In Search of A Unicorn*

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Abstract

The search of valuable investment opportunities is one of the fundamental responsibilities of corporate managers. Existing studies of this search process usually model the investment opportunity as a binary signal and the role of the manager ends when such a signal arrives. This paper studies a dynamic agency model in which investors delegate a manager to find valuable investment opportunities arriving stochastically with two novel features. First, investment targets arrive at different levels of quality that are only observable to the manager. Second, once the investment target is chosen, the same manager is also in charge of the ensuing production process and can continue to utilize his superior information about the target to extract rents from the investors. These novel features imply an adverse selection problem interacting with a moral hazard problem. The optimal contract features a progressively lower threshold for investment if a suitable target is not found in time. The investment threshold is always lower than the first-best along the equilibrium path, consistent with the over-investment behaviors observed in practice. The theoretical predictions of the model offer empirically relevant hypotheses regarding the strategies and returns of mergers and acquisitions, hedge function activism, or special purpose acquisition companies.

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1 Introduction

The ability to identify and exploit valuable investment opportunities is one of the defining characteristics of successful managers of modern businesses. Firm managers must regularly find and acquire new technologies in the market to maintain their growth. Managers of activist hedge funds are on constant lookout for undervalued targets for intervention. The massive uncertainty and market turmoil brought by the ongoing COVID-19 pandemic saw the rapid rise of attention to the managers (known as the *sponsors*) of special purpose acquisition companies, or SPACs, designated with the exclusive job of hunting for the next "unicorn", the most promising private firm, to merge with.[1] Existing studies (e.g., Green and Taylor, 2016) usually model the search for valuable information as a random discovery process. A principal, representing the investors, supplies the necessary resources to an agent, represents the manager, to conduct the search, and the valuable information arrives stochastically over time. This standard model framework acknowledges two important frictions. First, the search requires costly but unverifiable effort from the manager. Second, the result of the search, or the arrival of valuable information, is the manager's private knowledge. These features spawn a *moral hazard* problem, for which the solution requires the design of an incentive contract that ensures both the exertion of search effort and the timely report of the search result. Typically, this is done by promising the manager a reward when the valuable information is discovered and disclosed. The size of the promised reward decreases over time in the absent of the discovery, and the contract is terminated if the discovery is not made after sufficiently long time.

The standard framework and its resulting contract design, however, does not capture all the crucial features of the search for valuable information in practice. First, in the standard framework, information is modeled as a binary signal, for which only the timing of its arrival matters. This ignores the *content* of information, such as the quality of the investment opportunity. Second, the role of the manager ends when he discloses the arrival of the information. In practice, the manager can well be the key personnel delegated to utilize that information for production. In cases of merger and acquisition (M&A), the

[1] See, e.g, coverage from *Forbes* (2020) and how itself plans to go public via a SPAC.

manager of the acquirer is also the one operating the combined firm. The manager of an activist hedge fund is also the one overseeing the revamping of the undervalued targets. Even venture capitalists and SPAC sponsors are known to play critical roles in the growth and development of their investments by injecting their management expertise in addition to the capital. These realistic features have important implications on the optimal contract design for the search process. First, the fact that information can be of different quality generates an *adverse selection* problem. In addition to receiving sufficient incentives to exert the search effort, the manager must also be given proper incentives to truthfully reveal the quality of the information. Second, if the manager's role continues after the information has arrived, the manager can take advantage of his information superiority to extract personal benefits from the subsequent production process. These benefits in turn affect the return to the search process for the investors and the type of opportunities they are willing to invest in.

This paper explores the implications of agency frictions in the search of valuable investment opportunities in a dynamic model that incorporates these two critical features. The model has two distinct stages: in the first (search) stage, investors must supply resources to the managers to find investment opportunities, or targets, which arrive randomly over time via a Poisson process. The manager can shirk, diverting resources for his private benefit, which results in no target arrival. Importantly, the quality of the target is a random variable, of which the actual realization is only observable to the manager. The investors make the decision whether to end the search and begin production based on the manager's report of the arrival of targets and their quality. In the second (production) stage, the productivity of the target evolves over time subject to time-varying shocks, and its quality determines the initial productivity. The evolution path of productivity is once again the manager's private information, and the investors dynamically adjust their production policies based on the manager's reported productivity. The investors' objective is to design the optimal contract that maximizes their total return from the investment net of the manager's compensation, which provides incentives for the manager to exert the search effort and to report all private information truthfully in both stages.

Absent of any agency frictions, the investors' first-best strategy is to wait for and invest in the first target that clears a sufficiently-high bar of quality. This cutoff level of quality, or the investment threshold, is a constant under the assumption of zero discounting. The aforementioned information frictions, however, create both a moral hazard and an adverse selection problem that interact with each other. In the production stage where productivity is the manager's private information, the incentives for truthful reporting are provided in the form of excess compensation, known as the manager's *information rent*. This rent is increasing in the quality of the target, which determines the expected return investors can receive from a given investment policy. In the search stage, incentives are provided in the form of promised utility. The investors specify in the contract their policies for investment and the reward for reporting the arrival of suitable targets, which combined drive the evolution of the manager's promised utility given the path of the search results. In particular, the optimal investment policy takes the form of a *time-varying* cutoff level of target quality. As long as the manager does not report the arrival of a target the bar of investment, the promised utility to the manager drifts down to provide incentives for the manager to maintain the search effort. Early on in the search stage, the bar of investment is high, and the manager is just indifferent between exerting the search effort and shirking and consuming the search resources for private benefits. As the managers promised utility decreases over time in the absence of high-quality targets, the optimal cutoff declines, and the manager begins to strictly prefer to exert the effort to conduct the search. The decline continues until all targets regardless of their quality will trigger the start of production. If a target is still not found after sufficiently long search the manager's contract is terminated without pay. Critically, the optimal investment threshold induced by the agency frictions is always below first-best level, representing *overinvestment* by the investors.

The main results of the model and the their comparative statics generate empirically testable hypotheses regarding M&A, hedge fund activism (HFA), and SPAC, where there is extensive evidence of overinvestment.[2] Existing studies (e.g., Shleifer and Vishny, 1997; Franzoni, 2009; Malenko, 2019; etc.) often attribute overinvestment to the manager's empire-building preferences. This paper offers an alternative explanation based on optimal con-

[2]See, e.g., Andrade, Mitchell, and Stafford, 2001, Eckbo, 2008, Gahng, Ritter, and Zhang, 2021, and the references therein.

tracting under agency frictions. Moreover, the model predicts that overinvestment should be correlated with the frequencies of M&A, HFA, or SPAC activities; the incentive powers of the acquirer, activist fund managers, or the SPAC sponsors; and the average level of the search cost, which can be empirically measured by stock liquidity, analyst coverage, or institutional holdings. The model also predicts that the returns in these markets should be more dispersed when there are more targets available or when the incentive powers of the managerial contract is stronger.

This paper belongs to the rich literature of dynamic contracting models with Poisson jumps, including Biais, Mariotti, Rochet, and Villeneuve (2010), Hoffmann and Pfeil (2010, 2021), Piskorski and Tchistyi (2010, 2011), DeMarzo, Fishman, He, and Wang (2012), DeMarzo, Livdan, and Tchistyi (2013), Myerson (2015), Guo (2016), Li and Williams (2017), Sun and Tian (2018), Rivera (2020), and Feng (2021). The most closely-related studies are Green and Taylor (2016), Varas (2018), Curello and Sinander (2021), Madsen (2021), Mayer (2022), etc. These studies assume that through effort, the agent can observe a private signal that is valuable for the principal. The optimal contract provides incentives for the agent to exert the effort to uncover that signal and to report it as soon as it arrives. This paper differs from these studies in two new dimensions. First, in addition to the arrival time, the agent also privately observes the quality of the private information and must be incentivized to truthfully convey both the arrival and the quality to the principal. Second, the contracting relationship does not end with the disclosure of the private information. The value of the information manifests through a production process, during which the agent can continue to utilize his information superiority to extract rents from the investors.

The introduction of these new dimensions results in an adverse selection problem in addition to the moral hazard problem, a feature that can be seen in Sung (2005), Sannikov (2007), Cvitanić, Wan, and Yang (2013), and Che, Iossa, and Rey (2021), etc. In particular, Cvitanić, Wan, and Yang (2013) studies an extension of the well-known contracting problem in DeMarzo and Sannikov (2006) by assuming one of the agent's characteristics (such as the private utility he enjoys if he diverts the cash flows) is the agent's private information, and the principal must design a screening contract to infer that information *before* hiring the agent. This turns out to be analytically difficult, as the contract must always keep track of at least two state variables: one for each type of the agent if he accepts the contract. Consequently, the solution in Cvitanić, Wan, and Yang (2013) relies mainly on numerical simulation. In contrast, this paper assumes the adverse selection arises *after* the manager is hired, which allows the separation of the adverse selection from the moral hazard problem. Crucially, the screening of the manager's private information can be made relatively independent of the manager's moral hazard problem, which lowers the technical hurdles in deriving the optimal contract substantially. Meanwhile, there is still a meaningful interaction between the two frictions because the design of the screening contract affects the dynamics of the incentives during the search stage. Consequently, the model yields different but practically relevant predictions than those in models with either adverse selection or moral hazard only, which are discussed in details in Section 4.

More broadly speaking, our paper is also related to the literature of mechanism design in which the agent can also take private actions.[3] For example, Krähmer and Strausz (2011) and Liu and Lu (2018) study the optimal procurement policy in two-period models where the agent is able to influence the principal's screening outcome in the second period through private actions in the first period. The agent makes one report only about some private information and the contract either moves on to the next stage, if the report clears a pre-specified hurdle, or terminates altogether. In comparison, our search stage is fully dynamic and thus features endogenous termination and stochastic transition time to the second stage. The agent is allowed to make repeated reports about the arrival and quality of the targets until either transition or termination occurs. The dynamics also imply that the criterion for transition is type-varying as the result of previous reports. Meanwhile, Halac, Kartik, and Liu (2016) studies experiments in a learning model in which the agent's private effort is a necessary (but not sufficient) condition for success. The adverse selection of the underlying success likelihood results in a screening contract with different endogenous deadlines at which the contract is terminated if success has not been achieved. Our model shares the similarities that "success", the arrival of an investment target, is stochastic and only possible if the manager exerts effort. However, our model differs in that "success" carries the additional

[3]See, e.g., Garrett and Pavan (2012, 2015), Gershkov and Perry (2012), Chassang (2013), Gary-Bobo and Trannoy (2015), Gershkov, Li, and Schweinzer (2016), Duggan (2017), Shan (2017), in addition to the studies reviewed below.

information of quality and thus requires different levels of incentives under the optimal screening contract. This additional dimension of the agent's private information also implies that the definition of "success" in our model is time-varying: earlier targets must clear a higher hurdle in order to successfully trigger investment.

In terms of the empirical predictions, this paper complements the recent studies that also feature overinvestment as the result of dynamic agency frictions with different mechanisms, e.g., Bolton, Wang, and Yang (2019) and Ai, Kiku, Li, and Tong (2021) with limited commitment, Gryglewicz, Mayer, and Morellec (2020) with correlated short-run and long-run effort, Szydlowski (2019) with multi-tasking, Gryglewicz and Hartman-Glaser (2020) with real options, and Feng (2022) with persistent effect of moral hazard. All of these studies assume the output process is driven by Brownian shocks and the level of investment can be adjusted constantly and smoothly. Consequently, their models apply more naturally to operational investments such as capital expenditure or research and development expenses. In contrast, this paper focuses on the setting in which the investment targets arrive via a Poisson process if the manager exerts sufficient search effort. The theory is therefore more applicable to lumpy investments such as M&A, HFA, and SPAC.

