short-sale constraints cause a substitutive (negative β_1) or complementary effect (positive β_1) in put warrants trading. For the regression, we also control the variables related to the warrants market. $Stock\ volume_{i,t}$ is the trading volume for stock i on day *t*. $Stock\ return_{i,t}$ is the daily return for stock i on day *t*; $Index\ return_t$ is the Taiwan Weighted Stock Index return on day *t*; $Ln(Size)_{i,t}$ is the natural logarithm of market value for stock *i* on day *t*. $(Time\ to\ maturity)^{-1}{}_{j,i,t}$ is a reciprocal of time to maturity for put warrants. $Stock\ spread_{i,t}$ is defined as the closing ask price minus the closing bid price then divided by the midpoint of the closing bid and ask price.

In addition, to examine the overall impact of short-selling actions on warrants trading, we also investigate the influence of the 2008 short-selling ban on the put warrants market. To this end, we adopt extra variables to capture additional impacts during the global crisis period. Equation (2) is modified as follows:

$$
\begin{aligned}
&Put\ volume_{j,i,t} \\
&\qquad = \beta_0 + \beta_1 D_{i,t}^{Unrestricted} + \beta_2 D^{Crisis} + \beta_3 D^{PostCrisis} \\
&\qquad + \beta_4 D^{Crisis} \times D_{i,t}^{Unrestricted} + \beta_5 D^{PostCrisis} \times D_{i,t}^{Unrestricted} \\
&\qquad + \beta_6 Stock\ volume_{i,t} + \beta_7 Stock\ return_{i,t} + \beta_8 Stock\ return_{i,t-1} \\
&\qquad + \beta_9 Index\ return_t + \beta_{10} Index\ return_{t-1} + \beta_{11} Ln(Size)_{i,t} \\
&\qquad + \beta_{12} (Time\ to\ maturity)^{-1}{}_{j,i,t} + \beta_{13} Stock\ spread_{i,t} + \varepsilon_{j,i,t}
\end{aligned}
\tag{3}
$$

D_t^{Crisis} is a dummy variable that equals one if day *t* is between September 22, 2008 and December 31, 2008, and zero otherwise. The coefficient of D_t^{Crisis} of Equation (3) will show us how put warrants trading behaved in the short-selling ban period, while the interaction term $D_t^{Crisis} \times D_{i,t}^{Unresticted}$ of Equation (3) further examines how put warrants linked to the stocks that were supposed to be unrestricted from short selling reacted to the ban. The regression results for Equations (2) and (3) are reported in Table 3 with/without controlling fix effects of years and industries. The estimated standard errors are two-way clustered by firm and year.

As reported, the first two columns of Table 3 show that the coefficients of $D_{i,t}^{Unresticted}$ are significantly negative across different model specifications. This finding indicates that the stocks with no short-selling restriction tended to have lower put warrants trading volumes. In other words, when investors were not able to freely execute bearish trading in the spot market, they transferred to put warrants as an alternative trading instrument, suggesting the existence of the substitute effect between short selling and put warrant trading.

[Table 3 about here]

The next two columns show that during the ban period resulting from the 2008 global financial crisis, the overall put warrants trading volume decreased distinctly given that D_t^{Crisis} has a consistently significantly negative impact across different models. This result implies that there was a market-wide decline in all trading activities during the ban period. When facing the high uncertainty that accompanies a global financial crisis, investors tend to trade more conservatively. Interestingly, when $D_t^{Crisis} \times D_{i,t}^{Unresticted}$ is included in the regression, without changing the signs of the coefficients of $D_{i,t}^{Unresticted}$ and D_t^{Crisis}, the interaction term is significantly positive. For example, as column 6 shows, the ban period decreased daily put warrant trading volume by an average of 560 contracts for restricted stock and 477 contracts (i.e., -560+83) for unrestricted stock. This finding indicates when the entire put warrants market shrunk during the ban period, put warrants played a substitutive role only for stocks that were initially unrestricted in short selling. The ban made these *pro forma* unrestricted stocks restricted stocks *per se*, and forced investors to use put warrants as their alternative instrument to execute bearish trading strategies.

Combined with previous results, the findings in Table 3 show that put warrants essentially functioned as substitutes for bearish stock trading. The substitutive effect can be seen for pro forma unrestricted stocks when actually all stocks in the market were prohibited from short-selling activities. In addition, the Index return_t, $\text{Index return}_{t-1}$,

$\text{Stock return}_{i,t}$ and $\text{Stock return}_{i,t-1}$ coefficients are significantly negative, suggesting that investors trade more put warrants when the market or individual stock prices are going down. The positive coefficient of $\text{Stock spread}_{i,t}$ implies that investors move to the warrant market when the stock market is illiquid. The coefficient of $(\text{Time to maturity})^{-1}{}_{j,i,t}$ is significantly negative, meaning that investors tend to trade put warrants with longer times to maturity.

4.2 The effect of the short sales ban on put warrants bid-ask spreads

The reported findings in Section 4.1 show that put warrants substitute for stock short sales on the TWSE. This raises the question, when equity short-selling restrictions motivate stock investors to shift their transactions to the put warrants market, what is the concurrent impact on trading quality? Now we shift our focus to the bid-ask spreads of put warrants. To investigate the relationship between short sale restrictions and the bid-ask spreads of put warrants, we adopt a regression model similar to Equation (3), but replace the dependent variables with bid-ask spreads:

$$\begin{aligned} Put\ spread_{j,i,t} \\ &= \beta_0 + \beta_1 D_{i,t}^{Unrestricted} + \beta_2 D^{Crisis} + \beta_3 D^{PostCrisis} \\ &+ \beta_4 D^{Crisis} \times D_{i,t}^{Unrestricted} + \beta_5 D^{PostCrisis} \times D_{i,t}^{Unrestricted} \\ &+ \beta_6 Moneyness_{j,i,t} + \beta_7 Moneyness_{j,i,t}^2 \\ &+ \beta_8 (Time\ to\ maturity)^{-1}{}_{j,i,t} + \beta_9 Stock\ spread_{i,t} + \varepsilon_{j,i,t} \end{aligned} \tag{4}$$

$Put\ spread_{j,i,t}$ is the bid-ask spread of put warrant *j* written for stock *i* on day *t*. $Moneyness_{j,i,t}$ is the natural logarithm of the underlying stock price relative to the strike price. Other variables are defined as previous regression models. We report all the regression results in Table 4.

