Stress Tests and Model Monoculture*

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Abstract

We study whether regulators should reveal stress test results which contain imperfect information about banks' financial health. Although disclosure restores market confidence in banks, it misclassifies some healthy banks as risky. This encourages banks to choose portfolios that are deemed safe by regulators, leading to *model monoculture* and making the financial system less diversified. Optimal policy involves a commitment to a bang-bang policy that is non-monotonic in the severity of adverse selection problems: the regulator should fully reveal stress test results when adverse selection is very severe or very mild, but should never disclose the results otherwise.

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1 Introduction

Stress tests have become a key tool for supervisors since the 2007 – 2009 Global Financial Crisis. By providing credible information about the financial health of banks, stress tests can restore confidence in the banking system in a crisis; a testing regime can also guard against the accumulation of risks that can lead to a financial crisis. However, stress tests cannot perfectly predict how banks will react to all possible adverse shocks. Tests use only a few stress scenarios, so will inevitably leave out some sources of risk; they may not even correctly predict how bank capital evolves in the stress scenarios that do materialize.[1] This raises the concern that routine stress testing and full disclosure of test results may lead to *model monoculture* (Bernanke, 2013; Schuermann, 2013).[2] That is, banks may ignore their own, possibly superior risk management systems, and instead choose portfolios to game the regulators' models, leaving the whole banking system vulnerable to risks which are not well captured by these models – such as the ongoing effects of the COVID-19 pandemic.

The discussion above highlights a tradeoff in the design of supervisory stress test policies: disclosing (imperfect) stress test results can reduce uncertainty in financial markets, but may increase systemic risk by encouraging model monoculture. In this paper, we ask how regulators should optimally trade off these costs and benefits when deciding whether to release stress test results.

To address this question, we develop a stylized theoretical model to capture the tradeoff associated with supervisory stress tests. In our model, "good" banks choose ex ante whether to invest in either type γ or type δ financial project. Each project yields a positive return in a different state of the economy: project γ gives a positive return in state G but zero return in state D, and project δ gives a positive return in state D but yields zero return in the

[1] Frame, Gerardi and Willen (2015) present a cautionary example. Before the outbreak of the subprime mortgage crisis, the Office of Federal Housing Enterprise Oversight (OFHEO) conducted a stress test regarding the adequacy of the risk-based capital of government-sponsored enterprises (GSEs) such as Fannie Mae and Freddie Mac. However, the OFHEO's stress test underestimated both the downside risk to house prices – the adverse house price scenario used in the stress test (an 11 percent peak-to-trough decline) was less severe than the actual decline, ex post (18 percent) – and the GSEs' exposure to this risk. As a result, realized defaults were 4-5 times greater than those predicted by OFHEO's model, and Fannie Mae and Freddie Mac were both deemed insolvent in September 2008, even though OFHEO's stress test reported that the GSEs were well capitalized as late as the first quarter of 2008.

[2] Gillian Tett, a columnist in the Financial Times, also expressed a similar concern: "what (stress test) models cannot measure is the chance that banks all act as a herd — creating financial panics" ("Stress tests for banks are a predictable act of public theatre" *The Financial Times*, March 5, 2015).

other state.[3] After choosing which type of the project to undertake, each bank must borrow funds from competitive outside investors to continue the project. In addition, some measure of "bad" banks, which can only invest in type β projects yielding zero return in any state, also attempt to raise funds. The population of these bad banks is a random variable and realized when the capital market is open. The outside investors cannot observe what kind of financial project each bank chooses, which creates an adverse selection problem: if the fraction of bad banks is high enough, banks' cost of funding increases and they may be unable to continue their projects.[4]

To resolve the adverse selection problem, the regulator can conduct a stress test and disclose its result to outside investors. However, the regulator's information is imperfect – it cannot assess all the risks banks are exposed to. Specifically, the regulator can conduct a stress test based on state G, and determine whether each bank will have a positive return in this state, but she has no ability to assess whether banks will have a positive return in state D. As a result, the regulator can verify whether or not each bank invests in project type γ, but cannot distinguish type δ banks from type β banks. We assume that before the good (non-β) banks choose their projects and the fraction of type β banks is realized, the regulator can commit to a disclosure policy contingent on the fraction of type β banks, which specifies the probability $\alpha \in [0, 1]$ with which a randomly selected type γ bank is revealed to pass the test. Test results are revealed to outside investors before all banks issue bonds to fund their projects but after good banks choose their types and the fraction of type β banks is realized.

Although the regulator's information is imperfect, releasing this information can alleviate adverse selection problem in the capital market *ex post*, i.e., after all good banks choose their types and the fraction of bad banks is realized. Specifically, releasing information always weakly increases ex-post social welfare, for two reasons. First, disclosure fully eliminates adverse selection for type γ banks, allowing them to be fully funded. Second, there is less socially wasteful lending to type β banks: outside investors believe that failing banks are likely to be those of type β, and therefore curtail lending to them. In addition, these ex-post welfare benefits of disclosure are larger, the more severe the adverse selection problem in the capital market.

Despite these benefits, a policy of full disclosure of stress test results (i.e., $\alpha = 1$ with probability 1) makes the financial system more exposed to risks which are not captured by the

[3]As we discuss below, a bank's project can be alternatively interpreted as its choice of portfolio or risk model.