2 Model

This section introduces the model. Section 2.1 describes the basic structure of the model and defines the contract, followed by discussions of the key assumptions in Section 2.2, and the first-best solution in Section 2.3.

2.1 Basic Structure

Time is continuous. A principal (she), representing the investors, contracts with an agent (he), representing a manager, to find and manage an investment. Investors have deep pockets while the manager is protected by limited liability. Both parties are risk-neutral with no discounting, and their outside options are normalized to 0.[4] The contracting relationship

[4] The assumption of no discounting for both the investors and the manager is common among models in which the arrival of valuable information follows a Poisson process (e.g., Green and Taylor, 2016; Mayer,

has two distinct stages: the search stage, and the production stage.

2.1.1 The Search Stage

In the search stage, the manager is tasked with finding an investment opportunity, such as a promising startup as an acquisition target, or an undervalued firm as an activism target. The search incurs a flow cost δ that must be financed by the investors. These targets arrive via a Poisson jump process N_t with intensity λ if the manager exerts effort. Each target is characterized by its quality θ, which follows a known distribution with cumulative distribution function $F(\theta)$ and probability density function $f(\theta)$.

The agency frictions in this stage is two-fold: first, the manager's (search) effort is unobservable to the investors. Thus, the manager can shirk, which generates private benefit $\rho < \delta$ for the himself, and no target arrives while he shirks. Secondly, only the manager observes the arrival of targets and their true quality θ. The investors must make all their decisions based on reported information from the manager. The length of this stage is endogenous: the search ends either when the investors decide to invest and move on to the next (production) stage, or when the contract is terminated.

2.1.2 The Production Stage

In the production stage, the manager produces output based on the target chosen by the investors in the previous stage. The production technology is denoted by $y(\theta, e)$, where e is the manager's (production) effort subject to a quadratic personal cost $h(e) = e^2/2$. $y_\theta(\theta, e) > 0$, and $y_e(\theta, e) > 0$. That is, target quality and production effort both increase output.

Similar to the first stage, effort and target quality are both the manager's private information. The output y, however, is observable to the investors. This implies that the main agency friction in the production stage is adverse selection: investors can set their desired level of output and make compensation to the manager based on the reported target quality. The unobservable effort is only used to conceal the manager's report of θ. A manager with

2022; etc.). Discounting implies investors prefer earlier resolution, which distorts their investment threshold downward. Except for this result, discounting usually adds little economic intuition in these types of models but a substantial degree of algebraic complexity.

a low-quality target can always mimic the output of a manager with a high-quality target (or vice versa) by exerting higher (lower) effort. In contrast, the agency frictions in the first stage involve both moral hazard and adverse selection, because the manager can both shirk and lie about the arrival and the quality of the investment targets.

2.1.3 Contract

A contract $\mathcal{C}$ between the investors and the manager consists of the investors' investment and production policies, and the associated compensation to the manager. In the search stage, the contract specifies the set of targets that will be invested in, the reward for the manager for announcing the arrival of the target, and the condition under which the contract is terminated. In this production stage, the contract specifies the investors' desired level of output and the associated compensation to the manager if he produces the required output.[5] All policies are conditional on the manager's reports $\hat{\theta}_t$ and the filtration generated. A contract is incentive-compatible if the manager finds it optimal to always exert the desired search effort and announce his private information truthfully.

2.2 Simplifying Assumptions

The solution of the model relies on two assumptions – one for each stage – on the functional forms of certain key variables. These assumptions are made for analytical tractability only and their roles in the model will be discussed below. The first assumption pertains to the distribution of θ in the search stage:

Assumption 1 *θ follows a Pareto distribution with scale parameter $\theta_{\min}$ and shape parameter κ. i.e.,*

$$F(\theta) = 1 - \left(\frac{\theta_{\min}^{\kappa}}{\theta^{\kappa}}\right) \tag{1}$$

$$f(\theta) = \frac{\kappa\theta_{\min}^{\kappa}}{\theta^{\kappa+1}} \tag{2}$$

θ takes on positive values only, (i.e., $\theta_{\min} > 0$) and has a finite variance (i.e., $\kappa > 2$).

[5]Because there is no noise in the production technology, and the manager is risk-neutral, it is without the loss of generality to assume that the manager only gets paid if he produces the required level of output.

The main analytical advantages of the Pareto distribution are two-fold: first, the inverse hazard rate of a Pareto distribution $[1 - F(\theta)]/f(\theta) = \theta/\kappa$ is a linear function in θ. The proof of Proposition 1 and all subsequent analyses are much simplified as a result of this linearity. Secondly, a Pareto distribution truncated from below at an arbitrary point $x > \theta_{\min}$ is still a Pareto distribution with the same shape parameter and the new scale parameter x. As mentioned in the introduction (and will be explicitly shown soon), the optimal investment policy in the search stage is to invest in all targets with reported quality above a certain threshold. The fact that the value of the threshold does not change the shape of the distribution is vital for tractability. The requirement $\kappa > 2$ is a mere technical one. It implies a sufficiently thin right tail of the distribution to ensure a finite variance of θ and, as will be shown in Section 2.3 soon, a finite solution to the first-best.

In addition to its analytical convenience, as a member of the power-law family, Pareto distribution is also widely applicable in describing economic activities in practice. In particular, the extreme value theory (e.g, as in Gabaix and Landier, 2008) suggests that the tail distribution of many economic values (firm size, managerial talent, etc.) must follow the power-law. Targets that are worth considering for investment for most firms, especially the large ones, are likely highly promising opportunities in general. Consequently, despite being a special case mathematically, Pareto distribution is arguably a reasonable proxy for the actual distribution of the quality of valuable investment opportunities in practice.

The second assumption pertains to the production technology:

Assumption 2 *Managerial effort and target quality are mainly complements in the production stage, i.e., $y = \theta e$.*

This assumption achieves two useful simplifications: first, in the equilibrium, production effort will never be shut down regardless of target quality.[6] Second, the manager cannot produce any output without an actual target. Consequently, while the manager can misreport the quality of the arriving targets, he cannot fabricate their existence.[7]

[6] If $y = \theta + e$, the optimal contract may shut down effort completely when the target quality is sufficiently high. See the example in Feng, Wang, and Wu (2022).

[7] This assumption is intuitive in the context of M&A, HFA, and SPAC, because the success of the merger and the intervention require cooperation from the targets. While managers of the acquiring firm or the activist fund can exaggerate the true value of the targets to their investors, they are usually not able to create phony targets.

While Assumptions 1 and 2 are helpful in deriving various solutions in closed-form, they are not essential for delivering the main economic implications. They are also not the unique combination in maintaining tractability: for example, another distribution that retain its "shape" when truncated from below is the exponential distribution. The underlying mechanism of the paper is qualitatively intact if θ follows an exponential distribution in the search stage, or if θ and e display a higher degree of substitution in the production technology.

2.3 The First Best

If all information is public, the first-best effort and output in the production stage solve

$$\max_{e} \; y - h(e) = \theta e - h(e) \;, \tag{3}$$

The solution is

$$e^{FB} = \theta \tag{4}$$

$$y^{FB} = \theta^2 \tag{5}$$

The first-best effort and output are both increasing functions in θ. The manager receives compensation exactly equals his cost of effort, and the investors' payoff given the quality of the target is $V_2^{FB}(\theta) = \theta^2/2$.

The search stage under the first-best scenario represents a standard bandit problem. Let Θ^{FB} denote the set of targets that will be invested in and V_1^{FB} denote the investors' expected value at the outset of the research stage. The following lemma summarizes the first-best investment policy:

Lemma 1 *Under the first-best scenario when all information is public,* $\Theta^{FB} = \{\theta : \theta \geq x^{FB}\}$*, where*

$$x^{FB} = \left[\frac{\lambda \theta_{\min}^{\kappa}}{\delta(\kappa - 2)}\right]^{\frac{1}{\kappa-2}} \tag{6}$$

The investors' maximal expect payoff at the beginning of the search stage is

$$V_1^{FB}(x^{FB}) = \quad \frac{\kappa\left(x^{FB}\right)^2}{2(\kappa-2)} - \frac{\delta\left(x^{FB}\right)^\kappa}{\lambda\theta_{\min}^\kappa} \tag{7}$$

In sum, the optimal strategy of the investors under the first-best scenario is to finance the search with a *minimal* (cutoff) quality x^{FB} arrives.[8] The manager's utility equals his outside option (i.e., zero) because he always exerts effort during the search stage and thus receives no private benefit from shirking. Because x^{FB} also determines the expected duration of the search, Lemma 1 suggests that investors on average wait longer in the first-best scenario if there are more opportunities in the market (higher λ) or if the search cost (δ) is lower. These results yield useful empirical implications, which are discussed in Section 3.3.

3 Optimal Contract Under Asymmetric Information

This section solves the optimal contract when the manager's effort in both stages as well as the quality and productivity of the investment target are his private information. The contract is derived via backward induction: Section 3.1 solves the second (production) stage, and Section 3.2 solves the the first (search) stage. Section 3.3 derives some useful comparative statics and discuss their empirical implications.

3.1 The Production Stage

In this stage the investors mainly face an adverse selection problem. For ease of exposition, imagine the following reduced problem without the search stage: the manager is endowed with a target of quality θ that is unobservable to the investors. The manager has reservation utility W_{τ^-}, which later represents the utility carried over from the search stage but is exogenous in this reduced problem. The investors must design a screening contract that solicit the truthful reporting of θ while maximizing their payoff, which is the output minus the compensation to the manager.

[8] Equations (6) and (7) illustrate the need to impose $\kappa > 2$ in Assumption 1. If $\kappa \leq 2$, then x^{FB} and V_1^{FB} are not well-defined.

Formally, the screening contract consists of a menu of required output $y(\hat{\theta})$ and the associated compensation $w(\hat{\theta})$ if the output is produced, both as functions of the manager's reported quality $\hat{\theta}$. The manager's objective is to maximize his compensation minus his (production) effort cost from the contract:

$$R(\theta) = \max_{\hat{\theta}} \quad w(\hat{\theta}) - h(e) \tag{8}$$

subject to the constraint

$$e = y(\hat{\theta})/\theta \tag{9}$$

The contract is incentive compatible if and only if

$$\theta = \arg\max_{\hat{\theta}} \quad w(\hat{\theta}) - h\left(\frac{y(\hat{\theta})}{\theta}\right) \tag{10}$$

When (10) is satisfied, $R(\theta)$ is known as the manager's *information rent*: the amount of utility (in addition to his reservation utility) that he must receive in order to truthfully reveal his private information.

Meanwhile, because the investors do not observe θ, their objective is to maximize the *expected* output minus the manager's compensation from the contract. The expectation is taken over the distribution of θ, which in the equilibrium is the result of the investors' investment policy. We assume and later verify that like in the first-best scenario, the optimal investment policy is a cutoff strategy in which all targets with reported quality $\hat{\theta} \geq x$ trigger the investment. Given that, the investor's maximal payoff from the screening contract can be written as $V_2(x) - W_{\tau^-}$, where $V_2(x)$ solves

$$\begin{aligned} V_2(x) &= \max_{y(\hat{\theta}), w(\hat{\theta})} \quad \int_x^{+\infty} [y(\theta) - w(\theta)] \left(\frac{\kappa x^\kappa}{\theta^{\kappa+1}}\right) d\theta && (11) \\ &= \max_{y(\hat{\theta}), w(\hat{\theta})} \quad \int_x^{+\infty} [y(\theta) - h(e(\theta)) - R(\theta)] \left(\frac{\kappa x^\kappa}{\theta^{\kappa+1}}\right) d\theta && (12) \end{aligned}$$

subject to the IC condition (10).[9] In other words, given the investment policy x, $V_2(x)$ captures the investors' expected payoff from production under the incentive-compatible screening contract with optimally designed output-compensation combinations.