The first two columns show that $D_{i,t}^{Unresticted}$ is significantly and negatively related to $Put\ spread_{j,i,t}$, indicating that in general the bid-ask spreads of put warrants are narrower for the stocks without short-selling restrictions. On the TWSE, when warrant market makers write put warrants, they must hedge their positions and short sell underlying stocks.[11] Given that traders in the market are more willing to hold unrestricted stocks to avoid short-selling restrictions, the stocks without short-sale restrictions should be more liquid. Since market makers face lower liquidity costs when selling unrestricted stocks short, they charge less compensation when writing put warrants. This mechanism explains the narrower put warrant bid-ask spreads found in the stocks with no short-sale constraints.

[Table 4 about here]

The next two columns show that the influence of D_t^{Crisis} on $Put\ spread_{j,i,t}$ is significantly positive in each model specification, suggesting that during the global crisis period, the bid-ask spreads of put warrants tended to be wider than those in the unbanned period since all stocks were subject to the short-sale ban. This finding is consistent with our liquidity costs explanation for the narrower spreads of put warrants on short-selling-unrestricted stocks.[12] The last two columns of Table 4 show the additional influence of the short-selling ban to the stocks that were originally unrestricted for short sales in the unbanned period. The interaction term $D_t^{Crisis} \times D_{i,t}^{Unresticted}$ has a significantly positive impact on $Put\ spread_{j,i,t}$, suggesting that the bid-ask spreads of put warrants

[11] Unlike other types of investors, put warrant market makers on the TWSE are exclusively allowed to short sell stocks for hedging purposes regardless of whether the underlying stocks are restricted or unrestricted for short sales.

[12] This finding is also similar to Battalio and Schultz (2011) who attributed their results to the uncertainty caused by the regulatory change in the market.

for *pro forma* unrestricted stocks are even widened compared to restricted stocks in the ban period. Given that Table 3 shows that the *pro forma* unrestricted stocks tended to have more put warrants trading in the ban period, we argue that the imbalanced order flow from the demand side forces market makers to widen bid-ask spreads to compensate for the information risk when they trade with implicitly informed investors.

Our findings in Table 4 are consistent with Grundy et al. (2012) who show that short-sale bans increase spreads for put options. However, we explain that the increased put warrant bid-ask spreads is a concurrent outcome caused by increased demand for buying put warrants during the ban period, while Grundy et al. (2012) argue that the increased transaction costs in options market curb demand for put options trading and generate a complementary effect on it. Given Grundy et al.'s findings and explanations, we need to further examine the relationship between trading volume and put warrant bid-ask spreads to verify the substitutive effect we document in Section 4.1 and clarify whether the reduction in put trading volume during the ban period was mainly driven by the short-selling restriction itself but not via the increased trading costs in terms of wider put warrant spread. In later sections, we simultaneously control for trading volume, bid-ask spreads, and implied volatility to address this concern.

4.3 The effect of the short sales ban on implied volatility

In addition to bid-ask spreads, we examine the influence of short-sell constraints on the implied volatility of put warrants. Based on the theoretical definition, the implied volatility of put warrants is equivalent to their prices. We therefore expect that if the stocks underlying put warrants cannot be sold short because of restrictions, the implied volatility of put warrants should be higher because of the higher warrant price driven by increased demand for bearish trading. As in previous sections, we adopt the regression model shown in Equation (4), but replace the dependent variable with the implied volatility of put warrants.

$$
\begin{aligned}
&Implied\ volatility_{j,i,t} \\
&\qquad = \beta_0 + \beta_1 D_{i,t}^{Unrestricted} + \beta_2 D^{Crisis} + \beta_3 D^{PostCrisis} \\
&\qquad + \beta_4 D^{Crisis} \times D_{i,t}^{Unrestricted} + \beta_5 D^{PostCrisis} \times D_{i,t}^{Unrestricted} \\
&\qquad + \beta_6 Moneyness_{j,i,t} + \beta_7 Moneyness_{j,i,t}^2 \\
&\qquad + \beta_8 (Time\ to\ maturity)^{-1}{}_{j,i,t} + \beta_9 Stock\ spread_{i,t} + \varepsilon_{j,i,t}
\end{aligned}
\tag{5}
$$

$Implied\ volatility_{j,i,t}$ is the implied volatility of put warrant j written for stock i on day t, and other variables are defined as in previous equations. Again, we first execute the regressions by having $D_{i,t}^{Unresticted}$ and D_t^{Crisis} individually and then the analysis followed by having $D_{i,t}^{Unresticted}$ and D_t^{Crisis} together with their interaction terms in the model.

The first two columns of Table 5 show that, on average, the implied volatility of put warrants for unrestricted stocks is significantly lower than that for restricted stocks, while the next two columns show that the implied volatilities of put warrants during the ban period were significantly higher than during the unbanned period. These findings are consistent with our expectation that when stocks are prohibited from short-sale trading, investors seek put warrants as substitutes to realize their bearish strategies and, as a result, the implied volatility of put warrants tends to be higher in reflecting investors' prospects about spot prices. These results are also similar to the finding in Evans et al. (2009) that implied volatilities become higher for puts relative to calls under short-selling constraints.

[Table 5 about here]

The last two columns of Table 5 show that the coefficients of $D_t^{Crisis} \times D_{i,t}^{Unresticted}$ are significantly positive in each mode specification, indicating that the implied volatility of put warrants for *pro forma* unrestricted stocks were even higher than *pro forma* restricted stocks during the ban period. The results in Table 5 are comparable to previous tables, showing that the put warrants of *pro forma* unrestricted stocks tended to have a higher trading volume and wider bid-ask spreads during the ban period.

4.4 Simultaneous determination of put warrants spreads, implied volatility, and trading volume

To this point, we show that bid-ask spreads and the implied volatility of put warrants increased in short-selling ban period, while the put warrant trading volume increased if the underlying stocks were subject to short-sell constraints in the overall period—evidence of the substitutive effect for stock short selling. By contrast, Grundy et al. (2012) find that put options trading actually decreased during the ban period because increased spreads made trading costs higher for investors in the options market. Similarly, George and Longstaff (1993) show that bid-ask spreads and trading activity are simultaneously determined, and changes in trading activity influence option bid-ask spread and vice-versa. Also, given the reduction in options trading, increased spreads during the ban period reflect an illiquid options market and could generate higher market uncertainty, which is signaled by the higher implied volatility of options.

