Traders' Rule and the Long-term Options

Sol Kim* and In Jung Song†

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Key words: Options, Volatility Smile, Ad-hoc Black and Sholes Model, Absolute Smile Approach, Stochastic Volatility

JEL classification: G13, G14

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Abstract

Prior studies show that option traders prefer traders' rule to a theoretical mathematically sophisticated model with short-term options. By incorporating various time-to-maturity, this paper examines if traders' rule still outperforms the mathematically sophisticated models. With Standard & Poor's 500 index option contracts, the pricing and hedging performance are investigated using the extended time intervals. The results show evidence that traders' rule still enjoys the smallest pricing errors for short-term options. However, the mathematically sophisticated model performs the best for the options with longer time-to-maturity. While traders' rule yet can be useful for short-term options, option traders must consider longer time effect in the current options market.

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1. Introduction

Option traders use several option pricing models to obtain both theoretical and practical value of options. Although Black-Scholes model (BS model) in 1973 is still widely used for its simplicity, it has received numerous criticisms such as constant risk-free rates and volatility over the option's lifespan. Further, an implied volatility of BS model appears to be different across exercise prices and maturities thus results in mispricing the options.[1]

To mitigate these shortcomings of BS model, the mathematically sophisticated model such as stochastic volatility model (SV model) has been thoroughly examined (Hull & White, 1987; Johnson & Shanno, 1987; Stein & Stein, 1991; Heston, 1993; Duan, 1995; Heston & Nandi, 2000; Kim & Kim, 2004).[2] Amongst, Heston (1993) introduces the stochastic volatility models with a closed-form solution by accounting BS model's limitations that do not explain non-normal distribution of asset returns and mean-reverting volatility. Previous studies not only consider the mathematically sophisticated models with stochastic volatility factor but also stochastic interest rates or jumps. When a random jump is added, the model is significantly improved for short-term options (Bakshi et al. ,1997; Kim & Kim, 2005) while interest rate factor is essential to price the options with relatively longer time-to-maturity (Bakshi et al., 2000). However, Bakshi, Cao and Chen (1997, 2000) and Kim and Kim (2005) confirm that the most important impact factor is the stochastic volatility while the other factors marginally improve the performance of SV model.

The growing body of empirical evidence suggests that traders' rule (i.e. *ad-hoc* Black-Scholes model, AHBS model) is well-adopted by market practitioners[3] and has become a benchmark for

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[1] These phenomena are known as "volatility smile" and "volatility term structure".
[2] Other related mathematically sophisticated models are stochastic volatility with jumps model (SVJ), GARCH model, and variance gamma model (VG). A random jump can increase its complexity and does not necessarily provide better results than SV (Bakshi et al., 1997; Kim and Kim, 2005). Although GARCH and VG yield the closed-form solutions of an option, Kim and Kim (2004) find that GARCH and VG show the worst performance while SV model performs the best.
[3] Hull and White (1987) state that "traders allow the implied volatility to depend on time to maturity and strike prices. Also, volatility surfaces combine volatility smiles with the volatility term structure to tabulate the volatility appropriate for pricing an option with any strike price and any maturity". Further, Dumas et al. (1998) quote that "To account for the sneer patterns in Black-Scholes implied volatilities, many market makers simply smooth the implied volatility relation across exercise prices (and days to expiration) and then value options using the smoothed relation".

evaluating the option pricing models (Dumas et al., 1998; Jackwerth & Rubinstein, 2001; Li & Pearson, 2007; Kim, 2009; Choi & Ok, 2012). In practice, traders estimate the volatility surfaces for different underlying assets from the market prices of options and allow the implied volatility to be increasing or decreasing function of strike price and time-to-maturity. The current literature finds that AHBS models are superior to mathematically sophisticated models. However, most studies in this area focus on short-term options and there has been little attention paid to long-term options.

The purpose of this paper is to compare the distinct attributes between short-term and long-term options on traders' rule by examining the performance of options with longer time-to-maturity. In other words, if traders' rule still outperforms using long-term options can be checked. To the best of our knowledge, this paper provides the first evidence that documents the performance of AHBS models with long-term options.

There are several reasons to investigate the options with longer time-to-maturity. Firstly, option's time-to-expiration is an important feature that must be considered when finding the value of the options. Although the most traded contracts in the current options market are short-term options, managers often utilize more than one hedging instrument via longer-term contracts. It is crucial to study the options with longer time-to-maturity because option traders frequently use long-term options for hedging purposes whereas short-term options can be used for speculation purposes. Also, low volume of long-term options allows traders to have unique preferences on option pricing models. There exists relatively less noise involved in pricing long-term options and thus the mathematical sophisticated models can be expected to be more appropriate for the options with longer time-to-maturity. Next, by examining longer-term options, one can intuitively understand the over-the-counter (OTC) options market since options traded in OTC market are mostly long-term contracts. There is paucity in the current literature focusing on OTC market. Because the option contracts in OTC market last until its maturity and the market prices do not normally exist, finding an optimal option pricing model for OTC products has not been an interest of current researchers. Options in OTC markets and long-term options in S&P 500 are comparable since they share similar characteristics: longer time-to-maturity, low trading volume, and being traded by institutional investors for hedging purposes.

Despite this, the existing literature pays very little attention on longer-term options. Bakshi et al. (2000) argue that short-term and long-term options do contain distinguished information. This paper not only fills the gap in the current literature by suggesting the optimal option pricing model for long-term options but also enhances the understanding of longer time-to-expiration for option traders.

To preview the results, this paper finds evidence that traders' rule still outperforms other models in pricing errors with short-term options whose time-to-maturity is less than 60 days. However, mathematically sophisticated model shows the least pricing and hedging errors for long-term options with time-to-maturity more than 60 days. On average, SV model shows the overall best pricing and hedging performances regardless of time-to-maturity.

The remainder of this paper proceeds as follows. The next section introduces two models: stochastic volatility and ad-hoc Black Scholes models. The subsequent section discusses the data followed by the empirical results of pricing and hedging performance with respect to moneyness and time-to-maturity. For the robustness check, statistical validation and sub-periods analyses are reported, and the last section finally concludes.

2. Option Pricing Models

2.1. Stochastic Volatility Model

Heston (1993) introduces the mathematically sophisticated model referring to continuous-time stochastic volatility model (SV model) which assumes the mean-reverting square-root process. It is preferred over other similar SV models because it allows the correlation between asset returns and volatility in the closed-form solution. The model builds on the following stochastic process for underlying assets and the variance.

$$dS_t = \mu S_t dt + \sqrt{\nu_t} S_t dW_S \tag{1}$$

$$d\nu_t = \kappa(\theta - \nu_t)dt + \sigma_\nu \sqrt{\nu_t} dW_\nu \tag{2}$$

where dW_S and dW_v are the Wiener stochastic processes with an arbitrary correlation ρ, S_t is a spot price at time t, v_t is an instantaneous variance at time t, σ_v is a volatility of variance, μ is a drift of the underlying asset return, θ is a long run variance, and κ is a mean reversion rate. The closed-form pricing model of European call options with risk-neutral probability is shown whereas the price of European put options on the same stock can be obtained from the put-call parity.

$$C_t = S_t P_1 - K e^{-r\tau} P_2 \tag{3}$$

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{e^{-i\varphi\, ln[K]} f_j(x, v_t, \tau\,; \varphi)}{i\varphi}\right] d\varphi \;(j = 1, 2) \tag{4}$$

where K is a strike price, τ is a time-to-maturity, *i* is an imaginary number. $Re[\cdot]$ denotes the real part of complex variables. $f_j(x, v_t, \tau\,; \varphi) = exp[C(\tau\,; \varphi) + D(\tau\,; \varphi)v_t + i\varphi x]$ with $C(\tau\,; \varphi)$ and $D(\tau\,; \varphi)$ are functions of θ, κ, ρ, σ_v and v_t.

The closed-form option pricing equation makes it possible to derive comparative statics and hedge ratios analytically. There are two sources of stochastic variations over time, price risk S_t, and volatility risk v_t. Consequently, there are two deltas:

$$\Delta_{S,t} = \frac{\partial C_t}{\partial S_t} = P_1 \tag{5}$$

$$\Delta_{v,t} = \frac{\partial C_t}{\partial v_t} = S_t \frac{\partial P_1}{\partial v} - K e^{-r\tau} \frac{\partial P_2}{\partial v} \tag{6}$$

$$\frac{\partial P_j}{\partial v} = \frac{1}{\pi}\int_0^\infty Re\left[(i\varphi)^{-1} e^{-i\varphi\, ln[K]} \frac{\partial f_j}{\partial v}\right] d\varphi \;(j = 1,2) \tag{7}$$

For unobserved parameters, the model needs to be calibrated to find the parameters obtained from option pricing model very close to the market price of the option. Given the closed-form solutions, the parameters are estimated by minimizing the sum of squared errors between market and model prices for every single day in the sample.[4]

$$\min_{\emptyset_t} \sum_{i=1}^{N} [O_i^*(t,\tau\,;K) - O_i(t,\tau\,;K)]^2 \quad (t = 1, \cdots, T) \tag{8}$$

where $O_i^*(t,\tau\,;K)$ denotes the model price of the option i on day t and $O_i(t,\tau\,;K)$ denotes the market price of the option i on day t. $\emptyset_t$ refers to a set of model's parameters. N is the number of options on day t, and T is the number of days in the sample.

2.2. *Ad-hoc* Black Scholes Model

To deal with the drawbacks of BS model, Dumas et al. (1998) calibrate the volatility smile using implied volatility and find that AHBS model helps overcoming the traditional BS model's major criticism of constant volatility. A number of studies support that AHBS model tends to outperform other alternatives models in the options market (Jackwerth & Rubinstein, 2001; Li & Pearson, 2007; Kim, 2009).

Unlike BS model, AHBS model allows options to have different implied volatilities by strike price and time-to-expiration. Figure 1 describes the Black-Scholes implied volatility surface. It clearly shows a smile phenomenon as an indication of negatively skewed risk-neutral distribution with excess kurtosis. Therefore, the alternative option pricing models to BS model need to be considered.

AHBS model incorporates two smile approaches: "absolute smile" and "relative smile". The key difference is that the "absolute smile" approach treats its implied volatility as a function of strike price while implied volatility is treated as a function of moneyness in the "relative smile" approach. Dumas et al. (1998), Jackwerth and Rubinstein (2001), Li and Pearson (2007) and Kim (2009) conclude that the "absolute smile" approach of AHBS model significantly outperforms the "relative smile"

[4] Estimating the parameters from asset returns is an alternative method but it has limitations. Using historical data from asset returns only reflects what happened in the past and the risk premiums cannot be easily identified.

approach. Thus, this paper focuses only "absolute smile" approach with time-to-maturity. In addition, the main goal of this paper is comparing the best model among traders' rule (i.e. ABHS models) with the mathematically sophisticated model (i.e. SV model), so the results of "relative smile" approach are not reported but readily available upon request.