[4]Throughout, we call a bank running a type γ, δ, or β financial project a type γ, δ, or β bank, respectively.

regulator's model (i.e., the realization of state D). Banks know that the only way to pass the stress test and thus easily obtain external financing is to choose project γ. However, if banks choose project δ, they will definitely fail the test, increasing their cost of borrowing due to the severe adverse selection. Knowing this, an inefficiently large fraction of banks will choose project γ rather than δ, making the banking system relatively under-diversified compared to the case without disclosure.[5] As a result, the whole economy becomes more vulnerable to state D in which project γ gives zero return. We interpret this phenomenon as model monoculture: too many banks choose portfolios according to the regulator's model which focuses on the risk realized in state G, while relatively few banks adjust their portfolios to prepare for an alternative risk realized in state D.

Given this tradeoff, we next ask what disclosure policy maximizes social welfare *ex ante*. The optimal policy features corner solutions for α, i.e., $\alpha \in \{0, 1\}$ for each fraction of β banks. In particular, the optimal policy exhibits non-monotonicity with respect to the severity of adverse selection: the regulator should commit to reveal all the type γ banks ($\alpha = 1$) when the fraction of type β banks is either very high or very low, but should never disclose any type γ bank ($\alpha = 0$) otherwise.

To see why, suppose there are sufficiently many type β banks. When adverse selection problems are severe, disclosure increases welfare by reducing adverse selection, and this benefit is higher if the regulator discloses more information, or if adverse selection becomes more severe. On the other hand, the welfare cost of disclosure also arises because disclosure adversely incentivizes good banks to choose type γ. If the regulator were to disclose only a small fraction α of γ banks – confirming to outside investors that these are good banks – these good banks will borrow at very low interest rates, due to their scarcity in the capital market. Since being identified as a good bank is very profitable, even if unlikely, many good banks will choose project γ, worsening model monoculture. Thus, any disclosure policy with a small $\alpha > 0$ is strictly dominated by zero disclosure ($\alpha = 0$). However, if the regulator were to disclose a high fraction α of γ banks, these "confirmed-to-be-good" banks are no longer scarce, so their borrowing terms are only slightly favorable, relative to the banks whose type is not identified by the regulator. It follows that a marginal increase in α, starting from a high level, only slightly increases the incentive to choose project γ. In other words, the marginal welfare cost of disclosure decreases with α, which implies that the net marginal benefit of disclosure is

[5] This explains our choice of notation for the states G and D and project types γ and δ. While γ projects are known by the regulator to be *good* because they perform well under its stress scenario (state G), an additional δ project increases *diversification* in the financial system – since an inefficiently large fraction of banks choose project γ in equilibrium – but is *disfavored* by the regulator's model.

increasing in the degree of disclosure α increases as the regulator discloses more stress test results. Consequently, the optimal policy is bang-bang: the regulator either discloses all γ banks, or none of them. In particular, since the net benefit from disclosure is increasing in the severity of adverse selection b, full disclosure is optimal when adverse selection is sufficiently severe, and zero disclosure is optimal for moderate values of b.

Next, suppose there is a small number of type β banks. Under mild adverse selection, the good banks, regardless of their type, are fully funded even in the absence of the stress test. Thus, disclosing stress test results only influences the good banks' *ex-ante* incentive to choose project γ, and thus model monoculture; it does not affect the degree to which banks' projects are funded. Since type δ banks are scarce relative to type γ banks, outside investors are willing to pay a high price for bonds issued by type δ banks – if these banks can be identified as such – as a hedge against the risk associated with state D. With a small number of type β banks, banks whose types are not identified by the regulator are likely to be of a "non β" type. If the regulator increases α, i.e. they reveal a large fraction of type γ banks, investors will deem it more likely that a remaining "unclassified" bank is type δ. Consequently, these "unclassified" banks will enjoy a *scarcity* premium — a low borrowing cost relative to the banks identified as type γ — making project δ more profitable, relative to project γ. Thus from an ex ante perspective, fully disclosing the regulator's information when the fraction of β banks is low reduces the incentive to choose project γ, reducing the model monoculture problem.

The remainder of this paper proceeds as follows. Section 2 presents our model. Section 3 characterizes the partial equilibrium at date 1. In Section 4, we characterize the full equilibrium at date 0 and find an ex-ante optimal disclosure policy. In Section 5 we extend our baseline model to discuss how our results depend on investors' risk aversion and on the extent to which stress tests are predictably biased. Section 6 reviews related work. Finally, Section 7 concludes the paper. All proofs are relegated to Appendix.

2 Model

There are three dates: $t = 0, 1, 2$. At date 0, there is a continuum of "good" banks with measure 1 that undertake long-term financial projects that create stochastic returns at date 2. Moreover, a continuum of outside investors with measure 1 fund those projects at date 1 by purchasing bonds issued by the banks. Finally, a regulator, who seeks to maximize social welfare, decides whether to reveal information about banks at date 1, before outside investors purchase bonds. At date $t = 2$, the returns of the financial projects are realized and accrue to the banks and the outside investors.

At date 0, nature randomly draws a state $\omega \in \{G, D\}$, where $Pr(\omega = G) = p \in (0, 1)$; the two states represent different crisis scenarios. No one knows the true state until $t = 2$. After the underlying state of the economy is drawn, each "good" bank chooses one of two financial projects $\theta \in \{\gamma, \delta\}$, where project γ returns one unit of cash flow at $t = 2$ if $\omega = G$ and nothing if $\omega = D$, and project δ returns one unit of cash flow at $t = 2$ only in state $\omega = D$. As will become clear, state G represents a crisis scenario which is well-understood by the regulators and captured in their stress test models, while state D represents a scenario which is not. We prefer to think of the "project" as the bank's portfolio: project γ represents a portfolio robust to the risks captured by the regulators' models, while δ represents a portfolio vulnerable to these risks. One can also interpret the banks' choice of project as a choice of risk model. That is, choosing project γ can be interpreted as choosing the same risk model as the regulator (and choosing a portfolio based on that model), while choosing δ can be interpreted as choosing a different risk model.