Given the complementary effect documented in Grundy et al. (2012) and that put warrant bid-ask spreads, implied volatility, and trading volume may be simultaneously determined, our findings shown in preceding tables may be merely caused by a pseudo substitutive effect resulting from a failure to control endogenous variables. To further address this concern, we adopt a 2SLS regression to control the endogeneity.

Following Lin and Lu (2016), we use stock returns, lag stock returns, index returns, lag index return and firm size as the instrumental variables for put warrant trading volume, and use Moneyness, Moneyness^2, $(\text{Time to maturity})^{-1}$, and stock bid-ask spread as the instrumental variables for put warrant bid-ask spreads and implied volatility.

Table 6 shows the results of 2SLS regression. As the results in the table show, after adding the fitted value of the put warrants bid-ask price, implied volatility, and trading volume, which are estimated in the first stage regression in each respective regression model, we find that coefficients of $D_{i,t}^{Unresticted}$, D_t^{Crisis}, and $D_t^{Crisis} \times D_{i,t}^{Unresticted}$ are similar to those reported in Tables 3, 4, and 5. Our examination by 2SLS regression supports our previous findings, indicating that put warrants are substitutes for stock short selling and also lead to wider bid-ask spreads and higher implied volatility via increased trading volume.

[Table 6 about here]

4.5 Selection bias about market makers

The sample used in this study contains more than 400 put warrants issued by more than 20 issuers. The quality of market makers for warrants trading could be very diverse. Although the TWSE requires that all warrant issuers make the market to provide basic liquidity to warrant investors, quotation regulations on warrant issuers are very slack on the TWSE; hence, warrant issuers are not always responsible for the quality of market-making. For example, when a security firm issues a warrant, the issuer is only responsible for ensuring the bid-ask spread does not go higher than ten ticks. Usually, the bid-ask spreads for liquid stocks or warrants on the TWSE are within two ticks; thus, the requirement of ten ticks is obviously not very effective for market-making.

The slack quotation rules allow warrants issuers to manipulate warrant trading and warrant prices. Within our sample period, short-selling-restricted stocks are usually smaller in market capitalization, and their prices are more easily affected by market noises or manipulated by specific institutional investors. A warrant issuer with ulterior motives could be more willing to make the market for the warrants linked to these restricted stocks and profit at the retail investor's expenses. By taking advantage of slack regulations, if immoral warrant issuers selectively make the market for the put warrants of specific stocks, investors may be essentially misled to trade specific warrants. This would mean our existing findings would be subject to these selection biases.

We control for possible selection bias by excluding put warrants issued by issuers with low market-making rankings. Chan and Chih (2014) conduct a comprehensive comparison to evaluate warrant market makers' performance on the TWSE. They recommend a list of trustworthy warrant issuer with effective and qualified market-making performance. We refer to their recommendations to exclude put warrants not issued by the following six untrustworthy securities firms: Polaris Securities, Yuanta Securities, KGI Securities, Capital Securities, President Securities, and Industrial bank Securities. Since we include only put warrants issued by issuers that serve as good market makers after issuance, the subsample is largely free of the selection bias caused by ulterior market-making.

We reexamine the regression analyses in Tables 3–5 using a subsample of warrants and report the results in Table 7. As the reported results show, put warrant trading volume is higher when underlying stocks are subject to short-selling restrictions. The bid-ask spreads and implied volatility of these warrants are also higher. During the 2008 short-sale ban period, although short selling was prohibited for all stocks, the outcomes of more trading volume, wider bid-ask spread, and higher implied volatility were much distinct for the put warrants of *pro forma* unrestricted stocks. All these findings are consistent with the results reported in previous tables. We undertake this robustness test to further support our main findings in this study: put warrants are substitutes for stock short sales on the TWSE, while widening bid-ask spreads and higher implied volatility accompany increased substitutive trading in put warrants.

[Table 7 about here]

4.6 The hedge impact on dealer stock trading volume

Under short-selling restrictions, investors can still effectively take short positions by trading in the put warrant markets because ban regulations did not impose any direct restrictions on derivative trading. Although short-sale restrictions are directly implemented on underlying stocks to prevent short selling shares, the restrictions may also affect spot trading through the hedging behaviors of market makers in derivatives markets. Chung, Liu, and Tsai (2014) examine the impact of covered warrants hedging on the TWSE and find a significantly pervasive effect of hedging demand on trading volume during the lives of warrants. It is thus natural to ask whether the impacts of hedging are homogeneous across time or differently affect stocks during periods of short-selling restrictions. If the availability of the put warrants market allows traders to take short positions on underlying stocks during periods of short-selling restrictions, it should reinforce the impact of dynamic hedging on the trading volume of underlying stocks. To address this concern, we further investigate whether the impacts of hedging intensify under short-selling restrictions. If we observe a positive relationship between hedging activities of warrant market makers and trading activities on underlying stocks, we can further verify the substitutive effect we document in previous sections.

To quantify the impact of dynamic hedging on the trading volume of underlying stocks, we run a regression for dealer stock trading volume on net hedging positions of put warrants and report the results in Table 8. Since there can be more than one warrant issuer for the same underlying stock, we have to aggregate the delta-hedging positions of all put warrants. Thus, we define the net delta at time *t* of all put warrants on the same underlying stock *i* as follows:

$$netDelta_{i,t} = (\sum_{j=1}^{n_t} CS_j \times N_{j,t} \times \Delta_j(t, S_t))/M_{i,t}, \tag{6}$$

CS_j is the contract size, S_t is the underlying stock price, and $N_{j,t}$ and $\Delta_j(t, S_t)$ are the volume and the delta of the put warrant *j*, respectively. n_t is the number of warrants on the same underlying stock at time *t*. $M_{i,t}$ is the number of outstanding shares of the underlying stock *i* at time *t*. We then estimate the following regression model to examine the hedging effect on the trading volume of underlying stocks:

$$\begin{aligned} Volume_{i,t} = \beta_0 &+ \beta_1 \left|netDelta_{i,t} - netDelta_{i,t-1}\right| + \beta_2 D_{i,t}^{Unrestricted} \\ &+ \beta_3 D_{i,t}^{Unrestricted} \times \left|netDelta_{i,t} - netDelta_{i,t-1}\right| + \beta_4 Volume_{i,t-1} \\ &+ \beta_5 Volume_{i,t-2} + \beta_6 Volume_{i,t-3} + \beta_7 Volume_{i,t-4} + \beta_8 Volume_{i,t-5} \\ &+ \beta_9 \sigma_{i,t} + \varepsilon_{i,t} \end{aligned} \tag{7}$$

$Volume_{i,t}$, and $\sigma_{i,t}$ represents the dealer stock trading volume and historical volatility of the underlying stock *i* on day *t*, respectively. $|netDelta_{i,t} - netDelta_{i,t-1}|$ is trading volume due to hedge rebalance, and other variables are defined as in previous equations. We include lagged dealer stock trading volumes for the underlying stocks in Equation (7). If autocorrelation exists in the dealer stock trading volumes, then we expect that the coefficients of the lagged terms will be significantly positive.