In the "absolute smile" approach, its implied volatility is treated as a function of strike price. It means that the implied volatility of the "absolute smile" approach does not vary with the underlying assets given fixed strike prices. There are four models as shown below.[5]

$$\text{A1T2:} \quad \sigma_{i,j} = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot \tau_j + \beta_4 \cdot {\tau_j}^2 \tag{9}$$

$$\text{A1T2C:} \quad \sigma_{i,j} = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot \tau_j + \beta_4 \cdot {\tau_j}^2 + \beta_5 \cdot K_i \cdot \tau_j \tag{10}$$

$$\text{A2T1C:} \quad \sigma_{i,j} = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot {K_i}^2 + \beta_4 \cdot \tau_j + \beta_5 \cdot K_i \cdot \tau_j \tag{11}$$

$$\text{A2T2C:} \quad \sigma_{i,j} = \beta_1 + \beta_2 \cdot K_i + \beta_3 \cdot {K_i}^2 + \beta_4 \cdot \tau_j + \beta_5 \cdot {\tau_j}^2 + \beta_6 \cdot K_i \cdot \tau_j \tag{12}$$

where $\sigma_{i,j}$ is an implied volatility of option price, K_i is a strike price and τ_j is a time-to-maturity. The models are based on the "absolute smile" approach with a strike price (K_i) as the independent variables. A1T2 is the "absolute smile" model with constant (β_1), strike price (K_i), time-to-maturity (τ_j) and the square of the time-to-maturity (${\tau_j}^2$); A1T2C is the "absolute smile" model with constant (β_1), strike price (K_i), time-to-maturity (τ_j) and strike price (K_i) multiplied with time-to-maturity ($K_i \cdot \tau_j$); A2T1C is the "absolute smile" model with constant (β_1), strike price (K_i), the square of the strike price (${K_i}^2$), time-to-maturity (τ_j) and strike price multiplied with time-to-maturity ($K_i \cdot \tau_j$); A2T2C is the "absolute smile" model with constant (β_1), strike price (K_i), square of the strike price (${K_i}^2$), time-to-maturity (τ_j), the square of the time-to-maturity (${\tau_j}^2$) and strike price multiplied with time-to-maturity ($K_i \cdot \tau_j$). By comparing A1T2 with A1T2C, the interaction effects can be examined while the

[5] This paper originally had sixteen models in total including "absolute smile" and "relative smile" approaches with interactions and high-order terms. However, this set-up may increase the chance of AHBS models outperforming other models thus only the top four versions of AHBS models that perform the best are included to compare with the mathematically sophisticated models.

higher-order effects can be inspected among A1T2C, A2T1C, and A2T2C. In all models, implied volatility ($\sigma_{i,j}$) is the dependent variable.[6]

The AHBS model requires the following four steps to implement. First, BS model's implied volatility is generated from each option. Second, the ordinary least square regression model is run with implied volatility as an explained variable while strike price and time-to-maturity as explanatory variables to estimate the parameters. Third, using the obtained parameters from the second step, the volatility in each option's strike price and time-to-maturity is calibrated. Last, the calibrated implied volatility from the third step is used to theoretically price the options with BS formula:

$$C_t = S_t N(d_1) - Ke^{-r\tau} N(d_2) \tag{13}$$

$$P_t = Ke^{-r\tau} N(-d_2) - S_t N(-d_1) \tag{14}$$

$$d_1 = \frac{ln[S_t/K] + (r + \sigma^2/2)\tau}{\sigma\sqrt{T-t}}, \; d_2 = d_1 - \sigma\sqrt{\tau} \tag{15}$$

where C_t and P_t are the call and put option prices at time t. K is a strike price and τ is a time-to-maturity. r is a risk-free rate and σ is an implied volatility. S_t is a spot price at time t and $N(\cdot)$ denotes the standard normal cumulative distribution function. In gauging the hedging performance, deltas for BS and AHBS model are $N(d_1)$ for call options and $N(-d_1)$ for put options.

3. Data

The sample contains Standard & Poor's 500 (S&P 500) index European-style option contracts traded on the Chicago Board Options Exchange (CBOE) from Option Metrics L.L.C database. With a

[6] This paper does not include the higher degrees such as the third and the fourth power of the variables in the models because it only marginally explains the role of additional variables and does not improve the overall R^2.

twenty-year period[7] spanning from 1996 to 2015, daily information from the average of the best bid and ask price of each option contracts are used.[8]

This paper focuses on S&P 500 index option contracts for many reasons.[9] First, S&P 500 provides a higher volume and greater liquidity of both short-term and long-term options. Second, a style of exercising the rights of S&P 500 options is European which is appropriate to analyze the option pricing models. Third, the leading studies in this area use the data from S&P 500 options so this paper will offer a direct comparison with their work (Bakshi et al., 1997, 2000; Dumas et al., 1998, Li and Pearson, 2007).

The summary of the option price and the total number of observations are in Table 1 with six intervals of moneyness and five extended intervals of time-to-maturity.[10] Overall, there are 664,449 observations for calls and 1,233,983 observations for puts. The number of observations for out-of-the-money (OTM) options are higher than others which indicates that they are the most active traded options. The price of the option is calculated as the average of the best bid and ask quote at the end of each trading day using OTM calls and puts.[11] Put options are more frequently traded than call options indicating that there is a high demand for portfolio insurance. Although the trading volumes for shorter-term options are higher than longer-term options, S&P 500 options are the most frequently

[7] Twenty years-long data from S&P 500 index option contracts from 1996 to 2015 are used. This is a lengthy sample period that includes major financial crises in the world. For example, the Asian financial crisis in 1997, the global financial crisis in 2007-2008, the European sovereign debt crisis in 2010-2011. Consequently, sub-periods analyses are conducted for the robustness check.

[8] S&P 500 index option contracts offer higher frequency data such as intraday. However, the main purpose of this paper is to compare prior studies that used daily data for short-term options with daily data for long-term options. To compare apples to apples, daily information from S&P 500 is used since higher frequency data will not provide a significant implication.

[9] There exist studies on the performance of AHBS model using the Korea Composite Stock Price Index (KOSPI) 200 options (Kim, 2009; Choi and Ok, 2011; Choi et al., 2012) but their analyses are concentrated on short-term options only.

[10] Five categories of time-to-maturity are divided as follows: less than 60 days ($\tau < 60$), between 60 days and 120 days ($60 < \tau < 120$), between 120 days and 300 days ($120 < \tau < 300$), between 300 days and 600 days ($300 < \tau < 600$), and lastly more than 600 days ($600 < \tau$).

[11] In-the-money (ITM) options are not considered because the trading volumes are significantly low and thus any information regarding ITM can be doubtful. Moreover, it may cause possible duplicates from double counting since in-the-money (ITM) calls and out-of-the-money (OTM) puts are equivalent for a specified strike price according to put-call parity.

traded contracts for the options with longer time-to-expiration and nearly the only options market to investigate the performance of AHBS models on the long-term options.

In assembling the data, this paper follows the convention of previous studies (Bakshi et al., 1997, 2000). Any duplicates or missing values in bid or ask quote, the strike price or time-to-maturity are excluded. Options with less than 7 days are removed to avoid any liquidity bias and the prices lower than 3/8 are also excluded to control for the price discreteness. Finally, the prices that are not satisfying no-arbitrage conditions are eliminated.

Table 2 illustrates the implied volatility calculated by BS model which is used to obtain Figure 1. The implied volatility is an average value for each moneyness across different time-to-maturity. It is clear to observe the "volatility smile" or "volatility sneer" effects irrespective of time-to-maturity. In general, the implied volatility decreases to near-the-money (NTM) but increases to OTM. Also, the implied volatility is mostly higher for longer time-to-maturity with an exception of deep OTM options. Hence, it is important to consider the alternative option pricing models to mitigate these smiling or sneer phenomena.

4. Empirical Results

The theoretical prices derived from the previously discussed models, AHBS and SV, differ from the actual market prices of the options. The empirical results of both pricing and hedging performance must be compared consistently with the previous literature (Bakshi et al., 1997, 2000; Kim & Kim, 2004, 2005; Kim, 2009; Choi & Ok, 2012).

Two comparison measures across different models are used: mean absolute errors (MAE) and root mean squared errors (RMSE).

$$\text{MAE} = \frac{1}{T}\sum_{t=1}^{T}\frac{1}{N}\sum_{i=1}^{N}|O_i(t,\tau\,;K) - O_i^*(t,\tau\,;K)| \tag{16}$$

$$\text{RMSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\frac{1}{N}\sum_{i=1}^{N}[O_i(t,\tau\,;K) - O_i^*(t,\tau\,;K)]^2} \tag{17}$$

where T is the number of days in sample and N is the number of options on day t. $O_i^*(t,\tau\,;K)$ denotes the model price of the option i with the time-to-maturity τ and the strike price K on day t. $O_i(t,\tau\,;K)$ denotes the market price of the option i with the time-to-maturity τ and the strike price K on day t. The magnitude of the error is measured by MAE with an absolute value of difference between market prices and model prices whereas the volatility of the error is calculated by RMSE with its standard deviation. It is preferred as MAE gets closer to zero and RMSE gets smaller.

4.1. In-sample Pricing Performance

The spot volatility and structural parameters are unobservable in option pricing models thus it is necessary to estimate the parameters of each model. The ordinary least square procedure for AHBS models is used and for BS model and the sum of squared errors between market prices and model prices of options are minimized for SV model (Bakshi et al., 1997, 2000). Using these unconstrained parameters, in-sample analysis compares the market prices with model prices since an estimation procedure indicates the market sentiment on a daily basis.

Table 3 reports the mean and standard error of the parameter estimates for AHBS models in Panel A and for BS and SV models in Panel B. The majority of estimated parameters are statistically significant with small standard errors. Additionally, as the number of independent variables increase (i.e. models including higher-order terms and/or interaction terms), R^2 increases which may suggest an overfitting problem.[12] The correlation, ρ, for SV model has a negative value implying that the implied risk-neutral distribution is negatively skewed which supports the “volatility sneer” phenomenon.

The major contribution of this paper is that it incorporates the longer time-to-maturity in the option pricing models to observe the extended time effect. In this study, the five categories of time-to-maturity are divided as follows: less than 60 days (τ < 60), between 60 days and 120 days (60 < τ <

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[12] To solve an over-fitting problem, this paper examines the out-of-sample pricing performance in the next section.

120), between 120 days and 300 days ($120 < \tau < 300$), between 300 days and 600 days ($300 < \tau < 600$), and lastly more than 600 days ($600 < \tau$).

