Semivariance and Semiskew Risk Premiums in Currency Markets *

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Abstract

Using the model-free methodology proposed in the literature, variance and skew swaps are extracted from currency options for several foreign exchange rates. Moreover, these variables are decomposed into semivariance and semiskew swaps, which are conditional to the evolution of the foreign exchange rate, and it is shown to have higher explanatory power for currency excess return. These semivariances enable the definition of a variance-skew swap that also possesses a strong explanatory power for currency excess return. From these variables, higher moment semi-risk premiums can be computed and measure how tail risks are priced. These semivariance and semiskew swaps better explain the currency excess return than the standard or undecomposed ones. For semivariance swaps, both the up and down contracts are equally informative while for semiskew swaps only the down tail related one is. Down semivariance and semiskew swaps carry complementary information regarding the currency excess return. Trimming these variables enables us to show that extreme movements affecting the currency option market contain no information on the evolution of the currency. Lastly, forecasting tests further illustrate the importance of decomposing the variance and skew swaps into semi components as it improves significantly the results.

JEL Classification: G11; G12; G13
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1 Introduction

The explanation of the currency risk premium is an important problem in finance as it synthesizes how two economies interact, Engel (2013). Beyond the usual macroeconomic variables used as determinant of the currency risk premium, the option market offers an interesting route to investigate this problem. Since the seminal work of Bates (1991), many studies have shown that options could be used to extract market participants' expectations for the currency risk premium.

Using a model-free approach that has been extensively used during the recent years, a variance swap contract is extracted from currency options and used to build factor models for the foreign exchange excess return. This contract enables the computation of the variance risk premium that is a key quantity in asset pricing. Going one step farther than the literature, we decompose the variance swap contract into up and down components, the former depending on the upper tail of the currency distribution while the latter depends on the lower tail of the currency distribution. What is more, these semivariance swap contracts allow us to compute the variance-skew swap that captures the skewness of the currency distribution. To further investigate the importance of the currency skewness distribution we follow Kozhan et al. (2013) and compute a skew swap value from foreign exchange rate options and, similarly to the variance swap contract, it is decomposed into up and down semiskew swap contracts. This large set of option related variables allows us to build several factor models for the currency excess return and assess to which extent they can explain and predict its evolution.

We show that decomposed variance and skew variables have higher explanatory power for the currency excess return than undecomposed ones; that the variance-skew swap is highly informative for the currency excess return; that the down semiskew swap is as informative as the skew swap regarding the currency excess return; a combination of down semivariance and semiskew swaps have higher explanatory power for the currency excess return than an up one; for certain currencies third order moment related quantities, mainly down quantities, provide additional information to second order moment related quantities. Trimming these explanatory variables, by removing extreme movements, and re-estimating the same factor models allow us to show that the information extracted from the far ends of the tails contain no information on the evolution of the currency excess return. Lastly, the predictability of the 1-month, 3-month and

6-month currency excess return is improved when semivariance and semiskew swaps are used in place of undecomposed variance and skew swaps, it further illustrates the importance of the decomposition.

The paper is organized as follows. We present the key ingredients to obtain the quantities from option prices in Section 2. A description of the empirical data used in our analysis is provided in Section 3. Regression tests and analysis are performed in Section 4 and Section 5 concludes the paper.

2 Analytical results

The main purpose of this work is to analyze the variance, the variance-skew and the skew risk premiums for the foreign exchange option market using a model-free methodology based on call and put foreign exchange rate options. To this end, $c_{t,T}(k)$ and $p_{t,T}(k)$ denote the European call and put option prices at time t with maturity T and strike k on the spot foreign exchange rate denoted s_t, which represents the value at time t in the domestic currency of one unit of the foreign currency. It is often more convenient to use the forward foreign exchange rate (or forward price), so $f_{t,T}$ stands for the forward price value at time t with maturity T that is related to the spot value through the standard equality $f_{t,T} = s_t e^{(r_d - r_f)(T-t)}$, where r_d and r_f represent the risk free domestic and foreign rates, respectively. Lastly, it is convenient to define $r_{t,T} = \ln f_{T,T} - \ln f_{t,T}$, the log return of a position in the forward contract of maturity T for the period $[t\ ; T]$. The availability of these derivative products enables the computation of a variance swap contract, a variance-skew swap contract as well as a skew swap contract along with the risk premiums associated with those contracts in a model-free way. In this work, we use the approach proposed by Kozhan et al. (2013). Extracting distribution information from option prices, like higher moments, has a long history and has been performed in many works.

2.1 The variance and semivariance risk premiums

A variance swap contract, payer of the fixed leg and receiver of the floating leg, is a contract between two counterparties that involves the payment at maturity $t + \tau$ of an amount specified at the initiation date t, this amount is called the variance swap rate, and receiving at maturity (i.e. $t + \tau$) of the swap contract the realized variance of a given asset computed over the interval

$[t\ ; t+\tau]$. The amount specified at the initiation date t is called the fixed leg as it is known during the life of the contract while the amount received at maturity is called the floating leg of the swap as it is known only at the end of the contract, so during the life of the contract that quantity fluctuates. Following the literature, it is known that the variance swap rate is given by

$$var_{t,t+\tau} = \frac{2}{b_{t,t+\tau}} \int_{f_{t,t+\tau}}^{+\infty} \frac{c_{t,t+\tau}(k)}{k^2} dk + \frac{2}{b_{t,t+\tau}} \int_{0}^{f_{t,t+\tau}} \frac{p_{t,t+\tau}(k)}{k^2} dk \tag{1}$$

$$= var^u_{t,t+\tau} + var^d_{t,t+\tau} \tag{2}$$

with $b_{t,t+\tau}$ the zero-coupon value at time t with maturity $t+\tau$ that is expressed in the domestic currency. The quantity $var^u_{t,t+\tau}$ captures the second moment of the upper tail distribution of the underlying asset while $var^d_{t,t+\tau}$ captures the second moment of the lower tail distribution. More precisely, $var^u_{t,t+\tau}$ involves only call options with strikes larger or equal to the forward price $f_{t,t+\tau}$ and, as such, it depends on the second moment of the log return of the forward price (i.e. $r_{t,t+\tau}$) *conditional* on the event that it is positive, that is to say, conditional on $\mathbf{1}_{\{r_{t,t+\tau}>0\}}$. Similarly, $var^d_{t,t+\tau}$ depends on the second moment of the log return of the forward price (i.e. $r_{t,t+\tau}$) *conditional* on the event that it is negative, that is to say, conditional on $\mathbf{1}_{\{r_{t,t+\tau}<0\}}$. Also, as option prices are computed under the risk neutral probability, the variance swap rate is also called the risk neutral variance and hereafter we use interchangeably these names.

Notice that Eq.(1) involves a continuum of options and as in the market only a finite number of options are available the two integrals are approximated by sums, see Eq.(23) in Kozhan et al. (2013) for details.

The floating leg of the variance swap is the realized variance of the underlying currency computed over $[t\ ; t+\tau]$ using end-of-day values as it is the standard practice in the market and it is given by

$$rvar_{t,t+\tau} = \sum_{i=t}^{t+\tau-1} g^v(\bar{r}_{i,i+1}), \tag{3}$$

with $\bar{r}_{i,i+1} = \ln f_{i+1,t+\tau} - \ln f_{i,t+\tau}$ with $i \in \{t, t+1, \ldots, t+\tau-1\}$ the set of days covering the interval $[t\ ; t+\tau]$ and $g^v(x) = 2(e^x - 1 - x)$. Performing a Taylor expansion of the exponential function the terms in the sum turn out to be $\bar{r}^2_{i,i+1}$, hence justifying the expression of realized variance for $rvar_{t,t+\tau}$. The fixed and floating legs being defined, the realization of a variance swap, payer of the fixed leg and receiver of the floating leg, is given by

$$vs_{t,t+\tau} = rvar_{t,t+\tau} - var_{t,t+\tau}. \tag{4}$$

In practice this product is used to hedge against an increase of the currency's volatility. Indeed, at time t the amount $var_{t,t+\tau}$ is specified (and paid at time $t+\tau$) and if over the period $[t\ ; t+\tau]$ the currency's volatility increases substantially, the quantity $rvar_{t,t+\tau}$ received at time $t+\tau$ is larger than the amount specified at time t, the net value is positive. Conversely, if the currency's volatility remains low or decreases over the period $[t\ ; t+\tau]$, $rvar_{t,t+\tau}$ is smaller than $var_{t,t+\tau}$, overall the investor makes a loss. In this example, the point of view of a volatility protection buyer is taken, the counterparty in that contract acts as a volatility protection seller and the cash-flows are exactly the opposite.

Following the literature such as Carr and Wu (2009), taking the expectation (under the historical probability measure) of Eq.(4) leads to the variance risk premium, it is denoted

$$\overline{vs}_\tau = \mathbb{E}[vs_{t,t+\tau}].$$

This quantity is negative in practice, it is interpreted as the amount an investor is willing to pay in order to hedge against volatility risk. It is known that variance risk premiums for equity options and equity index options are negative, see Carr and Wu (2009).

It is of interest to compare an investment made in the variance swap contract with the return of an investment made in a risky asset and therefore it is convenient to introduce the excess return of an investment made in the variance swap contract, it is denoted by

$$xv_{t,t+\tau} = \frac{rvar_{t,t+\tau}}{var_{t,t+\tau}} - 1. \tag{5}$$

The qualifier "excess" follows from the fact that var in Eq.(1) involves at the denominator a zero-coupon bond, it makes that quantity a forward price, see Carr and Wu (2009) for details.

Similarly to the decomposition performed for the variance swap rate $var_{t,t+\tau}$ in Eq.(2), the realized variance can be decomposed into two components conditional on the return of the forward price $f_{t,t+\tau}$, it leads to

$$rvar_{t,t+\tau} = \sum_{i=t}^{t+\tau-1} g^v(\bar{r}_{i,i+1})\mathbf{1}_{\{r_{t,t+\tau}>0\}} + \sum_{i=t}^{t+\tau-1} g^v(\bar{r}_{i,i+1})\mathbf{1}_{\{r_{t,t+\tau}<0\}} \tag{6}$$

$$= rvar^u_{t,t+\tau} + rvar^d_{t,t+\tau}. \tag{7}$$

Combining the decompositions for the risk neutral variance $var_{t,t+\tau}$ and realized variance $rvar_{t,t+\tau}$, it is natural to define the semivariance swap contracts

$$vs^u_{t,t+\tau} = rvar^u_{t,t+\tau} - var^u_{t,t+\tau}, \tag{8}$$

$$vs^d_{t,t+\tau} = rvar^d_{t,t+\tau} - var^d_{t,t+\tau}, \tag{9}$$

as these swaps involve "half" (roughly speaking) of the underlying currency distribution. Moreover, as $vs^u_{t,t+\tau}$ depends on the upper part of the underlying currency distribution, it is tempting to name it the up semivariance swap contract while for obvious reasons $vs^d_{t,t+\tau}$ is named the down semivariance swap contract.

In full analogy with what is done for the variance swap, it is possible to define the up and down semivariance risk premiums, they are given by

$$\overline{vs}^u_\tau = \mathbb{E}[vs^u_{t,t+\tau}], \quad \overline{vs}^d_\tau = \mathbb{E}[vs^d_{t,t+\tau}],$$

as well as the excess returns of these semivariance swap contracts

$$xv^u_{t,t+\tau} = \frac{rvar^u_{t,t+\tau}}{var^u_{t,t+\tau}} - 1, \quad xv^d_{t,t+\tau} = \frac{rvar^d_{t,t+\tau}}{var^d_{t,t+\tau}} - 1. \tag{10}$$

Let us stress the fact that the realized variables $rvar_{t,t+\tau}$, $rvar^u_{t,t+\tau}$ and $rvar^d_{t,t+\tau}$ are known at time $t+\tau$ while the risk neutral quantities $var_{t,t+\tau}$, $var^u_{t,t+\tau}$ and $var^d_{t,t+\tau}$ are known at time t (i.e. one month earlier as we work with one-month maturity options).

Remark 2.1. In Kilic and Shaliastovich (2018), the authors work with S&P500 index options and decompose the second risk neutral moment into two components similar to Eq.(2) and name these two terms "good" and "bad" variances as the first one is related to upward evolutions of the equity index, which is often considered as a favorable outcome, while the second depends on downward movements and is therefore associated with a market turmoil. They also perform a decomposition of the realized variance into "good" and "bad" components as follows

$$\begin{aligned}\widetilde{rvar}_{t,t+\tau} &= \sum_{i=t}^{t+\tau-1} \bar{r}_{i,i+1}^2 \mathbf{1}_{\{\bar{r}_{i,i+1}>0\}} + \sum_{i=t}^{t+\tau-1} \bar{r}_{i,i+1}^2 \mathbf{1}_{\{\bar{r}_{i,i+1}<0\}} \\ &= \widetilde{rvar}^u_{t,t+\tau} + \widetilde{rvar}^d_{t,t+\tau}\end{aligned}$$

and by combining them with their corresponding risk neutral quantity they define "good" and "bad" variance risk premiums.[1] Notice the difference with the choice made in this work as here the sum is split depending on the return over the entire interval $[t\ ; t+\tau]$. As explained by these authors it is known that, thanks to Barndorff-Nielsen et al. (2008), under the hypothesis that $(\bar{r}_t)_{t\geq 0}$ satisfies the dynamic $\bar{r}_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dw_s + J_t$ with $(w_t)_{t\geq 0}$ a Brownian motion and J_t a pure jump process, the following convergences in probability hold

$$\begin{aligned}\widetilde{rvar}^u_{t,t+\tau} &\to \frac{1}{2}\int_t^{t+\tau} \sigma_s^2 ds + \sum_{t\leq s\leq t+\tau} (\Delta\bar{r}_s)^2 \mathbf{1}_{\{\Delta\bar{r}_s\geq 0\}}, \\ \widetilde{rvar}^d_{t,t+\tau} &\to \frac{1}{2}\int_t^{t+\tau} \sigma_s^2 ds + \sum_{t\leq s\leq t+\tau} (\Delta\bar{r}_s)^2 \mathbf{1}_{\{\Delta\bar{r}_s\leq 0\}}\end{aligned}$$

with $\Delta\bar{r}_s = \bar{r}_s - \bar{r}_{s-}$. These quantities are often called semivariances, see Patton and Sheppard (2015).

Notice that while the "good" risk neutral variance depends on positive evolutions of the underlying asset over the interval $[t\ ; t+\tau]$, the "good" realized variance depends only on positive daily returns (if we consider daily observations). A similar remark applies to the "bad" variance part as well. To properly define a risk premium, the same quantity has to be computed under the risk neutral and historical probabilities, it explains the different choice made in this work for the decomposition of the realized variances in Eq.(3), and at that level the current work departs from Kilic and Shaliastovich (2018). An alternative way to understand the potential mismatch between $var^u_{t,t+\tau}$ and $\widetilde{rvar}^u_{t,t+\tau}$ is that the former depends on the positive and negative underlying jump distributions while the latter only depends on the positive underlying jump distribution. Hence, the difficulty to properly define a variance risk premium using these two quantities.

[1] What is more, the interval $[t\ ; t+\tau]$, which is typically one month long, is decomposed into 5-min sub-intervals in Kilic and Shaliastovich (2018) while in this work daily data are used, but this difference is irrelevant regarding the issue we want to underline. As already mentioned, in practice the floating leg of a variance swap is always computed using daily data.

2.2 The variance-skew risk premium

An important contribution of Kozhan et al. (2013), see also Neuberger (2012), is the possibility to evaluate the skew risk premium associated with an asset log-return distribution. In the particular case of the S&P500 index options it was shown that the skew swap, from which the skew risk premium can be derived, is related to S&P500 index excess returns (see Eq.(34) in Kozhan et al. (2013)). In the case of foreign exchange options market there is a difficulty, which is explained in the data section, that prevents an immediate application of Kozhan et al. (2013)'s methodology, so we propose two alternatives. The first one involves the definition of a variance-skew swap and a variance-skew risk premium. For the second one, presented in the next section, we generate the data required by Kozhan et al. (2013)'s methodology by performing several interpolations and it is difficult to assess to which extent they affect the final results.

The variance-skew risk premium relies on a definition for the skewness proposed in the literature, see for example Feunou et al. (2017), and is based on the second order moments. More precisely, let us define a variance-skew swap as

$$vss_{t,t+\tau} = rvar^u_{t,t+\tau} - rvar^d_{t,t+\tau} - (var^u_{t,t+\tau} - var^d_{t,t+\tau}) \tag{11}$$

$$= vs^u_{t,t+\tau} - vs^d_{t,t+\tau}. \tag{12}$$

The Eq.(11) involves the differences between the up and down variance components for both, the realized and risk neutral variances, and as such they measure the discrepancies between the right (up) and left (down) tails of the underlying log-return distribution. It justifies our choice to qualify $vss_{t,t+\tau}$ as a variance-skew swap contract. However, as these quantities depend on the second order moment and not the third moment, it does not correspond to the mathematical concept of skewness. As the up semivariance swap depends on call options while the down semivariance swap depends on put options the variance-skew swap shares some similarities with the well-known risk-reversal option strategy that is sometimes called vol-skew and measures the distortion between the left and right tails of the underlying log-return distribution. Indeed, a risk-reversal volatility is the call option volatility for a given delta minus the put option volatility with same delta, this justifies our choice to name variance-skew swap the quantity Eq.(11). From the definition of the variance-skew swap and more precisely Eq.(12), it is possible to deduce that the variance-skew risk premium, which is the expectation of this equation, is given by:

$$\overline{vss}_\tau = \overline{vs}^u_\tau - \overline{vs}^d_\tau \tag{13}$$

but also that a variance-skew swap is a long position in an up semivariance swap contract and short position in a down semivariance swap contract. It is interesting to note that the delta (sensitivity with respect to the underlying asset) is one for the risk-reversal strategy, thanks to the call-put parity relation, while it is much lower for the variance-skew swap.