Deriving $R(\theta)$ and $V_2(x)$ represents a static mechanism design problem to which the solution is summarized as follows:

Proposition 1 *Let $\gamma \equiv \frac{\kappa}{\kappa+2} < 1$. Given any investment threshold $x \geq \theta_{\min}$ set in the search stage, the optimal contract in the production stage has the following properties:*

- *The manager's information rent from a target of quality $\theta \geq x$ is given by*

$$R(\theta) = \frac{\gamma^2}{2}(\theta^2 - x^2) \tag{13}$$

- *The investor's expected payoff from production is given by*

$$V_2(x) = \frac{\gamma\kappa}{2(\kappa-2)}x^2 \tag{14}$$

- *The optimal output $y^* = \gamma\theta^2$ and the implied equilibrium production effort is $e^* = \gamma\theta$.*

Proposition 1 shows that, at the outset of the production stage, the manager's information rent is a quadratic function of target quality θ and the investor's expected payoff is a quadratic function of the investment threshold x. Compared with the first-best level of effort e^{FB} in (4) and the first-best level of output y^{FB} in (5), adverse selection distorts both the optimal effort e^* and output y^* downward by a constant amount: $1 - \gamma = 2/(\kappa+2)$. Intuitively, a higher κ corresponds to a lower variance in θ and therefore, less information asymmetry.

Proposition 1 also implies the following assumption made earlier is indeed part of the investors' optimal strategy:

Corollary 1 *Under the optimal contract the optimal investment policy is equal to a threshold policy: $\Theta_t = \{\theta : \theta \geq x_t\}$.*

[9] The term $\frac{\kappa x^\kappa}{\theta^{\kappa+1}}$ represents the distribution of θ given the investment threshold x, utilizing the convenient property of the Pareto distribution truncated from below.

Intuitively, the investors' expected net return during the production stage is an increasing function of target quality. Thus, excluding high-quality targets is sub-optimal because it reduces the investors' expected payoff while increasing the expected wait time before the search ends and production begins. Given x, (13) implies that the conditional expectation of the information rent the manager can receive is

$$U(x) \equiv \mathrm{E}\left[R(\theta)|\theta \geq x\right] = \int_x^{+\infty} \frac{\gamma^2}{2}(\theta - x)^2 \left(\frac{\kappa x^\kappa}{\theta^{\kappa+1}}\right) d\theta = \frac{\gamma^2 x^2}{\kappa - 2} \tag{15}$$

The closed-form expression for $U(x)$ and $V_2(x)$ are the direct results of Assumptions 1 and 2 and greatly simplify the design of the optimal contract in the search stage in the next section.

3.2 The Search Stage

In this stage investor faces both a moral hazard problem and an adverse selection problem: to incentivize the search effort and to procure truthful and timely report of the quality of the arriving targets. While the interaction of the two problems can impose substantial analytical challenges in a general model, the setting in this paper allows them to be tackled sequentially. In particular, Proposition 1 shows that the solution to adverse selection requires giving the manager his information rent $R(\theta)$ for each target that receives investment. Consequently, the design of the optimal contract in the search stage can be simplified to focus on the incentives for the search effort only.

Similar to standard agency models with a sole moral hazard problem, incentives for search effort are provided in the form of promised future compensation to the manager. Specifically, let τ denote the stopping time either due to the transition to the production stage or contract termination, $\{a_t\}_{t\in[0,\tau]} \in \{0,1\}$ denote the agent's search effort, and $\{C_t\}_{t\in[0,\tau]}$ denote the compensation to the agent, the contract can be characterized using the agent's continuation utility W_t, defined as

$$W_t = \mathrm{E}\left[\int_t^\tau \rho(1-a_s)ds + \int_t^\tau dC_s + W_\tau\right]. \tag{16}$$

The first term inside the integral is the agent's private benefit if he shirks (i.e., $a_s = 0$). The last term W_τ represents his terminal compensation. The assumptions of risk-neutrality and zero-discounting imply that the contracting space can be simplified as follows:

Lemma 2 *The optimal contract always implements no shirking (i.e., $a_t = 1$) during the search stage. The manager is paid if and only if the contract moves to the production stage and the manager produces the required output (i.e., $dC_t = 0$ for all $t < \tau$ and $W_\tau = 0$ if the contract is terminated without production).*

The first result holds because the search requires flow cost δ but shirking produces $\rho < \delta$ flow utility to the manager while producing no target. Therefore, it is always more efficient to award the manager with compensation and the incentives to work than to induce shirking. The second result holds because both the investors and the manager are equally patient with zero-discounting, so any intermediate compensation can always be delayed at no cost. Given that the production stage is static without noise or risk, it is without the loss of generality to accrue all payments until the required output specified and analyzed in the previous section is produced.

With the contracting space simplified, the following proposition characterizes the dynamics of the manager's continuation utility and the IC condition in the search stage:

Proposition 2 *Given the investment policy x_t in the contract, the manager's continuation utility W_t evolves according to*

$$dW_t = U(x_t)\left[dN_t - \lambda\left(\frac{\theta_{\min}}{x_t}\right)^\kappa dt\right] \tag{17}$$

Manager exerts the search effort if and only if

$$\left[\lambda\left(\frac{\theta_{\min}}{x_t}\right)^\kappa\right]U(x_t) \geq \rho \tag{18}$$

The contract is terminated if $W_t = 0$.

Proposition 2 entails three results. First, W_t drifts down in the absence of the (reported) arrival of target. If a target arrives with sufficiently high quality, the manager is rewarded with an upward jump in W_t, and the contract moves on to the production stage. The speed of the downward drift must be commensurate with the size of the upward jump to maintain W_t as a martingale (eq. 17).

Second, unlike the standard models in which the role of the manager ends when the search result is revealed, the existence of the production stage implies that given the investment policy x_t at each point in time, $U(x_t)$ is the manager's expected utility reward – the *additional* utility he can expect – if a suitable investment target arrives. Here a target is *suitable* if its quality clears the threshold of investment, i.e., $\theta_t > x_t$, which happens at the rate of $\lambda \left(\frac{\theta_{\min}}{x_t}\right)^\kappa$ by the property of the Pareto distribution. Consequently, the manager faces a tradeoff between working and shirking: shirking yields flow benefits ρdt. However, because no target arrives while he shirks, his continuation utility drifts down at the rate of $U(x_t)\left[\lambda \left(\frac{\theta_{\min}}{x_t}\right)^\kappa\right] dt$. The manager prefers not to shirk if the latter cost exceeds the former benefit, which is captured by the IC condition (18).

Finally, given that W_t drifts downward in the absence of a suitable target, the contract terminates and the manager receives no payment if he does not find such a target after sufficiently long time has passed. Note that this pertains to a convenient property of the production technology in Assumption 2: the manager can only produce something out of an actual target. If not, the manager with a very low W_t may find it optimal to falsely announces the arrival of a suitable target and produce the required output using his effort only. The optimal contract in that case generally involves random termination (as in Green and Taylor, 2016).

Investors have two controls when designing their optimal contract: the investment threshold x_t, and the terminal compensation W_τ. Lemma 2 implies that their payoff under the optimal contract, denoted as $V_{1,t}$ solves

$$V_{1,t} = \mathrm{E}\left[\int_t^\tau -\delta ds + y_\tau - W_\tau\right], \tag{19}$$

subject to the IC constraints (10) and (18). $y_\tau = y$ if production takes place, and zero if the contract is terminated without production. The analyses so far have pinned down the optimal, incentive compatible terminal compensation: if a suitable target can be found, the manager receives an extra reward, which is $U(x)$ in expectation. Otherwise, he receives no payment if the contract terminates without production. Thus, the ensuing analyses focuses on characterizing the optimal investment policy x_t. Proposition 2 implies that the investor's payoff under the optimal contract can be summarized as a function of the manager's continuation utility, or $V_1(W)$, which solves the following Hamilton-Jacobi-Bellman (HJB) equation with x being the only control variable:

$$0 = \max_x \quad -\delta - \lambda \left(\frac{\theta_{\min}}{x}\right)^{\kappa} U(x)V_1'(W) + \lambda \left(\frac{\theta_{\min}}{x}\right)^{\kappa} [V_2(x) - W - V_1(W)] \tag{20}$$

subject to the IC constraint (18). The first term represents the search cost. The second term stems from the drift of dW_t, and the third term represents the change in the investor's payoff if a suitable target is found and the contract moves into production.[10]

Rearranging terms, the HJB equation can be equivalently written as

$$V_1(W) = \max_x \quad V_2(x) - W - U(x)V_1'(W) - \frac{\delta}{\lambda}\left(\frac{x}{\theta_{\min}}\right)^{\kappa} \tag{21}$$

which highlights the tradeoff investors face when setting the optimal investment threshold: a higher x yields a higher expected payoff for the investors, once a suitable target arrives ($V_2'(x) > 0$, as seen in 14). The cost, however, is two-fold: first, targets with sufficiently higher quality arrive at a lower rate and thus requires more search cost in expectation (the last term in 21). Second, once such targets arrive, the manager must also be given a higher reward to truthfully reveal their quality ($U'(x) > 0$, as seen in 15). This higher reward must be accompanied by a faster decline of W to maintain W as a martingale (eq. 17), which increases the likelihood of contract termination.

If the IC constraint (18) is slack, the optimal choice of x can be obtained by the first-order

[10]Conditional on moving into production, the investors' final payoff is the output y minus two terms: the wage $w(\theta)$ paid for production, and the manager's residual utility W carried over from the search stage. The former can be further divided into the compensation for the manager's production effort $h(e)$ and his information rent $R(\theta)$, all embedded in the definition of $V_2(x)$ in equation (12). Put differently, the investors' final payoff can be written as $y - R - h(e) - W$, where the first three terms are captured (in expectation) by V_2.

condition:

$$V_2'(x) - U'(x)V_1'(W) - \left(\frac{d}{dx}\right)\left[\frac{\delta}{\lambda}\left(\frac{x}{\theta_{\min}}\right)^{\kappa}\right] = 0 \tag{22}$$

The first term is positive (higher expected payoff from production) and the third term is negative (higher search cost). The middle term captures the marginal value of manager's continuation utility for the investors and its sign depends on W. When W is large, $V_1'(W) < 0$, because the promised compensation to the manager lowers the investors' payoff if a suitable target arrives and the search ends. When W is small, $V_1'(W) > 0$, because the primary concern for the investors is the likelihood of triggering probation and the subsequent contract termination. Substituting in $V_2(x)$ from (14) and $U(x)$ from (15) into (22) yields the optimal investment policy when the IC constraint is slack:

$$x(W) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right)V_1'(W)\right)\left(\frac{\lambda\gamma\theta_{\min}^{\kappa}}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}} \tag{23}$$

Clearly, $x'(W) > 0$ if $V''(W) < 0$. That is, if $V_1(W)$ is a concave function, which is verified later. Thus, the optimal choice of the investment threshold is an increasing function of the manager's continuation utility. Put differently, because W_t drifts down in time, the optimal investment strategy is to adopt a progressively lower threshold for the quality of the arriving targets worth investing in.[11]

However, the left-hand-side of the IC condition (18) is decreasing in x, because the arrival rate of high-quality targets decreases faster than the manager's expected rent from those targets. Therefore, there may exists $\overline{W}$ such that the IC condition is binding when $W \geq \overline{W}$. In that region, $x(W)$ is given by the IC condition, or

$$x(W) = \bar{x} \equiv \left[\frac{\lambda\gamma^2\theta_{\min}^{\kappa}}{\rho(\kappa-2)}\right]^{\frac{1}{\kappa-2}}, \quad \text{if } \ W \geq \overline{W} \tag{24}$$

[11]In a recent study, Luo and Sun (2021) develops a model that also features progressively lenient selection of targets albeit in a setting designed for SPAC only. The mechanisms differ in that the compensation structure and the contracting horizon in Luo and Sun (2021) are both given to match the standard practices of SPAC. In contrast, the optimal contract including its termination is endogenous in this paper, which allows a broader application to other markets such as M&A and HFA, etc.