In Table 4, the results for in-sample pricing errors by moneyness are presented. Using the estimated parameters, market prices with model prices are compared each day. There are three results. First, SV model performs the best with the smallest mean absolute errors (MAE) and root mean squared errors (RMSE) across six intervals for the degree of moneyness and in total. Among AHBS models, A1T2C (A2T1C) shows the least mean absolute errors (root mean squared errors) for in-sample pricing performance. When comparing SV model that has five parameters with ABHS model that has the same number of parameters (i.e. A2T2C model), SV model shows about half size of in-sample pricing errors than A2T2C model. So, the result is not simply driven by the number of parameters rather it concludes that SV model explains the current option's market better than ABHS models with respect to moneyness.

Second, MAE and RMSE are smaller for NTM options and increase as moneyness moves to OTM options for BS model. Since two measures of pricing errors, MAE and RMSE, are based on the absolute price of options, it is expected to see increased pricing errors as it moves to the options with a greater absolute value of price (i.e. NTM). SV and AHBS models generally experience the decreased pricing errors as moneyness moves from NTM to OTM. However, BS model shows that OTM errors are larger than those of NTM options. BS model still suffers from the lack of market explanation power. Further, AHBS models show the lowest in-sample pricing error in OTM put options implying that AHBS models are broadly used by professional traders and practitioners in the options market.

Third, it is necessary to include the option's time-to-maturity and its interaction terms as they demonstrate the smallest in-sample pricing errors. The model with an interaction term outperforms the one without it (A1T2 vs A1T2C). Likewise, the higher-order term with respect to time (A2T1C vs A2T2C*)* shows the least pricing error in total but the higher-order term with respect to strike price (A1T2C vs A2T2C) does not always offer smaller errors. This is because the volatility surface of time-to-maturity is nonlinear which can be adopted well in its square term. Overall, SV model explains the current S&P 500 options market well with the least in-sample pricing errors.

The results for in-sample pricing errors by time-to-maturity are shown in Table 5. In all five intervals of time periods, SV model shows the least mean-absolute-errors (MAE) except for the options whose time-to-maturity is less than 60 days ($\tau < 60$).[13] For short-term options less than 60 days ($\tau < 60$), A2T1C illustrates the least in-sample pricing errors. It indicates that traders' rule still shows the smallest in-sample pricing errors for short-term options but when it comes to the options with longer time-to-maturity, the mathematically sophisticated model performs the best.

These results reaffirm the previous studies that have shown the decent performance of AHBS models in short-term options. Nevertheless, SV model performs the best for the entire options, especially long-term options. When examining the out-of-sample pricing performance of long-term options, it can be predicted that SV model outperforms AHBS models as well.

4.2. Out-of-sample Pricing Performance

To solve an over-fitting problem from in-sample pricing performance, the model's out-of-sample pricing performance is. Having more parameters may improve the in-sample setting but also get penalized if the extra parameter does not enhance the model's overall fit. In this way, out-of-sample pricing performance controls for the stability of parameters over time and serves as an information constancy indicator. Using the current day's estimated parameters, the price of options in one-day-ahead and one-week-ahead is measured.

One-day-ahead and one-week-ahead out-of-sample pricing errors by moneyness are shown in Table 6. There are four outcomes to highlight. First, SV model has the smallest mean absolute errors (MAE) and root mean squared errors (RMSE). Unlike the prior studies, SV model outperforms AHBS models when time-to-maturity is taken into consideration. Second, among AHBS models, A1T2C (A2T1C) performs the best with mean absolute error (root mean squared errors) for one-day-ahead out-of-sample pricing errors. For one-week out-of-sample, A2T1C outperforms other models for both types of errors. Also, A1T2C outperforms A1T2 demonstrating that the models with time-to-maturity and

[13] RMSE results do not change MAE's ranks. The results for MAE are not shown in the paper for saving the space.

interaction terms show smaller out-of-sample pricing errors. Thus, time-to-maturity improves the options' pricing performance. Yet, the models with higher-order terms do not essentially enhance the model fit. Third, out-of-sample pricing performance results are similar to those of in-sample pricing performance. For BS model, the pricing errors are smaller for NTM options and increase as moneyness moves to OTM while the errors decrease as moneyness moves from NTM to OTM for SV and AHBS models. In addition, A1T2C model among AHBS models outperforms SV model for OTM put options where $1.00 < S/K$. This result can be interpreted as the trading volumes for put options are generally higher than those of call options. It indicates that AHBS models are used extensively by option traders and propose better performance (Bollen & Whaley, 2004; Bondarenko, 2014). Fourth, the pricing errors increase from in-sample to out-of-sample. For MAE, the average of in-sample pricing errors is increased by 0.093 for one-day-ahead and 0.434 for one-week-ahead out-of-sample pricing. On average, in-sample pricing errors for SV model are less than those of AHBS model which means that there exists the over-fitting problem in all models, although SV model suffers the least. To sum up, the mathematically sophisticated model (SV model) becomes the best model surpassing other models regardless of the moneyness and time-to-maturity.

Table 7 illustrates one-day-ahead and one-week-ahead out-of-sample pricing errors by time-to-maturity. For both one-day-ahead and one-week-ahead pricing errors, A2T1C outperforms other models for short-term options less than 60 days ($\tau < 60$) but SV model shows the least out-of-sample pricing errors for longer time-to-maturity and for all maturities combined. This result suggests that while traders' rule still can be useful for short-term options, option traders must consider longer time effect using the mathematically sophisticated models.

It is natural to expect that traders' rule fits the best for the options that are liquid. Because option traders follow traders' rule to trade short-term options, the prices of short-term options reflect traders' rule accurately. On the other hand, the trading volumes for long-term options are low which means that long-term options are not likely to be affected by traders' rule. It explains why AHBS models do not work well for long-term options unlike short-term options. This phenomenon is known as self-fulfilling prophecy. Self-fulfilling prophecy is defined as a situation that evokes a new behavior which makes the original concept to be truly real as people adapt their behavior to it (Merton, 1959). Azariadis (1981) applies the self-fulfilling prophecy to the price investors conjecture based on their simultaneous expectations. For example, a bank is forced to default and liquidate assets because asset prices are low or asset prices are low as a result of mass bankruptcy and thus associated liquidation of bank assets (Allen & Gale, 2004). According to Cherian and Jarrow (1998), the Black-Scholes formula can be a self-fulfilling prophecy in an incomplete market. Option traders trade short-term options more frequently than long-term options so unsurprisingly traders' rule works the best for short-term options. Longer-term options are illiquid because they are less frequently traded by traders therefore traders' rule cannot be well-applied. This paper argues that traders' rule should be more proper for short-term options with the most actively traded volume while the mathematically sophisticated model equips the best when pricing the illiquid long-term options.

So far, the previous studies that focus on AHBS models use short-term options and have not incorporated the effect of a longer period. The key contribution of this paper is that the extended intervals of time-to-maturity from S&P 500 index option contracts are used and conclude that traders' rule is no longer useful, and the mathematically sophisticated model should be considered when pricing long-term options.

4.3. Hedging Performance

In addition to the pricing performance, it is important to examine the hedging performance since it provides essential information for professional traders in the options market. Hedging performance can be used to forecast the volatility of options' price while pricing performance can be used to estimate and forecast the options' price level.

Following Baskhi et al. (1997), this paper employs the underlying asset as a single instrument for the hedging performance. This type of procedure allows the dimensions of uncertainty that move a target value of option but are not associated with the price of underlying stock to be uncontrolled. In practice, option traders mainly focus on the underlying asset price volatility thus hedging by an underlying stock as a single instrument is the most feasible for option traders to implement.

Assume a firm intends to hedge a short position in a call option with strike price, K, and τ periods to expiration. Let $X_{0,t}$ be the residual cash position and $X_{S,t}$ be the number of shares of the underlying asset to be purchased. This paper employs the delta hedging strategy solving for the standard minimum-variance hedging problem under SV model. The number of shares of the stock to be purchased as follows.

$$X_{S,t} = \frac{Cov[dS_t, dC_t]}{Var[dS_t]} = \Delta_{S,t} + \rho\sigma_v\Delta_{v,t}\frac{1}{S_t} \tag{18}$$

$$X_{0,t} = C_t - X_{S,t} \cdot S_t \tag{19}$$

Suppose that portfolio rebalancing takes place at intervals of length Δt. On day t, we take a short position in the call option, a long position in $X_{S,t}$ shares of the stock and invest the residual, $X_{0,t}$, in a risk-free bond maturing instantaneously. Then, on day $t + \Delta t$, we calculate the hedging error as follows.

$$Hedging\ Error = X_{S,t} \cdot S_{t+\Delta t} + X_{0,t}e^{r\Delta t} - C_{t+\Delta t} \tag{20}$$

This paper employs three steps. First, we estimate the parameters implied by all options of day $t-1$. Next, we use these parameters with the current day's spot index and interest rates on day t to construct the hedged position. Finally, the hedging errors as of day $t+1$ or $t+7$ are obtained. These steps are repeated for each option and every trading day in the sample.

Table 8 presents one-day-ahead and one-week-ahead hedging errors by moneyness. Similar to the pricing performance, SV model shows the smallest hedging errors while BS model shows the largest hedging errors for both one-day-ahead and one-week-ahead irrespective of moneyness. AHBS models are generally improved particularly for A2T2C which performs the best among AHBS models. Also, the hedging performance errors becomes greater as the time period for hedging shifts from one-day to one-week just like pricing performance outcomes. While the hedging error for BS model decreases as it moves from OTM call options (S/K < 0.97) to OTM put options (S/K > 1.06), the edging errors for SV and AHBS models are the least for both OTM call and OTM puts. In consideration of the most frequently traded OTM put options and the biggest pricing and hedging errors for BS model, SV and AHBS model must be adopted by the market participants.

One-day-ahead and one-week-ahead hedging errors by time-to-maturity are displayed in Table 9. Regardless of time-to-maturities, SV model has the lowest hedging errors while BS model has the highest hedging errors for both one-day-ahead and one-week-ahead. Although AHBS model outperforms SV model for short-term options with pricing performance, SV model is superior to other models in all six-time intervals.

Looking at different time-to-maturity, the hedging errors for short-term and long-term options are greater than mid-term options. Considering higher prices for long-term options, the hedging errors for short-terms options are relatively larger than options with longer time-to-maturity. This supports that short-term options suffer the most errors which makes it challenging to price and hedge in the options market. In short, SV model is superior to other models in hedging performance irrespective of moneyness and time-to-maturity.

5. Robustness Check

For the robustness check, statistical validation tests and sub-periods analyses are conducted by different time-to-maturity. Table 10 represents the t-statistics of the difference between the errors of each model from the pair-wise comparison results. Panel A (Panel B) reports the t-statistics of the difference between each model's one-day-ahead (one-week-ahead) out-of-sample absolute pricing errors. Likewise, Panel C (Panel D) reports the t-statistics of the difference between each model's one-day-ahead (one-week-ahead) hedging errors.