As shown in Table 8, the coefficient of $|netDelta_{i,t} - netDelta_{i,t-1}|$ is positive and statistically significant (β_1=0.44 and t-value=5.73), meaning that buying or selling one percentage of the underlying stock for dynamic hedging purposes will increase the trading volume by 0.44%. The coefficient on $D_{i,t}^{Unresticted} \times |netDelta_{i,t} - netDelta_{i,t-1}|$ is negative and significant (β_3= -0.29 and t-value= -4.67). Moreover, the coefficients on five lagged trading volumes are also positive and significant, signaling the existence of high serial correlation in daily dealer stock trading volumes.[13] Stocks with short-selling restrictions tend to have higher trading volumes due to hedge rebalancing, suggesting that when investors are not able to freely execute bearish trading in the spot market, they transfer to the put warrant market as an alternative solution for realizing a bearish strategy. This transfer leads to the active warrant trading and therefore forces warrant market makers to more frequently execute hedging transactions for underlying stocks. Again, the findings reported in Table 8 further support the substitutive role of put warrants.

[Table 8 about here]

5. Conclusion

This paper examines the relationship between stock short-sale restrictions and bearish equity derivatives. Our analysis focuses on whether the constraints on stock short selling drive investors to realize their bearish viewpoints via equity derivatives—a phenomenon we call the substitution effect in this paper. We examine Taiwan's active warrants market, which was ranked the second-largest warrants market in Asia and the

[13] The result is consistent with the Gallant, Rossi, and Tauchen (1992) and Campbell, Grossman, and Wang (1993) that the trading volumes of stocks exhibit positive autocorrelation.

fifth largest in the world during our sample period. We find that put warrants essentially serve as substitutes for stocks subject to short-selling restrictions for investors executing bearish strategies. Also, we show that widening bid-ask spreads and higher implied volatility go along with increased substitutive trading in put warrants. These findings suggest that although the put warrants market provides opportunities for pessimistic investors or speculators to execute transactions when they are subject to trading constraints, investors unavoidably pay higher trading costs and trade more expensive warrants. More interestingly, we show that the increased trading volume in put warrants resulting from the substitution effect significantly impacts dealers' stock positions even in the presence of short-selling restrictions, suggesting that derivatives hedging demand still plays an important role in stock market liquidity.

To the best of our knowledge, while previous studies focus solely on put options, this study is the first to expand the scope of related research to bearish warrants. In addition, our paper contributes to the literature with several advantages of empirical analysis. First, in Taiwan, individual equity warrants are traded in the same exchange (TWSE) as their underlying stocks; moreover, the TWSE has implemented short-selling restrictions for most listing stocks since 1998. These features allow us to examine the immediate influence of stock short sales on put warrants for a more extended period. In comparison, previous studies usually examine similar issues for shorter periods or merely focus on the impact of temporary short-selling bans surrounding ad hoc events, e.g., the 2008 financial crisis.

Next, Taiwanese warrants data has advantages over other markets when it comes to studying stock short selling during the financial crisis period. Surrounding the financial crisis, many countries adopted different types of short-sale bans to alleviate selling pressure on the stock market. However, among major stock markets with active stock options/warrants trading, Taiwan is the only market that launched a short-selling ban for all stocks but executed the ban for a relatively reasonable period. If the short-sale ban is not for all stocks, investors still can find unrestricted and related stocks to sell short instead of purchasing bearish derivatives. Furthermore, if the ban is long-lasting, investors may not just shift to the derivatives market as their short-selling substitute in the local market, but also consider other countries not subject to short-sale constraints. The uniqueness of the 2008 short-selling ban implemented on the TWSE enables us to analyze the substitution effect of derivatives trading without the concerns outlined above.

Our empirical findings show a significant increase in trading volume, bid-ask spread, and implied volatility in put warrants when their underlying stocks are constrained for short selling. The results stand for a 2SLS regression and a robustness test of the subsample accounting for warrant market maker quality. Our study serves as a counterpoint to previous studies that find that the short-sale ban on stocks also restricted trading in options in the U.S., e.g., Battalio and Schultz (2011) and Grundy et al. (2012). In different markets such as Taiwan, we suggest that equity derivatives could provide effective functions for investors and may benefit price discovery or alleviate illiquidity for restricted stocks in the market. Given our conclusion here, questions for future research include but are not limited to the following: Why did the bearish equity derivatives in the U.S. fail to serve as a solution for short-selling-restricted investors in the stock market? In addition to the TWSE, will the substitution effect be observed in other Asian stock markets? These questions warrant further attention. We leave these issues for future research.

Table 1

This table shows the descriptive statistics for warrants listed on the Taiwan Stock Exchange (TWSE) from May 16, 2005 to December 31, 2015. We obtain data on individual equity warrants from the Taiwan Economic Journal (TEJ) database. The statistics are reported on an annual basis, while the last row of the table presents the respective non-repeated cases for American/European puts and the number of underlying stocks for the whole period and the average daily variables (trading volume, bid-ask spread, and implied volatility) of all available trading days for all years.

Year	American Put	European Put	Number of underlying stocks	Average daily Trading volume	Average daily bid-ask spread	Average daily Implied volatility
2005	22	7	25	1395	0.0803	0.4622
2006	35	28	45	814	0.1798	0.6829
2007	20	63	56	778	0.1531	0.6604
2008	42	135	71	657	0.0666	0.7524
2009	305	593	144	143	0.1522	0.7073
2010	447	1080	195	185	0.1147	0.5741
2011	530	1599	205	258	0.0887	0.6215
2012	621	3547	225	143	0.0754	0.6908
2013	275	3113	203	135	0.1159	0.6041
2014	247	5232	289	332	0.1066	0.5965
2015	258	5821	269	260	0.0822	0.6710
All	2096	14802	439	234	0.0970	0.6395

Table 2

This table reports the number of unrestricted stocks and the number of unrestricted stocks with put warrants in the May 16, 2005 to December 31, 2015 sample period. The numbers are reported on an annual basis, while the last row of the table presents the non-repeated cases for entire whole period.