Lastly, it is convenient to normalize the variance-skew swap, it is denoted

$$xvs_{t,t+\tau} = \frac{vss_{t,t+\tau}}{var^u_{t,t+\tau} + var^d_{t,t+\tau}}. \tag{14}$$

Notice that there is a difference between trading separately in an up and a down semivariance swaps and trading in a variance-skew swap as this latter implies to be long and short by the same number of units in the two semivariance swaps.

2.3 The skew and semiskew risk premiums

A skew swap, receiver of the floating leg and payer of the fixed leg, is a contract between two counterparties that involves receiving at maturity $t+\tau$ of the contract the realized skewness of a given asset while paying at that date an amount specified at the initiation date t of the contract called the skew swap rate or fixed leg. The skew swap rate is given by

$$\begin{aligned} skew_{t,t+\tau} &= \frac{6}{b_{t,t+\tau}} \int_{f_{t,t+\tau}}^{+\infty} \frac{k - f_{t,t+\tau}}{k^2 f_{t,t+\tau}} c_{t,t+\tau}(k) dk - \frac{6}{b_{t,t+\tau}} \int_0^{f_{t,t+\tau}} \frac{f_{t,t+\tau} - k}{k^2 f_{t,t+\tau}} p_{t,t+\tau}(k) dk & (15) \\ &= skew^u_{t,t+\tau} - skew^d_{t,t+\tau} & (16) \end{aligned}$$

and as explained in Kozhan et al. (2013) it depends on the risk neutral skewness of the underlying asset distribution (i.e. the skewness of the log of foreign exchange rate distribution) and is implied from the options. Notice that $skew^u_{t,t+\tau}$ and $skew^d_{t,t+\tau}$ are by definition positive. If we define

$$\begin{aligned} ve_{t,t+\tau} &= \frac{2}{b_{t,t+\tau}} \int_{f_{t,t+\tau}}^{+\infty} \frac{c_{t,t+\tau}(k)}{k f_{t,t+\tau}} dk + \frac{2}{b_{t,t+\tau}} \int_0^{f_{t,t+\tau}} \frac{p_{t,t+\tau}(k)}{k f_{t,t+\tau}} dk & (17) \\ &= ve^u_{t,t+\tau} + ve^d_{t,t+\tau} & (18) \end{aligned}$$

then $ve^u_{t,t+\tau}$ only involves call options with strikes larger or equal to the forward price $f_{t,t+\tau}$ while $ve^d_{t,t+\tau}$ only depends on put options with strikes smaller or equal to the forward price $f_{t,t+\tau}$. The remarks made for $var^u_{t,t+\tau}$ and $var^d_{t,t+\tau}$ in the previous section apply here as well. As a result, the equality $skew_{t,t+\tau} = 3(ve_{t,t+\tau} - var_{t,t+\tau})$, the decomposition of $ve_{t,t+\tau}$ given by Eq.(18) and the decomposition of $var_{t,t+\tau}$ given by Eq.(2) lead to $skew^u_{t,t+\tau} = 3(ve^u_{t,t+\tau} - var^u_{t,t+\tau})$ and $skew^d_{t,t+\tau} = 3(var^d_{t,t+\tau} - ve^d_{t,t+\tau})$, the former depending on the distribution of the log return of the forward price (i.e. $r_{t,t+\tau}$) *conditional* on the event that it is positive, that is to say, conditional on $\mathbf{1}_{\{r_{t,t+\tau}>0\}}$ while the latter depends on the distribution of the log return of the forward price (i.e. $r_{t,t+\tau}$) *conditional* on the event that it is negative, that is to say, conditional on $\mathbf{1}_{\{r_{t,t+\tau}<0\}}$.

From an implementation point of view, as for $var_{t,t+\tau}$ that can be approximated by discrete sums, the quantity $skew_{t,t+\tau}$ given by Eq.(15) is obtained by discretization of the integrals, see Eqs.(23),(24) in Kozhan et al. (2013) for details.

Kozhan et al. (2013) shows that the floating leg of the skew swap is given by:

$$rskew_{t,t+\tau} = \sum_{i=t}^{t+\tau-1} 3\Delta v^e_{i,t+\tau}(e^{\bar{r}_{i,i+1}} - 1) + g^s(\bar{r}_{i,i+1}) \tag{19}$$

$$= rskew_{t,t+\tau}\mathbf{1}_{\{r_{t,t+\tau}>0\}} + rskew_{t,t+\tau}\mathbf{1}_{\{r_{t,t+\tau}\leq 0\}} \tag{20}$$

$$= rskew^u_{t,t+\tau} - rskew^d_{t,t+\tau} \tag{21}$$

with $g^s(x) = 6(2 - 2e^x + x + xe^x)$, $\Delta v^e_{i,t+\tau} = v^e_{i+1,t+\tau} - v^e_{i,t+\tau}$ the daily change of $v^e_{i,t+\tau}$ with $i \in \{t, t+1, \dots t+\tau-1\}$ the set of days covering the interval $[t\,; t+\tau]$ and the term $rskew^d_{t,t+\tau}$ equals to minus the last term of Eq.(20). A Taylor expansion of the function $g^s(x)$ shows that it behaves like x^3.

Remark 2.2. The implementation of formula Eq.(19), and more precisely the term $\Delta v^e_{i,t+\tau}$, requires options with a decreasing time to maturity and it is not an issue when dealing with options traded on exchange markets. Unfortunately, for the data considered in this work, the options are quoted with a fixed time to maturity, it prevents the application of this formula. In the data section a solution to overcome that problem is presented.

Combining Eq.(15) and Eq.(19), the value of a realization of the skew swap is

$$ss_{t,t+\tau} = rskew_{t,t+\tau} - skew_{t,t+\tau},$$

from which the skew risk premium can be obtained after averaging under the historical probability measure, it leads to $\overline{ss}_\tau = \mathbb{E}[ss_{t,t+\tau}]$. It is the amount an investor is willing to pay in order to hedge against a change in the skewness of the underlying log-return distribution over a time interval of length τ. Following the decompositions performed for the variance swap, the decompositions Eqs.(16),(21) suggest to rewrite the skew swap as

$$
\begin{aligned}
ss_{t,t+\tau} &= rskew^u_{t,t+\tau} - skew^u_{t,t+\tau} - (rskew^d_{t,t+\tau} - skew^d_{t,t+\tau}) \qquad (22)\\
&= ss^u_{t,t+\tau} - ss^d_{t,t+\tau} \qquad (23)
\end{aligned}
$$

where the first part on the right hand side of Eq.(22) hedged the up tail of the asset's distribution while the second part on the right hand side of that equation depends on the down tail of the asset's distribution. The decomposition can also be interpreted as a portfolio of two semiskew swaps. Indeed, the first term in Eq.(23) is a long position on a swap paying $skew^u_{t,t+\tau}$ and receiving $rskew^u_{t,t+\tau}$ while the second term in this equation is a short position on a swap paying $skew^d_{t,t+\tau}$ and receiving $rskew^d_{t,t+\tau}$ (or equivalently, a long position on a swap receiving $skew^d_{t,t+\tau}$ and paying $rskew^d_{t,t+\tau}$). Averaging the up and down semiskew swaps gives the corresponding semiskew risk premiums, mathematically they write as $\overline{ss}^u_\tau = \mathbb{E}[ss^u_{t,t+\tau}]$ and $\overline{ss}^d_\tau = \mathbb{E}[ss^d_{t,t+\tau}]$.

As for the previous swaps, it simplifies the analysis to normalize these (semi)skew swaps, they are denoted

$$
xs_{t,t+\tau} = \frac{ss_{t,t+\tau}}{(var_{t,t+\tau})^{\frac{3}{2}}}, \quad xs^u_{t,t+\tau} = \frac{ss^u_{t,t+\tau}}{(var^u_{t,t+\tau})^{\frac{3}{2}}}, \quad xs^d_{t,t+\tau} = \frac{ss^d_{t,t+\tau}}{(var^d_{t,t+\tau})^{\frac{3}{2}}}. \qquad (24)
$$

To lighten the notations the dependency with respect to the second time parameter $t+\tau$ in the different quantities related to either variance or skew swaps might be dropped if no confusion is possible.

Remark 2.3. Notice that Kozhan et al. (2013) define $xs_{t,t+\tau} = rskew_{t,t+\tau}/skew_{t,t+\tau} - 1$, it leads to a formula consistent with xv in Eq.(5) as it is the excess return made on the skew swap. In the foreign exchange case, that formula is problematic as $skew_{t,t+\tau}$ can be zero, or very small, thanks to the stochastic skewness of the smile in that market, the variable xs then becomes very large and is unsuitable as dependent variable for a regression. The normalization of the skew swap $ss_{t,t+\tau}$ presented in Eq.(24) follows Kozhan et al. (2013) (see their Eqs.(25),(28)) while for $ss^u_{t,t+\tau}$ and $ss^d_{t,t+\tau}$ we proceed by analogy. It amounts to make the formula closer to the mathematical definition of the skewness. In that vein, it would be natural to normalize by $var^u_{t,t+\tau} - var^d_{t,t+\tau}$ in Eq.(14) for xvs to obtain the excess return for that contract. Again, the stochastic skewness of the foreign exchange market can lead to a very small value for the normalizing factor that in turn leads to a large value for xvs. Lastly, for xs^u and xs^d in Eq.(24) we could normalize by $skew^u_{t,t+\tau}$ and $skew^d_{t,t+\tau}$ instead as these quantities do not vanish, it would lead to excess return of investments in those semiskew swaps and would be more in the spirit of Kozhan et al. (2013), the results for these variables are very close to those obtained for the choice made here.

2.4 Currency excess return

Let $xm_{t,t+\tau}$ be the currency excess return from t to $t+\tau$, it is given by

$$xm_{t,t+\tau} = \ln \frac{s_{t+\tau}}{f_{t,t+\tau}} = \ln \frac{s_{t+\tau}}{s_t} - (r_d - r_f)\tau. \tag{25}$$

The right hand side of the above equation is the (log) return for a position taken at time t on the forward contract with maturity $t+\tau$ and closed at time $t+\tau$ on the spot foreign exchange market. As it costs zero to enter in that position it is in fact the excess return for that currency pair.

In this work the time to maturity of the options in Eqs.(1),(15) is one month, so if t evolves at daily frequency there is an overlap between two consecutive observations for any variables presented above. It results in strongly autocorrelated variables and regressions based on them can be problematic. Although these aspects can be controlled for, we choose the rather stringent solution of keeping only monthly observations when it comes to compute statistical quantities and/or performing regressions.

3 Data and descriptive statistics

To compute the different quantities presented in the previous section European foreign exchange rate option prices provided by Bloomberg are used. In this work, the three pairs EURUSD, GBPUSD and USDJPY are considered. Spot and 1-month forward foreign exchange rates for these three pairs along with the 1-month overnight interest swaps (OIS) are used as risk free rates in the option pricing formulas. The sample frequency is daily and ranges from 05/01/2006 (DD/MM/YYYY) to 30/12/2016 for all the data[2]. As usual, these options are quoted per deltas,

[2] If the data sample ends on the 30/12/2016, Eq.(3) implies that $rvar_{t,t+\tau}$ (and thereby $xv_{t,t+\tau}$) for $\tau = 1$ month can only be defined up to 30/11/2016.

these are $\{10\Delta, 15\Delta, 25\Delta, 35\Delta\}$, for each delta there are butterfly and risk-reversal volatility quotes. An additional quote is given, it corresponds to the at-the-money (ATM) call, directly given as a volatility. From these quotes, using a standard procedure the implied volatilities for the puts (4 puts) and the calls (4 calls), all out of the money (OTM), are obtained that combined with the ATM call volatility give nine volatility points. For the time to maturity we restrict to the 1-month options. From now on, we only refer to these options and no longer mention the maturity any more. The foreign exchange option market has many distinctive particularities compared to the other option markets, one of them, which is related to its over-the-counter nature, is that only fixed time to maturities are quoted and not maturities, it means that every day there are options with a 1-month time to maturity, as time passes there is not a diminution of the time to maturity. This has a strong impact on the methodology we use as it prevents us to straightforwardly exploit the results of Kozhan et al. (2013) (see also Neuberger (2012)) and more precisely the term $\Delta v^e_{i,t+\tau}$ in Eq.(19) to evaluate the realized skew. To overcome that difficulty we generate butterfly and risk-reversal quotes for any maturity by interpolating butterfly and risk-reversal quotes with maturities one week, two weeks, three weeks and one month. For options with maturity less than one week we take the one-week butterfly and risk-reversal quotes. This difficulty explains our choice to also investigate the alternative of quantifying skew risk by defining the variance-skew swap using the difference between two semivariance distributions in Eq.(11) as the realized part of that contract does not involve any options.

Using standard transformation procedures, the option quotes (i.e. butterfly and risk-reversal quotes) are converted into standard implied volatilities per moneyness, which is defined as the option strike divided by the spot foreign exchange rate, and then these implied volatilities are transformed into option prices using Garman-Kohlhagen's formula, see Castagna and Mercurio (2006).[3] This last step requires the domestic and foreign zero rate curves that are built as follows. For a given currency pair, from the forward and the spot foreign exchange rates along with the USD OIS rate the corresponding interest rate for the other currency is inferred (either EUR, GBP or JPY). Once the two rates are available all the quantities to apply Garman-Kohlhagen's formula are available. As a result, the forward foreign exchange basis problem is avoided, see Chang and Schlögl (2012). Regarding missing option data, if for a given day not all the nine

[3]Notice that from one day to the next the inferred strikes will change, therefore in the following figures the average strike values are reported.

quotes are available, the latest complete set of quotes is used and scaled in a way that the ATM quote of this set of quotes matches the ATM quote of the given day that, in fact, is always available. The missing data are exclusively located at the extreme deltas. It is at that level that the computations of the realized skew can be problematic as there can be missing data for each time to maturity, thus inducing several extrapolations with the difficulty of assessing the impact on the final results. Once these option prices are computed the formulas presented in Section 2 can be evaluated. We first describe the result for the pair EURUSD and then consider the GBPUSD and USDJPY but only underline the specifics as many of the properties of the EURUSD pair are shared by the two others.

Remark 3.1. Regarding the missing option data, the strategy described above captures changes in the level of the smile, which are predominant, but not the other movements (i.e. changes in the slope and curvature). Indeed, a principal component analysis along the moneyness axis produces the usual level, slope and curvature factors, see Figure 1, and the corresponding eigenvalues are (as a percentage of the sum of the eigenvalues): 97.8%, 1.9% and 0.1%. Notice also that for a given day we use exclusively the nine quotes that are available, in particular we do not extrapolate or interpolate the quotes to create fictitious prices.

[Insert Figure 1 here]

3.1 The EURUSD

The evolution of the EURUSD spot foreign exchange rate, the value of 1 EUR in USD, is reported in Figure 2 where an increase of the exchange rate (i.e. a depreciation of the USD against the EUR) can be observed from beginning 2006 to beginning 2008 corresponding to the start of the global financial crisis (GFC) along with good economic prospects for the Euro zone. From that date onwards, there is a global down trend with two larges dips during beginning 2009 and mid 2010. It leads to the mean value for xm given by Eq.(25) and computed on a monthly basis equals to -1.491×10^{-3} (-1.79% per annum) and a standard deviation of 0.029, see Table I.

[Insert Figure 2 here]

[Insert Table I here]

As call options can be used to hedge against an increase of the foreign exchange rate while put options can be used to hedge against a decrease of the foreign exchange rate, it is possible to extract from the foreign option market information on market participants' expectations. Having built the daily time series for the option implied volatility or smile, the Figure 3 reports the average computed over all the sample. The smile is decreasing suggesting that put options, given by the moneynesses smaller than one, are more expensive than call options, given by the moneynesses larger than one. Overall, market participants value more a decrease of the EURUSD foreign exchange rate (i.e. a depreciation of the EUR against the USD) than the opposite. The shape of the smile is similar to the one observed in the equity index option market that is well known to be decreasing, puts are more expensive than calls, market participants mainly fear a drop of the index. Contrarily to the equity index market, the foreign exchange option market possesses the rather specific property that market participants can also fear an increase of the foreign exchange rate that is equivalent to say a depreciation of the USD against the EUR. It means that even if on average market participants fear a depreciation of the EUR against the USD it is possible to have the opposite, market participants can fear a depreciation of the USD against the EUR or, equivalently, an increase of the EURUSD foreign exchange rate. The point is well illustrated in Figure 4 giving the evolution of the daily smile. From this graph, it is apparent that the smile is most often decreasing; there is a surge of the option market volatility during the GFC; during mid 2016 the exchange rate dropped strongly; there are periods during which the smile is increasing.