Consistent with the results in Table 7, SV model performs the best with statistically significant t-statistics except for short-term options. For short-term options, AHBS models are superior than SV model. Among the intricate AHBS models with an exception of A1T2, there is no substantial difference especially for mid-term and long-term options. Additionally, the significance level for pricing errors reduces as it moves from one-day-ahead to one-week-ahead out-of-sample pricing.

Unlike pricing performance in Panel A and Panel B, hedging performance errors in Panel C and Panel D are not considerably different among models. The most statistically significant model is SV model regardless of time-to-maturity. Among the intricate AHBS models, there is no substantial difference except for A1T2. Also, the significance level for hedging errors reduces as it shifts from one-day-ahead to one-week-ahead hedging. In other words, the longer the forecasting period, the smaller the differences between pricing and hedging errors of each model.

Further, supplementary analyses using sub-periods by different time-to-maturity are conducted. With twenty years-long data spanning from 1996 to 2015 this paper confirms whether the findings are maintained yearly basis. The sample period includes the major financial crises such as the Asian financial crisis in 1997, the global financial crisis in 2007-2008, and the European sovereign debt crisis in 2010-2011.

In Table 11, one-day-ahead and one-week-ahead out-of-sample pricing errors are presented by sub-periods whereas Table 12 illustrates one-day-ahead and one-week-ahead hedging errors by sub-periods. In both tables, SV model shows the smallest pricing and hedging errors irrespective of years. For short-term options ($\tau < 60$), AHBS model performs slightly better than SV model in earlier years (1996 – 2000) but SV model continuously improves and outperforms AHBS model in recent times. This confirms that SV model is well-utilized in the current options market. The sub-periods analyses using the yearly data support that the results do not vary over time as shown in Table 11 and Table 12. The option pricing and hedging errors surge consistently in the market conditions such as Asian Financial Crisis in 1997, the global financial crisis in 2007-2008, and the European sovereign debt crisis in 2010-2011. In such highly volatile markets, this paper confirms that the pricing and hedging performance deteriorates, regardless of the option pricing model.

6. Conclusion

The option's time-to-maturity is an important factor that must be considered when finding the value of the options. Since the advent of *ad-hoc* Black-Scholes models, traders' rule is commonly viewed superior to the mathematically sophisticated models. However, most studies in this area focus only on short-term options and less is known about the options with longer time-to-maturity. Using various time-to-maturity intervals of S&P 500 index options from 1996 to 2015, this paper investigates if traders' rule still outperforms mathematically sophisticated models in the current options market with extended time-to-expiration.

With the pricing and hedging performance measures, this paper renders the following results. Consistent with previous studies, traders' rule is superior to price the short-term options. This is because of the liquid short-term options and AHBS models well-adapted by professional traders and practitioners. There is no doubt the prices of short-term options reflect the traders' rule accurately.

However, SV model shows the least pricing and hedging errors for the options with longer time-to-expiration. Since long-term options are limited in terms of their availability and popularity, the theoretical mathematically sophisticated model can be the most suitable. The results are statistically significant and remain consistent in sub-periods analyses with yearly data.

In a nutshell, the current literature finds that traders' rule is useful in the options market, but those findings only apply to the options with shorter time-to-maturity. This paper proposes that the mathematically sophisticated model should be considered when pricing and hedging the long-term options. Future researchers and option traders must be careful with various time-to-maturity when applying traders' rule in the current options market.

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Table 1. S&P 500 Options Data

This table describes the summary statistics. For each call and put options, the average option price and the total number of observations in six intervals of moneyness (S/K) and five extended intervals of time-to-maturity (τ) are reported. The sample period is from 1996 to 2015. S&P 500 option prices are calculated as the average of the best bid and ask quote of each option contracts.

		τ < 60		60 < τ < 120		120 < τ < 300		300 < τ < 600		600 < τ		
	Moneyness	Price	Number	Price	Number	Price	Number	Price	Number	Price	Number	Total
Calls	S/K < 0.94	2.9484	49,598	5.2724	73,915	13.1292	101,878	27.8338	108,887	46.5911	67,689	401,967
	0.94 < S/K < 0.97	6.1322	46,496	16.3747	33,428	42.2753	21,942	79.8294	15,958	126.1401	7,988	125,812
	0.97< S/K < 1.00	15.8007	56,520	33.0853	35,008	61.8184	21,138	101.7847	15,805	147.2193	8,199	136,670
	Total	8.2938	152,614	18.2441	142,351	39.0743	144,958	69.8160	140,650	106.6502	83,876	664,449
Puts	1.00< S/K < 1.03	18.4877	53,669	37.6650	32,394	62.3191	20,235	100.1988	15,222	149.5359	7,938	129,458
	1.03< S/K < 1.06	10.4095	47,304	26.7345	28,202	49.8719	18,598	85.4935	14,425	135.9664	7,137	115,666
	1.06 < S/K	3.5029	223,841	7.1706	226,254	15.2203	210,412	27.8658	209,337	50.4614	119,015	988,859
	Total	10.8000	324,814	23.8567	286,850	42.4704	249,245	71.1860	238,984	111.9879	134,090	1,233,983

Table 2. Black-Scholes Implied Volatility

This table reports the implied volatility calculated by Black-Scholes model using S&P 500 options from 1996 to 2015. The implied volatility of options is the average within each value in six intervals of moneyness (S/K) and across different time-to-maturity (τ).

		τ < 60	60 < τ < 120	120 < τ < 300	300 < τ < 600	600 < τ
Calls	S/K < 0.94	0.2230	0.1744	0.1660	0.1605	0.1570
	0.94 < S/K < 0.97	0.1510	0.1483	0.1591	0.1605	0.1588
	0.97< S/K < 1.00	0.1531	0.1583	0.1667	0.1641	0.1576
Puts	1.00< S/K < 1.03	0.1854	0.1939	0.2100	0.2194	0.2355
	1.03< S/K < 1.06	0.2083	0.2088	0.2194	0.2254	0.2405
	1.06 < S/K	0.3257	0.3117	0.3075	0.3068	0.3139

Table 3. Parameters

The table reports the mean and the standard error of the parameter estimates for each model. R^2s and the standard deviation for AHBS models are reported. In AHBS models, the implied volatility is treated as a function of strike price (K) and time-to-maturity (τ) with higher-order terms and/or interaction terms. For AHBS models, each parameter is estimated by the ordinary least squares every day. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with stochastic volatility. For BS and SV models, each parameter is estimated by minimizing the sum of squared errors between model and market prices every day. * p<0.05, ** p<0.01, *** p<0.00

Panel A. AHBS Models

	C	K	K^2	τ	τ^2	$K \cdot \tau$	R^2
A1T2	0.5573***	-2.909e-04***		-0.0121***	0.0049***		0.8137
	(0.0015)	(1.504e-06)		(0.0009)	(0.0003)		(0.0809)
A1T2C	0.7166***	-4.332e-04***		-0.1708***	0.0049***	1.441e-04***	0.8853
	(0.0019)	(2.046e-06)		(0.0013)	(0.0003)	(1.0727e-06)	(0.0743)
A2T1C	0.7733***	-5.166e-04***	1.569e-08***	-0.1530***		1.327e-04***	0.9021
	(0.0030)	(5.395e-06)	(4.364e-09)	(0.0010)		(1.0454e-06)	(0.0563)
A2T2C	0.7827***	-5.291e-04***	2.384e-08***	-0.1684***	0.0080***	1.314e-04***	0.9080
	(0.0030)	(5.186e-06)	(4.141e-09)	(0.0012)	(0.0003)	(1.0364e-06)	(0.0497)

Panel B. Other Models

BS	σ				
	0.1933***				
	(0.0007)				
SV	κ	θ	σ_v	ρ	v_t
	0.1694***	0.0403***	0.6143***	-0.7649***	0.0483***
	(0.0073)	(0.0005)	(0.0033)	(0.0015)	(0.0006)

Table 4. In-sample Pricing Errors by Moneyness

This table reports in-sample pricing errors for S&P 500 options with respect to moneyness. Each model is estimated every day during the sample period and in-sample pricing errors are computed using the estimated parameters from the current day. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with stochastic volatility. In AHBS models, the implied volatility is treated as a function of strike price (K) and time-to-maturity (τ) with higher-order terms and/or interaction terms.

Panel A. Mean Absolute Errors

	S/K < 0.94	0.94 < S/K < 0.97	0.97< S/K < 1.00	1.00< S/K < 1.03	1.03< S/K < 1.06	1.06 < S/K	Total
BS	12.5548	10.6309	9.5508	8.5704	8.9864	9.8939	10.3359
SV	2.3178	3.7641	6.4751	5.5188	4.1959	1.8727	2.8138
A1T2	5.6630	8.8355	10.6142	5.9612	4.2879	2.3697	4.4508
A1T2C	5.6555	7.1489	8.9826	4.4574	3.3818	1.7467	3.7378
A2T1C	4.1450	6.2920	8.0575	4.9952	3.9964	2.5223	3.7727
A2T2C	4.7484	6.0516	7.8355	5.4608	4.3640	2.2128	3.7614

Panel B. Root Mean Squared Errors

	S/K < 0.94	0.94 < S/K < 0.97	0.97< S/K < 1.00	1.00< S/K < 1.03	1.03< S/K < 1.06	1.06 < S/K	Total
BS	16.3150	13.6213	12.2454	13.7642	14.7018	15.9262	15.4162
SV	4.3208	7.2462	9.6610	9.3224	7.4736	3.4976	5.4638
A1T2	8.9463	11.2420	13.1924	10.3320	8.1716	4.7144	7.7959
A1T2C	11.7949	12.9103	14.0600	7.4747	6.0699	3.2317	8.1381
A2T1C	8.2510	9.8349	11.4340	9.3133	8.3314	5.5952	7.5291
A2T2C	10.5383	10.6361	11.8641	9.6725	8.4613	4.8287	8.0020

Table 5. In-sample Pricing Errors by Time-to-maturity

This table reports in-sample pricing errors for S&P 500 options with respect to time-to-maturity, measured by mean absolute errors. Each model is estimated every day during the sample period and in-sample pricing errors are computed using the estimated parameters from the current day. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with stochastic volatility. In AHBS models, the implied volatility is treated as a function of strike price (K) and time-to-maturity (τ) with higher-order terms and/or interaction terms.

	$\tau < 60$	$60 < \tau < 120$	$120 < \tau < 300$	$300 < \tau < 600$	$600 < \tau$	Total
BS	3.4269	5.5307	9.0209	15.7663	27.9278	12.3345
SV	1.5636	1.3070	1.7212	3.9056	8.6027	3.4200
A1T2	2.6070	2.6070	2.9351	5.1570	12.0983	5.0809
A1T2C	1.6396	1.8208	2.5101	4.9285	12.2698	4.6338
A2T1C	1.5286	1.6984	2.5954	4.8793	12.9937	4.7391
A2T2C	1.6305	1.7161	2.3004	5.0249	12.9110	4.7166

Table 6. Out-of-sample Pricing Errors by Moneyness

This table reports one-day-ahead and one-week-ahead out-of-sample pricing errors for S&P 500 options with respect to moneyness. Each model is estimated every day during the sample period and out-of-sample pricing errors are computed using the estimated parameters from the previous day or week. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with stochastic volatility. In AHBS models, the implied volatility is treated as a function of strike price (K) and time-to-maturity (τ) with higher-order terms and/or interaction terms.