In other words, the optimal investment threshold x is a constant for sufficiently high level of W. Setting $x = \bar{x}$ in (23) implies that $\overline{W}$ solves

$$V_1'(\overline{W}) = \frac{\kappa}{2}\left(\frac{1}{\gamma} - \frac{\delta}{\rho}\right) \tag{25}$$

Meanwhile, $\theta_{\min}$ represents the lowest investment threshold that the investors can set. Substituting $x = \theta_{\min}$ into (23) implies that $x(W) = \theta_{\min}$ for all $W \leq \underline{W}$, where $\underline{W}$ solves

$$V_1'(\underline{W}) = \left(1 - \frac{\delta(\kappa - 2)}{\gamma\lambda\theta_{\min}^2}\right)\frac{\kappa}{2\gamma} \tag{26}$$

That is, when W is sufficiently low, the optimal policy is to invest in the next target that arrives, regardless of its quality. This maximizes the probability that the contract moves to the production stage before it is terminated.[12]

Finally, we impose the following parameter assumption to maximize the economic values of the $\overline{W}$ and $\underline{W}$ derived above:

Assumption 3 *The parameters $\lambda, \theta_{\min}, \kappa, \delta, \rho$ satisfy the following conditions:*

$$\lambda\theta_{\min}^2 > \frac{\rho(\kappa - 2)}{\gamma^2} \tag{27}$$

$$\rho > \gamma\delta \tag{28}$$

The first condition ensures that $\underline{W} < \overline{W}$, so they both exist under the optimal contract. Intuitively, this condition says that the search is fairly valuable to the investors in terms of the arrival rate and quality of the targets. Therefore, the investors find it worthwhile to wait for a target with sufficient quality as long as W is not too low and termination is not imminent. If this condition is not satisfied, investors may find it optimal to minimize the

[12] The result that all targets triggers production when W is sufficiently low undoubtedly relies partially on the assumptions that all targets, regardless of their quality, generate positive return to the investors and require effort and time to be discovered. The former can be justified if the investors have a common sense of the basic properties of investment opportunities worth taken (e.g., firms with strong growth history and healthy balance sheet), and the latter can be understood as there is no "free lunch" in the financial market. If, instead, the $\theta_{\min}$ target represents a "default" option that is always immediately available, then when W is sufficiently low, investor will intuitively abandon the search by resorting to the default option in lieu of contract termination. If $\theta_{\min} < 0$, or if there is a substantial fixed cost for production, then the optimal investment policy may exclude some low-quality targets even when W is low and termination is imminent.

waiting time by investing in the first target regardless of its quality.

The second condition ensures that $V_1'(\overline{W}) > 0$. Intuitively, $V_1'(W) < 0$ if W is sufficiently high, because a larger promised utility to the manager diminishes the investors' payoff when a target is found. However, because W drifts downward in the absence of a suitable target, investors can set the manager's initial continuation utility at the outset of the search stage to be $W^* = \arg\max_W V_1(W)$, or $V_1'(W^*) = 0$. Once the search begins, W drifts down into the region in which $V_1'(W) > 0$ until either a suitable target is found or the contract is terminated. Condition (28) therefore ensures that $\overline{W}$ is on the equilibrium path taken into account this optimal initial continuation utility for the manager. Note that since $\gamma = \kappa/(\kappa+2) < 1$, condition (28) can be jointly satisfied with $\rho < \delta$, which ensures that providing the incentives for search is better than letting the manager shirk.

Altogether, the optimal contract during the search stage can be summarized in the following proposition:

Proposition 3 *Under the optimal contract, the investors' value function $V_1(W)$ solves the HJB equation (20) subject to the IC condition (18) and the boundary condition $V_1(0) = 0$. Under Assumption 3, there exist $\{\overline{W}, \underline{W}\}$ which solve (25) and (26), respectively, such that the optimal investment policy $x(W)$ is given by*

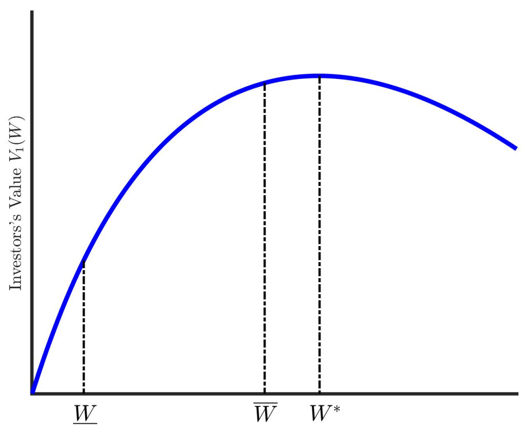
$$x(W) = \begin{cases} \theta_{\min}, & \textit{if } W < \underline{W} \\ \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'(W)\right)\left(\frac{\lambda\gamma\theta_{\min}^{\kappa}}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}}, & \textit{if } \underline{W} \le W < \overline{W} \\ \left[\frac{\lambda\gamma^2\theta_{\min}^{\kappa}}{\rho(\kappa-2)}\right]^{\frac{1}{\kappa-2}}, & \textit{if } W \ge \overline{W} \end{cases} \tag{29}$$

where $x'(W) > 0$ for all $W \in (\underline{W}, \overline{W})$.

Figure 1 plots the value function $V_1(W)$ and the three regions of the optimal investment policy $x(W)$. It also plots the first-best investment policy x^{FB} and illustrates the following result:

Corollary 3 *$x(W) < x^{FB}$ for all W.*

That is, the agency frictions in the model result in a lower bar for investment compared to that under the first-best, representing *overinvestment* by the investors. This occurs because investors lower the bar for investment to reduce the likelihood of contract termination. When W is sufficiently low, all targets trigger production regardless of their quality. When W is larger, the concern for termination is somewhat eased but is never eliminated. Hence, the optimal investment threshold in the presence of the agency frictions in always lower than that without the frictions.



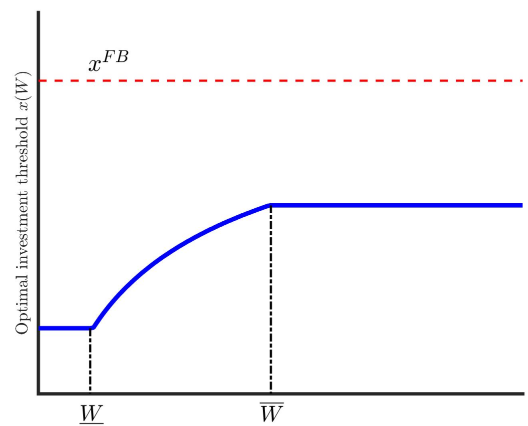


Figure 1: **Investors' Value Function and The Optimal Investment Policy**
The left panel of this figure plots the investors' value function $V_1(W)$ under the optimal contract. The right panel plots the optimal investment threshold $x(W)$ according to Proposition 3 and the first-best investment threshold x^{FB}. Parameter values are $\lambda = 2.5$, $\kappa = 4.25$, $\delta = 1.1$, $\rho = 0.8$.

3.3 Comparative Statics and Empirical Implications

The simple form of the optimal investment policies x^{FB} and $x(W)$ summarized in Lemma 1 and Proposition 3 allows the derivation of useful comparative statics. According to Corollary 3, agency frictions in the model lead to overinvestment, the degree of which can be measured in at least two ways: the ratio between $x(\underline{W})$ and x^{FB} (i.e., $\theta_{\min}/x^{FB}$), which represents the *maximal* degree of overinvestment; and the ratio between $x(\overline{W})$ and x^{FB} (i.e., $\bar{x}/x^{FB}$), which represents the *minimal* degree of overinvestment.[13] A change to a generic parameter is said to exacerbate overinvestment if it lowers either of the two ratios. Based on these definitions, Lemma 1 and Proposition 3 imply the following:

[13]Note that condition (28) ensures that $x(W^*) = \bar{x}$. Thus, the minimal degree of overinvestment is also equivalent to the initial level of overinvestment – the level of overinvestment at the outset of the search stage.

Proposition 4 *A higher λ or ρ and a lower δ all exacerbate overinvestment under the optimal contract.*

Intuitively, as discussed in Section 2.3, a higher target arrival rate λ and a lower search cost δ increases x^{FB}, which lower the ratio $\theta_{\min}/x^{FB}$ and thus exacerbates the maximal degree of overinvestment. δ also does not affect $\bar{x}$, which is pinned down by the IC condition. Therefore, a lower δ also lowers $\bar{x}/x^{FB}$ and exacerbates the minimal degree of overinvestment. Meanwhile, the tightness of the IC condition depends on the manager's private benefit from shirking ρ. A higher ρ tightens the IC condition and lower $\bar{x}$ but does not affect x^{FB}, thus exacerbating the minimal degree of overinvestment.

These comparative statics generate empirically relevant predictions that are testable in the markets of M&A, HFA, or SPAC, in which there is extensive empirical evidence for overinvestment. Existing studies (e.g., Shleifer and Vishny, 1997; Franzoni, 2009; Malenko, 2019; etc.) often attribute overinvestment to the manager's empire-building preferences. This paper offers an alternative explanation based on optimal contracting under agency frictions. In particular, Proposition 4 implies that overinvestment is positively correlated with λ and ρ and negatively correlated with δ. Empirically, λ can be proxied by the frequency of M&A, HFA, or SPAC activities. ρ can be measured by the incentive power of the managerial contract such as the fraction of inside equity, following the standard interpretation in the optimal contracting literature (e.g., DeMarzo and Sannikov, 2006). δ can be proxied by the standard measures of information frictions, such as stock liquidity, analyst coverage, or institutional holdings. Together, the results in Proposition 4 can be translated into the following testable hypothesis:

Hypothesis 1 *Overinvestment in M&A, HFA, or SPAC is positively associated with the frequency of deals in these markets, the average incentive powers of the bidder, activist hedge fund managers, or SPAC sponsors, and the stock liquidity, analyst coverage, and institutional holdings of their targets.*

Meanwhile, the model also predicts that along with the equilibrium path, investors optimally adopt a progressively lower investment threshold between $x(\overline{W})$ and $x(\underline{W})$. Because a higher investment threshold is associated with higher expected payoff for the investments

($V_2'(x) > 0$), the dynamics of x can be potentially proxied by the variations in the returns to M&A deals, activism targets, or SPAC combos. In particular, the gap between $x(\overline{W})$ and $x(\underline{W})$ can be interpreted as the return *dispersion* in those markets, which is arguably straightforward to measure. Thus, Propositions 3 and 4 suggest that such dispersion is wider if λ is higher or if ρ is lower. Therefore, Proposition 4 implies the following testable hypothesis:

Hypothesis 2 *The return dispersion of M&A deals, activism targets, or SPAC combos is positively associated with the frequency of those activities and negatively associated with the average incentive power of the managerial contract of the bidder, activist hedge fund managers, or SPAC sponsors.*

It is worth emphasizing that these hypotheses are formulated *ceteris paribus*. Empirical testing of these hypotheses thus requires identification to control for other confounding factors. Although rigorous empirical analysis is outside the scope of this paper, several identification strategies, such as using the decimalization on major stock exchanges as an exogenous shock to stock liquidity (e.g., Edmans, Fang, and Zur, 2013), already exist in the literature and may provide useful settings to explore the predictive power of the model in this paper.