[Insert Figure 3 here]

[Insert Figure 4 here]

To develop a bird's eye view of the smile, Figure 2 also displays the difference between the implied volatility for a moneyness of 0.96 (left-end part of the smile) and the implied volatility for a moneyess of 1.04 (right-end part of the smile). During most of the period ranging from 2006 to end of 2009, with a short break around October 2008, this difference is negative, implying an increasing smile and suggesting that market participants fear a depreciation of the USD against the EUR, through expensive calls, and it is consistent with the evolution of the exchange rate during that period. From beginning 2010 onwards, the smile is most of the time decreasing as the difference between the two implied volatilities is positive, market participants value more a drop of the EURUSD than the opposite, puts are more expensive than calls. Also evident from the figures, either Figure 2 or Figure 4, is the fact that the smile can change abruptly over a very short period of time and suggests that certain events are unexpected. The Figure 5 gives the smile for the dates 28/11/2006, 27/10/2008, 18/12/2008, 18/03/2009, 14/06/2016 and 8/11/2016 while Table II provides certain historical events occurring during those dates. For example, on the 14/06/2016 the smile is strongly decreasing, puts are much more expensive than calls, it corresponds to the Brexit, market participants fear a strong depreciation of the EUR. On the contrary, on the 8/11/2016, the smile is increasing, calls are more expensive than puts, market participants fear a depreciation of the USD and it corresponds to the U.S. presidential election. It is interesting to notice that these extreme shapes are in general short lived. The use of options to analyze market participants' expectations during certain punctual events, such as elections, was already performed long ago in Rockinger and Jondeau (2000).

[Insert Figure 5 here]

[Insert Table II here]

The fact that the smile can be increasing or decreasing has important consequences when using a parametric model. The most flexible framework is the affine model proposed by Heston (1993) and it leads to a smile with a constant slope, this property is related to the fact that in this model the correlation between the asset and its volatility is constant. If that correlation is negative, a decrease of the asset is associated with an increase of its volatility and as option prices depend on the asset's distribution it translates into put prices, which depend on the left tail asset's distribution, being more expensive than calls, which depend on the right tail asset's distribution, and a decreasing smile. A positive correlation has the opposite effect. As a result, the slope of the smile is controlled by the sign of the correlation between the asset and its volatility, it also gives an indication of the skewness of the asset's log-return distribution. Indeed, a negative correlation implies a thicker left tail than right tail for this distribution, so a negatively skewed distribution. As the slope of the smile in the foreign exchange option market can change, the Heston model is too rigid to capture these market data properties and a model that can generate a stochastic skew is needed. Such kind of parametric model was proposed by Carr and Wu (2007). An important consequence is that when puts are expensive and calls are not, the smile is decreasing, market participants fear a drop of the foreign exchange rate. When the opposite occurs, puts are not expensive while calls are, the smile is increasing, market participants fear an increase of the foreign exchange rate. This fact is rather specific to this market, it does not occur in the equity index option market. The third case, also specific to this market, is a smile that is symmetric, puts are as expensive as calls, certain market participants fear an increase while others fear a decrease of the foreign exchange rate. These considerations underline the usefulness of decomposing distribution related quantities, such as the second and third order moments, into left (down) and right (up) components as performed in the pricing section. Already at the descriptive statistic level it proves to be very instructive.

To gain some intuition on the evolution of the variables, Figure 6 reports the daily values for var of Eq.(1) and $rvar$ of Eq.(3). It can be checked, at least visually, that $rvar$ is most of the time lower than var, it implies that on average the variance swap protection buyer is willing to lose money to hedge against an increase of the volatility, the volatility risk premium is, as expected, negative. The decomposition of the risk neutral variance var into var^u and var^d, as defined in Eq.(2), leads to Figure 7 where it can be checked that the former is on average larger than the latter. In Figure 8, the realised and risk neutral skews are reported. A striking fact is that both can be either positive or negative. It is know from Kozhan et al. (2013) that a negative risk neutral skew is associated with a downward sloping implied volatility smile while Da Fonseca and Xu (2017) show a positive skew is associated with an upward implied volatility smile. This Figure illustrates convincingly the stochastic skew nature of the foreign exchange option market. Furthermore, Figure 8 distinctively shows a strong negative shock around 27/10/2008, it corresponds to a more skewed (to the left) distribution and a steeper downward sloping implied volatility smile, a fact that can be checked in Figure 5, with puts more expensive than calls to hedge against a depreciation of the EUR. In terms of event, it corresponds to the announcement by the ECB president J.-C. Trichet to commit to cut borrowing costs to mitigate liquidity risks, and around that period started the steady decline of key ECB rates (i.e. the interest rate on the main refinancing operations, on the deposit facility and on the marginal lending facility).

[Insert Figure 6 here]

[Insert Figure 7 here]

[Insert Figure 8 here]

As explained in the pricing section to reduce the impact of autocorrelation it is advisable to work at monthly frequency when performing regressions and the like. Indeed, it is shown in

Figure 9 that at this frequency the risk neutral variance var and realized variance $rvar$ are still autocorrelated, which is not surprising, but their difference is not and, as a result, xv given by Eq.(5) is not autocorrelated. These results are consistent with those reported in Table 4 of Jurek (2014). A similar remark applies to the variables $skew$ and $rskew$ but let us stress the fact that $rskew$ in Jurek (2014) differs from Kozhan et al. (2013)'s definition that follows Neuberger (2012). A byproduct of this weak autocorrelation is that in the regressions the correction proposed in Newey and West (1987) will mainly control for heteroskedasticity. As a consequence, all the statistical values given in Table I and all subsequent results will be obtained at that monthly frequency.

[Insert Figure 9 here]

Table I reports the descriptive statistics for xv, xv^u, xv^d, xvs, xs, xs^u and xs^d. The negative volatility risk premium implies a negative mean value for xv of -7.391×10^{-2} (i.e. -88.7% per annum) and a standard deviation of 0.470. The decomposition of the variance swap contract into up and down components leads to mean values of -0.409 and 0.480 for xv^u and xv^d, respectively. The first value indicates that an investor entering into such up semivariance swap contract makes on average a rather substantial loss. On the contrary, the second value implies that a down semivariance swap contract generates a positive return. These two numbers are consistent with the shape of the implied volatility smile and the return of the (undecomposed) variance swap contract. Indeed, with this latter contract an investor loses on average but makes a gain whenever the (received) realized variance volatility is higher than the (paid) risk neutral variance. On average the smile is downward sloping, a decrease of the foreign exchange rate implies an increase of its volatility, so as the purpose of a variance swap is to hedge against an increase of the volatility it has to generate a gain when the foreign exchange rate drops and a loss when it increases as in that case the realized variance is smaller than the risk neutral variance. This explains the signs for xv^u and xv^d. Regarding the standard deviations, the values are 0.700 and 1.766, the down related variable is more volatile.

The average value of xvs is -0.435 and its standard deviation is 1.019 thereby implying that an investor is willing to pay in order to hedge a decrease of the skewness. To understand this, it might be easier to consider the pseudo-skew risk premium $\overline{vss}_\tau = \overline{vs}^u_\tau - \overline{vs}^d_\tau$ of Eq.(13) that is equal to -1.074×10^{-4}, it means that the risk premium for the upper tail distribution is smaller than the risk premium for the lower tail distribution, an investor does not value similarly the two tails of the underlying log-return distribution.

Regarding the skew swap xs, the mean value is equal to -0.068 and the standard deviation is 0.483. It simplifies the analysis to look at the values of $rskew_{t,t+\tau}/(var_{t,t+\tau})^{\frac{3}{2}}$ and $skew_{t,t+\tau}/(var_{t,t+\tau})^{\frac{3}{2}}$, they are -0.173 and -0.105, so entering into a skew swap implies paying 0.105 and receiving 0.173, the skew risk premium is positive. It differs from the results of Kozhan et al. (2013) that finds a negative skew risk premium as in their case the amount paid is 1.808 while the amount received is 1.001 (see their Table 1 in page 2185). For the up and down semiskew swaps the mean values are -1.344 and -2.696, respectively, while the standard deviations are 0.341 and 2.017, the lower tail related contract shows a much higher variance value. Both semiskew swaps imply a loss and negative semiskew risk premiums with the down semiskew swap leading to a lower value (or higher absolute value). It is worth pointing out that according to Eq.(23), trading a skew swap is equivalent to being long on an up semiskew swap and short on a down semiskew swap, the former will generate a loss while the latter a gain. As previously explained, a bear market condition is associated with a negatively skewed distribution and more volatile market conditions.

Regarding the correlations between the variables $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$, they are reported in Table III. xv is weakly negatively correlated with xv^u while it is strongly positively correlated with xv^d (these results sharply contrast with those of Kilic and Shaliastovich (2018), it underlines the consequences of Remark 2.1); xs is positively correlated with xs^u and strongly negatively correlated with xs^d; xv is strongly negatively correlated with both xvs and xs (this latter correlation contrasts with the value obtained in Kozhan et al. (2013) for S&P500 options). xs and xvs are strongly positively correlated and as these two variables are supposed to carry a similar information this result is very appealing. To some extent, xs and xvs have correlations with the other variables that are comparable (in sign and magnitude). Lastly, it is interesting to notice that both xv and xs have a stronger correlation with the down component (xv^d for xv and xs^d for xs) than with the up component (xv^u for xv and xs^u for xs).

[Insert Table III here]

Overall, when restricted to the (undecomposed) variance swap, along with the associated risk premium, the results are in line with those of Kozhan et al. (2013) and consistent with the downward sloping shape of the smile as a decrease of the foreign exchange rates is associated with an increase of its volatility, the left tail of the underlying log-return distribution is thicker than the right tail, the distribution is negatively skewed. The semiswaps (and the associated risk premiums) carry the same information. For the skew related contracts, the down semiskew swap implies a higher risk premium than the up one, it is also consistent with the negatively skewed distribution and shape of the smile. As a result, second and third order semimoments carry consistent information.

3.2 The GBPUSD

For the GBPUSD foreign exchange rate, Figure 10 reports the evolution of the spot value along with the difference between the implied volatilities for the moneynesses 0.96 and 1.04. It can be seen in this figure the depreciation of the USD from beginning 2006 to beginning 2008, followed by a rapid drop of the exchange rate during the year 2008, a stable level from 2010 to 2015 and a steady decline from 2015 onwards. Regarding the slope of the smile it is most often downward sloping although during the beginning of the sample the smile was increasing (as for the EURUSD). Two spikes can be observed on the slope curve, the first is around mid 2014 and corresponds to the Scottish referendum while the second is around mid 2016 and is the Brexit, see Table II. Averaging the smile over the sample period gives Figure 11 where the downward shape of the smile is confirmed while Figure 12 clearly displays the changing nature of the implied volatility smile. Consistently with these figures, in Table I the descriptive statistics show that xm is on average equal to -2.726×10^{-3} with a standard deviation of 0.028. The mean value for xv is -2.443×10^{-2} (i.e. -29.3% per annum), the volatility risk premium is also negative as for the EURUSD, while for xv^u and xv^d the average values are -0.366 and 0.527, respectively. As for the EURUSD, investors are willing to pay in order to hedge against an increase of the volatility that will occur when the underlying asset declines due to the downward slope of the smile. The up semiskew swap will generate a loss while the down semiskew swap will produce a gain. The down tail related contract exhibits more variance as its standard deviation is 1.862 while the up one is 0.731. The interpretation developed for the pair EURUSD applies here as well.

[Insert Figure 10 here]

[Insert Figure 11 here]

[Insert Figure 12 here]

For xvs the average value is -0.425 while its standard deviation is 1.074. For the skew swap, the mean value is -0.123 and the standard deviation is 0.624, while for the up and down semiskew swaps the mean values turn out to be -1.415 and -2.483 with standard deviations equal to 0.318 and 2.725, respectively. As for the other currency pair, it is fruitful to look at $rskew_{t,t+\tau}/(var_{t,t+\tau})^{\frac{3}{2}}$ and $skew_{t,t+\tau}/(var_{t,t+\tau})^{\frac{3}{2}}$, their mean values are -0.263 and -0.140, and imply a positive skew risk premium (i.e. the skew swap involves paying 0.140 and receiving 0.263). Regarding the correlations between the variables $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$, they are also reported in Table III and are very similar to those of EURUSD. The main differences are the correlations between xvs and xs^u, which now is stronger and negative (-0.244 against 0.007 for the EURUSD), and xv^u and xs^u that is -0.203 while it is 0.097 for the currency pair EURUSD.

Overall, these values do not differ from those of the EURUSD and all the remarks made for the previous currency pair apply to this currency as well.

3.3 The USDJPY

For that currency pair the evolution of the foreign exchange rate and the implied volatility slope are given in Figure 13, the average implied volatility smile appears in Figure 14, Figure 15 reports the evolution of the smile while Table I contains the descriptive statistics. For the foreign exchange rate evolution, there is a steady decline from 2006 to end 2012 and the trend reversed from that date onwards. The slope of the smile is decreasing, market participants value more a depreciation of the USD (against the JPY) than the opposite as puts are more expensive than calls. On that aspect, it is interesting to note the difference with the two other pairs. The evolution of smile only shows one turbulent period corresponding to the GFC. Notice also that there are few periods during which the smile is increasing, for example around January/February 2013. From Table II, it is known that it corresponds to the announcement by the government to approve a stimulus package and the Bank of Japan to commit to buy assets to raise the inflation rate. The average value for xm is 1.273×10^{-3} (i.e. 1.52% per annum) and the standard deviation is 0.033 while for xv the mean value is -1.641×10^{-2} (i.e. -19.69% per annum) implying, as for the other pairs, a negative volatility risk premium. Regarding xv^u and xv^d their mean values are -0.283 and 0.356, respectively, while the corresponding standard deviation are 0.921 and 2.014. These results are qualitatively similar to those for the other currency pairs.

For xvs the average value is -0.313 and the standard deviation is 1.198, these values are in line to those for the EURUSD or the GBPUSD. As Figure 15 shows only one turbulent period, the GFC, the small negative average value for xvs is mainly due to that event and it is worth pointing out the fact that during that period the slope of the smile is around 0.1 (see Figure 14) while for the two others they are around 0.06. As previously mentioned, a steeper slope implies a stronger distortion of the underlying log-return distribution that is captured by xvs.

For the skew swap, the mean value is -0.198 and the standard deviation is 1.571 (the average values for $rskew_{t,t+\tau}/(var_{t,t+\tau})^{\frac{3}{2}}$ and $skew_{t,t+\tau}/(var_{t,t+\tau})^{\frac{3}{2}}$ are -0.393 and -0.195 implying again a positive skew risk premium). For the up and down semiskew swaps, the mean values are -1.385 and -0.1966 while the standard deviations are 1.556 and 5.678, these values are similar to those obtained for the other pairs although the standard deviations appear to be a bit larger. As for the other currencies the down semivariance and semiskew swaps have larger (absolute) mean values and standard deviations than their up counterparts. Regarding the correlations between the variables $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$, they are also reported in Table III and are very similar to those of EURUSD. The main differences are the correlations between xvs and xs^u, which now is stronger and negative (-0.244 against 0.007 for the EURUSD), and xv^u and xs^u that is -0.203 while it is 0.097 for the currency pair EURUSD. Regarding the correlations between the variables $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$, they are reported in Table III and are very similar to those for the two other currency pairs. The main differences are the correlation between xv and xv^u that is equal to 0.223 while for the other currency pairs the values are -0.056 for EURUSD and -0.087 for USDJPY; the correlation between xv and xs^u that is now weaker (0.078 against 0.193 and 0.168 for the two others). Notice that the down tail corresponds to a depreciation of the USD against the JPY.

[Insert Figure 13 here]

[Insert Figure 14 here]

[Insert Figure 15 here]

4 Empirical results

Having defined various kind of swaps the next step is to analyze to which extent these variables can explain the foreign exchange risk premium. For the equity index market the importance of the variance risk premium to explain the market excess return was established, long ago, in Carr and Wu (2009). More recently, for the same market the importance of skew risk premium was proved in Kozhan et al. (2013) while for the foreign exchange market it was carried out in Broll (2016). Still for the currency market, Della Corte et al. (2017) and Londono and Zhou (2017) showed that the variance risk premium is an important determinant of the carry trade[4].

We push further the analysis of the interaction between the variance risk premium and the foreign exchange rate excess return by performing conditional decompositions of the variance swap into semivariance swaps and show their relevance in explaining the currency excess return. We show the equal importance of the variance-skew swap, a result that appears here for the first time. Lastly, we consider third moment related contracts such as skew swap and, more importantly, their conditional decompositions named the up and down semiskew swaps and show that they contain relevant information for the currency excess return. The importance of semivariance measures, such as realized semivariances, is shown in Patton and Sheppard (2015) and we push further the analysis at the risk premium level as well as at the third moment level. As mentioned, the interesting work of Kilic and Shaliastovich (2018) also performs a decomposition for the variance risk premium but their decomposition is different than ours and also they study a different market (i.e. the equity index option market). Despite these differences, comparing our results with Kilic and Shaliastovich (2018)'s underlines the specific of the currency market.

As explained in the previous sections the decomposition of the variance swap into up and down components along with their negative combination, the variance-skew swap, is particularly relevant for the currency market as it possesses the rather distinctive feature of stochastic skewness. Indeed, this latter property leads to the serious problems mentioned in the data section when dealing with the skew swap that are solved by the decomposition into up and down components.

[4]A very large body of the foreign exchange literature is devoted to the carry trade problem and in addition to the works already quoted we can mention Jurek (2014) and Bekaert and Panayotov (2017). Despite the importance of that subject this work does not pretend to contribute to that literature but instead it can be considered as an empirical extension of Kozhan et al. (2013), which consider the equity index market, with the aim of analyzing the specifics of the currency market.

The results will confirm that intuition.

In a first part, we present factor models for the EURUSD foreign exchange rate excess return based on various swaps then the other two pairs (i.e. GBPUSD, USDJPY) are presented but most of the results are in line with those of the EURUSD. However, any significant difference will be underlined. In a second part, we propose a simple strategy to assess the importance of extreme movements affecting (semi)variance and (semi)skew swaps to explain the currency excess return. It helps to determine whether strong shocks occurring in the foreign exchange option market carry any long-term view on the evolution of the foreign exchange rate. Lastly, a forecasting test is performed and shows the importance of the decomposition extends to the prediction of the currency excess return.