After the good banks choose their projects, a continuum of "bad" banks with measure $b \geq 0$ enters. Their project – henceforth called type β – produces zero output in every state, but yields a private benefit to each bad bank. Thus these banks always strictly prefer to continue their projects, if they can do so at zero cost.[6] The measure of these bad banks $b \geq 0$ is a random variable with a continuous distribution function $F(\cdot)$ over the support $[0, \infty)$. This captures the idea that adverse selection problems may be more or less severe at different times and allows us to ask how disclosure policy should optimally adjust to high- and low-stress episodes.[7] We call each bank's financial project $\theta \in \{\gamma, \delta, \beta\}$ selected at $t = 0$ its "type."

At $t = 1$, banks – including the bad ones – must raise $x \in (0, 1)$ dollars to continue their financial projects. They raise these funds by issuing bonds to a continuum of outside investors with measure one. We assume

$$x < \min\{p, 1 - p\}, \tag{1}$$

[6] When we study optimal policy, we will assume the private benefit accruing to bad banks is not added to the regulator's social welfare function. The reader is welcome to assume that the bad banks' private benefit converges to zero.

[7] Flannery, Kwan and Nimalendran (2013) document empirically that bank opacity, or the accuracy with which outsiders can assess a bank's value, varies substantially over time and that it increased substantially during the 2007-2009 Global Financial Crisis.

which guarantees that investing in any good project $\theta \in \{\gamma, \delta\}$ has a positive net present value. The bidding game is described in detail in Appendix B. In brief, the bonds issued by the banks are sold as follows. Each investor $i \in [0, 1]$ places a buy order (q_i, R_i) such that the investor offers to purchase q_i units of bonds for repayment terms R_i per unit of capital. Whenever possible, trade takes place at a market-clearing interest rate R such that the supply of bonds issued by banks equals the amount demanded by investors requesting a rate of return R or lower. At this market-clearing interest rate, each bank is matched with a representative sample of investors, who lend to the banks at the rates promised. However, if the total supply of bonds exceeds the demand at the highest interest rate R_i offered by investors, banks are stochastically rationed such that only some fraction of banks are matched with investors. Finally, at date 2, each bond-holder receives R_i per bond issued by a solvent bank, i.e. a bank whose payoffs from its project exceed its liabilities. If the liabilities of a bank exceed the return from its project, we assume for simplicity a 100% bankruptcy cost, i.e., the bank's equity holders receive nothing. Appendix B shows that all investors place the same buy order, i.e., $(q_i, R_i) = (q, R)$ for all $i \in [0, 1]$ in any equilibrium.

Every investor has preferences represented by

$$U(c_1, c_2) = c_1 + \mathbb{E}[u(c_2)] = c_1 + pu(c_G) + (1 - p)u(c_D),$$

where c_1 is consumption at date 1, and c_G, c_D denote consumption at date 2, in states G and D respectively. Our baseline model assumes $u(x) = \log x$; thus investors' utility becomes[8]

$$U(c_1, c_2) = c_1 + p \log c_G + (1 - p) \log c_D. \tag{2}$$

Note that since the investors are risk averse, they would ideally like to hold bonds issued by both γ and δ banks to smooth consumption across states G and D at date 2. Each investor is endowed with a sufficiently large amount of cash to fund all the banks' projects at date 1. For analytical simplicity, we further assume that each investor owns an equal amount of shares issued by every bank. Under this assumption, all residual returns of the banks after repaying the bonds are equally distributed to the investors; thus, social welfare is simply calculated as the aggregated expected utility of the investors. All the good banks, whether type γ or δ, behave in the best interest of their equity holders (i.e., the investors). Finally, each bank's type is private information, which creates an adverse selection problem.

[8] In Section 5.1, we show that our results are not qualitatively changed if investors have CRRA utility with risk aversion greater than log.

A *regulator* maximizes the social welfare by deciding whether to reveal its superior but imperfect information about the banks' private types before the capital market opens at date 1. Specifically, the regulator can conduct a stress test to determine whether each bank's project pays off in state G (a crisis scenario which is well-understood by the regulator and captured in its models), but cannot determine whether the bank's project pays off in state D (which is not well-understood). All banks are subject to the regulator's stress test. Thus, after conducting the stress test, the regulator can distinguish type γ banks from the others, but cannot distinguish between type δ and β banks. Formally, we assume that for each individual bank indexed by $k \in [0, 1+b]$, the regulator has access to a signal $\tau_k \in \{0, 1\}$ about bank k's type $\theta_k \in \{\gamma, \delta, \beta\}$ with the following conditional probabilities:

$$Pr(\tau_k = 1|\theta_k = \gamma) = 1,\ Pr(\tau_k = 1|\theta_k = \delta) = 0,\ Pr(\tau_k = 1|\theta_k = \beta) = 0. \tag{3}$$

We assume that the regulator can choose the probability α with which she discloses type γ banks. Formally, the regulator chooses a disclosure policy $\alpha \in [0, 1]$ such that the regulator reveals its information on each type-γ bank with probability α. The regulator receives this information before the banks enter the capital market. Hence, if the regulator discloses its information on bank k's type θ_k, the investors update their belief about θ_k and place their buy orders accordingly.