4 Discussions

The core problem analyzed in this paper is the interaction of two agency frictions: moral hazard and adverse selection. To further illustrate the effect of such interaction, it is useful to compare the results of this model to models with one of the frictions only.

First, imagine there is a single type of target available for investment. The gross return of the target to the investors is V_2 (now a constant), which is realized immediately when the target is found and revealed to the investors. The the required search cost (δ) and the arrival intensity of the target if the manager works (λ) are the same as before. This represents a problem of moral hazard (regarding the unobservable search effort) only, and the following proposition summarizes the optimal contract under this alternative setting:

Proposition 5 *Suppose there is a single target worth V_2 arriving via a Poisson process with intensity λ when the manager exerts the search effort. Then, under the optimal contract,*

$$dW_t = \frac{\rho}{\lambda}(dN_t - \lambda dt) \tag{30}$$

The investors' value function $V_1(W)$ solves the HJB equation

$$0 = -\delta - \rho V_1'(W) + \lambda[V_2 - W - V_1(W)] \tag{31}$$

plus the boundary conditions $V_1(0) = 0$.

The optimal contract described in Proposition 5 has a number of different properties than that described in Proposition 3. Most crucially, without the distribution of target quality, investors do not have the choice of the investment threshold x, and the search ends as soon as the target arrives. The resulting IC constraint for search effort is always binding along the equilibrium path. This is because the rate at which W drifts down is subject to the IC constraint only, and a higher rate of the drift carries two costs: first, it expedites termination in the absence of the target; secondly, it must be compensated with a larger upward jump in W when the target does arrive, which lowers the investors' payoff.

Meanwhile, a benchmark setting with adverse selection only can be found in Malenko (2019), who studies the optimal design of a dynamic capital allocation process in which the division manager privately observes the arrival and quality of investment projects. There is no moral hazard because project arrivals are stochastic and exogenous without the manager's effort. The optimal contract focuses on soliciting truthful report of the manager's private information, which is transient: each project is a take-it-or-leave-it opportunity with instant return if invested by the headquarters, and there is an auditing technology that (for the most part) perfectly reveals the quality of the project at a cost. The agency friction arises from the manager's empire-building preference: he is inclined to exaggerate the quality of the project in order to induce a larger investment. As a result, his continuation utility drifts upward in the absence of any reported project and jumps downward when an investment project is taken without auditing to cancel out his private benefit from the investment. There is no termination under the optimal contract, because the headquarters can always prevent W from dropping too low by shrinking the size of the investment. Finally, the optimal second-best contract always induces underinvestment. That is, investment is always lower than the first-best level that maximizes the headquarters and the manager's joint surplus, .

In comparison, in this study investors face a moral hazard problem of unobservable search effort in addition to unobservable target arrival and quality. This means that the manager's continuation utility must drift downward in the absence of a suitable target and jump upward if a target triggers production.[14] In particular, because the manager can divert the search resources to generate private benefits, the search stage must end at some point, otherwise the manager will never report the arrival of any target and enjoy unlimited utility. Moreover, with his private information regarding the target quality, the manager can continue to extract rents from the investors during the production stage, which imposes an additional constraint on the size of the reward for the report of a target and likewise, the punishment for the lack of such report. Finally, different from Malenko (2019), the optimal contract always induces overinvestment along the same equilibrium path, which yields different empirical implications.

5 Dynamic Adverse Selection

So far, the model assumes that production is a one-time decision. Once the investment decision is made and the manager exerts the production effort e, a single output y is realized, and the contracting relationship ends. This implies a static adverse selection problem and simplifies the derivation of the optimal screening contract. However, the assumption of a one-time production is not indispensable. This section demonstrates the robustness of the model when the production stage is also dynamic and the manager's private information evolves stochastically.

Let τ represent the end of the search stage. Consider the following extension: If invest-

[14] In that regard this result resembles Che, Iossa, and Rey (2021) where an uninformed principal uses another follow-on contract to induce the truthful report from an informed agent about the cost of implementing an innovation idea. However, both the generation and the implementation of the idea in Che, Iossa, and Rey (2021) are static problems. In contrast, the search stage in this paper involves a dynamic problem with endogenous termination.

ment is triggered, the production stage lasts an exogenous period of $T > 0$ (i.e., from τ to $\tau + T$), during which the manager continuously produces outputs based on the target chosen by the investors in the previous stage. The production technology is given by

$$y_t = e_t \xi_t \tag{32}$$

where y_t is the output; e_t is the manager's (production) effort, which subject to a quadratic personal cost $h(e_t) = e_t^2/2$; ξ_t is the *productivity* of the target, which now evolves over time. For tractability, we assume the following law of motion for ξ_t:

Assumption 4 *ξ_t follows to a standard geometric Brownian motion (GBM)*

$$d\xi_t = \xi_t \left(\mu dt + \sigma dZ_t\right) \tag{33}$$

with publicly-known parameters μ and σ. θ determines the initial value of ξ_t, i.e., $\theta = \xi_\tau$.

The main advantage of this assumption is that, when ξ_t follows a GBM, $\xi_t = \xi_\tau \nu_t$, where

$$\nu_t = \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma Z_t\right], \quad (\nu_\tau = 1) \tag{34}$$

is an exogenous stochastic process with known distribution for any given t. Therefore, $d\xi_t/d\theta = \xi_t/\xi_\tau = \nu_t$. In other words, the marginal value of target quality θ on its subsequent productivity at any time during the production stage depends on the path of exogenous shocks only, a property that greatly simplifies the analysis below.

If all information is public, the first-best effort and output in the production stage solve

$$\max_{e_t} \; y_t - h(e_t) = e_t \xi_t - h(e_t) \tag{35}$$

The solution is

$$e_t^{FB} = \xi_t \tag{36}$$

$$y_t^{FB} = \xi_t^2 \tag{37}$$

The investors' expected payoff given the quality of the target, denoted by $V_2^{FB}(\theta)$, is given by

$$V_2^{FB}(\theta) = \mathrm{E}\int_{\tau}^{\tau+T}\left(y_t^{FB} - \frac{1}{2}\left(e_t^{FB}\right)^2\right)dt = \frac{\phi}{2}\theta^2 \tag{38}$$

where

$$\phi = \mathrm{E}\int_{\tau}^{\tau+T}\nu_t^2 dt = \frac{e^{(2\mu+\sigma^2)T} - 1}{2\mu+\sigma^2} \tag{39}$$

is a constant. ϕ thus measures the marginal value of target quality summarizing the joint effect of μ, σ, and T, and will be treated as a known parameter in the subsequent analyses. The first-best investment policy is still a cutoff quality

$$x^{FB} = \left[\frac{\lambda\phi\theta_{\min}^{\kappa}}{\delta(\kappa-2)}\right]^{\frac{1}{\kappa-2}} \tag{40}$$

Investors never abandon the search, and production begins only when a target that clears that bar arrives. Their maximal expect payoff at the beginning of the search stage is

$$V_1^{FB}(x^{FB}) = \frac{\kappa\left(x^{FB}\right)^2}{2(\kappa-2)} - \frac{\delta\left(x^{FB}\right)^{\kappa}}{\lambda\theta_{\min}^{\kappa}} \tag{41}$$

Similar to the baseline model, an adverse selection problem arises if effort and productivity are both the manager's private information (while the output y_t is still observable to the investors). However, this adverse selection is now dynamic in nature, because the investors must solicit the truthful reporting of ξ_t for the entire duration of the production stage. The resulting screening contract now involves a series of the output target $\{y_t\}_{t\in[\tau,\tau+T]}$ and the corresponding wage $\{w_t\}_{t\in[\tau,\tau+T]}$ if the required output is produced.

Let $\hat{\theta}$ and $\{\hat{\xi}_t\}_{t\in[\tau,\tau+T]}$ represent the manager's *reported* target quality and time-t productivity, respectively. Conditional on any utility W_{τ^-} carried over from the search stage, the manager's objective is to maximize his expected wage minus his (production) effort cost from the contract – his *information rent* – which is given by

$$R(\hat{\theta}) = \max_{\hat{\theta}} \; \mathrm{E}\left[\int_{\tau}^{\tau+T} (w_t - h(e_t))dt\right] \tag{42}$$

subject to the constraint (32). That is, at any time t, if the manager reports $\hat{\xi}_t$, he must exert effort $e_t = y_t/\hat{\xi}_t$ to produce the required level of output y_t, similar to that in the baseline model. The investors' objective is to maximize the expected output minus the manager's wage from the contract. Following a similar argument as that used in the baseline model, we assume and verify later that Θ is a open set bounded from below, i.e.,

$$\Theta = \{\theta : \theta \geq x\} \tag{43}$$

for some cutoff threshold x. Then, the investors' objective is to maximize their payoff at the outset of the production stage, which is $V_2(x) - W_{\tau^-}$, where

$$V_2(x) = \max_{y_t, w_t} \int_{x}^{+\infty} \mathrm{E}\left[\int_{\tau}^{\tau+T} (y_t - w_t)dt\right] dF(\theta) \tag{44}$$

subject to the IC constraint $\hat{\theta} = \theta$ and $\hat{\xi}_t = \xi_t$ for all $t \in [\tau, \tau + T]$.

Comparing to the existing literature, a theoretical innovation (and challenge) of this setting is that the manager's private information ξ_t is *persistent*, which implies the adverse selection problem the investors face is *dynamic*. Studies exploring persistent private information in the context of dynamic moral hazard including Williams (2011, 2015), He, Wei, Yu, and Gao (2017), Marinovic and Varas (2019), and Feng (2022). In comparison, studies of persistent private information in the context of adverse selection are rare, as it is known to be a challenging problem.

Fortunately, the specific structures of this model, in particular Assumption 4, ensures that the dynamic adverse selection problem in this model can be solved using the *revenue-maximizing direct mechanism* technique developed in Bergemann and Strack (2015), which builds on the general Myersonian mechanism of Eső and Szentes (2007) and Pavan, Segal, and Toikka (2014) but extended to continuous time. In particular, the technique of Bergemann and Strack (2015) is sufficient to uniquely pin down $R(\theta)$ and $V_2(x)$ under any incentive compatible contract (and thus the optimal contract as well), which is all that is needed to feed back into the first, search stage. The resulting optimal contract under this extension is summarized as follows:

Proposition 6 *Under Assumption 4, for any given investment policy x, the optimal contract in the production stage has the following properties:*

- *The manager's information rent from a target of quality $\theta > x$ at the beginning of the production stage is given by*

$$R(\theta) = \frac{\phi\gamma^2}{2}(\theta^2 - x^2) \tag{45}$$

- *The investor's expected payoff at the beginning of the production stage is*

$$V_2(x) = \frac{\phi\gamma\kappa}{2(\kappa-2)}x^2 \tag{46}$$

- *During the production stage, the investors' optimal output target $\{y_t^*\}_{t\in[0,T]}$ is given by $y_t^* = \gamma\xi_t^2$ and the implied equilibrium production effort is $e_t^* = \gamma\xi_t$*

The optimal contract during the search stage can be summarized by the investors' value function $V_1(W)$, which solves an HJB equation analogous to (20) subject to the boundary condition $V_2(0) = 0$. In particular, if Assumption 3 holds, and $\phi \geq 1$, then there exist $\{\overline{W}, \underline{W}\}$ such that the optimal investment policy $x(W)$ is given by

$$x(W) = \begin{cases} \theta_{\min}, & \text{if } W < \underline{W} \\ \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'(W)\right)\left(\frac{\phi\lambda\gamma\theta_{\min}^{\kappa}}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}}, & \text{if } \underline{W} \leq W < \overline{W} \\ \left[\frac{\phi\lambda\gamma^2\theta_{\min}^{\kappa}}{\rho(\kappa-2)}\right]^{\frac{1}{\kappa-2}}, & \text{if } W \geq \overline{W} \end{cases} \tag{47}$$

$x(W) < x^{FB}$, and $x'(W) > 0$ for all $W \in (\underline{W}, \overline{W})$.