4.1 Factor models for the currency excess return

4.1.1 The EURUSD

First, we consider factor models based on the variance swap Eq.(26), the semivariance swaps considered individually in Eqs.(27),(28) and then a two-factor model based on the up and down semivariance swaps Eq.(29), it leads to the equations

$$xm_t = \alpha_0 + \alpha_1 xv_t + \epsilon_t^{\alpha}, \tag{26}$$

$$xm_t = \beta_0 + \beta_1 xv_t^u + \epsilon_t^{\beta}, \tag{27}$$

$$xm_t = \gamma_0 + \gamma_1 xv_t^d + \epsilon_t^{\gamma}, \tag{28}$$

$$xm_t = \theta_0 + \theta_1 xv_t^u + \theta_2 xv_t^d + \epsilon_t^{\theta}, \tag{29}$$

with the results reported in Table IV in columns (1)-(4).

[Insert Table IV here]

The first model, given by Eq.(26), leads to a coefficient for xv that is negative and equal to -0.018, it is significant. This result is consistent with the purpose of a variance swap and the shape of the implied volatility smile of that market. Indeed, the implied volatility smile is decreasing implying that a decrease of the exchange rate is associated with an increase of its volatility. As a result, a variance swap should provide a positive return in a bear market condition, so the coefficient has to be negative. The constant term is not significant and the R^2 is 8.32%. For the factor model based on xv^u, given by Eq.(27), the coefficient is equal to 0.031 and highly significant. The positive sign is consistent with the definition of xv^u given by Eq.(10) as the up semivariance swap can generate a positive return only when xm is positive. The constant coefficient is positive and significant. More importantly, the R^2 is large and equal to 55.19%, it shows that the upper tail distribution, as the up semivariance swap only depends on it, contains more information than the aggregation of up and down tails that constitutes the variance swap. Similar conclusions can be drawn for the third factor model given by Eq.(28) based on xv^d as the coefficient for this variable is negative and highly significant while the R^2 is large and equal to 47.62%. As for the previous model, the negative sign of the coefficient is consistent with the definition of a down semivariance swap, the product generates a gain only if the foreign exchange rate drops. Regarding the constant term, it is positive and significant. Building a factor model using xv^u and xv^d (i.e. Eq.(29)) further underlines the interest of decomposing the variance swap into two components as the regression based on these variables leads to coefficients that are significant, whose signs are consistent with those of univariate regressions (i.e. Eq.(27) and Eq.(28)), and a R^2 of 60.44% implying that the two variables provide complementary information for the foreign exchange rate excess return. The trading strategy associated with Eq.(29) amounts to trade separately on the up and down semivariance swaps while Eq.(26) involves trading simultaneously on the up and the down semivarance swaps with the same position (long) on both contracts while Eq.(29) suggests that it is better to go long in the up semivariance swap contract and short in the down one. Here also, the constant term is significant and positive.

The regression specified in Eq.(30) uses as dependent variable the variance-skew swap xvs_t

$$xm_t = \nu_0 + \nu_1 xvs_t + \epsilon_t^{\nu}, \tag{30}$$

with the results reported in column (5) of Table IV that lead to a positive significant coefficient of 0.22, a R^2 of 58.58% and a constant term that is positive and significant. The R^2 is close to what is achieved when regressing on both xv^u and xv^d, it suggests that the important feature of the variance-skew swap is having opposite direction in the up and down components and it underlines, once more, that being long in both, the up and the down semivariance swaps, is not optimal (what happens with xv). The positiveness of the xvs regression coefficient can be better understood by considering the term $rvar^u_{t,t+\tau} - rvar^d_{t,t+\tau}$ in Eq.(11) and $xm_{t,t+\tau}$ as by definition an increase for the latter is associated with an increased value for the former. If we consider xvs as a proxy for the skewness of the foreign exchange rate (log)return distribution, this last regression clearly shows that for this currency pair this quantity is highly important to explain currency returns.

The other contracts related to the skewness of the distribution are the skew swap, the semiskew swaps and their combination, they lead to consider the factor models:

$$xm_t = \mu_0 + \mu_1 xs_t + \epsilon^\mu_t, \tag{31}$$

$$xm_t = \eta_0 + \eta_1 xs^u_t + \epsilon^\eta_t, \tag{32}$$

$$xm_t = \lambda_0 + \lambda_1 xs^d_t + \epsilon^\lambda_t, \tag{33}$$

$$xm_t = \kappa_0 + \kappa_1 xs^u_t + \kappa_2 xs^d_t + \epsilon^\kappa_t, \tag{34}$$

with the results reported in Table IV in columns (6)-(9). The model based in xs produces a significant regression coefficient for the dependent variable and the R^2 is high at 27.05%, it suggests that the skew swap excess return strongly explains the foreign exchange rate excess return. As for the variance swap, the conditional decomposition of the skew swap proves to be very fruitful. In Eq.(32), a factor model based on xs^u with results in column (7) shows that the up skew swap explains poorly the currency excess return, the R^2 is low at 1.54% and the regression coefficients are not significant. In sharp contrast, the model based on the down skew swap xs^d, with results in column (8), produces strong results, the R^2 reaches a 27.45% and both coefficients are highly significant. The negative coefficient sign for the variable xs^d, given by Eq.(24), dwells from the expression of $rskew^d$ from which it can be checked that an increase of xm leads to a decrease of $rskew^d$ and, therefore, to a decrease of xs^d. Hence, the negative sign for the regression coefficient. In conclusion, the decomposition of the skew swap gives strong results with the down component proving to be much more informative with respect to the currency excess return as in fact it contains all the information of the skew swap. The fact that the left part of smile, which depends on puts and therefore on the likelihood of a decrease of the currency rate, is more expensive than the right part of the smile, which depends on calls and therefore on the likelihood of an increase of the currency rate, certainly explains the importance of the down semiskew swap.

The next step is to consider the combination of xs^u and xs^d, it leads to the model Eq.(34) and the results are reported in column (9) of Table IV, it allows us to assess whether these two variables have complementary information. We already know that it is the case for xv^u and xv^d. The factor model based on this pair of variables produces a R^2 of 29.63%, which is close to the sum of the R^2 of the two univariate regressions on xs^u and xs^d, but it can be checked that the results are essentially similar to those obtained when xs^d is considered alone as the xs^u variable is still not significant. The coefficient signs of the dependent variables are consistent with the natural decomposition of the skew swap, positive for xs^u (albeit this coefficient is not significant at 10% level) and negative for xs^d as explained in the pricing section with the Eq.(22). The similarity with the factor model based on xs (i.e. Eq.(31)) underlines the fact that regarding the currency excess return, the down component carries the essential information contained in the (undecomposed) skew swap.

The next step is to consider factor models based on the pairs (xv, xs) in Eq.(35), (xv^u, xs^u) in Eq.(36) and (xv^d, xs^d) in Eq.(37) with the results in columns (10), (11) and (12) of Table IV, respectively. As our purpose is to analyze the significance of the tails, only combinations involving two up variables or two down variables are considered. It leads to the regressions

$$xm_t = \iota_0 + \iota_1 xv_t + \iota_2 xs_t + \epsilon_t^{\iota}, \tag{35}$$

$$xm_t = \varsigma_0 + \varsigma_1 xv_t^u + \varsigma_2 xs_t^u + \epsilon_t^{\varsigma}, \tag{36}$$

$$xm_t = \omega_0 + \xi_1 xv_t^d + \omega_2 xs_t^d + \epsilon_t^{\omega}. \tag{37}$$

The first regression gives a R^2 of 27.39% that is close to the value obtained when regressing on xs alone, only this variable is significant implying that the skew and variance swaps are redundant (w.r.t. the currency excess return). In Eq.(36), the regression on the up semivariance and semiskew swaps produces a R^2 of 55.46% and significant coefficients only for the constant term and the up semivariance swap. The model is essentially similar to the univariate model based on the first dependent variable, the second variable (i.e. xs^u) remains insignificant as in the univariate case. The factor model based on the down variables, that is to say, xv^d and xs^d, put in perspective with the two univariate regressions on these variables show that they share quite a lot of information as the R^2 is equal to 50.5% and is marginally higher than the R^2 achieved when xv^d is considered alone (i.e. 47.62%). Still, the variable xs^d is highly significant, it conveys information not available in the down semiskew swap. Lastly, the regression coefficients' signs are consistent with those of the univariate regressions (and in line with intuition).

Overall, the regression results convincingly show that the decomposition is crucial to obtain strong results. The contrast in terms of performance between undecomposed and decomposed swaps, whether the variance swap or the skew swap, is striking. The up and down semivariance swap components are nearly equally informative regarding the currency excess return while for the semiskew swap only the down component is informative. The variance-skew swap strongly explains the currency excess return.

4.1.2 The GBPUSD

For that currency pair the same factor models are estimated and the results, reported in Table V, are very much in line with those of EURUSD, that is to say, the coefficients' signs and coefficients' significances are identical for both currencies across the regressions. Therefore, only differences will be analyzed and they are mainly related to the R^2. The factor model based on xv leads to a R^2 of 19.08%, twice the value found for EURUSD, while regressions on xv^u and xv^d produce R^2 of 43.53% and 51.29%, respectively. Combining the two variables (xv^u, xv^d) leads to a regression with both coefficients significant and a R^2 of 55.72%, there is quite an overlap between these variables in terms of information content but, still, they contain complementary and significant information. These values are comparable to those obtained for the EURUSD. The R^2 of the regression on the variance-skew swap excess return xvs is high at 56.63% and its regression coefficient is positive and highly significant, the results are similar to those for the EURUSD.

The factor model based on the skew swap excess return xs provides results much in line with those of the EURUSD as this variable is significant and the R^2 is high at 18.45%. As for the EURUSD, xs^u is weakly significant and barely explains the currency excess return (the R^2 is around 3%) while xs^d carries significant information for this variable as the R^2 is 21.99%. The other factor models lead to similar conclusions to those derived for the EURUSD case and confirm the importance of the decomposition to obtain strong results.

A difference with the currency pair EURUSD appears when considering a factor model on xv and xs, the R^2 is 22.62%, marginally higher than when regressing on xv or xs alone but xs becomes insignificant when combined with xv (the opposite to the EURUSD case). Another difference is the factor model based on (xv^d, xs^d), the second variable is not significant, the results are similar to those achieved when regressing on xv^d alone.

[Insert Table V here]

4.1.3 The USDJPY

For this currency pair the results are reported in Table VI and for all the coefficients but the coefficient of xv in Eq.(26) and the coefficient of xs^d in Eq.(33), the coefficients' signs and the coefficients' significances are similar to those of EURUSD. Again, the discussion will focus on the R^2 values. The R^2 for the factor model based on xv (i.e. Eq.(26)) is low at 2.76% and contrasts with the values for two other currency pairs. For the univariate regressions on xv^u and xv^d the R^2 are 51.3% and 41.92%, respectively. The magnitude is comparable to the other currency pairs and is closer to the EURUSD pair in the sense that the up semivariance swap has more explanatory power than the down one. The regression on the two variables xv^u and xv^d leads to an R^2 of 61.37%. It is for this currency that there is the largest discrepancy between the regression based on aggregated variable (i.e. xv) and the disaggregated variables (xv^u and xv^d). As for the EURUSD and GBPUSD, the regression on the variance-skew swap produces a large R^2 of 59.81% and the regression coefficient is highly significant and close to those for the other currency pairs.

Regarding the variable xs and the associated regression Eq.(31), the regression coefficient is positive and significant but the R^2 is lower (i.e. 9.96%) than for the other currencies. Similarly, xs^u explains little of the currency excess return while xs^d, the lower tail related semiskew swap, contains most of the information that is in xs and underlines, once again, the importance of the lower tail compared to the upper tail. For the factor model based on (xv^d, xs^d), the regression coefficient of xs^d is significant but positive while it is negative in the other regressions (i.e. xs^d alone or xs^u and xs^d) and, more importantly, there is a strong complementarity between xv^d and xs^d as the R^2 when regressing of these two variables is close to the sum of the univariate regressions' R^2 based on xv^d and xs^d, this result contrasts with those obtained for the other currency pairs.

Overall, the results confirm the importance of the lower tail. Notice that it is associated with a depreciation of the USD against the JPY, in opposition to what is observed for the other currency pairs. The singularity is related, or maybe we should add probably, to the fact that JPY is traditionally a funding currency in carry trade activity, thus puts are the natural products to hedge against an adverse evolution of the foreign exchange rate that can jeopardize this kind of trading activity[5].

[Insert Table VI here]

4.2 Trimming the tails

A rather puzzling fact in the smile evolution is that certain shocks can be extreme and short lived. Whether we consider the Brexit for the EURUSD and GBPUSD or the U.S. presidential election for the EURUSD, see Figures (2), (10), these events were associated with strong changes of the smile (level and slope) but a few weeks after they occurred, foreign exchange option prices were back to their average levels. These figures also show that the spot foreign exchange rate did not display such violent movements. As a result, we can question whether extreme shocks affecting the option market, which are by nature short lived, convey any relevant information regarding the evolution of the currency excess return. To assess that statement, we propose a very simple modification of our data. Instead of trying to extract rare events from option prices, which by definition are few and is problematic from a statistical point of view, and see whether they can explain the currency excess return, we reverse the strategy, that is, we remove extreme events by "trimming" the explanatory variables and check if their explanatory power is improved or diminished.

To implement the aforementioned analysis, we proceed as follows. Given a variable $(x_t)_{t \geq 0}$ and denote by m_x and σ_x its mean value and standard deviation, respectively, then its "trimmed" version consists in replacing (centered) values that are larger in absolute value terms than 3

[5] In addition to carry trade and as pointed out in Bank of Japan (2009) p. 71, Japanese institutional investors hedge their foreign exchange risks associated with their foreign currency-denominated bonds as well as exporters for their foreign currency-denominated payments.

standard deviations. Mathematically, it reads as

$$x_t \mathbf{1}_{\{m_x - 3\sigma_x < x_t < m_x + 3\sigma_x\}} + (m_x - 3\sigma_x)\mathbf{1}_{\{x_t < m_x - 3\sigma_x\}} + (m_x + 3\sigma_x)\mathbf{1}_{\{x_t > m_x + 3\sigma_x\}} \tag{38}$$

and we perform the regressions of the previous section using these trimmed variables as dependent variables. Any improvements in the regressions' R^2 illustrate the fact that extreme and rare events affecting the currency option market do not carry information on the evolution of the currency. In the analysis we only focus on salient differences with the results already presented.

4.2.1 The EURUSD

For the EURUSD, the results are reported in Table VII and show no important differences with those reported in Table IV. Indeed, the regressions lead to R^2 that are very close to those obtained for untransformed variables. More precisely, the R^2 are slightly larger, with a maximum of 300 basis points when considering xv^d and xs^d as explanatory variables, but are never smaller. As a result, we conclude that extreme movements, which are short lived shocks, carry no information on the evolution of the currency excess return.

[Insert Table VII here]

4.2.2 The GBPUSD

For the GBPUSD, the regressions on xv, xv^u, xv^d and xvs are barely affected by the transformation, removing the extreme tails of the variables do not deteriorate their explanatory power. On the contrary, trimming xs significantly improves the regression's R^2 as it jumps from 18.45%, for the untransformed case, to 28.08%. The regression coefficient of xs keeps the same sign and becomes even more significant. The regression on xs^d is significantly improved as the R^2 increases from 21.99% to 35.88% and the regression coefficient for that variable is -0.008 with a significant level of -5.13 (compared to -0.004 and -2.10 for the untransformed case). A byproduct of these improvements is that the regression on the pair (xs^u, xs^d) displays now a R^2 of 37.14% compared to 23.63% in the previous case. Consistently with the improvement of the explanatory power of the variable xs, the regression on (xv, xs) produces a R^2 of 29.49% (thus similar to the univariate regression on the trimmed xs) with the noticeable difference with the previous case that now xv is not significant (i.e. compare with column (10) of Table V). Lastly, regressing on the trimmed variables xv^d and xs^d does not differ significantly than regressing on the (untransformed) variable xv^d alone as xs^d is not significant and trimming the variable xv^d does change significantly the results. As for the EURUSD, regression results show that removing the tails induces no loss of information and, on the contrary, slightly improves the results. However, in contrast with that currency pair, removing the tails does significantly improve the explanatory power of xs and xs^d. A possible explanation of why trimming skew related variables improves the results is certainly due to the fact that skewness is more sensitive to extreme values. The fact that trimming xs^u does not affect the result is worth noticing.

[Insert Table VIII here]

4.2.3 The USDJPY

For the USDJPY, univariate regressions on the variables xv, xv^u, xvs and xs^u are barely affected. For xv^d, trimming the variable improves the R^2 of the regression from 41.92% to 46.2% (the regression coefficient is not altered). Similar conclusion applies to the univariate regressions on xs and xs^d as the R^2 increase from 9.96% to 13.51% and from 7.59% to 13.67%, respectively. The improvement for the former variable translates into a higher R^2 when regressing on both xv and xs (i.e. 13.71% compared to 9.98%) while for the latter (i.e. xs^d) the regression on xs^u and xs^d leads to a R^2 of 15.72%, a value larger than the 9.25% achieved with the untransformed variables (the improvement is uniquely due to xs^d as xs^u is not significant). As for the two other currency pairs, trimming the tails improves the explanatory power of the dependent variables. It underlines the fact that extreme movements in the currency option market does not convey long term views for the currency evolution.