Year	Numbers of unrestricting stocks	Numbers of unrestricting stocks with put warrants
2005	55	13
2006	56	19
2007	151	30
2008	165	50
2009	169	106
2010	169	116
2011	176	118
2012	165	111
2013	718	139
2014	753	204
2015	802	182
All	834	281

Table 3

This table shows the regression results for put warrants trading volume under the short-sale restrictions. Put volume$_{j,i,t}$ is the daily trading volume for put warrant *j* written for stock *i* on day *t*. $D_{i,t}^{Unrestricted}$ is a dummy variable that equals one if stock *i* does not suffer short-sale restrictions on day *t* and equals zero otherwise. D_t^{Crisis} is a dummy variable that equals one if the observation date is between September 22, 2008 and December 31, 2008 inclusive, and zero otherwise. $D_t^{PostCrisis}$ is a dummy variable that equals one if the observation date is between January 1, 2009 and September 22, 2013 inclusive, and zero otherwise. Stock volume$_{i,t}$ is the trading volume for stock *i* on day *t*. Stock return$_{i,t}$ is the daily return for stock *i* on day *t*. Lag stock return$_{i,t-1}$ is the daily return for stock *i* on day *t*-1; Index return$_t$ is the Taiwan Weighted Stock Index return on day *t*; Lag index return$_{t-1}$ is the Taiwan Weighted Stock Index return on day *t*-1. Ln(Size)$_{i,t}$ is the natural logarithm of market value for stock *i* on day *t*. Time to maturity is the number of days to the put warrant's expiration date. Stock spread$_{i,t}$ is defined as the closing ask price minus the closing bid price then divided by the midpoint of closing bid and ask price for stock *i* on day *t*. The estimated standard errors are two-way clustered by firm and year. *, **, *** indicating the significance at the 10%, 5%, and 1% levels.

	Put volume$_{i,j,t}$				
Constant	284.933***	530.3390***	537.6602***	431.8175***	487.4231***
	(14.05)	(2.84)	(8.31)	(3.02)	(8.86)
$D_i^{Unrestricted}$	-93.0016***	-25.5763***	-15.1882***	-48.9330***	-44.8165***
	(-2.91)	(-3.07)	(-2.71)	(-2.90)	(-2.72)
D_t^{Crisis}				-628.7377***	-573.6584***
				(-2.49)	(-4.41)
$D_t^{PostCrisis}$				-566.7913***	-768.9981***
				(-4.03)	(-3.89)
$D_i^{Unrestricted} \times D_t^{Crisis}$				148.7450***	72.6865***
				(5.83)	(2.92)
$D_i^{Unrestricted} \times D_t^{PostCrisis}$				-69.5502**	-58.8504**
				(-2.25)	(-2.27)
Stock volume$_{i,t}$		0.0026***	0.0029***	0.0022***	0.0029***
		(3.01)	(4.76)	(2.84)	(4.75)
Stock return$_{i,t}$		-1.9402	-2.8388***	-2.4424**	-2.8205***
		(-1.47)	(-2.46)	(-1.99)	(-2.44)
Stock return$_{i,t-1}$		-1.0935	-1.4212	-1.3325	-1.4516
		(-1.04)	(-1.33)	(-1.18)	(-1.34)
Index return$_t$		-629.6150**	-300.1188**	-431.4245***	-308.1219**
		(-2.12)	(-2.23)	(-3.05)	(-2.28)
Index return$_{t-1}$		-967.4560***	-657.0961*	-774.4059**	-641.4177*
		(-3.29)	(-1.87)	(-2.02)	(-1.78)
Ln(Size)$_{i,t}$		41.5283***	40.5948***	41.1851***	40.1524***

		(2.43)	(3.95)	(2.54)	(3.95)
Stock spread$_{i,t}$		19,683.6400***	18,608.6406***	17,530.8189***	18,590.4523***
		(5.80)	(2.42)	(4.41)	(2.43)
(Time to maturity)$^{-1}$		-5,285.0100*	-5,176.3151*	-5,258.2531*	-5,179.2527*
		(-1.83)	(-1.89)	(-1.83)	(-1.89)
N	393,179	381,155	381,155	381,155	381,155
Adj. R^2(%)	0.13%	1.90%	5.00%	3.76%	5.23%
Year	N	N	Y	N	Y
Industry	N	N	Y	N	Y

Table 4

This table shows the regression results for put warrants bid-ask spread under the short-sale restrictions. Put $\text{spread}_{j,i,t}$ is the bid-ask spread of put warrant *j* written for stock *i* on day *t*. $D_{i,t}^{Unrestricted}$ is a dummy variable that equals one if stock *i* does not suffer short-sale restrictions on day *t* and equals zero otherwise. D_t^{Crisis} is a dummy variable that equals one if the observation date is between September 22, 2008 and December 31, 2008 inclusive, and zero otherwise. $D_t^{PostCrisis}$ is a dummy variable that equals one if the observation date is between January 1, 2009 and September 22, 2013 inclusive, and zero otherwise. $\text{Moneyness}_{j,i,t}$ is the natural logarithm of the underlying stock price relative to the strike price for put warrant *j* written for stock *i* on day *t*. Time to maturity is the number of days to the put warrant's expiration date. Stock $\text{spread}_{i,t}$ is defined as the closing ask price minus the closing bid price then divided by the midpoint of closing bid and ask price for stock *i* on day *t*. The estimated standard errors are two-way clustered by firm and year. *, **, *** indicating the significance at the 10%, 5%, and 1% levels.