To study how different policy regimes affect diversification in the financial system, we assume that the regulator can commit to a *disclosure policy* $\alpha : [0, \infty) \to [0, 1]$ before banks choose their types at $t = 0$, where $\alpha(b)$ is the probability that a type γ bank is disclosed given that the measure of bad banks is b. A policy in which the regulator always discloses her information ($\alpha(b) = 1$ for all $b > 0$) is analogous to routinized stress tests, such as the DFAST and CCAR for large-sized U.S. financial firms. Conversely, the stress tests conducted contingently in the event of financial crises can be thought of as a policy of revealing information only when adverse selection at financial market are very severe (i.e., b is high).[9] Throughout, we abbreviate θ_k, τ_k and σ_k to θ, τ and σ, respectively, since all banks with the same type behave symmetrically in equilibrium, as we will show later. We occasionally refer to the banks whose types are not revealed by the regulator as the ones "failing" the stress

[9] Our assumption that the regulator observes the fraction of bad banks b, but not which banks are bad, is intended to capture the idea that policymakers have some information about the severity of adverse selection and the funding pressures facing banks, so that it is feasible to follow a policy of only disclosing stress test results when adverse selection is especially severe. Indeed, the Board of Governors chose to disclose information on bank-level losses under the 2009 SCAP precisely in order to reduce uncertainty during the financial crisis:

test, and refer to type γ banks whose type is revealed by the regulator as "passing" the stress test.

Our stylized model abstracts from a number of features of real-world stress test for ease of exposition. First, in reality, banks are directly penalized for failing stress tests: for example, under CCAR, the Fed forbids failing banks from making dividend payments or stock repurchases. We abstract from these direct consequences, to focus on the indirect costs that arise because investors are less willing to lend to failing banks. Incorporating direct costs would only strengthen our main result that disclosure can reduce diversification in the financial system.

Another simplifying assumption is that some banks fail in every state of the world, and the regulator's information concerns whether a bank's project will pay off unconditionally. In reality, banks generally only fail in a crisis, and thus stress tests concern the probability of banks' capital shortfall conditional on a crisis. To model this explicitly, we could introduce a shock $s \in \{0, 1\}$ realized at date 1, where $s = 0$ represents "normal times" and $s = 1$ represents a rare, adverse event. If $s = 0$, all projects pay off 1 for sure and banks do not require funding to continue their projects. If $s = 1$, the model proceeds as described earlier, and projects may not pay off at date 2 depending on the value of $\omega \in \{G, D\}$. Clearly, maximizing the ex-ante bank profits (respectively, social welfare) will be equivalent to maximizing profits (respectively, social welfare) in the adverse state. It is thus without loss of generality to assume that the adverse scenario always realizes.[10] This simplifies the model and restricts our attention to outcomes in a crisis scenario, which are relevant to stress tests.

Lastly, we assume that the total measure of the good banks is fixed but the measure of the bad banks is random. One may argue that a more plausible source of variations in the severity of adverse selection problems would be fluctuations in the *proportion* of bad banks, keeping the total measure fixed. We do not follow this approach in our model, because, as we describe in Appendix C, such an alternative framework turns out to be less tractable

"The unprecedented nature of the SCAP, together with the extraordinary economic and financial conditions that precipitated it, has led supervisors to take the unusual step of publicly reporting the findings of this supervisory exercise. The decision to depart from the standard practice of keeping examination information confidential stemmed from the belief that greater clarity around the SCAP process and findings will make the exercise more effective at reducing uncertainty and restoring confidence in our financial institutions." (Board of Governors of the Federal Reserve System, 2009, p.1)

[10] Another possibility is that the regulator can disclose information in "normal times," before the adverse scenario is realized. In this case, it may no longer literally be true that maximizing ex ante profits or welfare is equivalent to maximizing profits or welfare in the crisis state.

while delivering qualitatively similar results only in a restricted setting of the investors' risk appetite.

3 Ex-post Equilibrium at Date 1

In this section, we describe the partial equilibrium at date 1 after the good banks choose their types and the fraction of type β banks $b \geq 0$ is realized.

3.1 Partial Equilibrium Absent Stress Tests

As a benchmark, we first characterize the partial equilibrium at date 1 when the regulator does not release any information. Let $g \in [0, 1]$ and $d := 1 - g$ be the fraction of the banks with type γ and type δ, respectively. After choosing their types at date 0, the banks with types γ and δ, whose total measure is 1, enter the capital market at date 1. If a bank with type γ (respectively, δ) receives the funds x required to continue its project, then its project produces a unit return in state G (respectively, D) at date 2. Hence, the return on the individual bank's project per dollar is either $\overline{R} := 1/x > 1$ or 0.

We analyze how the investors strategically place their buy orders at the capital market. Since investors do not know whether a bank is type γ, δ, or β, an investor who buys bonds will end up meeting type γ banks with probability $\pi_g = \frac{g}{1+b}$ and type δ banks with probability $\pi_d = \frac{d}{1+b}$, respectively. An investor who succeeds in executing the buy order (q_i, R_i) receives expected utility

$$p \log(y_G + q_i \pi_g R_i x) + (1 - p) \log(y_D + q_i \pi_d R_i x) - q_i x,$$

where $y_G := \int_0^1 (q_i \pi_g (1 - R_i x))di$ and $y_D := \int_0^1 (q_i \pi_d (1 - R_i x))di$ are state-dependent dividends equally distributed to each investor in states $\omega \in \{G, D\}$, respectively. At $t = 1$, the investor pays $q_i x$ for purchasing the bonds from q_i measure of banks. If state G is realized with probability p in $t = 2$, then each investor i receives the debt repayments $R_i x$ from type γ banks out of which sell the bonds to this investor. Hence, the total amount of the debt repayments is $q_i \pi_g R_i x$. In addition, the same investor, as an equity holder, also receives the residual returns y_G as dividends from every bank with type γ. Similarly, if state D is realized with probability $1 - p$, each investor i receives the debt repayments with the total amount of $q_i \pi_d R_i x$ and the dividends y_D from the banks with type δ. As mentioned above, Appendix B shows that all investors place the same buy order in the trading mechanism described in Section 2: in any equilibrium, $(q_i, R_i) = (q, R)$ for all $i \in [0, 1]$.