Panel A. One-day-ahead Out-of-sample Pricing Errors

	Mean Absolute Errors							Root Mean Squared Errors							
	S/K < 0.94	0.94 < S/K < 0.97	0.97< S/K < 1.00	1.00< S/K < 1.03	1.03< S/K < 1.06	1.06 < S/K	Total		S/K < 0.94	0.94 < S/K < 0.97	0.97< S/K < 1.00	1.00< S/K < 1.03	1.03< S/K < 1.06	1.06 < S/K	Total
BS	12.5624	10.6955	9.6846	8.6658	9.0113	9.8884	10.3566	BS	16.3907	13.7355	12.4227	13.9001	14.7569	15.9388	15.4683
SV	2.6847	4.1277	6.7085	5.9076	4.5212	2.0425	3.0671	SV	4.7702	7.5874	10.0387	9.6700	7.7408	3.7742	5.7736
A1T2	5.6965	8.8123	10.6226	6.0973	4.4443	2.4425	4.5137	A1T2	9.0475	11.2889	13.2707	10.4616	8.3284	4.7925	7.8809
A1T2C	5.7197	7.1447	9.0006	4.6233	3.5558	1.8268	3.8160	A1T2C	12.0371	13.0122	14.1713	7.6527	6.2587	3.3712	8.2860
A2T1C	4.2265	6.3030	8.0880	5.1571	4.1627	2.5835	3.8458	A2T1C	8.4087	9.8998	11.5253	9.4131	8.4309	5.6544	7.6194
A2T2C	4.8384	6.0660	7.8714	5.6230	4.5338	2.2883	3.8447	A2T2C	10.8772	10.7769	12.0043	9.7999	8.6003	4.9142	8.1704

Panel B. One-week-ahead Out-of-sample Pricing Errors

	Mean Absolute Errors							Root Mean Squared Errors							
	S/K < 0.94	0.94 < S/K < 0.97	0.97< S/K < 1.00	1.00< S/K < 1.03	1.03< S/K < 1.06	1.06 < S/K	Total		S/K < 0.94	0.94 < S/K < 0.97	0.97< S/K < 1.00	1.00< S/K < 1.03	1.03< S/K < 1.06	1.06 < S/K	Total
BS	12.5811	10.8666	9.9401	8.8956	9.1453	9.8717	10.4055	BS	16.6057	14.0993	12.8243	14.2140	15.0214	15.9943	15.6263
SV	3.2975	4.8777	7.2325	6.6443	5.2322	2.4710	3.6008	SV	5.7164	8.4492	10.7873	10.3864	8.4864	4.4176	6.4755
A1T2	5.7987	8.7356	10.5620	6.4898	4.8957	2.6817	4.7045	A1T2	9.2975	11.3856	13.3824	10.9140	8.8126	5.1485	8.1513
A1T2C	5.9189	7.1323	8.9471	5.0776	4.0359	2.0910	4.0511	A1T2C	13.2170	13.4200	14.4869	8.3252	6.9993	3.9603	8.9392
A2T1C	4.4481	6.2973	8.0436	5.5665	4.5908	2.7798	4.0453	A2T1C	8.9099	10.0385	11.6092	9.7577	8.7443	5.8766	7.8947
A2T2C	5.1186	6.1219	7.8659	6.0326	4.9638	2.5222	4.0831	A2T2C	12.5039	11.3521	12.4279	10.2984	9.1025	5.2936	8.9240

Table 7. Out-of-sample Pricing Errors by Time-to-maturity

This table reports one-day-ahead and one-week-ahead out-of-sample pricing errors for S&P 500 options with respect to time-to-maturity, measured by mean absolute errors. Each model is estimated every day during the sample period and out-of-sample pricing errors are computed using the estimated parameters from the previous day or week. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with stochastic volatility. In AHBS models, the implied volatility is treated as a function of strike price (K) and time-to-maturity (τ) with higher-order terms and/or interaction terms.

Panel A. One-day-ahead Out-of-sample Pricing

	τ < 60	60 < τ < 120	120 < τ < 300	300 < τ < 600	600 < τ	Total
BS	3.4545	5.5617	9.0463	15.7767	27.9214	12.3521
SV	1.7585	1.6353	2.0386	4.1193	8.7890	3.6681
A1T2	2.6750	3.4395	3.0050	5.2144	5.2144	3.9097
A1T2C	1.7297	1.9111	2.5725	4.9867	12.3604	4.7121
A2T1C	1.6149	1.7965	2.6522	4.9324	13.0524	4.8097
A2T2C	1.7228	1.8144	2.3758	5.0823	13.0037	4.7998

Panel B. One-week-ahead Out-of-sample Pricing

	τ < 60	60 < τ < 120	120 < τ < 300	300 < τ < 600	600 < τ	Total
BS	3.5367	5.6559	9.1058	15.7794	27.8666	12.3889
SV	2.1332	2.2623	2.6984	4.6688	9.2317	4.1989
A1T2	2.8368	3.5844	3.2379	5.4265	12.3871	5.4945
A1T2C	1.9129	2.1301	2.8076	5.2010	12.7768	4.9657
A2T1C	1.7922	2.0333	2.8558	5.1056	13.2632	5.0100
A2T2C	1.9093	2.0543	2.6225	5.2756	13.4150	5.0553

Table 8. Hedging Errors by Moneyness

This table reports one-day-ahead and one-week-ahead hedging errors for S&P 500 options with respect to moneyness. Following Bakshi et al. (1997), the underlying asset is used as a single hedging instrument. Parameters implied by all options of the previous day are used to establish the current day's hedging portfolio, which are then liquidated the following day or week later. For each target option, the hedging error is the difference between its market price and the replicating portfolio value. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with stochastic volatility. In AHBS models, the implied volatility is treated as a function of strike price (K) and time-to-maturity (τ) with higher-order terms and/or interaction terms.

Panel A. One-day-ahead Hedging

	Mean Absolute Errors								Root Mean Squared Errors						
	S/K < 0.94	0.94 < S/K < 0.97	0.97< S/K < 1.00	1.00< S/K < 1.03	1.03< S/K < 1.06	1.06 < S/K	Total		S/K < 0.94	0.94 < S/K < 0.97	0.97< S/K < 1.00	1.00< S/K < 1.03	1.03< S/K < 1.06	1.06 < S/K	Total
BS	1.3957	1.3960	1.1301	1.1296	1.0566	0.7531	1.0056	BS	2.2695	2.1255	1.6597	1.8905	1.8152	1.4187	1.7576
SV	0.8403	1.2023	1.1056	1.0670	0.9329	0.4952	0.7225	SV	1.6557	1.9416	1.6266	1.6955	1.5160	0.9408	1.3462
A1T2	1.0686	1.3626	1.1149	1.0557	0.9415	0.5247	0.7942	A1T2	2.0100	2.1008	1.6293	1.6663	1.5228	0.9954	1.4713
A1T2C	0.9893	1.2648	1.0940	1.0643	0.9446	0.5159	0.7665	A1T2C	1.9248	1.9798	1.6057	1.6802	1.5264	0.9845	1.4330
A2T1C	0.9825	1.2648	1.0918	1.0693	0.9507	0.5206	0.7683	A2T1C	1.8973	1.9777	1.6025	1.6971	1.5418	0.9975	1.4320
A2T2C	0.9745	1.2653	1.0918	1.0667	0.9497	0.5220	0.7672	A2T2C	1.8734	1.9796	1.6020	1.6885	1.5396	0.9998	1.4258

Panel B. One-week-ahead Hedging

	Mean Absolute Errors								Root Mean Squared Errors						
	S/K < 0.94	0.94 < S/K < 0.97	0.97< S/K < 1.00	1.00< S/K < 1.03	1.03< S/K < 1.06	1.06 < S/K	Total		S/K < 0.94	0.94 < S/K < 0.97	0.97< S/K < 1.00	1.00< S/K < 1.03	1.03< S/K < 1.06	1.06 < S/K	Total
BS	3.2655	2.4626	1.7438	2.5857	2.3029	1.6273	2.1970	BS	5.7682	3.6773	2.5090	5.0008	4.2970	3.2066	4.1665
SV	2.0553	2.2026	1.7263	2.2909	1.9419	1.0281	1.5136	SV	4.4851	3.3978	2.4977	4.1389	3.4224	2.0280	3.1160
A1T2	2.5209	2.4051	1.7097	2.3156	1.9978	1.1244	1.7023	A1T2	5.1528	3.6229	2.4593	4.2656	3.5716	2.2543	3.4864
A1T2C	2.3296	2.2580	1.6837	2.3303	2.0004	1.1010	1.6327	A1T2C	4.9103	3.4332	2.4325	4.2766	3.5653	2.2294	3.3743
A2T1C	2.3171	2.2549	1.6812	2.3433	2.0110	1.1054	1.6326	A2T1C	4.8782	3.4276	2.4244	4.3244	3.6025	2.2524	3.3742
A2T2C	2.3095	2.2611	1.6840	2.3422	2.0132	1.1078	1.6333	A2T2C	4.8478	3.4346	2.4321	4.3222	3.6095	2.2629	3.3694

Table 9. Hedging Errors by Time-to-maturity

This table reports one-day-ahead and one-week-ahead hedging errors for S&P 500 options with respect to time-to-maturity, measured by mean absolute errors. Following Bakshi et al. (1997), the underlying asset is used as a single hedging instrument. Parameters implied by all options of the previous day are used to establish the current day's hedging portfolio, which are then liquidated the following day or week later. For each target option, the hedging error is the difference between its market price and the replicating portfolio value. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with stochastic volatility. In AHBS models, the implied volatility is treated as a function of strike price (K) and time-to-maturity (τ) with higher-order terms and/or interaction terms.