	Put $\text{spread}_{i,j,t}$				
Constant	0.0349	-0.0124***	0.0123**	-0.011*	-0.0005**
	(18.89)	(-4.14)	(1.98)	(-1.73)	(-2.07)
$D_i^{Unrestricted}$	-0.0046***	-0.0034***	-0.0026*	-0.0205***	-0.0164***
	(-1.93)	(-2.77)	(-1.95)	(-2.72)	(-2.41)
D_t^{Crisis}				0.0295***	0.0285***
				(6.24)	(5.32)
$D_t^{PostCrisis}$				-0.0016***	-0.0177***
				(-3.37)	(-4.39)
$D_i^{Unrestricted} \times D_t^{Crisis}$				0.0334**	0.0301***
				(2.21)	(2.65)
$D_i^{Unrestricted} \times D_t^{PostCrisis}$				-0.0176***	-0.0142***
				(-3.71)	(-2.72)
Moneyness		0.0813***	0.0866***	0.0846***	0.0866***
		(9.62)	(8.74)	(9.05)	(8.52)
Moneyness^2		0.1998***	0.1931***	0.192***	0.1933***
		(5.54)	(4.73)	(4.94)	(4.78)
(Time to maturity)$^{-1}$		2.8829***	2.8789***	2.8945***	2.8791***
		(13.41)	(12.96)	(13.34)	(12.91)
Stock $\text{spread}_{i,t}$		0.6810	0.8047	0.6255	0.8145
		(1.14)	(1.41)	(1.01)	(1.43)
N	380,585	368,962	368,962	368,962	368,962
Adj. R^2	0.05%	23.41%	24.63%	23.67%	24.65%
Year	N	N	Y	N	Y
Industry	N	N	Y	N	Y

Table 5

This table shows the regression results for put warrant implied volatility under the short-sale restrictions. Implied volatility$_{j,i,t}$ is the implied volatility of put warrant *j* written for stock *i* on day *t*. $D_{i,t}^{Unrestricted}$ is a dummy variable that equals one if stock *i* does not suffer short-sale restrictions on day *t* and equals zero otherwise. D_t^{Crisis} is a dummy variable that equals one if the observation date is between September 22, 2008 and December 31, 2008 inclusive, and zero otherwise. $D_t^{PostCrisis}$ is a dummy variable that equals one if the observation date is between January 1, 2009 and September 22, 2013 inclusive, and zero otherwise. Moneyness$_{j,i,t}$ is the natural logarithm of the underlying stock price relative to the strike price for put warrant *j* written for stock *i* on day *t*. Time to maturity is the number of days to the put warrant's expiration date. Stock spread$_{i,t}$ is defined as the closing ask price minus the closing bid price then divided by the midpoint of closing bid and ask price for stock *i* on day *t*. The estimated standard errors are two-way clustered by firm and year. *, **, *** indicating the significance at the 10%, 5%, and 1% levels.

	Put implied volatility$_{i,j,t}$				
Constant	0.6728***	0.6395***	0.6657***	0.5936***	0.5860***
	(17.76)	(31.91)	(16.46)	(27.64)	(14.72)
$D_i^{Unrestricted}$	-0.1271***	-0.1254***	-0.1151***	-0.0871**	-0.0279**
	-(7.42)	(-7.41)	(-7.42)	(-2.11)	(-2.07)
D_t^{Crisis}				0.2739***	0.3463***
				(8.63)	(7.48)
$D_t^{PostCrisis}$				0.0460**	0.1240*
				(2.00)	(1.94)
$D_i^{Unrestricted} \times D_t^{Crisis}$				0.1761***	0.2351***
				(8.34)	(5.52)
$D_i^{Unrestricted} \times D_t^{PostCrisis}$				-0.0398***	-0.0899***
				(-2.86)	(-3.87)
Moneyness		-0.1381***	-0.0932***	-0.1242***	-0.0788***
		(-4.69)	(-2.71)	(-4.55)	(-2.36)
Moneyness^2		0.7713***	0.5383***	0.7439***	0.5116***
		(8.74)	(9.79)	(8.09)	(9.28)
(Time to maturity)$^{-1}$		0.5944	1.0915	0.6344	1.1350
		(0.66)	(1.14)	(0.71)	(1.18)
Stock spread$_{i,t}$		2.9852	0.8650	2.9811	0.9202
		(0.68)	(0.32)	(0.66)	(0.34)
N	393,179	381,156	381,156	381,156	381,156
Adj. R^2	4.58%	11.23%	42.61%	11.44%	42.92%
Year	N	N	Y	N	Y
Industry	N	N	Y	N	Y

Table 6

This table shows the two-stage least squares (2SLS) regression used to mitigate the concern that put warrant trading volumes, bid-ask spreads, and implied volatility are simultaneously determined. Put $volume_{j,i,t}$ is the daily trading volume for put warrant *j* written for stock *i* on day *t*. Put $spread_{j,i,t}$ is the bid-ask spread of put warrant *j* written for stock *i* on day *t*. Implied $volatility_{j,i,t}$ is the implied volatility of put warrant *j* written for stock *i* on day *t*. $D_{i,t}^{Unrestricted}$ is a dummy variable that equals one if stock *i* does not suffer short-sale restrictions on day *t* and equals zero otherwise. D_t^{Crisis} is a dummy variable that equals one if the observation date is between September 22, 2008 and December 31, 2008 inclusive, and zero otherwise. $D_t^{PostCrisis}$ is a dummy variable that equals one if the observation date is between January 1, 2009 and September 22, 2013 inclusive, and zero otherwise. Stock $volume_{i,t}$ is the trading volume for stock *i* on day *t*. Stock $return_{i,t}$ is the daily return for stock *i* on day *t*. Lag stock $return_{i,t-1}$ is the daily return for stock *i* on day *t*-1; Index $return_t$ is the Taiwan Weighted Stock Index return on day *t*; Lag index $return_{t-1}$ is the Taiwan Weighted Stock Index return on day *t*-1. $Ln(Size)_{i,t}$ is the natural logarithm of market value for stock *i* on day *t*. $Moneyness_{j,i,t}$ is the natural logarithm of the underlying stock price relative to the strike price for put warrant *j* written for stock *i* on day *t*. Time to maturity is the number of days to the put warrant's expiration date. Stock $spread_{i,t}$ is defined as the closing ask price minus the closing bid price then divided by the midpoint of closing bid and ask price for stock *i* on day *t*. *, **, *** indicating the significance at the 10%, 5%, and 1% levels.