Given a symmetric buy order (q, R), it is convenient to work with $\phi = \frac{q}{1+b} \in [0, 1]$, the proportion of the banks that obtain funding, rather than q, the total measure of the banks who obtain funding. Then, each investor, as a debt-holder, receives the following expected payoff:

$$p \log(y_G + \phi(1+b)\pi_g Rx) + (1-p)\log(y_D + \phi(1+b)\pi_d Rx) - \phi(1+b)x.$$

Since $\pi_g = \frac{g}{1+b}$ and $\pi_d = \frac{1-g}{1+b}$, this can be rewritten as

$$p \log(y_G + \phi g Rx) + (1-p)\log(y_D + \phi(1-g)Rx) - \phi(1+b)x. \tag{4}$$

Given the unit repayment R, each individual investor (weakly) prefers buying $\phi(1+b)$ bonds if and only if the marginal expected utility of investing in the banks is non-negative:

$$p\frac{gRx}{y_G + \phi g Rx} + (1-p)\frac{(1-g)Rx}{y_D + \phi(1-g)Rx} - (1+b)x \geq 0. \tag{5}$$

Since $y_G = \phi g(1 - Rx)$ and $y_D = \phi(1-g)(1-Rx)$, the equation (5) can be simplified as

$$\frac{R}{\phi} - (1+b) \geq 0. \tag{6}$$

The first term in (6) captures the marginal benefit of funding an extra bank at given (ϕ, R) per unit of capital; this is decreasing in ϕ because investors exhibit diminishing marginal utility from the date-2 consumption. The second term captures the marginal cost of funding an additional bank per unit of capital, which is increasing in b: the larger b is, the more likely a lender is to find herself lending to an unproductive type β bank, and the less willing she is to buy a bond. These properties allow us to characterize partial equilibrium at date 1.

Theorem 1. *If the regulator does not reveal her information at $t = 1$, there exists a unique partial equilibrium in which every investor places the same buy order (ϕ^*, R^*), where*

$$(\phi^*, R^*) = \begin{cases} (1, 1+b) & \text{if} \quad b \leq \frac{1}{x} - 1, \\ \left(\frac{1}{x(1+b)}, \overline{R}\right) & \text{if} \quad b > \frac{1}{x} - 1. \end{cases} \tag{7}$$

Proof. See Appendix A.1. *Q.E.D.*

Theorem 1 reveals that adverse selection in the capital market can lead to credit rationing. If the adverse selection problem is mild (i.e., $b \leq \frac{1}{x} - 1$), investors are unlikely to buy bonds issued by the bad banks, which makes the marginal cost of the bond purchase relatively low. Thus, the investors fund every bank ($\phi^* = 1$) and are willing to pay a relatively high price for the bank bonds ($R^* \leq 1/x$). However, as the fraction of the bad banks increases, the banks must offer investors a higher return to compensate for the increased risk of funding bad banks. If the adverse selection becomes severe (i.e., $b > \frac{1}{x} - 1$), the marginal cost of the bond purchase becomes too high for the investors to fund every bank even at a hefty premium ($R^* = 1/x$). In this case, only a fraction $\phi^* < 1$ of the banks receive external funding. In particular, credit rationing worsens as the proportion of the bad banks increases.

3.2 The Ex-Post Effects of the Regulator's Information

In this section, we study how the regulator can alleviate adverse selection by providing its information to the market. Recall that, under the regulator's disclosure policy $\alpha \in [0, 1]$, banks labeled $\sigma = 1$ are known to be type γ for sure, whereas banks labeled $\sigma = 0$ may be type γ with probability $\frac{(1-\alpha)g}{1-\alpha g+b}$, type δ with probability $\frac{1-g}{1-\alpha g+b}$, or type β with probability $\frac{b}{1-\alpha g+b}$. The regulator's information effectively segments the capital market into two submarkets: one for the fraction α of type γ banks (called "$\sigma = 1$ market") and the other for the remaining γ banks and all δ and β banks (called "$\sigma = 0$ market"). Now, each investor $i \in [0, 1]$ places buy orders $\{(\phi_{\sigma,i}, R_{\sigma,i})\}_{\sigma\in\{0,1\}}$ to the banks at the $\sigma = 0$ and $\sigma = 1$ markets, respectively. The results in Appendix B imply, as above, that every investor $i \in [0, 1]$ places the same buy orders in equilibrium: $\{(\phi_{\sigma,i}, R_{\sigma,i})\}_{\sigma\in\{0,1\}} = \{(\phi_\sigma, R_\sigma)\}_{\sigma\in\{0,1\}}$ for all investors $i \in [0, 1]$.