[Insert Table IX here]

4.3 On the predictability of the currency excess return

In this section we consider the problem of predicting the currency excess return by following the strategy proposed in Corsi (2009) (see also Patton and Sheppard (2015) for a similar approach). For a given variable $(x_n)_{n=1...N}$ and a given time t, the h-month past (to time t) average is defined as

$$x_{t,hm} = \bar{x}_{hm} = \frac{1}{h}\sum_{i=1}^{h} x_{t-i}. \tag{39}$$

To quantify the power of a given set of variables $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$, Corsi (2009) proposes to preform the regressions

$$\begin{aligned} xm_t &= \mu_0 + \mu_1 x_{t,1m} + \mu_6 x_{t,6m} + \mu_{12} x_{t,12m} + \epsilon^\mu \\ &= \mu_0 + \mu_1 \bar{x}_{1m} + \mu_6 \bar{x}_{6m} + \mu_{12} \bar{x}_{12m} + \epsilon^\mu \end{aligned} \tag{40}$$

when using a single explanatory variable or

$$\begin{aligned} xm_t &= \alpha_0 + \alpha_1 x^a_{t,1m} + \alpha_6 x^a_{t,6m} + \alpha_{12} x^a_{t,12m} + \beta_1 x^b_{t,1m} + \beta_6 x^b_{t,6m} + \beta_{12} x^b_{t,12m} + \epsilon^\alpha \\ &= \alpha_0 + \alpha_1 \bar{x}^a_{1m} + \alpha_6 \bar{x}^a_{6m} + \alpha_{12} \bar{x}^a_{12m} + \beta_1 \bar{x}^b_{1m} + \beta_6 \bar{x}^b_{6m} + \beta_{12} \bar{x}^b_{12m} + \epsilon^\alpha \end{aligned} \tag{41}$$

with (x^a_t, x^b_t) a two-dimensional variable that is equal to either (xv^u_t, xv^d_t), (xs^u_t, xs^d_t), (xv_t, xs_t), (xv^u_t, xs^u_t) or (xv^d_t, xs^d_t).

Remark 4.1. Notice that $xm_t = xm_{t,t+\tau}$ in Eq.(25) where the dependency of the variable to the current time and the maturity is explicitly stated, and as the time variable τ is one month, then xm_t is return over the time interval $[t\ ; t+\tau]$ while $x_{t,1m} = x_{t-1}$ and if the explanatory variable x_t is, for example, xv_t then $x_{t,1m} = xv_{t,1m} = xv_{t-1} = xv_{t-1,t}$ is the variance swap return computed over the time interval $[t-1\ ; t]$. As a result, xm_t and $xv_{t,1m}$ are computed over disjoint intervals. A similar remark applies to the other explanatory variables as well as to the pairs $(xm_t, x_{t,6m})$ and $(xm_t, x_{t,12m})$. Notice that this choice is different from Kilic and Shaliastovich (2018)'s, where their Eq.(17) implies that the explanatory variables and the dependent variable are computed over intervals that overlap.

We focus on the prediction of the 1-month, 3-month and 6-month currency excess returns using the 1-month past return(s), 6-month past return(s) and 12-month past return(s). We comment the results for the 1-month currency excess return for the currency pair EURUSD and summarize the salient differences with the 3-month and the 6-month currency excess returns. The tables for these last two cases are provided in the supplementary appendix for brevity.[6]

Table X contains the results when the average lagged variables $\bar{xv}, \bar{xv}^u, \bar{xv}^d, (\bar{xv}^u, \bar{xv}^d)$ and $\bar{xvs}$ for $h = 1, 6$ and 12 months are used as explanatory variables in the regressions Eq.(40) and

[6]The results for the other currency pairs are available upon request.

Eq.(41). Compared to regressions on contemporaneous variables, the R^2 are substantially lower. Regressing on the lags of xv leads to a R^2 of 6.3% and only the 1-year (past average) variable is significant and positive (see column (1) in Table X). Notice that the sign is different than when regressing on the contemporaneous variable xv. Using (average) lags of the up semivariance swap produces a regression with a low R^2 of 3.41% and none of the variables are significant (see column (2) in the aforementioned table). The lags of the down semivariance swap lead to a R^2 of 6.79% and both the constant term and the one year (past) average are significant. For this latter variable, the coefficient is positive, it is opposite to the sign obtained when regressing on the contemporaneous variable xv^d. Combining lagged up and down semivariance swaps, with results reported in column (4) of the table, increases the R^2 to 9.56%; the only significant variable is $\bar{xv}^d_{1y}$ with a positive coefficient while the constant term is no longer significant. The lagged values of the variance-skew swap xvs gives a R^2 of 5.57%; a constant term that is significant; a significant and positive coefficient for the variable $x\bar{v}s_{1y}$ (as for the previous regressions the sign is opposite of sign obtained for the contemporaneous regression). Regressing on the up and down variables or on $x\bar{v}s$, which combines up and down variables in a specific way, give R^2 that are close while the coefficients' signs are consistent across factor models.

[Insert Table X here]

The factor model based on the skew swap related variables $(\bar{xs}_{1m}, \bar{xs}_{6m}, \bar{xs}_{1y})$ leads to a very low R^2 of 2.2% and all the variables are insignificant, see column (1) in Table XI. In comparison, results for the up semiskew swap variables reported in column (2) of this table improve the R^2 to 6.33% with the constant term strongly significant while $\bar{xs}^u_{1y}$ is mildly significant. In case of the down semiskew swap, reported in the third column of that table, the R^2 is 3.7% and only the variable $\bar{xs}^d_{1y}$ is significant and positive, the sign is also the opposite of the contemporaneous case. On the last column, a regression on a combination of up and down semiskew swaps produces a high R^2 of 11.17% with a significant constant term; a significant and positive coefficient for $\bar{xs}^u_{1y}$ (it is not significant in the contemporaneous case); a significant and positive sign for $\bar{xs}^d_{1y}$ that is negative in the contemporaneous case. The R^2 is larger than the sum of the up R^2 and down R^2, the variables contain complementary information as there is an overall improvement in the t-stats.

[Insert Table XI here]

Combining second and third order related moments leads to the regression results presented in Table XII. The first column contains the regression coefficients for $(\bar{xv}_{1m}, \bar{xv}_{6m}, \bar{xv}_{1y}, \bar{xs}_{1m}, \bar{xs}_{6m}, \bar{xs}_{1y})$ and show that only the $\bar{xv}_{1y}$'s coefficient is significant and positive, a result that is consistent with those reported in Table X. Regarding the regression on the (lagged) up semivariance and semiskew swaps, the R^2 is 10.06% and close to the sum of R^2s in Table X (i.e. only xv^u related variables) and Table XI (i.e. only xs^u related variables); the constant term is significant; the coefficient of $\bar{xv}^u_{1y}$ is significant and negative while $\bar{xs}^u_{1y}$ is also significant but with a positive sign. Notice that while $\bar{xv}^u_{1y}$ is not significant in Table X and $\bar{xs}^u_{1y}$ is mildly significant in Table XI, they are now strongly significant. Lastly, regressing on the down semivariance and semiskew swaps leads to a R^2 of 7.47% with only one significant coefficient, namely, $\bar{xv}^d_{1y}$'s and the results for this variable are in line with those reported in Table X where only down semivariance variables are considered. Also, it is interesting to notice that $\bar{xs}^d_{1y}$ is no longer significant whereas it is significant for the regression involving only down semiskew swap variables (i.e. column (3) in Table XI).

[Insert Table XII here]

Overall, the results clearly show that 1-year past average variables (i.e. $\bar{xv}_{1y}, \bar{xv}^u_{1y}, \bar{xv}^d_{1y}, \bar{xs}^u_{1y}, \bar{xs}^d_{1y}$) better predict the 1-month currency excess return; that disaggregated variables (i.e. past average semivariance and semiskew swaps) better predict the 1-month currency excess return than aggregated variables (i.e. past average variance and skew swaps); up and down variables contain complementary information (with respect to the prediction of the 1-month currency excess return); second and third semimoments also contain complementary information.

The next step is to assess the robustness or dependency of the results to the parameter h in Eq.(39), we consider the 3-month and 6-month averages and for conciseness only report the salient differences.

The results corresponding to Table X show an overall increase of R^2 by a factor 2 or 3 when the averaging is performed over a longer period. It is as expected as the longer the averaging is, the smaller standard deviation of the forecasted variable is. The 1-year variables (i.e.

$\bar{xv}_{1y}, \bar{xv}^u_{1y}, \bar{xv}^d_{1y}, x\bar{v}s_{1y}$) tend to lose their explanatory power in favor of short term (average) variables. All the constant terms are not significant when the forecasted variable is either the 3-month average currency excess return or the 6-month average currency excess return.

The results corresponding to Table XI show an overall increase of R^2 when the forecasted variable is computed over a longer period. However, this improvement is minor for the skew and down skew related variables while it is substantial for the up skew swap related variables as the R^2 for $h = 6$ is four times the one for $h = 1$. As for the previous case, forecasting longer term variables leads to long term explanatory variables becoming less significant whilst short term ones become significant. Tails related explanatory variables often lead to significant constant terms, it reflects the fact that a factor is missing. Interestingly, combining up and down skew swap explanatory variables produces a significant constant term.

As for the two previous cases, the results corresponding to Table XII show that forecasting a variable computed by averaging over a longer period produces a higher R^2; short term explanatory variables become more significant while long term ones become less. Regressing on up related variables (variance and skew) leads to a constant term that is significant, a factor is missing. Interestingly enough, the constant term is never significant when forecasting the average currency excess return using a combination of down semivariance and semiskew swaps.

In conclusion, the decomposition of the variance and skew swaps into semi components along with the averaging technique proposed in Corsi (2009) helps to predict the evolution of the currency excess return as the improvement in R^2 value is substantial.

5 Conclusion

In this work, using currency options and a model-free methodology we extract variance and skew swaps and decompose them into up and down semivariance and semiskew swaps that capture the higher moments of the tails of the currency log-return distribution. These (semi)variance and (semi)skew swaps enable us to compute (semi)variance and (semi)skew risk premiums, they quantify how higher moment risk premiums are priced by the market. We also define the variance-skew swap that depends on the distortion between the left and right log-return currency tails. We develop several factor models for the currency excess return based on these (semi)variance and (semi)skew swaps that we apply to the three currency pairs EURUSD, GBPUSD and USDJPY.

Results show that the decomposition of the variance and skew swaps into up and down components is crucial to obtain significant results as these decomposed variables explain much more strongly the currency excess return. For the semivariance contracts both of them, the up and the down, contain relevant information for the foreign exchange rate while for the semiskew swaps all the information is carried out by the down semiskew swap. Combining down semivariance and semiskew swaps shows that they carry complementary information for the currency excess return. Lastly, the variance-skew swap strongly explains the evolution of the currency excess return.

We evaluate the importance of the tails by trimming the explanatory variables, that is, we remove the extreme values. Regressions on these transformed variables show R^2 that are always larger than those achieved when regressing on the untransformed ones. For certain variables the gain can be substantial. The results imply that extreme values extracted from foreign exchange options convey no information on the evolution of currency excess returns.

Lastly, prediction of the 1-month currency excess return following Corsi (2009)'s strategy also illustrates the importance of disaggregating the variance and skew swaps into semivariance and semiskew swaps as these variables are more informative (i.e. the R^2 improves significantly) and contain complementary views on the evolution of the currency. The results also show that long term past averages (i.e. 1-year past average) carry more information than short term past averages (i.e. either 1-month or 6 months) when forecasting the 1-month currency excess return. This property reverses when forecasting long term average currency excess return. The constant terms are often significant when tails related variables are used as explanatory variables reflecting the fact a factor is missing. In all the cases, the predictability of the currency excess return is enhanced by the tail decomposition of the risk premiums.

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A Tables

Table I: Descriptive statistics

	EURUSD		GBPUSD		USDJPY	
	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
xm	-1.491×10^{-3}	0.029	-2.726×10^{-3}	0.028	1.273×10^{-3}	0.033
xv	-7.391×10^{-2}	0.470	-2.443×10^{-2}	0.491	-1.641×10^{-2}	0.715
xv^u	-0.409	0.700	-0.366	0.731	-0.283	0.921
xv^d	0.480	1.766	0.527	1.862	0.356	2.014
xvs	-0.435	1.019	-0.425	1.074	-0.313	1.198
xs	-0.068	0.483	-0.123	0.624	-0.198	1.571
xs^u	-1.344	0.341	-1.415	0.318	-1.385	1.556
xs^d	-2.696	2.017	-2.483	2.725	-1.966	5.678

Note: Descriptive statistics such as mean, standard deviation for the variables. The variables are sampled at monthly frequency from January 2006 to November 2016.

Table II: Events

Date	Event	Market
28/11/2006	Concerns on the U.S. mortgage market.	EURUSD, GBPUSD, USDJPY
	Good prospects for the European economy.	EURUSD
27/10/2008	ECB President (J.-C. Trichet) announces that he may cut borrowing costs.	EURUSD
18/12/2008	ECB said it will encourage lending.	EURUSD
	The FED lowered the federal funds rate target to the range [0 ; 0.25%] (on the 16/12).	EURUSD, GBPUSD, USDJPY
18/03/2009	The FED agreed to buy Treasuries and mortgage bonds and to expand the balance sheet up to \$1.15 trillion.	EURUSD, GBPUSD, USDJPY
01/2013	Jap. Prime Minister Shinzo Abe announces a fiscal stimulus package.	USDJPY
	Bank of Japan commits to purchase assets.	USDJPY
18/09/2014	Scottish independence referendum.	GBPUSD
14/06/2016	Brexit result.	EURUSD, GBPUSD
8/11/2016	U.S. presidential election result.	EURUSD, GBPUSD, USDJPY

Note: Events for certain dates/periods as well as markets that are likely to be affected.

Table III: Correlation matrices

EURUSD

	xv	xv^u	xv^d	xvs	xs	xs^u	xs^d
xv	1.000	−0.056	0.733	−0.506	−0.454	0.193	0.533
xv^u		1.000	−0.712	0.888	0.310	0.097	−0.296
xv^d			1.000	−0.954	−0.521	0.049	0.553
xvs				1.000	0.477	0.007	−0.491
xs					1.000	0.260	−0.919
xs^u						1.000	0.044
xs^d							1.000

GBPUSD

	xv	xv^u	xv^d	xvs	xs	xs^u	xs^d
xv	1.000	−0.087	0.751	−0.528	−0.659	0.168	0.674
xv^u		1.000	−0.716	0.890	0.229	−0.203	−0.297
xv^d			1.000	−0.954	−0.608	0.236	0.661
xvs				1.000	0.495	−0.244	−0.557
xs					1.000	0.119	−0.957
xs^u						1.000	0.106
xs^d							1.000

USDJPY

	xv	xv^u	xv^d	xvs	xs	xs^u	xs^d
xv	1.000	0.223	0.699	−0.365	−0.558	0.078	0.645
xv^u		1.000	−0.527	0.824	0.188	0.048	−0.199
xv^d			1.000	−0.913	−0.624	0.028	0.717
xvs				1.000	0.509	0.003	−0.569
xs					1.000	0.479	−0.853
xs^u						1.000	0.016
xs^d							1.000

Note: Correlation matrices for the variables $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$. The variables are sampled at monthly frequency from January 2006 to November 2016.