Proposition 6 shows that, despite the dynamic nature of the adverse selection problem, the main results from the baseline model remain qualitatively intact. In particular, in the production stage, the manager's information rent is still a quadratic function of target quality

θ and the investor's expected payoff is still a quadratic function of the investment threshold x. In the search stage, W drifts downward in the absence of a suitable target, and the optimal investment threshold x is progressively lower, leading to overinvestment by the investors.

The empirical implications are likewise similar to those in the baseline model except for one additional prediction pertaining to the parameter ϕ, which can be interpreted as the industry or regional average return and frequency of M&A, activism, or SPAC activities. It is straightforward to see that a higher ϕ increases both x^{FB} and $x(\overline{W})$ while $x(\underline{W}) = \theta_{\min}$ is unchanged, leading to the following testable hypothesis:

Hypothesis 3 *The overinvestment and return dispersion in M&A, HFA, or SPAC are positively associated with the average returns in those markets.*

Overall, results in this section demonstrate the robustness of the baseline model and its practical implications. Nevertheless, the solution technique of this dynamic version of the adverse selection problem is far more involved than the one used in the static version and is potentially applicable to a broad class of questions involving persistent and time-varying private information. The technique utilizes a critical result in Bergemann and Strack (2015) that, despite the large set of possible deviations of the manager, it is without the loss of generality to establish the IC condition for a particular type of deviation from the manager: if he misreports θ, the *initial productivity*, to be $\hat{\theta} \neq \theta$, his follow-up strategy is to continue misreporting productivity *as if the true quality was $\hat{\theta}$ and he had reported that truthfully.* More formally, at any time, the manager's reported productivity satisfies the following so-called *consistent deviation*:

$$\hat{\xi}_t = \hat{\theta} v_t = \hat{\theta} \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma Z_t\right] \tag{48}$$

where Z_t represents the true productivity shocks the mis-reporting manager experiences. This implies that although the manager's private information regarding ξ_t is persistent, it is without the loss of generality to label each manager by the quality of his target θ only. Consequently, $R(\theta)$ and $V_2(x)$ under any incentive compatible contract can be uniquely pinned down without solving the usual dynamic programming problem involving differential equations. The original problem considered in Bergemann and Strack (2015) is a repeated sale problem in which the preference of the buyers (agents) evolve exogenously over time and remain their private information. Despite the topical different, this critical result of Bergemann and Strack (2015) applies in the setting in this paper because private information (productivity ξ_t) also evolves exogenously.

The details of the solution technique is left in the Appendix in the interest of space. To our knowledge, such technique has yet seen much utilization in the finance literature. The only exceptions are Gao and Wong (2017), which adopts this method on a capital budgeting problem, and Feng, Wang, and Wu (2022), which adopts the method on internal governance. Compared to these two studies of adverse selection problems only, this paper considers both dynamic adverse selection and dynamic moral hazard, and thus provides a unified framework to study these two important agency frictions in the same model.

6 Conclusions

Valuable information hardly comes without a cost. The efficient collection and utilization of valuable information is both an indispensable ingredient for successful business operations in practice and an important subject for academic research. This paper studies a model in which investors delegate the search of investment opportunities to a manger with information superiority in several dimensions. Unlike the existing framework, the model of this paper highlights two novel yet realistic features in the search process. First, the precise value of the information (i.e., its quality) is private knowledge of the manager, who must receive proper incentives to maximize the likelihood of discovering the investment targets and to disclose their value to the investors truthfully. Second, the investors-manager relationship does not end when the relevant information is disclosed. The manager is also delegated with generating returns out of the investment target he uncovers, a process in which the manager's information superiority about the target remains. These features lead to the interaction of an adverse selection and a moral hazard problem. The resulting optimal investment policies exhibit overinvestment relative to the first-best, which sheds light on understanding and predicting the returns to M&A deals, HFA targets, and/or SPAC business combinations.

This paper can be extended in several directions. For simplicity, the model assumes zero discounting, so that the results are not driven by the time value of money. Extending the model to allow (potentially different) discounting for both the manager and investors can be a fruitful exercise. The model also does not allow the manager to revert to the search stage once production begins. In practice, the search and production processes do not always move forward linearly. Letting the manager conduct both search and production repeatedly or simultaneously may yield interesting insights on firms' optimal internal organization and/or resource allocation. We leave these topics for future studies.

Appendix

Proof of Lemma 1

First, $V_2^{FB\prime}(\theta) = \theta > 0$. Therefore, if $x \in \Theta^{FB}$ for some x, then $\theta \in \Theta^{FB}$ for all $\theta > x$. i.e., $\Theta^{FB} = \{\theta : \theta \geq x^{FB}\}$. Thus, the conditional expectation of the investors' payoff from setting an arbitrary cutoff investment quality x is

$$\mathcal{V}_2^{FB}(x) \equiv \mathrm{E}\left[V_2^{FB}(\theta)|\theta \geq x\right] = \int_x^{+\infty} \frac{\theta^2}{2}\left(\frac{\kappa x^\kappa}{\theta^{\kappa+1}}\right) d\theta = \frac{\kappa x^2}{2(\kappa-2)} \tag{49}$$

(49) utilizes the fact that a Pareto distribution truncated from below at some $x > \theta_{\min}$ is still a Pareto distribution with the same shape parameter κ and scale parameter x. (49) also reveals why $\kappa > 2$ is needed for the first-best to exist. Let $V_1^{FB}(x)$ be the investors' value function at the outset of the search stage associated with cutoff policy x, then

$$V_1^{FB}(x) = \max_x \int_0^{+\infty} \left[-\delta t + F(x)V_1^{FB}(x) + (1-F(x))\mathcal{V}_2^{FB}(x)\right] \lambda e^{-\lambda t} dt \tag{50}$$

The three terms inside the square brackets represent the cost of search, the payoff from a target with quality lower than x, and the expected payoff from the arrival of a target with quality x or above, respectively. Using the fact that $F(x) = 1 - (\theta_{\min}/x)^\kappa$ for Pareto distribution, $V_1^{FB}(x)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$0 = \max_x \;\; -\delta + \lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa \left[\mathcal{V}_2^{FB}(x) - V_1^{FB}(x)\right] \tag{51}$$

Substituting (49) into the HJB equation and re-arranging the terms yields

$$V_1^{FB}(x) = \max_x \;\; \frac{\kappa x^2}{2(\kappa-2)} - \frac{\delta x^\kappa}{\lambda \theta_{\min}^k} \tag{52}$$

The first order condition with respect to x yields x^{FB} as in (6). Substituting x^{FB} into (52) yields $V_1^{FB}(x^{FB})$ as in (7). $\square$

Proof of Proposition 1

Let $y(\theta, e) = \theta e$ denote the production function. The principal offers a screening contract $\{w(\hat{\theta}), y(\hat{\theta})\}$. The agent reports his type $\hat{\theta}$, produces the required level of output, and receives the associated compensation. With a slight abuse of notation, define

$$R(\theta, \hat{\theta}) = w(\hat{\theta}) - h(e) \tag{53}$$

as the information rent of the agent with type-θ reporting $\hat{\theta}$, subject to the constraint that $y(\theta, e) = y(\hat{\theta})$. i.e., he must produce the level of output designed for the type-$\hat{\theta}$ agent. Let $e(y, \theta)$ represent the necessary effort required by a type-θ agent to produce output y. Then, one can define $R(\theta) = R(\theta, \theta)$ as the agent's equilibrium rent under truthful reporting, and

$$\hat{\theta}^*(\theta) = \arg\max_{\hat{\theta}} \; R(\theta, \hat{\theta}) \tag{54}$$

as the optimal reported type chosen by a type-θ agent. This optimality implies the following *envelope condition*

$$R_{\hat{\theta}}(\theta, \hat{\theta}^*(\theta)) = 0 \tag{55}$$

Therefore, in the equilibrium

$$R'(\theta) = \frac{\partial R(\theta, \hat{\theta})}{\partial \theta} = R_\theta + R_{\hat{\theta}}(\theta, \hat{\theta}^*(\theta)) \frac{d\hat{\theta}^*(\theta)}{d\theta} = R_\theta = -h'(e)e_\theta(y, \theta) \tag{56}$$

based on the envelope condition. Note that y depends on the agent's report and thus is *not* a function of θ (i.e., e_θ takes y as given.)

The investors' payoff in the production stage is therefore $V_2(x) - W_{t^-}$, where

$$V_2(x) = \max_{y,w} \int_x^\infty [y(\theta) - w(\theta)]dF(\theta) = \max_{y,w} \int_x^\infty [y(\theta) - h(e(\theta, y)) - R(\theta)]dF(\theta) \tag{57}$$

where

$$F(\theta) = 1 - \left(\frac{x}{\theta}\right)^\kappa \tag{58}$$

$$f(\theta) = \frac{\kappa x^\kappa}{\theta^{\kappa+1}} \tag{59}$$

Applying integration by parts to the last term inside the integral of $V_2(x)$ yields

$$\int_x^\infty R(\theta)dF(\theta) = \int_x^\infty R'(\theta)(1 - F(\theta))d\theta + R(x) \tag{60}$$

Substituting this into $V_2(x)$ above yields

$$V_2(x) = \max_y \int_x^\infty [y - h(e) - R'(\theta)g(\theta)]f(\theta)d\theta \tag{61}$$

where

$$g(\theta) \equiv \frac{1 - F(\theta)}{f(\theta)}$$

represents the inverse hazard function of θ. Replacing $R'(\theta)$ with (56), point-wise maximization with respect to y yields the following optimality condition:

$$1 - h'(e)e_y(e, \theta) + \frac{dh'(e)e_\theta(y, \theta)}{dy} g(\theta) = 0 \tag{62}$$

which yields the optimal target y for each type θ. Because $y = \theta e$ and $h(e) = e^2/2$,

$e(y, \theta) = y/\theta$, $e_\theta = -y/\theta^2$, and

$$\frac{dh'(e)e_\theta(y, \theta)}{dy} = -\frac{dy^2/\theta^3}{dy} = -\frac{2y}{\theta^3}$$