Panel A. One-day-ahead Hedging

	$\tau < 60$	$60 < \tau < 120$	$120 < \tau < 300$	$300 < \tau < 600$	$600 < \tau$	Total
BS	0.8992	0.8593	0.9294	1.0725	1.3415	1.0056
SV	0.6989	0.5969	0.6397	0.7629	0.9908	0.7225
A1T2	0.7933	0.7078	0.7078	0.8039	1.0275	0.7942
A1T2C	0.7237	0.6315	0.6660	0.8010	1.1114	0.7665
A2T1C	0.7238	0.6363	0.6712	0.8035	1.1031	0.7683
A2T2C	0.7268	0.6361	0.6643	0.7995	1.1112	0.7672

Panel B. One-week-ahead Hedging

	$\tau < 60$	$60 < \tau < 120$	$120 < \tau < 300$	$300 < \tau < 600$	$600 < \tau$	Total
BS	2.2398	2.0266	1.9603	2.1728	2.6829	2.1970
SV	1.7441	1.3171	1.2546	1.4625	1.9378	1.5136
A1T2	2.0933	1.6830	1.4235	1.5500	2.0127	1.7023
A1T2C	1.8904	1.4451	1.3188	1.5457	2.1737	1.6327
A2T1C	1.8915	1.4604	1.3269	1.5402	2.1591	1.6326
A2T2C	1.9032	1.4559	1.3138	1.5350	2.1819	1.6333

Table 10. Differences between the Errors of Each Model by Time-to-maturity

This table reports the t-statistics of the difference between the errors by time-to-maturity in each model. Panel A reports the t-statistics of the difference between one-day-ahead out-of-sample pricing errors of each model. Panel B reports the t-statistics of the difference between one-week-ahead out-of-sample pricing errors of each model. Panel C reports t-statistics of the difference between one-day-ahead hedging errors of each model. Panel D reports t-statistics of the difference between one-week-ahead hedging errors of each model. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with stochastic volatility. In AHBS models, the implied volatility is treated as a function of strike price (K) and time-to-maturity (τ) with higher-order terms and/or interaction terms.

	Panel A. One-day-ahead Out-of-sample Pricing					Panel B. One-week-ahead Out-of-sample Pricing					Panel C. One-day-ahead Hedging					Panel D. One-week-ahead Hedging				
	BS	SV	A1T2	A1T2C	A2T1C	BS	SV	A1T2	A1T2C	A2T1C	BS	SV	A1T2	A1T2C	A2T1C	BS	SV	A1T2	A1T2C	A2T1C
											τ<60									
SV	266.33					197.47					69.16					56.91				
A1T2	105.73	-161.57				89.57	-107.6				33.10	-36.16				24.48	-31.59			
A1T2C	267.69	6.56	164.21			235.13	40.69	146.33			59.10	-9.97	26.12			47.57	-9.02	22.53		
A2T1C	293.62	34.77	190.78	27.03		259.82	65.98	171.15	24.69		60.84	-8.64	27.69	1.40		49.28	-7.54	24.12	1.52	
A2T2C	269.87	8.20	166.26	1.54	-25.65	236.92	41.70	147.85	0.69	-24.23	58.30	-10.95	25.24	-0.95	-2.36	46.74	-9.86	21.71	-0.83	-2.35
											60<τ<120									
SV	469.82					366.07					112.56					96.87				
A1T2	207.58	-245.06				194.96	-161.0				57.86	-52.45				51.61	-43.39			
A1T2C	413.89	-52.61	193.86			380.06	20.71	176.77			91.90	-19.06	33.03			80.84	-14.83	28.32		
A2T1C	435.21	-32.54	213.53	20.11		397.40	37.27	192.81	15.74		91.15	-20.43	32.00	-1.23		79.46	-16.32	26.91	-1.45	
A2T2C	432.28	-35.93	210.67	16.90	-3.27	394.04	33.65	189.55	12.25	-3.54	91.02	-20.51	31.90	-1.32	-0.09	79.66	-16.11	27.11	-1.25	0.20
											120<τ<300									
SV	604.86					509.76					125.01					122.82				
A1T2	474.52	-123.15				442.93	-61.15				88.98	-34.98				91.38	-30.12			
A1T2C	543.77	-83.15	52.03			503.19	-14.22	49.20			108.52	-15.45	19.35			109.04	-12.73	17.25		
A2T1C	535.43	-94.56	42.18	-11.30		499.79	-20.54	43.77	-6.36		104.79	-18.35	16.20	-3.00		106.05	-15.17	14.68	-2.49	
A2T2C	571.01	-56.24	78.75	29.83	41.49	527.34	10.39	73.05	25.65	32.40	110.77	-13.83	21.16	1.70	4.70	109.78	-12.16	17.87	0.59	3.08
											300<τ<600									
SV	643.78					582.02					122.94					123.90				
A1T2	553.98	-87.82				523.76	-56.58				104.33	-18.60				106.22	-17.17			
A1T2C	571.54	-71.21	16.79			540.24	-40.58	16.01			104.94	-17.71	0.84			106.13	-17.14	0.01		
A2T1C	585.65	-70.03	21.59	4.25		556.79	-34.96	23.76	7.21		103.32	-18.92	-0.45	-1.28		104.94	-17.96	-0.87	-0.88	
A2T2C	558.72	-76.54	9.48	-7.00	-11.39	530.40	-45.13	10.48	-5.28	-12.54	106.74	-16.01	2.56	1.71	2.99	107.13	-16.34	0.84	0.83	1.71
											τ>600									
SV	453.48					430.57					90.64					86.51				
A1T2	356.17	-103.97				341.60	-94.07				79.57	-10.77				76.16	-10.36			
A1T2C	304.76	-86.74	-4.76			277.36	-78.65	-8.29			53.88	-35.44	-24.73			53.69	-31.84	-21.63		
A2T1C	319.03	-120.09	-23.67	-15.15		305.42	-109.4	-22.34	-9.84		58.88	-31.27	-20.45	4.47		56.67	-29.63	-19.31	2.51	
A2T2C	300.84	-107.16	-20.44	-13.20	1.11	269.26	-94.67	-22.26	-11.58	-3.12	55.28	-34.86	-24.03	0.93	-3.57	53.10	-33.02	-22.72	-0.92	-3.45

Table 11. Out-of-sample Pricing Errors for Sub-periods by Time-to-maturity

This table reports one-day-ahead and one-week-ahead out-of-sample pricing errors by time-to-maturity for sub-periods from 1996 to 2015, measured by mean absolute errors. Each model is estimated every day during the sample period and out-of-sample pricing errors are computed using the estimated parameters from the previous day or week. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with stochastic volatility. In AHBS models, the implied volatility is treated as a function of strike price (K) and time-to-maturity (τ) with higher-order terms and/or interaction terms.

Panel A. One-day-ahead Out-of-sample Pricing

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
										τ<60										
BS	1.329	2.060	3.160	4.003	2.870	2.932	2.979	1.927	2.152	1.934	2.196	3.270	5.359	2.828	3.242	3.908	3.721	3.757	3.903	4.535
SV	0.672	1.206	1.969	2.514	1.613	1.283	1.236	1.031	1.459	1.301	1.347	1.779	2.311	1.575	1.986	2.067	1.809	1.452	1.846	2.127
A1T2	0.809	1.349	2.312	2.370	1.956	2.170	1.989	1.528	1.485	1.512	1.835	2.517	2.805	2.590	2.728	3.119	2.728	2.592	3.747	4.302
A1T2C	0.603	0.967	1.550	1.595	1.560	1.636	1.435	1.093	1.069	1.060	1.245	1.721	2.177	1.741	1.819	1.974	1.556	1.657	2.163	2.396
A2T1C	0.533	0.903	1.520	1.797	1.477	1.288	1.221	1.025	1.147	1.128	1.178	1.585	1.950	1.487	1.770	1.815	1.571	1.685	1.939	2.147
A2T2C	0.544	0.938	1.535	1.610	1.502	1.439	1.334	1.056	1.079	1.073	1.217	1.669	2.030	1.577	1.856	1.934	1.648	1.814	2.236	2.486
										60<τ<120										
BS	2.489	3.246	5.604	6.264	4.727	4.369	4.209	3.162	3.407	3.029	3.609	5.829	7.436	4.886	5.221	6.274	5.886	5.463	6.151	7.006
SV	0.710	1.113	1.743	1.728	1.569	1.239	1.184	0.956	0.920	1.038	1.314	2.020	2.464	1.517	1.705	1.995	1.620	1.286	1.707	1.906
A1T2	0.964	1.283	2.364	2.299	2.402	2.710	2.480	1.846	1.562	1.852	2.416	3.248	4.419	3.145	3.316	4.151	3.403	3.049	4.487	4.933
A1T2C	0.725	0.957	1.330	1.311	1.805	2.051	1.665	1.151	0.963	1.211	1.511	2.020	3.417	1.770	1.885	2.471	1.661	1.605	2.065	2.295
A2T1C	0.677	0.951	1.319	1.356	1.612	1.470	1.264	1.035	0.952	1.207	1.441	1.912	3.055	1.407	1.749	2.093	1.655	1.748	2.030	2.325
A2T2C	0.670	0.946	1.311	1.323	1.620	1.515	1.305	1.045	0.961	1.207	1.459	1.919	2.995	1.427	1.787	2.112	1.681	1.749	2.077	2.370
										120<τ<300										
BS	3.906	5.053	9.554	11.461	8.640	6.616	6.243	5.805	6.911	5.824	6.603	10.210	10.478	8.581	9.494	10.652	10.500	9.167	10.019	12.525
SV	1.255	1.497	1.955	2.318	1.942	1.518	1.502	1.274	1.455	1.607	2.117	2.820	3.038	2.045	2.092	2.429	2.128	1.948	1.989	2.253
A1T2	1.404	1.803	2.181	2.279	2.450	2.736	2.673	2.277	2.033	1.990	2.939	3.325	4.667	3.573	3.032	3.896	3.324	2.852	2.927	4.035
A1T2C	1.406	1.803	2.140	2.171	2.378	2.701	2.511	1.847	1.707	1.688	2.390	3.096	3.939	3.086	2.517	3.376	2.420	2.255	2.725	3.036
A2T1C	1.370	1.873	2.169	2.593	2.267	2.425	2.377	1.693	1.728	1.720	2.341	3.239	4.501	2.506	2.383	3.110	2.544	2.500	2.919	3.739
A2T2C	1.344	1.841	2.085	2.136	1.946	1.904	1.833	1.553	1.651	1.671	2.342	3.027	3.388	2.291	2.169	2.624	2.500	2.412	2.776	3.221
										300<τ<600										
BS	7.486	8.682	15.364	18.738	15.181	11.369	9.736	9.886	12.039	10.753	11.247	16.373	17.066	13.665	15.264	18.688	19.902	15.735	18.745	21.270
SV	2.881	3.184	3.773	3.636	3.311	2.475	2.569	2.733	2.982	3.520	4.510	5.720	5.284	3.613	3.837	4.703	5.191	4.071	4.735	4.638
A1T2	3.447	4.348	6.127	6.446	4.520	4.236	3.984	3.579	3.607	4.010	4.909	6.429	6.618	4.818	4.836	6.030	5.904	4.634	5.989	5.733
A1T2C	3.330	3.809	4.913	4.364	4.049	4.123	4.218	3.679	3.478	4.171	4.932	6.329	6.267	4.590	4.669	5.871	5.661	4.442	5.893	5.786
A2T1C	2.995	3.489	4.846	4.543	3.841	3.250	3.220	3.606	3.726	4.286	4.737	5.887	6.359	3.897	4.227	5.495	6.052	4.818	5.856	6.491
A2T2C	3.021	3.608	4.909	4.391	3.994	3.520	3.196	3.596	3.688	4.261	4.782	6.103	6.663	3.937	4.363	5.596	6.161	4.973	6.361	6.607
										τ>600										
BS	11.094	12.647	20.019	25.678	20.000	15.655	14.361	14.956	18.069	16.482	17.713	25.691	27.427	19.835	26.762	30.832	33.524	29.311	31.727	38.554
SV	4.967	5.047	6.066	6.148	5.186	4.074	4.136	5.122	5.214	6.579	9.017	10.934	9.087	6.348	7.441	8.976	10.058	9.621	10.000	11.221
A1T2	5.478	5.645	10.900	12.701	7.425	8.155	8.420	6.752	6.753	8.048	10.723	14.691	14.253	8.845	9.835	11.442	11.852	11.768	14.335	18.304
A1T2C	6.505	7.499	11.312	11.537	8.980	10.964	10.174	8.905	7.887	9.546	11.452	15.281	15.634	11.099	10.709	13.684	13.208	11.011	11.960	15.683
A2T1C	6.011	6.377	10.838	12.814	9.940	8.456	7.846	8.112	8.720	9.851	10.975	13.775	15.556	9.387	11.657	14.295	15.487	13.670	13.417	16.546
A2T2C	5.796	6.373	10.747	11.982	9.215	6.533	6.619	8.047	8.716	9.699	11.083	13.898	13.569	9.309	11.641	14.659	15.517	13.543	13.580	17.236