Table IV: Factor models based on (semi)variance and (semi)skew swaps for EURUSD

	EURUSD											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Const.	−0.002	0.011	0.004	0.009	0.008	0.000	0.013	−0.022	−0.005	0.002	0.017	−0.005
	(−0.99)	(4.55)	(2.10)	(4.19)	(4.17)	(0.30)	(0.98)	(−4.87)	(−0.39)	(0.103)	(2.17)	(−1.06)
xv	−0.018									−0.004		
	(−2.99)									(−0.64)		
xv^u		0.031		0.021							0.031	
		(9.69)		(5.55)							(9.91)	
xv^d			−0.011	−0.005								−0.009
			(−7.15)	(−3.16)								(−6.15)
xvs					0.022							
					(9.18)							
xs						0.031				0.030		
						(5.96)				(5.37)		
xs^u							0.010		0.017		0.004	
							(1.05)		(1.56)		(0.85)	
xs^d								−0.007	−0.026			−0.003
								(−6.06)	(−6.11)			(−2.39)
Adj. R^2(%)	8.32	55.19	47.62	60.44	58.58	27.05	1.54	27.45	29.82	27.39	55.46	50.5

Note: Regressions of xm, the foreign exchange rate excess return given by Eq.(25), on explanatory variables based on $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$ defined in Eqs.(5),(10),(14) and (24). Column (1) corresponds to Eq.(26), column (2) to Eq.(27), column (3) to Eq.(28), column (4) to Eq.(29), column (5) to Eq.(30), column (6) to Eq.(31), column (7) to Eq.(32), column (8) to Eq.(33), column (9) to Eq.(34), column (10) to Eq.(35), column (11) to Eq.(36) and column (12) to Eq.(37). The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Table V: Factor models based on (semi)variance and (semi)skew swaps for GBPUSD

	GBPUSD											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Const.	−0.003	0.006	0.003	0.005	0.005	−0.001	−0.025	−0.015	−0.030	−0.001	0.001	0.003
	(−1.32)	(3.17)	(2.05)	(3.03)	(3.94)	(−0.13)	(−1.95)	(−2.15)	(−2.72)	(−0.64)	(0.14)	(0.68)
xv	−0.025									−0.015		
	(−4.23)									(2.32)		
xv^u		0.025		0.011							0.025	
		(8.12)		(3.52)							(8.44)	
xv^d			−0.010	−0.007								−0.011
			(−8.23)	(−4.33)								(−9.34)
xvs					0.019							
					(9.76)							
xs						0.019				0.011		
						(2.22)				(1.27)		
xs^u							−0.015		−0.011		−0.004	
							(−1.95)		(−1.68)		(−0.86)	
xs^d								−0.004	−0.004			0.000
								(−2.10)	(−2.07)			(0.05)
Adj. R^2(%)	19.08	43.53	51.29	55.72	56.63	18.45	3.13	21.99	23.63	22.62	43.72	51.29

Note: Regressions of xm, the foreign exchange rate excess return given by Eq.(25), on explanatory variables based on $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$ defined in Eqs.(5),(10),(14) and (24). Column (1) corresponds to Eq.(26), column (2) to Eq.(27), column (3) to Eq.(28), column (4) to Eq.(29), column (5) to Eq.(30), column (6) to Eq.(31), column (7) to Eq.(32), column (8) to Eq.(33), column (9) to Eq.(34), column (10) to Eq.(35), column (11) to Eq.(36) and column (12) to Eq.(37). The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Table VI: Factor models based on (semi)variance and (semi)skew swaps for USDJPY

	USDJPY											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Const.	0.001	0.008	0.005	0.008	0.007	0.002	0.005	−0.001	0.002	0.002	0.011	0.011
	(0.34)	(3.81)	(2.06)	(4.40)	(4.24)	(0.86)	(0.82)	(−0.56)	(0.32)	(0.81)	(3.38)	(3.89)
xv	−0.007									0.001		
	(−1.22)									(0.09)		
xv^u		0.025		0.018							0.025	
		(9.82)		(9.02)							(9.23)	
xv^d			−0.010	−0.006								−0.015
			(−5.50)	(−4.05)								(−7.10)
xvs					0.021							
					(14.06)							
xs						0.006				0.006		
						(2.86)				(2.03)		
xs^u							0.002		0.003		0.002	
							(0.85)		(0.92)		(0.98)	
xs^d								−0.001	−0.001			0.002
								(−2.36)	(−2.36)			(4.71)
Adj. R^2(%)	2.76	51.3	41.92	61.37	59.81	9.96	1.51	7.59	9.25	9.98	52.09	49.27

Note: Regressions of xm, the foreign exchange rate excess return given by Eq.(25), on explanatory variables based on $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$ defined in Eqs.(5),(10),(14) and (24). Column (1) corresponds to Eq.(26), column (2) to Eq.(27), column (3) to Eq.(28), column (4) to Eq.(29), column (5) to Eq.(30), column (6) to Eq.(31), column (7) to Eq.(32), column (8) to Eq.(33), column (9) to Eq.(34), column (10) to Eq.(35), column (11) to Eq.(36) and column (12) to Eq.(37). The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Table VII: Factor models based on “trimmed” (semi)variance and (semi)skew swaps for EURUSD

	EURUSD											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Const.	−0.003	0.011	0.004	0.009	0.008	0.000	0.014	−0.027	−0.008	−0.000	0.016	−0.007
	(−1.08)	(4.55)	(2.24)	(3.99)	(4.26)	(0.23)	(1.31)	(−5.66)	(−0.61)	(−0.14)	(1.99)	(−1.38)
xv	−0.021									−0.008		
	(−2.84)									(−1.16)		
xv^u		0.031		0.020							0.031	
		(9.69)		(4.85)							(9.97)	
xv^d			−0.012	−0.006								−0.010
			(−9.28)	(−3.38)								(−8.20)
xvs					0.022							
					(9.88)							
xs						0.038				0.035		
						(6.37)				(5.67)		
xs^u							0.011		0.014		0.004	
							(1.49)		(1.52)		(0.72)	
xs^d								−0.009	−0.009			−0.003
								(−6.57)	(−6.58)			(−2.58)
Adj. R^2(%)	8.76	55.19	50.39	61.09	59.35	28.02	1.64	28.93	31.36	29.15	55.38	53.69

Note: Regressions of xm, the foreign exchange rate excess return given by Eq.(25), on “trimmed” explanatory variables based on $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$ defined in Eqs.(5),(10),(14) and (24). Column (1) corresponds to Eq.(26), column (2) to Eq.(27), column (3) to Eq.(28), column (4) to Eq.(29), column (5) to Eq.(30), column (6) to Eq.(31), column (7) to Eq.(32), column (8) to Eq.(33), column (9) to Eq.(34), column (10) to Eq.(35), column (11) to Eq.(36) and column (12) to Eq.(37). The variables are “trimmed” according to Eq.(38). The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Table VIII: Factor models based on “trimmed” (semi)variance and (semi)skew swaps for GBPUSD

	GBPUSD											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Const.	−0.003	0.006	0.003	0.005	0.005	0.004	−0.029	−0.024	−0.040	−0.000	−0.002	−0.003
	(−1.42)	(3.17)	(2.11)	(2.99)	(3.94)	(0.18)	(−1.89)	(−4.51)	(−3.51)	(−0.23)	(−0.38)	(−0.58)
xv	−0.027									−0.009		
	(−4.31)									(−1.73)		
xv^u		0.025		0.010							0.025	
		(8.12)		(3.33)							(8.45)	
xv^d			−0.011	−0.008								−0.009
			(−8.82)	(−4.54)								(−8.67)
xvs					0.019							
					(9.76)							
xs						0.031				0.025		
						(4.79)				(3.34)		
xs^u							−0.018		−0.011		−0.006	
							(−1.89)		(−1.51)		(−1.29)	
xs^d								−0.008	−0.008			−0.002
								(−5.13)	(−4.95)			(−1.17)
Adj. R^2(%)	19.75	43.53	52.37	56.20	56.63	28.09	3.54	35.88	37.14	29.49	43.94	53.66

Note: Regressions of xm, the foreign exchange rate excess return given by Eq.(25), on “trimmed” explanatory variables based on $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$ defined in Eqs.(5),(10),(14) and (24). Column (1) corresponds to Eq.(26), column (2) to Eq.(27), column (3) to Eq.(28), column (4) to Eq.(29), column (5) to Eq.(30), column (6) to Eq.(31), column (7) to Eq.(32), column (8) to Eq.(33), column (9) to Eq.(34), column (10) to Eq.(35), column (11) to Eq.(36) and column (12) to Eq.(37). The variables are “trimmed” according to Eq.(38). The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Table IX: Factor models based on "trimmed" (semi)variance and (semi)skew swaps for USDJPY

	USDJPY											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Const.	0.001	0.009	0.005	0.009	0.008	0.002	0.007	−0.006	0.002	0.002	0.015	0.014
	(0.32)	(4.41)	(2.13)	(4.68)	(4.13)	(0.91)	(0.85)	(−2.20)	(0.03)	(0.92)	(4.65)	(3.40)
xv	−0.007									0.002		
	(−1.17)									(0.39)		
xv^u		0.027		0.018							0.027	
		(11.10)		(8.57)							(11.31)	
xv^d			−0.012	−0.007								−0.016
			(−7.05)	(−4.77)								(−7.16)
xvs					0.021							
					(9.43)							
xs						0.010				0.011		
						(4.16)				(3.26)		
xs^u							0.004		0.005		0.004	
							(0.87)		(1.08)		(1.58)	
xs^d								−0.003	−0.003			0.003
								(−6.14)	(−6.13)			(2.99)
Adj. R^2(%)	2.68	52.16	46.2	62.68	59.83	13.51	1.24	13.67	15.27	13.71	53.33	51.35

Note: Regressions of xm, the foreign exchange rate excess return given by Eq.(25), on "trimmed" explanatory variables based on $(xv, xv^u, xv^d, xvs, xs, xs^u, xs^d)$ defined in Eqs.(5),(10),(14) and (24). Column (1) corresponds to Eq.(26), column (2) to Eq.(27), column (3) to Eq.(28), column (4) to Eq.(29), column (5) to Eq.(30), column (6) to Eq.(31), column (7) to Eq.(32), column (8) to Eq.(33), column (9) to Eq.(34), column (10) to Eq.(35), column (11) to Eq.(36) and column (12) to Eq.(37). The variables are "trimmed" according to Eq.(38). The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Table X: Factor models based on lagged (semi)variance swaps for EURUSD

	EURUSD				
	(1)	(2)	(3)	(4)	(5)
Const.	−0.001	0.006	−0.006	−0.010	−0.008
	(−0.38)	(−0.87)	(−1.82)	(−1.22)	(−1.72)
$\bar{xv}_{1m}$	−0.011				
	(−1.41)				
$\bar{xv}_{6m}$	−0.011				
	(−0.67)				
$\bar{xv}_{1y}$	0.043				
	(2.00)				
$\bar{xv}^u_{1m}$		−0.003		−0.011	
		(−0.65)		(−1.48)	
$\bar{xv}^u_{6m}$		0.025		0.001	
		(1.56)		(0.10)	
$\bar{xv}^u_{1y}$		−0.033		−0.001	
		(−1.44)		(−0.07)	
$\bar{xv}^d_{1m}$			−0.001	−0.004	
			(−0.85)	(−1.62)	
$\bar{xv}^d_{6m}$			−0.007	−0.006	
			(−1.49)	(−0.94)	
$\bar{xv}^d_{1y}$			0.016	0.017	
			(2.45)	(2.42)	
$\bar{xvs}_{1m}$					0.000
					(0.31)
$\bar{xvs}_{6m}$					0.015
					(1.54)
$\bar{xvs}_{1y}$					−0.029
					(−2.27)
Adj. R^2(%)	6.31	3.41	6.79	9.56	5.57

Note: Regressions of xm, the foreign exchange rate excess return given by Eq.(25), on explanatory lagged variables based on (xv, xv^u, xv^d, xvs) defined in Eqs.(5),(10),(14). The variables are computed according to formula Eq.(39) for $h = 1, 6, 12$ months. Column (1) corresponds to Eq.(40) with $(\bar{xv}_{1m}, \bar{xv}_{6m}, \bar{xv}_{1y})$ as explanatory variables, column (2) corresponds to Eq.(40) with $(\bar{xv}^u_{1m}, \bar{xv}^u_{6m}, \bar{xv}^u_{1y})$ as explanatory variables, column (3) corresponds to Eq.(40) with $(\bar{xv}^d_{1m}, \bar{xv}^d_{6m}, \bar{xv}^d_{1y})$ as explanatory variables, column (4) corresponds to Eq.(41) with $(\bar{xv}^u_{1m}, \bar{xv}^u_{6m}, \bar{xv}^u_{1y}, \bar{xv}^d_{1m}, \bar{xv}^d_{6m}, \bar{xv}^d_{1y})$ as explanatory variables and column (5) corresponds to Eq.(40) with $(\bar{xvs}_{1m}, \bar{xvs}_{6m}, \bar{xvs}_{1y})$ as explanatory variables. The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Table XI: Factor models based on lagged (semi)skew swaps for EURUSD

	EURUSD			
	(1)	(2)	(3)	(4)
Const.	−0.003	0.076	0.013	0.111
	(−1.13)	(3.52)	(1.36)	(4.02)
$\bar{x}s_{1m}$	0.004			
	(1.20)			
$\bar{x}s_{6m}$	0.015			
	(0.78)			
$\bar{x}s_{1y}$	−0.037			
$\bar{x}s^{u}_{1m}$		−0.002		−0.000
		(−0.19)		(−0.08)
$\bar{x}s^{u}_{6m}$		−0.000		−0.019
		(−0.00)		(−0.66)
$\bar{x}s^{u}_{1y}$		0.060		0.085
		(1.61)		(2.56)
$\bar{x}s^{d}_{1m}$			−0.001	−0.001
			(−1.46)	(−1.34)
$\bar{x}s^{d}_{6m}$			−0.002	−0.000
			(−0.62)	(−0.16)
$\bar{x}s^{d}_{1y}$			0.010	0.011
			(1.79)	(2.25)
Adj. R^2(%)	2.20	6.33	3.70	11.17

Note: Regressions of xm, the foreign exchange rate excess return given by Eq.(25), on explanatory lagged variables based on (xs, xs^u, xs^d) defined in Eqs.(24). The variables are computed according to formula Eq.(39) for $h = 1, 6, 12$ months. Column (1) corresponds to Eq.(40) with $(\bar{x}s_{1m}, \bar{x}s_{6m}, \bar{x}s_{1y})$ as explanatory variables, column (2) corresponds to Eq.(40) with $(\bar{x}s^u_{1m}, \bar{x}s^u_{6m}, \bar{x}s^u_{1y})$ as explanatory variables, column (3) corresponds to Eq.(40) with $(\bar{x}s^d_{1m}, \bar{x}s^d_{6m}, \bar{x}s^d_{1y})$ as explanatory variables and column (4) corresponds to Eq.(41) with $(\bar{x}s^u_{1m}, \bar{x}s^u_{6m}, \bar{x}s^u_{1y}, \bar{x}s^d_{1m}, \bar{x}s^d_{6m}, \bar{x}s^d_{1y})$ as explanatory variables. The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

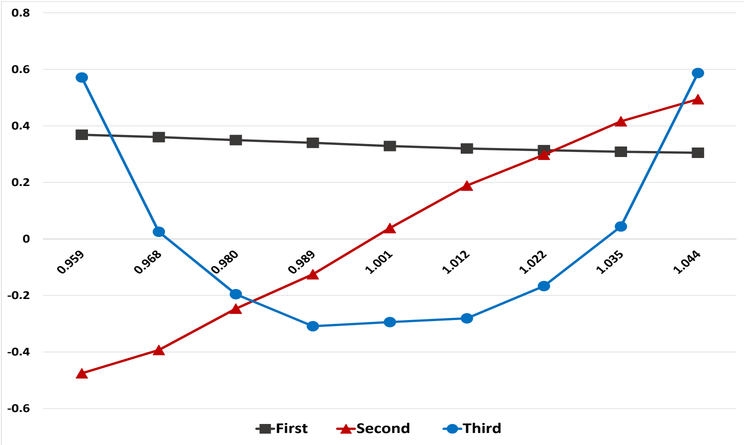
Table XII: Factor models based on lagged (semi)variance and (semi)skew swaps for EURUSD

	EURUSD				
	(1)		(2)		(3)
Const.	-0.003		0.073		0.003
	(-0.94)		(3.01)		(0.20)
$\bar{x}v_{1m}$	-0.011	$\bar{x}v^u_{1m}$	-0.003	$\bar{x}v^d_{1m}$	-0.000
	(-1.32)		(-0.69)		(-0.46)
$\bar{x}v_{6m}$	-0.023	$\bar{x}v^u_{6m}$	0.017	$\bar{x}v^d_{6m}$	-0.011
	(-0.86)		(1.03)		(-1.67)
$\bar{x}v_{1y}$	0.052	$\bar{x}v^u_{1y}$	-0.040	$\bar{x}v^d_{1y}$	0.018
	(1.82)		(-1.98)		(2.24)
$\bar{x}s_{1m}$	-0.001	$\bar{x}s^u_{1m}$	-0.000	$\bar{x}s^d_{1m}$	-0.000
	(-0.22)		(-0.08)		(-0.48)
$\bar{x}s_{6m}$	-0.006	$\bar{x}s^u_{6m}$	-0.015	$\bar{x}s^d_{6m}$	0.004
	(-0.22)		(-0.49)		(0.78)
$\bar{x}s_{1y}$	-0.018	$\bar{x}s^u_{1y}$	0.080	$\bar{x}s^d_{1y}$	-0.000
	(-0.72)		(2.13)		(-0.11)
Adj. $R^2(\%)$	7.24		10.06		7.47

Note: Regressions of xm, the foreign exchange rate excess return given by Eq.(25), on explanatory lagged variables based on $(xv, xv^u, xv^d, xs, xs^u, xs^d)$ defined in Eqs.(5),(10),(24). The variables are computed according to formula Eq.(39) for $h = 1, 6, 12$ months. Column (1) corresponds to Eq.(41) with $(\bar{x}v_{1m}, \bar{x}v_{6m}, \bar{x}v_{1y}, \bar{x}s_{1m}, \bar{x}s_{6m}, \bar{x}s_{1y})$ as explanatory variables, column (2) corresponds to Eq.(41) with $(\bar{x}v^u_{1m}, \bar{x}v^u_{6m}, \bar{x}v^u_{1y}, \bar{x}s^u_{1m}, \bar{x}s^u_{6m}, \bar{x}s^u_{1y})$ as explanatory variables, column (3) corresponds to Eq.(41) with $(\bar{x}v^d_{1m}, \bar{x}v^d_{6m}, \bar{x}v^d_{1y}, \bar{x}s^d_{1m}, \bar{x}s^d_{6m}, \bar{x}s^d_{1y})$ as explanatory variables. The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

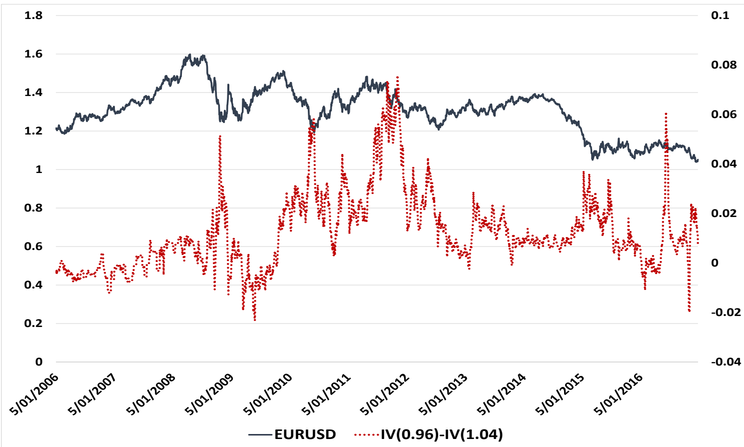
B Figures

Figure 1: Principal component factors for the EURUSD smile



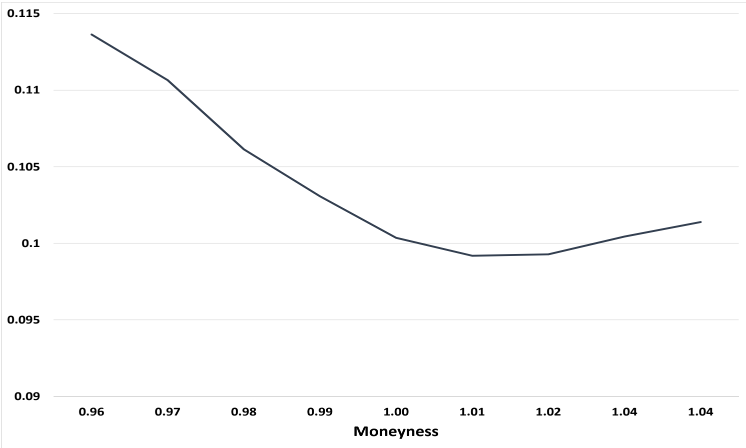
Principal component factors for the EURUSD smile. The first factor (black line with squares) is the level factor, the second factor (red line with triangles) is the slope factor and the third factor (blue line with circles) is the curvature factor. The x-axis is for the moneyess (0.96, 1.04). The moneyness is defined as k/s_t with k the option strike and s_t the spot foreign exchange rate at time t. Daily data from 05/01/2006 to 30/12/2016.