The fact that θ follows a Pareto distribution implies that

$$g(\theta) = \frac{1 - F(\theta)}{f(\theta)} = \frac{\theta}{\kappa} \tag{63}$$

which is a function of κ only. Substituting these results into (62) yields

$$1 - \frac{y}{\theta^2} - \frac{2y}{\kappa\theta^2} = 0 \tag{64}$$

which implies $y = \gamma\theta^2$, where

$$\gamma = \frac{\kappa}{\kappa + 2} \tag{65}$$

Substituting $y = \gamma\theta^2$ into (53). The IC constraint $\hat{\theta} = \theta$ and the fact that $R(x) = 0$ under the optimal screening contract yields

$$R(\theta) = \frac{\gamma^2}{2}(\theta^2 - x^2) \tag{66}$$

Combine this with $y = \gamma\theta^2$ implies that

$$V_2(x) = \frac{\gamma\kappa}{2(\kappa - 2)}x^2 \tag{67}$$

□

Proof of Corollary 1

Suppose there is an incentive-compatible optimal contract $\mathcal{C}$ under which an open set H exists with the following property: there exists θ' such that $\theta' \in \Theta$ but $\theta' < \tilde{\theta}$ for all $\tilde{\theta} \in H$. Let $\bar{\theta} \equiv \max\{\theta : \theta \in \Theta, \theta < \tilde{\theta}\}$. Clearly, $y(\bar{\theta}) - w(\bar{\theta}) > 0$ and $w(\bar{\theta}) \geq h(e(\bar{\theta}))$ if $\mathcal{C}$ is optimal. Now consider a contract $\mathcal{C}'$ that is otherwise identical to $\mathcal{C}$ except for the following: for any report $\hat{\theta} \in H$, the required output $y(\hat{\theta}) = y(\bar{\theta})$ and the associated compensation is $w(\hat{\theta}) = w(\bar{\theta})$. This contract is incentive-compatible because for all $\tilde{\theta} \in H$, $e(\tilde{\theta}) = y(\bar{\theta})/\tilde{\theta} < e(\bar{\theta})$ and is independent of the report $\hat{\theta}$. Thus, $w(\tilde{\theta}) = w(\bar{\theta}) > h(e(\tilde{\theta}))$ for all $\tilde{\theta}$. However, $y(\bar{\theta}) - w(\bar{\theta}) > 0$ implies that $\mathcal{C}'$ generates the same payoff as $\mathcal{C}$ for all $\theta \notin H$ but positive (higher) payoff for all $\tilde{\theta} \in H$, contradicting the assumption that $\mathcal{C}$ is optimal. Therefore, it must be that such H does not exist under the optimal contract.

Proof of Proposition 2

Let $\mathcal{F}_t$ denote the filtration generated by the manager's report θ_t (where $\theta_t = 0$ if no investment opportunity arrives). W_t is an $\mathcal{F}_t$-martingale and thus, by the martingale representation theorem for jump processes, there exists a $\mathcal{F}_t$-predictable, integrable process β_t such that

$$dW_t = a_t\beta_t(dN_t - \lambda(1 - F(x_t))dt) \tag{68}$$

Incentive compatibility of the search effort requires that $\lambda(1 - F(x_t))\beta_t \geq \rho$. Incentive compatibility of truthful reporting of θ requires that $W_\tau - W_{\tau^-} = R(\theta_\tau)$ if the contract moves to the next stage, which implies $\beta_t = \mathrm{E}[R(\theta_t)|\theta_t \geq x_t] = U(x_t)$ by the property of a martingale.[15] Thus, given the investment policy x_t,

$$dW_t = U(x_t)(dN_t - \lambda(1 - F(x_t))dt) \tag{69}$$

where $U(x_t)\lambda(1 - F(x_t)) \geq \rho$. Substituting in $1 - F(x_t) = \left(\frac{\theta_{\min}}{x_t}\right)^\kappa$ yields equations (17) and (18). $\square$

Proof of Proposition 3

Applying Ito's lemma to dW_t implies the investors' value function in the search stage solves the following HJB equation:

$$0 = \max_x \quad -\delta - \lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa U(x)V_1'(W) + \lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa [V_2(x) - W - V_1(W)] \tag{70}$$

subject to the IC constraint (18). Substituting $U(x)$ from (15) and $V_2(x)$ from (14) into the HJB equation and rearrange terms yields:

$$V_1(W) = \max_x \quad \left(\frac{\gamma\kappa}{2(\kappa-2)}\right)x^2 - \left(\frac{\gamma^2x^2}{\kappa-2}\right)V_1'(W) - W - \frac{\delta}{\lambda}x^\kappa\theta_{\min}^{-\kappa} \tag{71}$$

Suppose the IC constraint is slack, then the first order condition implies

$$\frac{\gamma}{\kappa-2}\left[\kappa - 2\gamma V_1'(W)\right]x = \frac{\delta}{\lambda}\kappa x^{\kappa-1}\theta_{\min}^{-\kappa} \tag{72}$$

The solution is

$$x(W) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right)V_1'(W)\right)\left(\frac{\lambda\gamma\theta_{\min}^\kappa}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}} \tag{73}$$

[15] Note that this also follows the definition of $R(\theta)$ in Section 3.1 which is the information rent the manager must be given to reveal θ truthfully *in addition to* any utility W_{τ^-} carried over from the search stage.

If (18) is binding, then

$$\lambda \left(\frac{\theta_{\min}}{x}\right)^{\kappa} U(x) = \frac{\lambda\theta_{\min}^{\kappa}}{x^{\kappa}} \left(\frac{\gamma^2 x^2}{\kappa - 2}\right) = \rho \tag{74}$$

yields the solution

$$x(W) = \bar{x} \equiv \left[\frac{\lambda\gamma^2\theta_{\min}^{\kappa}}{\rho(\kappa - 2)}\right]^{\frac{1}{\kappa-2}} \tag{75}$$

Substituting (75) into (73) implies that there exists $\overline{W}$ such that

$$x(\overline{W}) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'(\overline{W})\right) \left(\frac{\lambda\gamma\theta_{\min}^{\kappa}}{\delta(\kappa - 2)}\right)\right]^{\frac{1}{\kappa-2}} = \bar{x} \tag{76}$$

That is, $\overline{W}$ solves

$$V_1'(\overline{W}) = \frac{\kappa}{2}\left(\frac{1}{\gamma} - \frac{\delta}{\rho}\right) \tag{77}$$

Meanwhile, substituting $x = \theta_{\min}$ into (73) implies that there exists $\underline{W}$ such that

$$\theta_{\min} = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'(\underline{W})\right) \left(\frac{\lambda\gamma\theta_{\min}^{\kappa}}{\delta(\kappa - 2)}\right)\right]^{\frac{1}{\kappa-2}} \tag{78}$$

That is, $\underline{W}$ solves

$$V_1'(\underline{W}) = \left(1 - \frac{\delta(\kappa - 2)}{\gamma\lambda\theta_{\min}^2}\right) \frac{\kappa}{2\gamma} \tag{79}$$

The existence of both $\{\underline{W}, \overline{W}\}$ requires that $\overline{W} > \underline{W}$, which is equivalent to

$$\frac{\kappa}{2}\left(\frac{1}{\gamma} - \frac{\delta}{\rho}\right) < \left(1 - \frac{\delta(\kappa - 2)}{\gamma\lambda\theta_{\min}^2}\right) \frac{\kappa}{2\gamma} \tag{80}$$

which simplifies to (27). Finally, $V_1'(\overline{W}) > 0$ requires that

$$\frac{\kappa}{2}\left(\frac{1}{\gamma} - \frac{\delta}{\rho}\right) > 0 \tag{81}$$

which implies (28). □

Proof of Corollary 3

We can prove this result regardless of whether Assumption 3 holds, i.e., whether the IC constraint is binding for some $\overline{W}$. Note that under the optimal contract, $V'(W) > -1$ for all W. This is because investors can always make a cash transfer to the manager, which lowers W and $V(W)$ by the exact same amount. Therefore, the marginal value of building W inside the firm can never be lower than the marginal value of cash transfer, which is -1. Substituting $V'(W) = -1$ into the first-order condition (23) implies that

$$\lim_{W \to +\infty} x(W) = \left[\gamma \left(1 + \frac{2\gamma}{\kappa} \right) \left(\frac{\lambda \theta_{\min}^{\kappa}}{\delta(\kappa - 2)} \right) \right]^{\frac{1}{\kappa-2}} \tag{82}$$

if the IC constraint (18) is never binding. Because

$$\frac{2\gamma}{\kappa} = \frac{2\kappa}{\kappa(\kappa+2)} < 1 \tag{83}$$

when $\kappa > 2$, this limit is always smaller than x^{FB}. Thus, $x(W) < x^{FB}$ for all W.

Proof of Proposition 4

Equations (6) and (24) imply the following comparative statics regarding x^{FB} and $x(\overline{W})$.

$$\frac{\partial x^{FB}}{\partial \lambda} > 0, \quad \frac{\partial x(\overline{W})}{\partial \lambda} > 0 \tag{84}$$

$$\frac{\partial x^{FB}}{\partial \rho} = 0, \quad \frac{\partial x(\overline{W})}{\partial \rho} < 0 \tag{85}$$

$$\frac{\partial x^{FB}}{\partial \delta} < 0, \quad \frac{\partial x(\overline{W})}{\partial \delta} = 0 \tag{86}$$

The first and third lines of these results imply that

$$\frac{\partial}{\partial \lambda} \left(\frac{x(\underline{W})}{x^{FB}} \right) < 0 \tag{87}$$

$$\frac{\partial}{\partial \delta} \left(\frac{x(\underline{W})}{x^{FB}} \right) > 0 \tag{88}$$

given that $x(\underline{W}) = \theta_{\min}$ is a constant, while the second and the third lines imply that

$$\frac{\partial}{\partial \rho} \left(\frac{x(\overline{W})}{x^{FB}} \right) < 0 \tag{89}$$

$$\frac{\partial}{\partial \delta} \left(\frac{x(\overline{W})}{x^{FB}} \right) > 0 \tag{90}$$

□

Proof of Proposition 5

When only a single target is available, by the martingale representation theorem, there exists β_t such that

$$dW_t = \beta_t(dN_t - \lambda dt) \tag{91}$$

Incentive compatibility requires that $\lambda\beta_t \geq \rho$, or $\beta_t \geq \rho/\lambda$. Applying Ito's lemma on (91) implies that the investors' value function solves the HJB equation

$$0 = \max_{\beta \geq \rho/\lambda} \quad -\delta - \lambda\beta V_1'(W) + \lambda(V_2 - W - V_1(W)) \tag{92}$$

subject to boundary conditions $V_1(0) = 0$. Differentiating (92) with respect to β yields

$$-\lambda(V_1'(W) + 1) \leq 0 \tag{93}$$

Therefore it is optimal to always set $\beta_t = \rho/\lambda$, i.e., the IC constraint is always binding. □

Proof of Proposition 6

A. The Production Stage

This section solves the optimal screening contract in the production stage. The proof begins with deriving the manager's information rent for *any* incentive compatible contract. Consider any report $\hat{\theta}$ made by a manager possessing an arbitrary θ-quality target. Based on this report, the manager is assigned the contract $\mathcal{C}(\hat{\theta})$, which imposes output target $y_t^{\mathcal{C}(\hat{\theta})}$ and with the recommended effort $\hat{e}_t \equiv y_t^{\mathcal{C}(\hat{\theta})}/\hat{\xi}_t$ for all t, where $\hat{\xi}_t$ is the manager's reported productivity. The constraint (48) implies that

$$e_t\xi_t = \hat{e}_t\hat{\xi}_t \tag{94}$$

The payoff for the manager is

$$R(\theta; \hat{\theta}) = \mathrm{E}^{\theta}\left[\int_{\tau}^{\tau+T}\Big(w(\hat{\theta}, \hat{\xi}_t) - h(e_t)\Big)\, dt\right] \tag{95}$$

Differentiating with respect to θ yields

$$\begin{aligned} \frac{\partial}{\partial\theta}R(\theta; \hat{\theta}) &= \mathrm{E}\left[\int_{\tau}^{\tau+T}\left(-\frac{\partial}{\partial\theta}h(e_t)\right) dt\right] &\quad (96)\\ &= \mathrm{E}\left[\int_{\tau}^{\tau+T}\left(\frac{e_t\hat{e}_t\hat{\xi}_t}{\theta\xi_t}\right) dt\right] &\quad (97) \end{aligned}$$

where the second line utilizes the constraint (94) and the property of GMB $\xi_t = \theta\nu_t$. Evaluating (97) at the equilibrium ($\hat{e}_t = e_t, \hat{\xi}_t = \xi_t$) and substituting e_t with y_t/ξ_t implies

$$R'(\theta) = \mathrm{E}\left[\int_{\tau}^{\tau+T} \frac{1}{\theta}\left(\frac{y_t}{\xi_t}\right)^2 dt\right] \tag{98}$$

which is the *dynamic envelop condition* analogous to that derived in the proof of Proposition 1 above. Integrating (98) from x up to $\tilde{\theta}$ yields the information rent for any given $\tilde{\theta}$-quality target, which is given by

$$R(\tilde{\theta}) = \int_{x}^{\tilde{\theta}} \mathrm{E}\left[\int_{\tau}^{\tau+T} \frac{1}{\theta}\left(\frac{y_t}{\xi_t}\right)^2 dt\right] d\theta + R(x) \tag{99}$$

The notation $\tilde{\theta}$ is used because θ also represents the variable of integration.