	Put $volume_{i,j,t}$	**Put $spread_{i,j,t}$**	**Put implied $volatility_{i,j,t}$**	**Put $volume_{i,j,t}$**	**Put $spread_{i,j,t}$**	**Put implied $volatility_{i,j,t}$**
Constant	-324.2390***	-0.0033***	0.5477***	152.952	-0.0132***	0.7414***
	(-3.71)	(-5.12)	(4.26)	(5.78)	(-6.76)	(4.20)
$D_{i,t}^{Unrestricted}$	-72.8896***	-0.0005***	-0.11722	-29.2756***	-0.0142***	-0.0246***
	(-6.41)	(-2.71)	(-3.55)	(-4.23)	(-2.76)	(-4.95)
D_t^{Crisis}				-503.9700	0.0299***	0.2737***
				(-5.50)	(4.20)	(4.20)
$D_t^{PostCrisis}$				-444.1970	-0.0109***	0.1638***
				(-6.46)	(-6.00)	(7.33)
$D_{i,t}^{Unrestricted} \times D_t^{Crisis}$				64.6070	0.0333***	0.2091**
				(4.07)	(4.62)	(5.69)
$D_{i,t}^{Unrestricted} \times D_t^{PostCrisis}$				-32.6545***	-0.0149***	-0.0773***
				(-2.36)	(-8.02)	(-5.33)
Stock $volume_{i,t}$	0.0026***			0.0022***		
	(4.15)			(4.61)		

(continued)

Table 6 (continued)

	Put volume$_{i,j,t}$	Put spread$_{i,j,t}$	Put implied volatility$_{i,j,t}$	Put volume$_{i,j,t}$	Put spread$_{i,j,t}$	Put implied volatility$_{i,j,t}$
Stock return$_{i,t}$	-3.8050*			-3.8289*		
	(-1.87)			(-1.89)		
Stock return$_{i,t-1}$	-1.8517			-1.7727		
	(-0.91)			(-0.88)		
Index return$_{t}$	-582.7520***			-438.7700***		
	(-3.11)			(-2.36)		
Index return$_{t-1}$	-1,139.9800***			-946.5400***		
	(-6.14)			(-5.13)		
Ln(Size)$_{i,t}$	60.8905***			58.4856***		
	(6.63)			(5.51)		
Moneyness		0.0818***	0.5592***		0.0817***	0.6631***
		(12.05)	(13.15)		(11.80)	(14.91)
Moneyness^2		0.2128***	2.4986***		0.2133***	2.5945***
		(11.71)	(12.98)		(11.10)	(12.74)
(Time to maturity)$^{-1}$	-9,684.9400***	3.0749***	25.5854***	-8758.0500***	3.0632***	27.3931***
	(-4.54)	(5.58)	(6.10)	(-3.62)	(5.64)	(6.65)
Stock spread$_{i,t}$	19,047.3201***	0.2969***	8.6429***	17,877.7303***	0.3381***	11.0092***
	(3.19)	(4.65)	(4.61)	(2.88)	(5.27)	(3.20)
Fitted put volume$_{i,j,t}$		0.0001***	0.0001***		0.0001***	0.0002***
		(5.47)	(2.81)		(6.50)	(2.67)
Fitted put spread$_{i,j,t}$	1,405.0190***		-8.611***	1,072.2710***		-9.4913**
	(2.17)		(-2.47)	(2.52)		(-2.20)
Fitted implied volatility$_{i,j,t}$	13.8084***	-0.0324***		11.2028	-0.0326***	
	(1.22)	(-5.87)		(0.99)	(-6.00)	
N		368,962			368,962	
Year		Y			Y	
Industry		Y			Y	

Table 7

This table shows the regression used to mitigate for market making quality. Put volume$_{j,i,t}$ is the daily trading volume for put warrant *j* written for stock *i* on day *t*. Put spread$_{j,i,t}$ is the bid-ask spread of put warrant *j* written for stock *i* on day *t*. Implied volatility$_{j,i,t}$ is the implied volatility of put warrant *j* written for stock *i* on day *t*. $D_{i,t}^{Unrestricted}$ is a dummy variable that equals one if stock *i* does not suffer short-sale restrictions on day *t* and equals zero otherwise. D_t^{Crisis} is a dummy variable that equals one if the observation date is between September 22, 2008 and December 31, 2008 inclusive, and zero otherwise. $D_t^{PostCrisis}$ is a dummy variable that equals one if the observation date is between January 1, 2009 and September 22, 2013 inclusive, and zero otherwise. Stock volume$_{i,t}$ is the trading volume for stock *i* on day *t*. Stock return$_{i,t}$ is the daily return for stock *i* on day *t*. Lag stock return$_{i,t-1}$ is the daily return for stock *i* on day *t*-1; Index return$_t$ is the Taiwan Weighted Stock Index return on day *t*; Lag index return$_{t-1}$ is the Taiwan Weighted Stock Index return on day *t*-1. Ln(Size)$_{i,t}$ is the natural logarithm of market value for stock *i* on day *t*. Moneyness$_{j,i,t}$ is the natural logarithm of the underlying stock price relative to the strike price for put warrant *j* written for stock *i* on day *t*. Time to maturity is the number of days to the put warrant's expiration date. Stock spread$_{i,t}$ is defined as the closing ask price minus the closing bid price then divided by the midpoint of closing bid and ask price for stock *i* on day *t*. The estimated standard errors are two-way clustered by firm and year. *, **, *** indicating the significance at the 10%, 5%, and 1% levels.

	Put volume$_{i,j,t}$		**Put spread$_{i,j,t}$**		**Put implied volatility$_{i,j,t}$**	
Constant	478.8625***	468.3492***	0.0160***	0.0056	0.6498***	0.5489***
	(2.57)	(2.92)	(3.27)	(1.03)	(17.44)	(15.00)
$D_{i,t}^{Unrestricted}$	-15.1663**	-17.2459**	-0.0017**	-0.0129**	-0.1028***	-0.0071***
	(-2.17)	(-2.16)	(-2.04)	(-2.05)	(-2.47)	(-2.64)
D_t^{Crisis}		-470.5207***		0.0252***		0.3952***
		(-4.45)		(6.46)		(7.52)
$D_t^{PostCrisis}$		-842.4927***		-0.0221***		0.1411***
		(-5.44)		(-3.91)		(5.52)
$D_{i,t}^{Unrestricted} \times D_t^{Crisis}$		39.5718***		0.0249***		0.2703***
		(3.01)		(4.24)		(5.56)
$D_{i,t}^{Unrestricted} \times D_t^{PostCrisis}$		-19.5230**		-0.0114***		-0.1130***
		(-2.28)		(-5.72)		(-8.69)
Stock volume$_{i,t}$	0.0031***	0.0031***				
	(3.94)	(3.90)				

(continued)

Table 7 (continued)