We next study how investors' buy orders are determined in equilibrium. Each investor receives the expected payoff

$$p \log\left(y_G + q_1 R_1 x + \frac{(1-\alpha)g}{1-\alpha g+b} q_0 R_0 x\right) + (1-p)\log\left(y_D + \frac{1-g}{1-\alpha g+b} q_0 R_0 x\right) - q_0 x - q_1 x.$$

Again, rather than working with q_0 and q_1, it is convenient to work with the fraction of $\sigma = 0$ and $\sigma = 1$ banks, respectively, that obtain funding. Defining $\phi_0 = \frac{q_0}{1-\alpha g+b}$, $\phi_1 = \frac{q_1}{\alpha g}$ for $\phi_0, \phi_1 \in [0, 1]$, we can rewrite investors' expected utility as

$$p \log(y_G + \phi_1 \alpha g R_1 x + \phi_0(1-\alpha)g R_0 x) + (1-p)\log(y_D + \phi_0(1-g)R_0 x) - \phi_0(1-\alpha g+b)x - \phi_1 \alpha g x,$$

where $y_G := \phi_1 g(1-R_1x) + \phi_0(1-\alpha)g(1-R_0x)$ and $y_D := (1-g)\phi_0(1-R_0x)$ are the residual income paid as dividends to investors in states G and D, respectively. At the $\sigma = 0$ market, investors, as bond-holders, are willing to place the buy order (q_0, R_0) if and only if the marginal expected utility of purchasing bonds purchase is non-negative:

$$p\frac{(1-\alpha)gR_0}{\alpha g + (1-\alpha)g\phi_0} + (1-p)\frac{(1-g)R_0}{(1-g)\phi_0} - (1-\alpha g + b) \geq 0 \tag{8}$$

Similarly, at the $\sigma = 1$ market, investors are willing to place the buy order (q_1, R_1) if and only if

$$p\frac{\alpha g R_1 x}{\alpha g\phi_1 + (1-\alpha)g\phi_0} - \alpha g x \geq 0 \tag{9}$$

From the first-order conditions (8) and (9), we can characterize the date-1 partial equilibrium after the regulator discloses the information as follows.

Theorem 2. *Suppose that the regulator discloses γ banks with probability α. Then, there exists a unique partial equilibrium in which the investors place the buy orders (ϕ_0^*, R_0^*) to the banks labelled $\sigma = 0$ and (ϕ_1^*, R_1^*) to the banks labelled $\sigma = 1$. Specifically, (ϕ_1^*, R_1^*) is determined by*

$$(R_1^*, \phi_1^*) = \left(\frac{(\alpha + (1-\alpha)\phi_0^*)g}{p}, 1\right). \tag{10}$$

Furthermore, (ϕ_0^*, R_0^*) is determined by

$$(R_0^*, \phi_0^*) = \left(\frac{(1-\alpha)g + (1-g) + b}{(1-\alpha)p + (1-p)}, 1\right) \tag{11}$$

if $\frac{(1-\alpha)g+(1-g)+b}{(1-\alpha)p+(1-p)} \leq \frac{1}{x}$, but $R_0^* = \frac{1}{x}$ and ϕ_0^* is implicitly determined by the following indifference condition

$$p\frac{1}{\frac{\alpha}{1-\alpha} + \phi_0^*} + (1-p)\frac{1}{\phi_0^*} = ((1-\alpha)g + (1-g) + b)x. \tag{12}$$

if $\frac{(1-\alpha)g+(1-g)+b}{(1-\alpha)p+(1-p)} > \frac{1}{x}$.

Proof. See Appendix A.1. *Q.E.D.*

Since the regulator's information is imperfect, releasing it only benefits type γ banks, whereas it worsens the adverse selection problem faced by the other banks with type δ. Specifically, the adverse selection problem is fully eliminated for the fraction α of type γ banks whose type is disclosed: all these banks successfully sell their bonds to the investors (i.e., $\phi_1^* = 1$ for all $b \geq 0$) without paying any lemons premium (i.e., $R_1^* < \frac{1}{x}$). However, the banks consisting of the remaining type γ banks (with the fraction $1-\alpha$) as well as all type δ banks, are lumped together with the bad banks. Thus, these "failing" banks suffer a more severe adverse selection problem compared to the case with no disclosure. Indeed, these undisclosed banks face even tougher financing conditions as the regulator discloses more information to the market (i.e., α increases). This can be observed by applying the Implicit Function Theorem to (12), which yields

$$\frac{d\phi_0^*}{d\alpha} = -\frac{p\left(\frac{1}{\alpha+(1-\alpha)\phi_0^*}\right)^2 - gx}{p\frac{1}{\left(\frac{\alpha}{1-\alpha}+\phi_0^*\right)^2} + (1-p)\frac{1}{\left(\phi_0^*\right)^2}} < 0. \tag{13}$$

Since $\phi^* = \lim_{\alpha\to 0}\phi_0^*$, (13) also implies $\phi^* \geq \phi_0^*$ for every $\alpha > 0$. Furthermore, for a fixed disclosure policy α, these "undisclosed" banks suffer a more severe adverse selection problem as the population of type β banks increases:

$$\frac{d\phi_0^*}{db} = -\frac{x}{p\frac{1}{\left(\frac{\alpha}{1-\alpha}+\phi_0^*\right)^2} + (1-p)\frac{1}{\left(\phi_0^*\right)^2}} < 0. \tag{14}$$

One noteworthy feature from (10) and (14) is that the type-γ banks passing the stress test enjoy more favorable borrowing conditions as b increases. On the flip side, disclosure of the regulator's information worsens the adverse selection problem faced by the failing banks. In other words, buyers' willingness to purchase the high-quality asset certified by the regulator grows stronger as the alternative asset is believed to be a lemon with a higher likelihood.