Next, deriving the investors' expected payoff given the distribution of θ. With a slight abuse of notation, let $\int_x^\infty(\cdot)dF(\theta;x)$ denote the expectation of θ taken under the support Θ taking into account how the distribution of $F(\theta)$ shifts with x, then the investor's payoff at the outset of the production stage under any incentive compatible contract is $V_2(x) - W_{\tau^-}$, where

$$V_2(x) = \max_{y_t,w_t} \int_{x}^{\infty} \mathrm{E}\left[\int_{\tau}^{\tau+T} (y_t - w_t)dt\right] dF(\theta;x) \tag{100}$$

The definition of information rent R (Eq. 42) implies that

$$\mathrm{E}\left[\int_{\tau}^{\tau+T} w_t dt\right] = R(\theta) + \mathrm{E}\left[\int_{\tau}^{\tau+T} h(e_t)dt\right] \tag{101}$$

Substituting this into the definition of $V_2(x)$ (Eq. 100) yields

$$V_2(x) = \max_{y_t} \quad \int_{x}^{\infty} \mathrm{E}\left[\int_{\tau}^{\tau+T} \left(y_t - \frac{1}{2}\left(\frac{y_t}{\xi_t}\right)^2\right) dt\right] dF(\theta;x) - \int_{x}^{\infty} R(\theta)dF(\theta;x) \tag{102}$$

Applying integration by parts and the fundamental theorem of calculus to the last term yields

$$\int_{x}^{\infty} R(\theta)dF(\theta;x) = \int_{x}^{\infty} R'(\theta)\left(\frac{1-F(\theta;x)}{f(\theta;x)}\right) dF(\theta;x) + R(x) \tag{103}$$

$$= \int_{x}^{\infty} R'(\theta)\left(\frac{\theta}{\kappa}\right) dF(\theta;x) + R(x) \tag{104}$$

where the second line comes from the property of the Pareto distribution. Clearly, $R(x) = 0$ under the optimal contract. Replacing $R'(\theta)$ with (98) and substituting the above term back to (102) yields

$$V_2(x) = \max_{y_t} \int_x^{+\infty} \mathrm{E}\left[\int_\tau^{\tau+T} \left(y_t - \left(\frac{1}{2}+\frac{1}{\kappa}\right)\left(\frac{y_t}{\xi_t}\right)^2\right) dt\right] dF(\theta; x) \tag{105}$$

Point-wise maximization of (105) with respect to y_t yields the optimal output target y_t^* and effort e_t^*:

$$y_t^* = \gamma \xi_t^2 \tag{106}$$
$$e_t^* = \gamma \xi_t \tag{107}$$

where $\gamma = \kappa/(\kappa+2)$. Substituting (107) and (106) back into (99) yields the following information rent under the optimal contract:

$$R(\tilde{\theta}) = \int_x^{\tilde{\theta}} \mathrm{E}\left[\int_\tau^{\tau+T} \frac{1}{\theta}\left(\gamma\xi\right)^2 dt\right] d\theta \tag{108}$$

$$R(\tilde{\theta}) = \int_x^{\tilde{\theta}} \gamma^2 \theta \mathrm{E}\left[\int_\tau^{\tau+T} \nu_t^2 dt\right] d\theta \tag{109}$$

$$= \int_x^{\tilde{\theta}} \phi\gamma^2\theta d\theta = \frac{\phi\gamma^2}{2}\left(\tilde{\theta}^2 - x^2\right) \tag{110}$$

for a project with quality $\tilde{\theta}$. Again, the notation $\tilde{\theta}$ is used because θ also represents the generic variable being integrated. Substituting (106) back into (105) yields

$$V_2(x) = \int_x^{+\infty} \mathrm{E}\left[\int_\tau^{\tau+T} \frac{\gamma\xi_t^2}{2} dt\right] dF(\theta; x) \tag{111}$$

$$= \int_x^{+\infty} \frac{\phi\gamma\kappa x^\kappa}{2}\theta^{1-\kappa} d\theta = \frac{\phi\gamma\kappa}{2(\kappa-2)} x^2 \tag{112}$$

Note that, similar to the baseline model, because the investors and the manager share the same discount rate (both 0), and there is no endogenous turnover during the production stage, all wage payments $\{w_t\}$ can be postponed until the end of the production period. Any W_{τ^-} carried over to the production stage can also be paid at the end of the production stage together with all the accrued wage payments.

B. The Search Stage

Let $a_s \in \{0, 1\}$ denote the manager's shirking and working actions, respectively. Let τ and W_τ denote the stopping time of the search stage (either due to progress to the next stage or contract termination) and the associated promised utility to the manager. Because the investors and the manager share the same discount rate (both 0) and $\rho < \delta$, Lemma 2 still applies. That is, $a_t = 1$ for all $t < \tau$, and there is no intermediate payment during the search stage. The manager's continuation utility in this stage thus can be written as:

$$W_t = E\left[\int_t^\tau \rho(1-a_s)ds + W_\tau\right]. \tag{113}$$

By the martingale representation theorem for jump processes, given any investment strategy x of the investors, there exists a $\mathcal{F}_t$-predictable, integrable process β_t such that

$$dW_t = a_t\beta_t(dN_t - \lambda(1-F(x_t))dt) \tag{114}$$

Incentive compatibility of the search effort requires that $\lambda(1-F(x_t))\beta_t \geq \rho$. Incentive compatibility of truthful reporting of θ requires that $K_\tau - W_\tau = R(\theta_\tau)$ if the contract moves to the next stage, which implies $\beta_t = \mathrm{E}[R(\theta_t)|\theta_t \geq x_t] = U(x_t)$ by the property of a martingale, where

$$U(x) \equiv \mathrm{E}\left[R(\theta)|\theta \geq x\right] = \int_x^{+\infty} \frac{\phi\gamma^2}{2}(\theta^2 - x^2)\left(\frac{\kappa x^\kappa}{\theta^{\kappa+1}}\right)d\theta = \frac{\phi\gamma^2 x^2}{\kappa-2} \tag{115}$$

Therefore, under an incentive compatible contract with investment policy x_t,

$$dW_t = U(x_t)(dN_t - \lambda(1-F(x_t))dt) \tag{116}$$

where $\lambda(1-F(x_t)) \geq \rho$. Then, Ito's lemma implies the investors' value function in the search stage solves the HJB equation:

$$0 = \max_x \quad -\delta - \lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa U(x)V_1'(W) + \lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa [V_2(x) - W - V_1(W)] \tag{117}$$

subject to the IC constraint (18). Substituting $U(x)$ from (115) and $V_2(x)$ from (46) into the HJB equation and rearrange terms yields:

$$V_1(W) = \max_x \quad \left(\frac{\phi\gamma\kappa}{2(\kappa-2)}\right)x^2 - \left(\frac{\phi\gamma^2}{\kappa-2}\right)x^2 V_1'(W) - W - \frac{\delta}{\lambda}x^\kappa\theta_{\min}^{-\kappa} \tag{118}$$

Suppose the IC constraint is slack, then the first order condition implies

$$\frac{\phi\gamma}{\kappa-2}\left[\kappa - 2\gamma V_1'(W)\right]x = \frac{\delta}{\lambda}\kappa x^{\kappa-1}\theta_{\min}^{-\kappa} \tag{119}$$

The solution is

$$x(W) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right)V_1'(W)\right)\left(\frac{\lambda\phi\gamma\theta_{\min}^\kappa}{\delta(\kappa-2)}\right)\right]^{\frac{1}{\kappa-2}} \tag{120}$$

If (18) is binding, then

$$\lambda\left(\frac{\theta_{\min}}{x}\right)^\kappa U(x) = \frac{\lambda\theta_{\min}^\kappa}{x^\kappa}\left(\frac{\phi\gamma^2 x^2}{\kappa-2}\right) = \rho \tag{121}$$

yields the solution

$$x = \bar{x} \equiv \left[\frac{\lambda\phi\gamma^2\theta_{\min}^{\kappa}}{\rho(\kappa-2)} \right]^{\frac{1}{\kappa-2}} \tag{122}$$

Substituting (75) into (73) implies that there exists $\overline{W}$ such that

$$x(\overline{W}) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'((\overline{W}) \right) \left(\frac{\lambda\phi\gamma\theta_{\min}^{\kappa}}{\delta(\kappa-2)} \right) \right]^{\frac{1}{\kappa-2}} = \bar{x} \tag{123}$$

That is, $\overline{W}$ solves

$$V_1'(\overline{W}) = \frac{\kappa}{2}\left(\frac{1}{\gamma} - \frac{\delta}{\rho} \right) \tag{124}$$

Meanwhile, substituting $x = \theta_{\min}$ into (73) implies that there exists $\underline{W}$ such that

$$\theta_{\min} = x(W) = \left[\left(1 - \left(\frac{2\gamma}{\kappa}\right) V_1'(\underline{W}) \right) \left(\frac{\lambda\phi\gamma\theta_{\min}^{\kappa}}{\delta(\kappa-2)} \right) \right]^{\frac{1}{\kappa-2}} \tag{125}$$

That is, $\underline{W}$ solves

$$V_1'(\underline{W}) = \left(1 - \frac{\delta(\kappa-2)}{\lambda\phi\gamma\theta_{\min}^2} \right) \frac{\kappa}{2\gamma} \tag{126}$$

The existence of both $\{\underline{W}, \overline{W}\}$ requires that $\overline{W} > \underline{W}$, which is equivalent to

$$\frac{\kappa}{2}\left(\frac{1}{\gamma} - \frac{\delta}{\rho} \right) < \left(1 - \frac{\delta(\kappa-2)}{\lambda\phi\gamma\theta_{\min}^2} \right) \frac{\kappa}{2\gamma} \tag{127}$$

or

$$\lambda\phi\theta_{\min}^2 > \frac{\rho(\kappa-2)}{\gamma^2}$$

while (28) ensures $V_1'(\overline{W}) > 0$. $\square$

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