Figure 2: EURUSD foreign exchange rate evolution



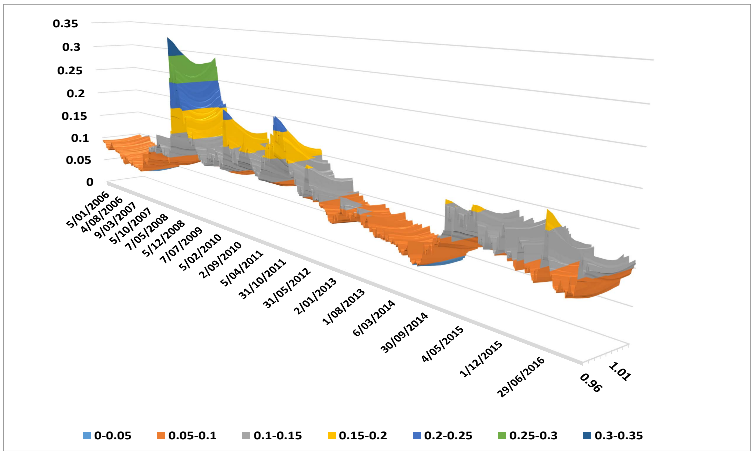
Foreign exchange rate EURUSD evolution (black solid line) with the y-axis on the left hand side and evolution of the difference between the one-month implied volatility smiles (red dot curve) for two moneyesses (0.96, 1.04) with the y-axis on the right hand side. The moneyness is k/s_t with k the option strike and s_t the spot foreign exchange rate at time t. Daily data from 05/01/2006 to 30/12/2016. The foreign exchange rate EURUSD is the value in USD of 1 EUR.

Figure 3: EURUSD 1-month average smile



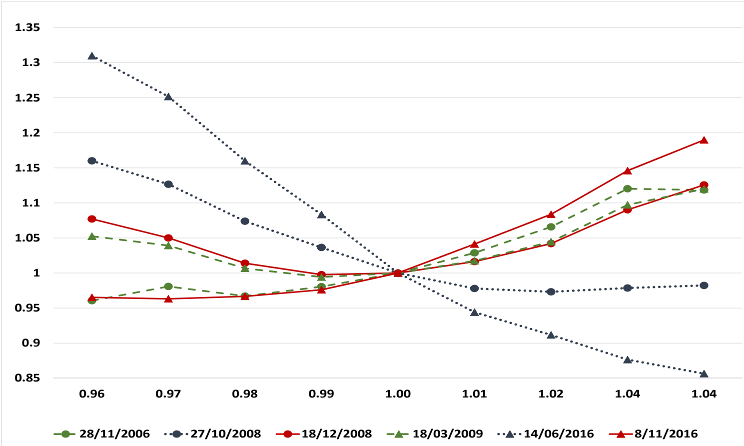
Average 1-month implied volatility smile for EURUSD options expressed as a function of the moneyness (the moneyness is k/s_t with k the option strike and s_t the spot foreign exchange rate at time t). Average computed using daily data from 05/01/2006 to 30/12/2016.

Figure 4: EURUSD 1-month smile evolution



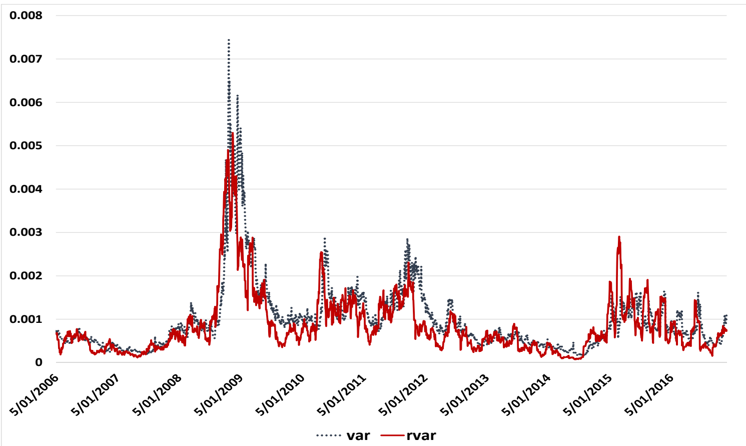
Time series for the 1-month implied volatility smile for EURUSD options expressed as a function of the moneyness (the moneyness is k/s_t with k the option strike and s_t the spot foreign exchange rate at time t). Daily data from 05/01/2006 to 30/12/2016.

Figure 5: EURUSD 1-month smiles



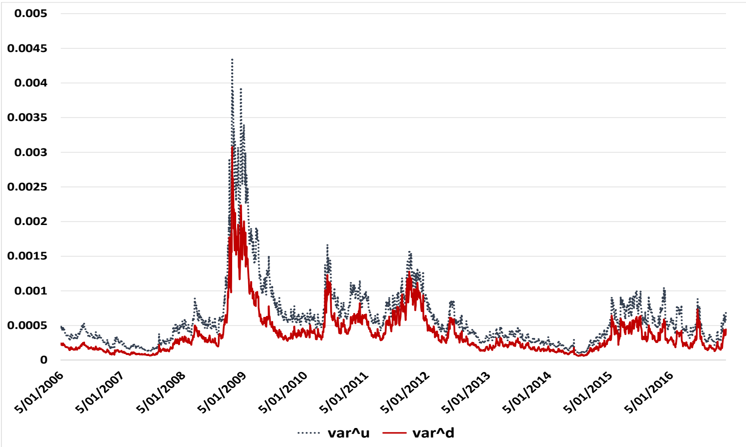
For a given set of dates, the 1-month implied volatility smiles for EURUSD options expressed as a function of the moneyness (the moneyness is k/s_t with k the option strike and s_t the spot foreign exchange rate at time t). For each day the smile is scaled by its ATM value.

Figure 6: EURUSD risk neutral and realized variances



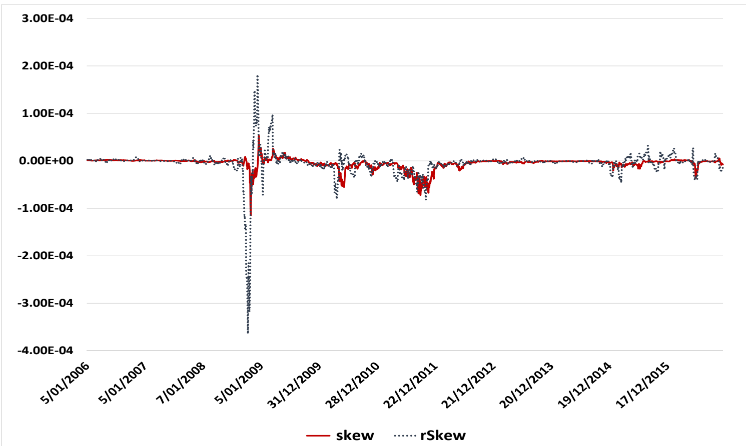
Time series for the risk neutral variance $var_{t,t+\tau}$ (black dot curve) given by Eq.(1) and realized variance $rvar_{t,t+\tau}$ (solid red curve) given by Eq.(3) for the EURUSD market ($\tau = 1$ month). Daily data from 05/01/2006 to 30/11/2016.

Figure 7: EURUSD risk neutral up and down variances



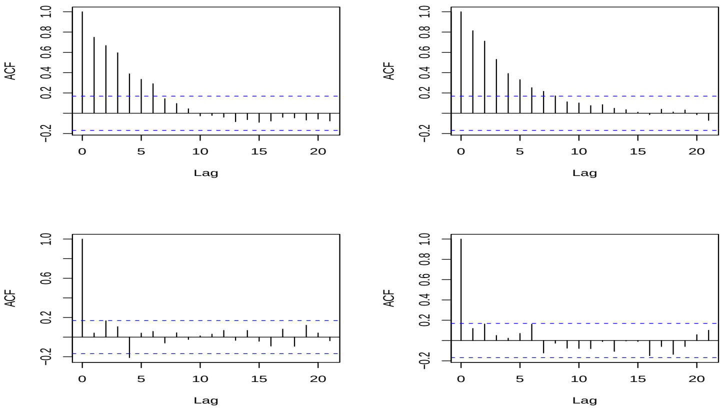
Time series for the risk neutral up variance $var^u_{t,t+\tau}$ (black dot curve) given by Eq.(2) and risk neutral down variance $var^d_{t,t+\tau}$ (solid red curve) given by Eq.(2) for the EURUSD market (τ = 1 month). Daily data from 05/01/2006 to 30/11/2016.

Figure 8: EURUSD risk neutral and realized skews



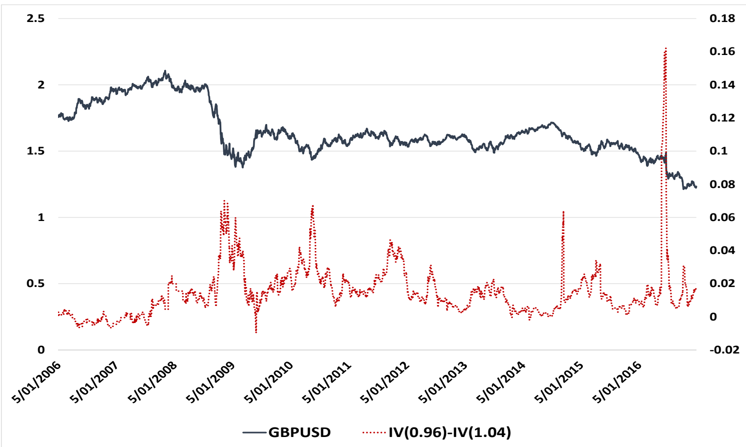
Time series for the risk neutral skew $skew_{t,t+\tau}$ (solid red curve) given by Eq.(15) and realized variance $rskew_{t,t+\tau}$ (black dot curve) given by Eq.(19) for the EURUSD market (τ = 1 month). Daily data from 05/01/2006 to 30/11/2016.

Figure 9: Autocorrelation functions



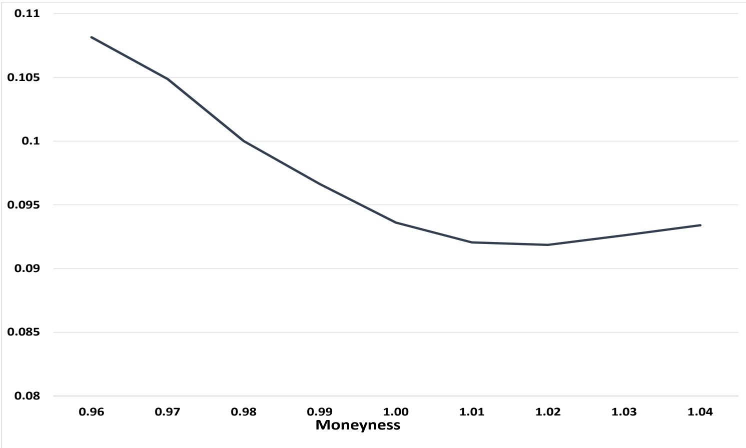
Autocorrelation functions for $rvar$ (upper left), var (upper right), $rvar - var$ (lower left) and $xvar$ (lower right) for the EURUSD. Monthly data January 2006 to November 2016.

Figure 10: GBPUSD foreign exchange rate evolution



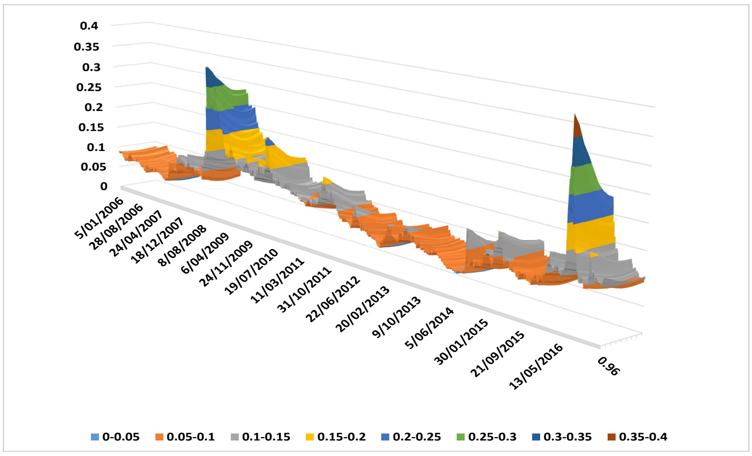
Foreign exchange rate GBPUSD evolution (black solid line) with the y-axis on the left hand side and evolution of the difference between the one-month implied volatility smiles (red dot curve) for two moneyesses (0.96, 1.04) with the y-axis on the right hand side. The moneyness is k/s_t with k the option strike and s_t the spot foreign exchange rate at time t. Daily data from 05/01/2006 to 30/12/2016. The foreign exchange rate GBPUSD is the value in USD of 1 GBP.

Figure 11: GBPUSD 1-month average smile



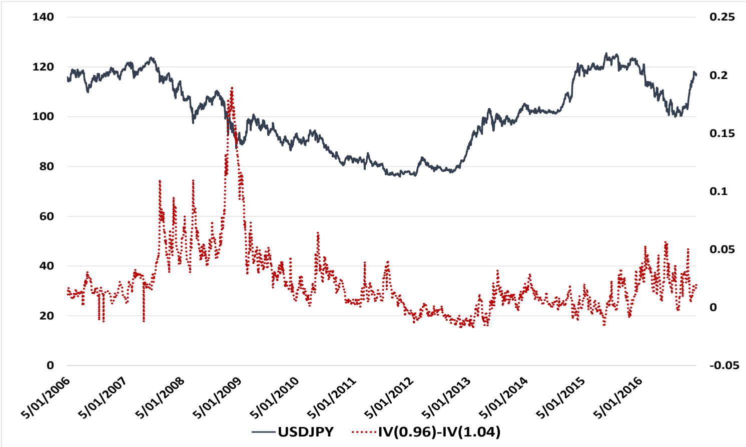
Average 1-month implied volatility smile for GBPUSD options expressed as a function of the moneyness (the moneyness is k/s_t with k the option strike and s_t the spot foreign exchange rate at time t). Average computed using daily data from 05/01/2006 to 30/12/2016.

Figure 12: GBPUSD 1-month smile evolution



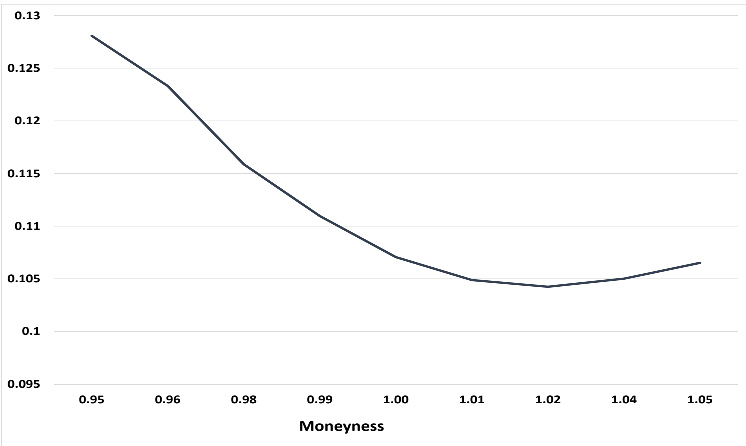
Time series for the 1-month implied volatility smile for GBPUSD options expressed as a function of the moneyness (the moneyness is k/s_t with k the option strike and s_t the spot foreign exchange rate at time t). Daily data from 05/01/2006 to 30/12/2016.