Panel B. One-week-ahead Out-of-sample Pricing

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
τ<60																				
BS	1.358	2.145	3.253	4.082	2.972	3.010	3.051	1.959	2.209	1.964	2.223	3.317	5.533	2.872	3.346	4.032	3.805	3.814	3.955	4.657
SV	0.767	1.479	2.385	2.774	1.988	1.853	1.762	1.223	1.618	1.386	1.463	2.162	3.135	1.853	2.339	2.624	2.053	1.662	2.148	2.740
A1T2	0.835	1.506	2.395	2.470	2.197	2.556	2.196	1.613	1.600	1.561	1.875	2.626	3.286	2.742	2.857	3.383	2.803	2.638	3.777	4.600
A1T2C	0.666	1.185	1.652	1.739	1.858	2.019	1.683	1.185	1.180	1.105	1.270	1.853	2.728	1.888	1.989	2.284	1.632	1.678	2.159	2.788
A2T1C	0.618	1.126	1.622	1.908	1.769	1.634	1.459	1.117	1.240	1.160	1.194	1.715	2.469	1.636	1.938	2.114	1.651	1.711	1.959	2.506
A2T2C	0.628	1.159	1.640	1.755	1.798	1.830	1.590	1.156	1.192	1.119	1.243	1.807	2.569	1.739	2.031	2.242	1.729	1.843	2.248	2.864
60<τ<120																				
BS	2.527	3.353	5.740	6.416	4.854	4.514	4.335	3.192	3.457	3.030	3.655	5.809	7.717	4.933	5.316	6.391	5.986	5.544	6.219	7.093
SV	0.967	1.440	2.622	2.503	2.249	2.099	1.896	1.247	1.250	1.229	1.621	2.662	3.608	2.047	2.468	2.874	2.089	1.730	2.343	2.546
A1T2	1.063	1.499	2.562	2.428	2.534	3.064	2.655	1.955	1.652	1.885	2.464	3.407	4.671	3.392	3.551	4.355	3.535	3.128	4.505	5.032
A1T2C	0.886	1.230	1.640	1.630	2.079	2.398	1.898	1.307	1.075	1.267	1.587	2.258	3.762	2.033	2.212	2.777	1.849	1.712	2.178	2.518
A2T1C	0.865	1.229	1.626	1.666	1.911	1.836	1.537	1.190	1.062	1.264	1.522	2.189	3.443	1.731	2.079	2.451	1.832	1.854	2.170	2.547
A2T2C	0.863	1.220	1.636	1.644	1.915	1.892	1.595	1.201	1.070	1.266	1.541	2.187	3.406	1.749	2.119	2.470	1.857	1.861	2.199	2.617
120<τ<300																				
BS	3.910	5.123	9.648	11.514	8.725	6.786	6.322	5.840	6.913	5.820	6.619	10.220	10.685	8.618	9.581	10.696	10.527	9.228	10.045	12.567
SV	1.455	1.830	3.128	3.263	2.765	2.359	2.241	1.653	1.827	1.814	2.350	3.659	4.014	2.557	3.003	3.416	2.670	2.386	2.594	2.955
A1T2	1.550	2.081	2.558	2.603	2.766	3.099	2.945	2.466	2.101	2.062	2.973	3.537	4.889	3.967	3.433	4.225	3.578	2.981	3.102	4.130
A1T2C	1.567	2.084	2.530	2.567	2.698	3.023	2.791	2.072	1.788	1.779	2.481	3.388	4.213	3.434	2.900	3.695	2.676	2.349	2.906	3.075
A2T1C	1.492	2.084	2.562	2.934	2.610	2.819	2.686	1.882	1.821	1.806	2.432	3.455	4.732	2.854	2.802	3.319	2.734	2.613	2.981	3.707
A2T2C	1.505	2.116	2.548	2.543	2.331	2.193	2.200	1.733	1.762	1.762	2.438	3.335	3.723	2.641	2.553	2.932	2.680	2.539	2.951	3.312
300<τ<600																				
BS	7.499	8.658	15.421	18.860	15.305	11.535	9.684	9.899	12.033	10.789	11.227	16.275	16.951	13.776	15.273	18.658	19.896	15.725	18.716	21.221
SV	3.027	3.458	5.145	4.752	4.191	3.365	3.165	3.050	3.286	3.708	4.648	6.396	6.224	3.952	4.654	5.470	5.602	4.327	5.243	5.099
A1T2	3.614	4.577	6.634	6.874	4.889	4.648	4.264	3.757	3.646	4.120	4.978	6.720	6.921	5.159	4.958	6.469	6.122	4.693	6.194	5.702
A1T2C	3.452	4.033	5.449	4.725	4.379	4.502	4.426	3.861	3.567	4.272	5.019	6.714	6.638	4.905	4.817	6.315	5.819	4.495	6.100	5.815
A2T1C	3.144	3.690	5.380	4.919	4.274	3.750	3.524	3.743	3.822	4.355	4.822	6.109	6.561	4.121	4.523	5.653	6.159	4.908	5.889	6.500
A2T2C	3.197	3.861	5.480	4.730	4.340	3.771	3.446	3.694	3.784	4.334	4.871	6.489	7.194	4.119	4.618	5.851	6.230	5.077	6.568	6.497
τ>600																				
BS	11.044	12.685	19.795	25.820	20.171	15.693	14.251	15.012	18.076	16.453	17.717	25.514	26.867	20.001	26.736	30.713	33.512	29.297	31.676	38.467
SV	5.091	5.178	8.061	7.440	5.918	5.347	4.811	5.465	5.612	6.773	9.077	11.358	9.715	6.735	8.161	9.636	10.402	9.867	10.323	11.478
A1T2	5.617	5.871	11.487	13.248	7.763	8.642	8.713	6.984	7.026	8.155	10.823	15.089	14.554	9.205	10.112	11.608	12.072	11.976	14.696	18.200
A1T2C	6.514	7.560	11.818	11.646	9.042	11.055	10.363	9.176	8.082	9.741	11.569	15.291	15.621	11.446	11.154	14.085	13.555	11.842	13.382	15.327
A2T1C	6.194	6.441	11.555	13.642	9.912	8.401	8.162	8.378	8.955	10.119	11.066	13.985	15.823	9.718	11.949	14.675	15.544	14.013	13.849	16.160
A2T2C	5.946	6.413	11.339	12.164	9.267	7.191	7.052	8.352	8.895	9.891	11.243	14.010	13.794	9.658	11.935	15.064	15.617	14.421	15.068	16.773

Table 12. Hedging Errors for Sub-periods by Time-to-maturity

This table reports one-day-ahead and one-week-ahead hedging errors by time-to-maturity for sub-periods from 1996 to 2015, measured by mean absolute errors. Following Bakshi et al. (1997), the underlying asset is used as a single hedging instrument. Parameters implied by all options of the previous day are used to establish the current day's hedging portfolio, which are then liquidated the following day or week later. For each target option, the hedging error is the difference between its market price and the replicating portfolio value. BS is the Black-Scholes (1973) option pricing model. SV is Heston's (1993) option pricing model with stochastic volatility. In AHBS models, the implied volatility is treated as a function of strike price (K) and time-to-maturity (τ) with higher-order terms and/or interaction terms.