	Put volume$_{i,j,t}$		Put spread$_{i,j,t}$		Put implied volatility$_{i,j,t}$	
Stock return$_{i,t}$	-3.4852	-3.5036				
	(-2.26)	(-2.27)				
Stock return$_{i,t-1}$	-0.8235	-0.8323				
	(-0.64)	(-0.64)				
Index return$_t$	-421.2704	-412.2931				
	(-2.34)	(-2.35)				
Index return$_{t-1}$	-844.8899	-835.6183				
	(-2.22)	(-2.15)				
Ln(Size)$_{i,t}$	49.371	48.4654				
	(2.74)	(2.72)				
Moneyness			0.0868***	0.0870***	-0.1061***	-0.0873***
			(8.50)	(8.29)	(-3.01)	(-2.91)
Moneyness^2			0.1752***	0.1749***	0.5280***	0.4928***
			(5.08)	(5.13)	(8.69)	(9.48)
(Time to maturity)$^{-1}$	-4,986.8983	-4,972.9465	2.7896***	2.7905***	1.1130	1.1677
	(-1.64)	(-1.64)	(12.38)	(12.34)	(1.19)	(1.25)
Stock spread$_{i,t}$	19,927.062	19,991.2145	0.4789	0.4858	1.2835	1.3077
	(2.16)	(2.19)	(0.88)	(0.88)	(0.88)	(0.86)
N	221,390	221,390	214,840	214,840	221,391	221,391
Adj. R^2	4.88%	5.07%	24.25%	24.26%	40.66%	41.25%
Year	Y	Y	Y	Y	Y	Y
Industry	Y	Y	Y	Y	Y	Y

Table 8

This table shows the regression results for dealer stock trading volume under the short-sale restrictions and the absolute changes in net delta ($|netDelta_{i,t}-netDelta_{i,t-1}|$). Dealer stock trading $volume_{j,i,t}$ is the dealer daily trading volume for stock *i* on day *t*. $|netDelta_{i,t}-netDelta_{i,t-1}|$ is the trading volume due to the hedging rebalance for stock *i* on day *t*. $D_{i,t}^{Unrestricted}$ is a dummy variable that equals one if stock *i* does not suffer short-sale restrictions on day *t* and equals zero otherwise. Historical $volatility_{i,t}$ is estimated as the annualized return volatility of the stock *i* over the most recent 250 days on day *t*. The estimated standard errors are two-way clustered by firm and year. *, **, *** indicating the significance at the 10%, 5%, and 1% levels.

	Dealer stock trading $volume_{i,j,t}$
C	0.0001***
	(3.93)
$\|netDelta_{i,t}-netDelta_{i,t-1}\|$	0.5322***
	(5.19)
$D_{i,t}^{Unrestricted}$	0.0001***
	(6.65)
$D_{i,t}^{Unrestricted} \times \|netDelta_{i,t}-netDelta_{i,t-1}\|$	-0.4533***
	(-5.52)
Dealer stock trading $volume_{t-1}$	0.3488***
	(12.08)
Dealer stock trading $volume_{t-2}$	0.1597***
	(4.05)
Dealer stock trading $volume_{t-3}$	0.0856***
	(11.01)
Dealer stock trading $volume_{t-4}$	0.0962***
	(5.43)
Dealer stock trading $volume_{t-5}$	0.0449
	(1.27)
Historical $voltaility_{i,t}$	0.0003***
	(5.93)
N	131,439
Adj. R^2	39.60%

Appendix A: Using Daily High and Low Prices to Estimate Spread

Following Corwin and Schultz (2012), we use daily high and low prices to estimate the put warrant bid-ask spread.

$$[Ln\left(H_t^o / L_t^o\right)]^2 = \left[Ln\left(\frac{H_t^A\left(1+S/2\right)}{L_t^A\left(1-S/2\right)}\right)\right]^2 \tag{A1}$$

S is a spread. Assuming that actual stock prices follow a diffusion process and the daily high price may be buyer-initiated trades and increases by half of the spread, daily low price may be seller-initiated trades and decreases by half of the spread. $H_t^o(L_t^o)$ is the observed high (low) price on day t and $H_t^A(L_t^A)$ is the actual high (low) price on day t.

Following Parkinson (1980) and Garman and Klass (1980) we design an equation as follows:

$$\mathrm{E}\left\{\frac{1}{T}\sum_{t=1}^{T}\left[ln\left(\frac{H_t^A}{L_t^A}\right)\right]^2\right\} = k_1\sigma_{HL}^2 \ and \ E\left\{\frac{1}{T}\sum_{t=1}^{T}\left[ln\left(\frac{H_t^A}{L_T^A}\right)\right]\right\} = k_2\sigma_{HL}^2 \tag{A2}$$

First, we rearrange equation (A1) and sum it over two single days:

$$\sum_{j=0}^{1}[Ln\left(H_{t+j}^o / L_{t+j}^o\right)]^2 = \sum_{j=0}^{1}\left[ln\left(\frac{H_{t+j}^A}{L_{t+j}^A}\right)\right]^2\left[1+2\left[ln\left(\frac{2+S}{2-S}\right)\right]\right]+\left[ln\left(\frac{2+S}{2-S}\right)\right]^2 \tag{A3}$$

Second, we use high-low price ratio in a two-day period and set it to simple notation:

$$[Ln\left(H_{t,t+1}^o / L_{t,t+1}^o\right)]^2 = \left[ln\left(\frac{H_{t,t+1}^A}{L_{t,t+1}^A}\right)\right]^2 + 2\left[ln\left(\frac{H_{t,t+1}^A}{L_{t,t+1}^A}\right)\right]\left[ln\left(\frac{2+S}{2-S}\right)\right]+\left[ln\left(\frac{2+S}{2-S}\right)\right]^2 \tag{A.4}$$

where $H_{t,t+1}^o\left(L_{t,t+1}^o\right)$ is the high (low) price in a two-day period from t to t + 1

Taking the expectation of (A3), we set:

$$\alpha = \left[ln\left(\frac{2+S}{2-S}\right)\right], \beta = \mathrm{E}\left\{\sum_{j=0}^{1}\left[ln\left(\frac{H_{t+j}^{0}}{L_{t+j}^{o}}\right)\right]\right\} and\ \gamma = \left[ln\left(\frac{H_{t,t+1}^{o}}{L_{t,t+1}^{o}}\right)\right]^{2}$$

We rearrange:

$$\mathrm{S} = \frac{2\left(e^{\alpha}-1\right)}{1+e^{\alpha}} \tag{A.5}$$

Substituting (A.3) from (A.2) and solving (A.3) and (A.4) together yields:

$$\alpha = \frac{\sqrt{2\beta}-\sqrt{\beta}}{3-2\sqrt{2}} - \sqrt{\frac{\gamma}{3-2\sqrt{2}}} \tag{A.6}$$

Finally, we can get our spread after inserting (A.6) into (A.5).