3.3 The Ex-Post Optimal Disclosure Policy

Finally, we study whether disclosure improves social welfare from an ex post perspective (i.e., holding the respective fractions g of type γ banks and b of bad banks fixed). Ex-post social welfare equals the sum of all investors' expected utilities; since every investor places the same buy order in equilibrium, this equals the expected utility of each investor, which can be defined as a function of (g, α, b):

$$U(g,\alpha,b) := p\log(\alpha+(1-\alpha)\phi_0^*)g + (1-p)\log\phi_0^*(1-g) - gx(\alpha+(1-\alpha)\phi_0^*) + ((1-g)+b)\phi_0^* x.$$

This can be further simplified to

$$U(g,\alpha,b) = p\log g + (1-p)\log(1-g) \\ + \left[p\log(\alpha+(1-\alpha)\phi_0^*) - gx(\alpha+(1-\alpha)\phi_0^*)\right] \\ + \left[(1-p)\log\phi_0^* - ((1-g)+b)\phi_0^* x\right]. \tag{15}$$

In particular, ex-post welfare is equal to $U(g,0,b)$ if the regulator does not disclose any information. Thus, the net gain from the disclosure policy α is $U(g,\alpha,b) - U(g,0,b)$, which is expressed as

$$U(g,\alpha,b) - U(g,0,b) = \left[p\log(\alpha+(1-\alpha)\phi_0^*) - gx(\alpha+(1-\alpha)\phi_0^*)\right] \\ + \left[(1-p)\log\phi_0^* - ((1-g)+b)\phi_0^* x\right] \\ - \left[p\log\phi^* + (1-p)\log\phi^* - (1+b)\phi^* x\right], \tag{16}$$

Recall from (13) that $\frac{d\phi_0^*}{d\alpha} \leq 0$, where the inequality is strict for every $b > (1-\alpha)\left[\frac{1}{x}p - g\right] + \left[\frac{1}{x}(1-p) - (1-g)\right]$.

We first analyze the welfare impact of disclosure policies $\alpha \in [0,1]$. By applying the standard Envelope Theorem, we have

$$\frac{d}{db}\left[U(g,\alpha,b) - U(g,0,b)\right] = x(\overline{\phi}_0^* - \phi_0^*) \geq 0, \tag{17}$$

where the inequality, once again, is strict for all $b > (1-\alpha)\left[\frac{1}{x}p - g\right] + \left[\frac{1}{x}(1-p) - (1-g)\right]$ from (14). Hence, for any given disclosure rule $\alpha > 0$, the regulator weakly prefers disclosing its information $\alpha > 0$ for every $b \geq 0$ to disclosing zero information. In particular, the net gain from information disclosure is strictly positive when adverse selection at the capital market is relatively severe.

Next, we characterize the ex-post optimal disclosure policy α^* for every degree of adverse selection $b \geq 0$. By applying the standard Envelope Theorem again, we have

$$\frac{d}{d\alpha}\left[U(g,\alpha,b) - U(g,0,b)\right] = (1-\phi_0^*)\left[\frac{1}{\alpha+(1-\alpha)\phi_0^*}p - gx\right] \geq 0. \tag{18}$$

The bracketed term on the right side in (18) is strictly positive, given the assumption $p - x > 0$ and the facts that $\frac{1}{\alpha+(1-\alpha)\phi_0^*} \geq 1$ and $\phi_0^*, g \in [0,1]$. In particular, the whole inequality in (18) is strict if and only if $\phi_0^* < 1$, i.e., $b > (1-\alpha)\left[\frac{1}{x}p - g\right] + \left[\frac{1}{x}(1-p) - (1-g)\right]$. Thus, full

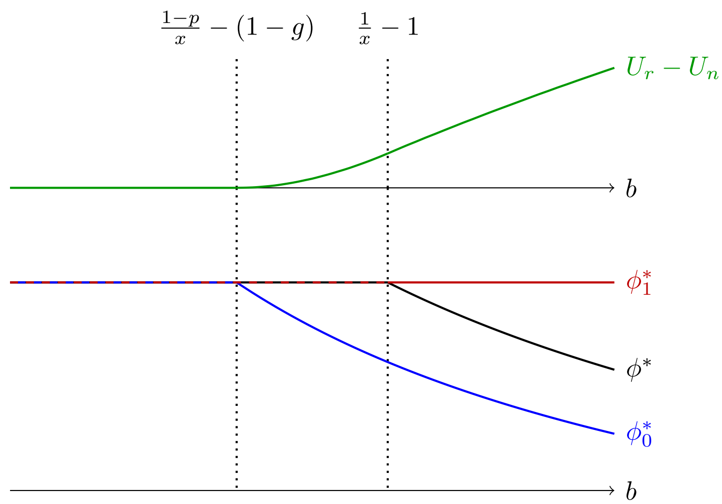


Figure 1 – The ex-post welfare gain from the full disclosure policy ($\alpha = 1$)

disclosure ($\alpha^* = 1$) is (weakly) ex-post optimal for every $b \geq 0$, and strictly optimal when b is sufficiently high. In words, information disclosure strictly improves ex-post welfare when the adverse selection problem is very severe. Furthermore, this welfare benefit is maximized by full disclosure.

Theorem 3. *The regulator maximizes ex-post welfare by choosing $\alpha^* = 1$ for every $b \geq 0$. In particular, the gain from disclosure is strictly positive if and only if $b \geq \frac{1}{x}(1-p) - (1-g)$.*

Proof. See Appendix A.1. *Q.E.D.*

As shown in Theorem 2, releasing the regulator's information can eliminate adverse selection for type γ banks (with fraction α), whereas it worsens adverse selection for type δ banks. Nevertheless, information disclosure always increases the ex-post welfare through two channels. First, type γ banks are fully funded even though there is a large population of type β banks at the capital market. That is, $\phi_1^* \geq \phi^*$ for all $b \geq 0$. Second, releasing information also reduces socially wasteful lending to unproductive type β banks. That is, after observing that a bank is labeled as $\sigma = 0$, the investors believe that the bank is more likely to be of type β and curtail lending to it. Figure 1 depicts how these two effects improve the ex-post social welfare for the case of full disclosure $\alpha = 1$. In particular, one can observe that $U(g,1,b) - U(g,0,b) > 0$ for all $b \in \left(\frac{1-p}{x} - (1-g), \frac{1}{x} - 1\right]$. In this range of b, the regulator's information lowers the fraction of type β banks that sell bonds from $\phi^* = 1$ to $\phi_0^* = \frac{1-p}{((1-g)+b)x} < 1$, thereby reducing wasteful lending to type β banks.