Panel A. One-day-ahead Hedging

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
τ<60																				
BS	0.401	0.783	0.958	1.112	1.320	0.984	0.911	0.572	0.592	0.526	0.559	1.161	1.701	0.795	0.884	1.136	0.644	0.660	0.734	1.148
SV	0.349	0.711	0.819	0.969	1.178	0.864	0.772	0.513	0.523	0.427	0.465	0.927	1.407	0.655	0.703	0.853	0.491	0.471	0.544	0.832
A1T2	0.359	0.712	0.867	0.981	1.225	0.947	0.877	0.560	0.539	0.477	0.518	1.026	1.648	0.748	0.777	0.962	0.556	0.562	0.639	0.999
A1T2C	0.352	0.695	0.819	0.936	1.186	0.906	0.814	0.530	0.512	0.440	0.477	0.947	1.536	0.672	0.711	0.863	0.495	0.504	0.565	0.893
A2T1C	0.349	0.693	0.820	0.948	1.185	0.882	0.790	0.526	0.517	0.446	0.473	0.940	1.481	0.661	0.719	0.869	0.503	0.512	0.569	0.891
A2T2C	0.350	0.694	0.821	0.937	1.186	0.895	0.809	0.529	0.513	0.442	0.475	0.944	1.517	0.666	0.722	0.872	0.505	0.517	0.571	0.903
60<τ<120																				
BS	0.488	0.749	1.111	1.101	1.357	1.002	0.833	0.548	0.571	0.494	0.596	1.285	1.729	0.838	0.905	1.171	0.765	0.680	0.853	1.077
SV	0.414	0.681	0.898	0.852	1.143	0.827	0.642	0.423	0.417	0.351	0.459	0.948	1.367	0.603	0.612	0.768	0.512	0.426	0.531	0.638
A1T2	0.417	0.674	0.924	0.895	1.188	0.922	0.766	0.478	0.460	0.411	0.526	1.069	1.758	0.712	0.740	0.941	0.611	0.522	0.662	0.833
A1T2C	0.412	0.670	0.890	0.858	1.158	0.874	0.698	0.442	0.427	0.377	0.482	0.977	1.623	0.619	0.651	0.816	0.538	0.456	0.560	0.713
A2T1C	0.409	0.670	0.892	0.861	1.155	0.854	0.678	0.437	0.429	0.378	0.479	0.974	1.584	0.611	0.661	0.822	0.550	0.471	0.572	0.725
A2T2C	0.409	0.670	0.892	0.860	1.156	0.859	0.685	0.438	0.429	0.378	0.479	0.974	1.592	0.612	0.663	0.823	0.550	0.469	0.569	0.722
120<τ<300																				
BS	0.453	0.748	1.162	1.241	1.426	1.048	0.922	0.608	0.668	0.580	0.679	1.435	1.751	0.963	1.052	1.303	0.876	0.768	0.840	1.124
SV	0.405	0.695	0.952	1.013	1.188	0.832	0.663	0.431	0.471	0.358	0.475	0.977	1.159	0.635	0.659	0.751	0.560	0.469	0.465	0.597
A1T2	0.384	0.669	0.939	1.020	1.184	0.874	0.757	0.485	0.512	0.404	0.523	1.089	1.427	0.762	0.782	0.908	0.643	0.534	0.546	0.747
A1T2C	0.383	0.668	0.935	1.016	1.180	0.864	0.720	0.462	0.494	0.377	0.486	1.028	1.329	0.681	0.716	0.820	0.591	0.491	0.488	0.654
A2T1C	0.382	0.667	0.933	1.015	1.178	0.857	0.711	0.457	0.494	0.377	0.484	1.032	1.382	0.657	0.725	0.831	0.600	0.504	0.500	0.685
A2T2C	0.383	0.669	0.934	1.015	1.175	0.839	0.692	0.452	0.496	0.379	0.484	1.026	1.273	0.652	0.720	0.819	0.598	0.501	0.495	0.665
300<τ<600																				
BS	0.571	0.890	1.469	1.598	1.770	1.267	1.095	0.699	0.759	0.726	0.796	1.524	2.173	1.020	1.225	1.508	1.092	0.802	1.030	1.298
SV	0.519	0.848	1.346	1.394	1.525	1.050	0.800	0.503	0.564	0.472	0.581	1.087	1.378	0.620	0.785	0.881	0.749	0.513	0.616	0.707
A1T2	0.502	0.815	1.341	1.402	1.526	1.053	0.825	0.538	0.576	0.512	0.600	1.157	1.457	0.706	0.874	0.996	0.821	0.552	0.669	0.777
A1T2C	0.502	0.816	1.329	1.422	1.529	1.059	0.844	0.546	0.589	0.519	0.595	1.168	1.456	0.695	0.873	0.998	0.811	0.544	0.662	0.758
A2T1C	0.498	0.815	1.328	1.422	1.530	1.051	0.817	0.537	0.592	0.522	0.592	1.163	1.500	0.653	0.880	1.001	0.813	0.557	0.673	0.781
A2T2C	0.499	0.815	1.328	1.424	1.530	1.044	0.798	0.535	0.592	0.522	0.592	1.164	1.408	0.644	0.863	0.983	0.810	0.554	0.674	0.777
τ>600																				
BS	0.714	0.926	2.105	1.886	2.114	1.776	1.470	0.914	1.007	0.896	0.894	1.715	2.467	1.168	1.385	1.734	1.288	1.121	1.285	1.596
SV	0.650	0.902	1.963	1.705	1.921	1.602	1.183	0.712	0.764	0.604	0.688	1.276	1.610	0.751	1.002	1.112	0.983	0.819	0.866	1.042
A1T2	0.647	0.863	1.988	1.701	1.931	1.605	1.191	0.735	0.796	0.646	0.705	1.366	1.755	0.813	1.045	1.180	1.006	0.852	0.908	1.069
A1T2C	0.658	0.869	1.999	1.777	1.988	1.670	1.298	0.780	0.860	0.695	0.743	1.476	1.915	0.950	1.167	1.385	1.091	0.918	0.998	1.221
A2T1C	0.653	0.868	1.999	1.797	1.996	1.609	1.180	0.766	0.876	0.709	0.733	1.457	1.841	0.849	1.164	1.343	1.098	0.931	0.998	1.192
A2T2C	0.652	0.867	2.002	1.782	1.997	1.625	1.222	0.768	0.873	0.705	0.734	1.459	1.818	0.855	1.176	1.387	1.102	0.942	1.020	1.246

Panel B. One-week-ahead Hedging

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
										τ<60										
BS	0.843	1.753	2.223	2.601	2.677	2.426	2.072	1.569	1.394	1.191	1.274	2.337	3.613	1.865	2.057	2.247	1.686	1.467	1.738	2.875
SV	0.758	1.629	1.879	2.340	2.446	2.174	1.899	1.461	1.272	1.055	1.067	1.900	3.352	1.638	1.662	1.775	1.346	1.039	1.253	2.313
A1T2	0.770	1.632	1.984	2.355	2.542	2.357	2.106	1.534	1.289	1.119	1.172	2.102	3.809	1.809	1.838	1.978	1.501	1.239	1.522	2.596
A1T2C	0.760	1.594	1.868	2.280	2.471	2.271	2.011	1.480	1.244	1.061	1.096	1.953	3.637	1.669	1.697	1.791	1.359	1.109	1.327	2.384
A2T1C	0.755	1.583	1.870	2.301	2.463	2.226	1.960	1.478	1.252	1.072	1.090	1.934	3.511	1.648	1.711	1.794	1.373	1.126	1.320	2.403
A2T2C	0.758	1.588	1.872	2.282	2.469	2.251	1.998	1.481	1.247	1.064	1.091	1.943	3.589	1.659	1.720	1.806	1.380	1.136	1.339	2.412
										60<τ<120										
BS	0.942	1.388	2.132	2.273	2.500	2.233	1.825	1.200	1.223	1.017	1.312	2.335	3.751	1.824	2.185	2.417	1.796	1.486	1.935	2.485
SV	0.779	1.254	1.495	1.703	2.136	1.772	1.452	0.954	0.918	0.796	0.960	1.667	3.164	1.415	1.445	1.670	1.206	0.892	1.180	1.706
A1T2	0.776	1.242	1.612	1.789	2.197	2.016	1.731	1.033	0.993	0.860	1.129	1.898	3.832	1.581	1.757	1.969	1.448	1.097	1.497	2.076
A1T2C	0.767	1.230	1.482	1.709	2.135	1.885	1.603	0.966	0.933	0.809	1.026	1.732	3.608	1.430	1.549	1.725	1.267	0.954	1.274	1.829
A2T1C	0.764	1.231	1.488	1.714	2.130	1.849	1.561	0.966	0.938	0.810	1.019	1.722	3.555	1.419	1.575	1.738	1.295	0.985	1.282	1.864
A2T2C	0.764	1.231	1.489	1.711	2.132	1.856	1.578	0.965	0.938	0.810	1.019	1.720	3.556	1.419	1.578	1.738	1.296	0.980	1.275	1.858
										120<τ<300										
BS	0.865	1.329	2.454	2.551	2.620	2.199	1.974	1.332	1.333	1.138	1.402	2.603	3.435	2.059	2.534	2.684	2.067	1.696	1.851	2.444
SV	0.758	1.262	1.705	1.905	2.090	1.639	1.428	0.930	0.944	0.772	0.971	1.728	2.493	1.569	1.592	1.651	1.281	0.995	1.034	1.445
A1T2	0.677	1.201	1.752	1.935	2.054	1.750	1.613	1.027	0.993	0.819	1.054	1.891	2.870	1.688	1.867	1.912	1.507	1.130	1.205	1.755
A1T2C	0.681	1.202	1.721	1.922	2.033	1.717	1.545	0.974	0.965	0.778	0.977	1.790	2.732	1.585	1.724	1.761	1.371	1.042	1.104	1.566
A2T1C	0.682	1.200	1.725	1.927	2.038	1.697	1.528	0.966	0.969	0.781	0.973	1.787	2.820	1.551	1.747	1.772	1.397	1.069	1.110	1.617
A2T2C	0.683	1.202	1.727	1.923	2.026	1.666	1.489	0.963	0.970	0.782	0.972	1.782	2.666	1.547	1.738	1.754	1.394	1.061	1.112	1.596
										300<τ<600										
BS	1.076	1.506	2.978	3.111	3.078	2.517	2.232	1.515	1.499	1.484	1.660	2.871	4.296	2.184	3.076	3.096	2.504	1.760	2.336	2.594
SV	0.917	1.500	2.428	2.540	2.500	1.985	1.619	1.034	1.113	1.013	1.218	2.084	3.050	1.522	2.020	1.895	1.709	1.128	1.363	1.493
A1T2	0.846	1.409	2.424	2.548	2.462	1.978	1.655	1.111	1.111	1.078	1.216	2.184	3.151	1.598	2.253	2.091	1.867	1.204	1.504	1.665
A1T2C	0.846	1.388	2.479	2.591	2.478	2.017	1.709	1.144	1.133	1.094	1.207	2.199	3.149	1.586	2.255	2.092	1.845	1.189	1.482	1.637
A2T1C	0.836	1.382	2.486	2.595	2.485	2.002	1.662	1.131	1.140	1.102	1.201	2.179	3.225	1.527	2.267	2.086	1.853	1.214	1.491	1.670
A2T2C	0.837	1.383	2.486	2.597	2.484	1.998	1.625	1.131	1.140	1.102	1.201	2.192	3.096	1.526	2.242	2.060	1.846	1.208	1.504	1.662
										τ>600										
BS	1.275	1.309	4.582	3.639	3.382	3.633	3.093	1.952	2.162	1.799	1.854	3.579	4.636	2.358	3.626	3.734	3.020	2.413	2.760	3.154
SV	1.100	1.293	3.898	3.082	3.035	3.106	2.489	1.512	1.615	1.265	1.403	2.718	3.361	1.707	2.680	2.446	2.327	1.687	1.812	2.123
A1T2	1.105	1.222	3.909	3.080	2.995	3.133	2.399	1.553	1.704	1.336	1.434	2.883	3.496	1.718	2.790	2.602	2.380	1.788	1.929	2.258
A1T2C	1.124	1.217	4.183	3.258	3.063	3.379	2.689	1.655	1.834	1.413	1.494	3.071	3.751	1.921	3.088	2.983	2.581	1.928	2.101	2.437
A2T1C	1.107	1.216	4.190	3.304	3.078	3.172	2.467	1.644	1.865	1.443	1.478	3.023	3.633	1.846	3.088	2.892	2.601	1.955	2.102	2.418
A2T2C	1.108	1.217	4.198	3.275	3.071	3.236	2.516	1.643	1.858	1.434	1.477	3.024	3.674	1.863	3.112	2.977	2.611	1.981	2.147	2.489

Figure 1. Volatility Surface

This figure illustrates S&P 500 volatility surface using Black-Scholes implied volatility from the sample period from 1996 to 2015. With six intervals of moneyness (S/K) and five intervals of time-to-maturity (τ), the implied volatility is the average value for each moneyness across different time-to-maturity.