Figure 13: USDJPY foreign exchange rate evolution



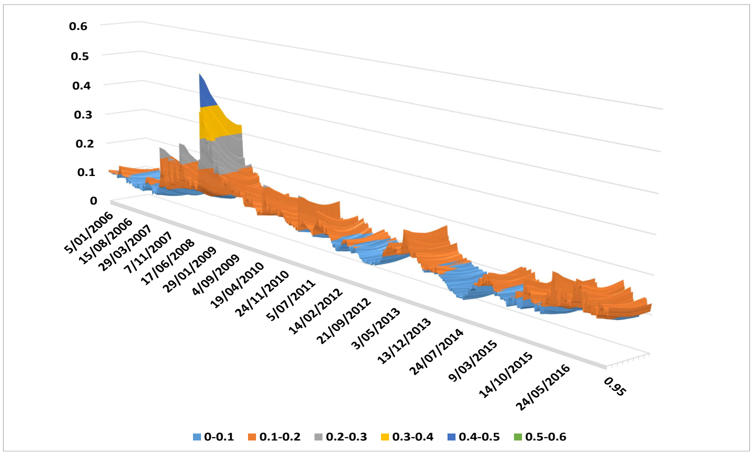
Foreign exchange rate USDJPY evolution (black solid line) with the y-axis on the left hand side and evolution of the difference between the one-month implied volatility smiles (red dot curve) for two moneyesses (0.96, 1.04) with the y-axis on the right hand side. The moneyness is k/s_t with k the option strike and s_t the spot foreign exchange rate at time t. Daily data from 05/01/2006 to 30/12/2016. The foreign exchange rate USDJPY is the value in JPY of 1 USD.

Figure 14: USDJPY 1-month average smile



Average 1-month implied volatility smile for USDJPY options expressed as a function of the moneyness (the moneyness is k/s_t with k the option strike and s_t the spot foreign exchange rate at time t). Average computed using daily data from 05/01/2006 to 30/12/2016.

Figure 15: USDJPY 1-month smile evolution



Time series for the 1-month implied volatility smile for USDJPY options expressed as a function of the moneyness (the moneyness is k/s_t with k the option strike and s_t the spot foreign exchange rate at time t). Daily data from 05/01/2006 to 30/12/2016.

Prediction of the 3-month average currency excess return for the EURUSD: This section reports the prediction of the EURUSD 3-month average currency excess return. In Eqs.(40),(41) the left hand side xm_t is replaced with $x\bar{m}_t^+ = \frac{1}{3}(xm_t + xm_{t+1} + xm_{t+2})$.

Table XIII: Factor models based on lagged (semi)variance swaps for EURUSD

	EURUSD				
	(1)	(2)	(3)	(4)	(5)
Const.	−0.000	−0.008	−0.006	−0.009	−0.009
	(−0.15)	(−0.95)	(−1.51)	(−0.95)	(−1.57)
$x\bar{v}_{1m}$	−0.007				
	(−1.67)				
$x\bar{v}_{6m}$	0.003				
	(0.25)				
$x\bar{v}_{1y}$	0.028				
	(1.27)				
$x\bar{v}^u_{1m}$		0.001		−0.004	
		(0.52)		(−1.33)	
$x\bar{v}^u_{6m}$		0.013		0.002	
		(0.96)		(0.14)	
$x\bar{v}^u_{1y}$		−0.030		−0.004	
		(−1.29)		(−0.22)	
$x\bar{v}^d_{1m}$			−0.001	−0.003	
			(−1.51)	(−1.82)	
$x\bar{v}^d_{6m}$			−0.002	−0.001	
			(−0.46)	(−0.18)	
$x\bar{v}^d_{1y}$			0.013	0.012	
			(1.77)	(1.54)	
$x\bar{v}s_{1m}$					0.002
					(1.43)
$x\bar{v}s_{6m}$					0.005
					(0.71)
$x\bar{v}s_{1y}$					−0.025
					(−1.87)
Adj. R^2(%)	7.18	3.44	10.7	9.67	9.49

Note: Regressions of $x\bar{m}_t^+$, the 3-month average currency excess return, on explanatory lagged variables based on (xv, xv^u, xv^d, xvs) defined in Eqs.(5),(10),(14). The variables are computed according to formula Eq.(39) for $h = 1, 6, 12$ months. Column (1) corresponds to Eq.(40) with $(x\bar{v}_{1m}, x\bar{v}_{6m}, x\bar{v}_{1y})$ as explanatory variables, column (2) corresponds to Eq.(40) with $(x\bar{v}^u_{1m}, x\bar{v}^u_{6m}, x\bar{v}^u_{1y})$ as explanatory variables, column (3) corresponds to Eq.(40) with $(x\bar{v}^d_{1m}, x\bar{v}^d_{6m}, x\bar{v}^d_{1y})$ as explanatory variables, column (4) corresponds to Eq.(41) with $(x\bar{v}^u_{1m}, x\bar{v}^u_{6m}, x\bar{v}^u_{1y}, x\bar{v}^d_{1m}, x\bar{v}^d_{6m}, x\bar{v}^d_{1y})$ as explanatory variables and column (5) corresponds to Eq.(40) with $(x\bar{v}s_{1m}, x\bar{v}s_{6m}, x\bar{v}s_{1y})$ as explanatory variables. The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Table XIV: Factor models based on lagged (semi)skew swaps for EURUSD

	EURUSD			
	(1)	(2)	(3)	(4)
Const.	−0.003	0.077	0.015	0.111
	(−0.87)	(3.20)	(1.48)	(3.64)
$\bar{xs}_{1m}$	0.003			
	(1.08)			
$\bar{xs}_{6m}$	0.015			
	(1.22)			
$\bar{xs}_{1y}$	−0.039			
	(−1.65)			
$\bar{xs}^u_{1m}$		−0.003		−0.003
		(−0.86)		(−0.86)
$\bar{xs}^u_{6m}$		0.03		0.013
		(0.96)		(0.49)
$\bar{xs}^u_{1y}$		0.031		0.056
		(0.77)		(1.67)
$\bar{xs}^d_{1m}$			−0.000	−0.000
			(−1.14)	(−0.92)
$\bar{xs}^d_{6m}$			−0.000	0.001
			(−0.33)	(0.46)
$\bar{xs}^d_{1y}$			0.008	0.008
			(1.70)	(2.28)
Adj. R^2(%)	2.67	16.46	4.40	23.32

Note: Regressions of $\bar{xm}_t^+$, the 3-month average currency excess return, on explanatory lagged variables based on (xs, xs^u, xs^d) defined in Eq.(24). The variables are computed according to formula Eq.(39) for $h = 1, 6, 12$ months. Column (1) corresponds to Eq.(40) with $(\bar{xs}_{1m}, \bar{xs}_{6m}, \bar{xs}_{1y})$ as explanatory variables, column (2) corresponds to Eq.(40) with $(\bar{xs}^u_{1m}, \bar{xs}^u_{6m}, \bar{xs}^u_{1y})$ as explanatory variables, column (3) corresponds to Eq.(40) with $(\bar{xs}^d_{1m}, \bar{xs}^d_{6m}, \bar{xs}^d_{1y})$ as explanatory variables and column (4) corresponds to Eq.(41) with $(\bar{xs}^u_{1m}, \bar{xs}^u_{6m}, \bar{xs}^u_{1y}, \bar{xs}^d_{1m}, \bar{xs}^d_{6m}, \bar{xs}^d_{1y})$ as explanatory variables. The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Table XV: Factor models based on lagged (semi)variance and (semi)skew swaps for EURUSD

	EURUSD				
	(1)		(2)		(3)
Const.	−0.001		0.076		−0.003
	(−0.52)		(3.23)		(−0.22)
$\bar{xv}_{1m}$	−0.008	$\bar{xv}^u_{1m}$	0.000	$\bar{xv}^d_{1m}$	−0.001
	(−2.13)		(0.28)		(−1.7)
$\bar{xv}_{6m}$	0.009	$\bar{xv}^u_{6m}$	0.004	$\bar{xv}^d_{6m}$	−0.003
	(0.43)		(0.44)		(−0.46)
$\bar{xv}_{1y}$	0.018	$\bar{xv}^u_{1y}$	−0.032	$\bar{xv}^d_{1y}$	0.01
	(0.64)		(−1.66)		(1.35)
$\bar{xs}_{1m}$	−0.000	$\bar{xs}^u_{1m}$	−0.002	$\bar{xs}^d_{1m}$	−0.000
	(−0.35)		(−0.74)		(−0.02)
$\bar{xs}_{6m}$	0.015	$\bar{xs}^u_{6m}$	0.013	$\bar{xs}^d_{6m}$	0.001
	(0.74)		(0.47)		(0.21)
$\bar{xs}_{1y}$	−0.027	$\bar{xs}^u_{1y}$	0.056	$\bar{xs}^d_{1y}$	0.000
	(−1.18)		(1.48)		(0.00)
Adj. R^2(%)	6.54		21.21		8.59

Note: Regressions of $\bar{xm}_t^+$, the 3-month average currency excess return, on explanatory lagged variables based on $(xv, xv^u, xv^d, xs, xs^u, xs^d)$ defined in Eqs.(5),(10),(24). The variables are computed according to formula Eq.(39) for $h = 1, 6, 12$ months. Column (1) corresponds to Eq.(41) with $(\bar{xv}_{1m}, \bar{xv}_{6m}, \bar{xv}_{1y}, \bar{xs}_{1m}, \bar{xs}_{6m}, \bar{xs}_{1y})$ as explanatory variables, column (2) corresponds to Eq.(41) with $(\bar{xv}^u_{1m}, \bar{xv}^u_{6m}, \bar{xv}^u_{1y}, \bar{xs}^u_{1m}, \bar{xs}^u_{6m}, \bar{xs}^u_{1y})$ as explanatory variables, column (3) corresponds to Eq.(41) with $(\bar{xv}^d_{1m}, \bar{xv}^d_{6m}, \bar{xv}^d_{1y}, \bar{xs}^d_{1m}, \bar{xs}^d_{6m}, \bar{xs}^d_{1y})$ as explanatory variables. The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Prediction of the 6-month average currency excess return for the EURUSD: This section reports the prediction of the EURUSD 6-month average currency excess return. In Eqs.(40),(41) the left hand side xm_t is replaced with $x\bar{m}_t^{++} = \frac{1}{6}(xm_t + xm_{t+1} + xm_{t+2} + xm_{t+3} + xm_{t+4} + xm_{t+5})$.

Table XVI: Factor models based on lagged (semi)variance swaps for EURUSD

	EURUSD				
	(1)	(2)	(3)	(4)	(5)
Const.	−0.000	−0.011	−0.007	−0.009	−0.010
	(−0.14)	(−1.02)	(−1.29)	(−0.86)	(−1.42)
$x\bar{v}_{1m}$	−0.005				
	(−1.28)				
$x\bar{v}_{6m}$	0.017				
	(1.5)				
$x\bar{v}_{1y}$	0.011				
	(0.77)				
$x\bar{v}^u_{1m}$		0.002		−0.002	
		(1.34)		(−0.88)	
$x\bar{v}^u_{6m}$		0.001		0.002	
		(0.14)		(0.18)	
$x\bar{v}^u_{1y}$		−0.026		−0.007	
		(−1.19)		(−0.44)	
$x\bar{v}^d_{1m}$			−0.001	−0.002	
			(−1.72)	(−1.88)	
$x\bar{v}^d_{6m}$			0.002	0.003	
			(1.14)	(0.66)	
$x\bar{v}^d_{1y}$			0.008	0.007	
			(1.64)	(1.16)	
$x\bar{v}s_{1m}$					0.002
					(1.72)
$x\bar{v}s_{6m}$					−0.002
					(−0.58)
$x\bar{v}s_{1y}$					−0.019
					(−1.69)
Adj. R^2(%)	11.7	8.77	19.67	18.5	18.79

Note: Regressions of $x\bar{m}_t^{++}$, the 6-month average foreign exchange rate excess return, on explanatory lagged variables based on (xv, xv^u, xv^d, xvs) defined in Eqs.(5),(10),(14). The variables are computed according to formula Eq.(39) for $h = 1, 6, 12$ months. Column (1) corresponds to Eq.(40) with $(x\bar{v}_{1m}, x\bar{v}_{6m}, x\bar{v}_{1y})$ as explanatory variables, column (2) corresponds to Eq.(40) with $(x\bar{v}^u_{1m}, x\bar{v}^u_{6m}, x\bar{v}^u_{1y})$ as explanatory variables, column (3) corresponds to Eq.(40) with $(x\bar{v}^d_{1m}, x\bar{v}^d_{6m}, x\bar{v}^d_{1y})$ as explanatory variables, column (4) corresponds to Eq.(41) with $(x\bar{v}^u_{1m}, x\bar{v}^u_{6m}, x\bar{v}^u_{1y}, x\bar{v}^d_{1m}, x\bar{v}^d_{6m}, x\bar{v}^d_{1y})$ as explanatory variables and column (5) corresponds to Eq.(40) with $(x\bar{v}s_{1m}, x\bar{v}s_{6m}, x\bar{v}s_{1y})$ as explanatory variables. The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Table XVII: Factor models based on lagged (semi)skew swaps for EURUSD

	EURUSD			
	(1)	(2)	(3)	(4)
Const.	−0.004	0.063	0.015	0.098
	(−0.66)	(2.31)	(1.78)	(3.72)
$\bar{xs}_{1m}$	0.002			
	(0.82)			
$\bar{xs}_{6m}$	0.004			
	(0.36)			
$\bar{xs}_{1y}$	−0.033			
	(−1.67)			
$\bar{xs}^u_{1m}$		−0.003		−0.003
		(−1.27)		(−1.74)
$\bar{xs}^u_{6m}$		0.042		0.028
		(1.53)		(1.20)
$\bar{xs}^u_{1y}$		0.009		0.033
		(0.20)		(0.97)
$\bar{xs}^d_{1m}$			−0.000	−0.000
			(−0.90)	(−0.51)
$\bar{xs}^d_{6m}$			0.001	0.003
			(0.69)	(1.42)
$\bar{xs}^d_{1y}$			0.005	0.004
			(1.01)	(1.6)
Adj. R^2(%)	4.14	25.9	7.79	38.14

Note: Regressions of $\bar{xm}_t^{++}$, the 6-month average foreign exchange rate excess return, on explanatory lagged variables based on (xs, xs^u, xs^d) defined in Eqs.(24). The variables are computed according to formula Eq.(39) for $h = 1, 6, 12$ months. Column (1) corresponds to Eq.(40) with $(\bar{xs}_{1m}, \bar{xs}_{6m}, \bar{xs}_{1y})$ as explanatory variables, column (2) corresponds to Eq.(40) with $(\bar{xs}^u_{1m}, \bar{xs}^u_{6m}, \bar{xs}^u_{1y})$ as explanatory variables, column (3) corresponds to Eq.(40) with $(\bar{xs}^d_{1m}, \bar{xs}^d_{6m}, \bar{xs}^d_{1y})$ as explanatory variables and column (4) corresponds to Eq.(41) with $(\bar{xs}^u_{1m}, \bar{xs}^u_{6m}, \bar{xs}^u_{1y}, \bar{xs}^d_{1m}, \bar{xs}^d_{6m}, \bar{xs}^d_{1y})$ as explanatory variables. The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.

Table XVIII: Factor models based on lagged (semi)variance and (semi)skew swaps for EURUSD

	EURUSD				
	(1)		(2)		(3)
Const.	−0.002		0.067		−0.007
	(−0.47)		(2.58)		(−0.61)
$\bar{xv}_{1m}$	−0.006	$\bar{xv}^u_{1m}$	0.002	$\bar{xv}^d_{1m}$	−0.001
	(−2.05)		(1.13)		(−1.93)
$\bar{xv}_{6m}$	0.023	$\bar{xv}^u_{6m}$	−0.0055	$\bar{xv}^d_{6m}$	0.0018
	(1.44)		(−0.83)		(0.37)
$\bar{xv}_{1y}$	0.0014	$\bar{xv}^u_{1y}$	−0.026	$\bar{xv}^d_{1y}$	0.010
	(0.06)		(−1.49)		(1.20)
$\bar{xs}_{1m}$	−0.0014	$\bar{xs}^u_{1m}$	−0.002	$\bar{xs}^d_{1m}$	0.0002
	(−0.64)		(−1.44)		(0.38)
$\bar{xs}_{6m}$	0.014	$\bar{xs}^u_{6m}$	0.02	$\bar{xs}^d_{6m}$	0.000
	(0.86)		(0.79)		(0.22)
$\bar{xs}_{1y}$	−0.025	$\bar{xs}^u_{1y}$	0.042	$\bar{xs}^d_{1y}$	−0.001
	(−1.26)		(1.00)		(−0.19)
Adj. R^2(%)	12.09		38.15		17.73

Note: Regressions of $\bar{xm}_t^{++}$, the 6-month average foreign exchange rate excess return, on explanatory lagged variables based on $(xv, xv^u, xv^d, xs, xs^u, xs^d)$ defined in Eqs.(5),(10),(24). The variables are computed according to formula Eq.(39) for $h = 1, 6, 12$ months. Column (1) corresponds to Eq.(41) with $(\bar{xv}_{1m}, \bar{xv}_{6m}, \bar{xv}_{1y}, \bar{xs}_{1m}, \bar{xs}_{6m}, \bar{xs}_{1y})$ as explanatory variables, column (2) corresponds to Eq.(41) with $(\bar{xv}^u_{1m}, \bar{xv}^u_{6m}, \bar{xv}^u_{1y}, \bar{xs}^u_{1m}, \bar{xs}^u_{6m}, \bar{xs}^u_{1y})$ as explanatory variables, column (3) corresponds to Eq.(41) with $(\bar{xv}^d_{1m}, \bar{xv}^d_{6m}, \bar{xv}^d_{1y}, \bar{xs}^d_{1m}, \bar{xs}^d_{6m}, \bar{xs}^d_{1y})$ as explanatory variables. The t-statistics are computed according to Newey and West (1987). The variables are sampled at monthly frequency from January 2006 to November 2016.