4 Ex-Ante Equilibrium and Optimal Disclosure Policy

We have shown that disclosing stress test results can improve social welfare by alleviating ex-post adverse selection in the capital market. However, disclosure also affects the relative return to γ and δ projects, and thus influences good banks' project choice *ex ante*. Good banks will find project δ less attractive when it is expected to suffer severe adverse selection problems relative to project γ. As a result, disclosure of the regulator's information may lead to *under-diversification* in the overall economy; there will be too many type γ banks and too few type δ banks, leaving investors with fewer projects that pay off, and lower consumption, in state D relative to state G. Thus, when designing an optimal disclosure policy, the regulator should take into account its effects on aggregate portfolio diversification.

In this section, we analyze how the regulator's *disclosure policy* $\alpha(b)$ influences each bank's ex-ante choice of financial project. Specifically, before banks choose their type $\theta \in \{\gamma, \delta\}$, the regulator commits to the fraction $\alpha(b)$ of type γ banks to be disclosed when $b \geq 0$, the degree of adverse selection, is realized at date 1. We investigate how each disclosure policy $(\alpha(b))_{\{b\geq 0\}}$ contributes to (in-)efficiency in diversification. Then we find the optimal disclosure policy given this tradeoff between ex-post adverse selection and ex-ante diversification.

We first study how banks choose their types $\theta \in \{\gamma, \delta\}$ at date 0 *ex ante*. Recall that every outside investor owns an equal share, i.e. is an *equity holder*, of each bank. Furthermore, there is no moral hazard between good banks and their equity holders: each bank aims to maximize the expected utility of its equity holders. Given a particular realization of $b \geq 0$ and a disclosure policy $\alpha \in [0, 1]$, the expected utility of each outside investor is

$$p \log(\alpha g(1 - R_1^* x) + (1 - \alpha) g\phi_0^*(1 - R_0^* x) + z_G) + (1 - p) \log((1 - g)\phi_0^*(1 - R_0^* x) + z_D) - C^*,$$

where $z_G := \alpha g R_1^* x + (1 - \alpha) g\phi_0^* R_0^* x$ is the ex-post debt repayment from type γ banks in state G, $z_D := (1 - g)\phi_0^* R_0^* x$ is the ex-post debt repayment from type δ banks in state D, and $C^* = \alpha g x + (1 - \alpha g + b)\phi_0^* x$ is the total investment to be made at date 1.

If a marginal bank chooses project γ rather than δ, equity holders' expected utility in state G will increase by the value of the type γ bank's dividend, multiplied by marginal utility in state G; their expected utility in state D will decrease by the value of the δ bank's dividend weighted by the marginal utility in state D. Thus, given a realization of b and the degree of disclosure in that state α, the *net* benefit to outside investors from an additional type γ bank, denoted by $\Delta(g, \alpha, b)$, is the difference between the probability-weighted marginal gains and losses from an extra type γ bank. Equivalently, $\Delta(g, \alpha, b)$ equals the derivative of expected utility with respect to g:

$$\Delta(g, \alpha, b) := \alpha \frac{p}{g} \left(\frac{1}{\alpha + (1-\alpha)\phi_0^*} - \frac{g}{p}x \right) + \left[\left(\frac{p}{g} \right) \frac{(1-\alpha)\phi_0^*}{\alpha + (1-\alpha)\phi_0^*} - \frac{1-p}{1-g} \right] (1 - R_0^* x). \quad (19)$$

The ex-ante marginal net benefit to the equity-holders from an extra type γ bank equals the expected value of $\Delta(g, \alpha, b)$ for $b \geq 0$. In any equilibrium arising from a disclosure policy $(\alpha(b))_{\{b \geq 0\}}$, the fraction $g \in (0, 1)$ of type γ banks adjusts to satisfy the following condition:

$$\int_0^\infty \Delta(g, \alpha(b), b) dF(b) = 0. \quad (20)$$

In equilibrium, every individual bank must be indifferent between choosing projects γ and δ *ex ante*. It is worth noting that $\Delta(g, \alpha, b)$ is decreasing in g, which implies that equity holders, like the regulator, value diversification. Specifically, a higher fraction of γ banks increases consumption of equity holders in state G relative to state D, and thus lowers the marginal value of having another bank that pays dividends out in state G. Note also that we cannot have a corner solution in which all banks strictly prefer the same project – either $g = 1$ and $\int_0^\infty \Delta(g, \alpha(b), b) dF(b) > 0$, or $g = 0$ and $\int_0^\infty \Delta(g, \alpha(b), b) dF(b) < 0$ – given our assumption that investors have log utility over date-2 consumption, which implies $\lim_{c \to 0} \frac{d}{dc} \log c = \infty$.

Crucially, any change in disclosure policies $(\alpha(b))$ shifts $\Delta(g, \alpha(b), b)$, and therefore influences overall diversification. For example, disclosing test results for high values of b pools type δ banks together with bad banks, thereby increasing the average borrowing cost of type δ banks as well as reducing their gross dividend payouts. Such a policy thus encourages good banks to choose project γ rather than δ, or equivalently, increases $\int_0^\infty \Delta(g, \alpha(b), b) dF(b)$. As a result, more good banks strategically respond by choosing project γ, increasing g so that (20) continues to hold. In this context, the equilibrium condition (20) summarizes how the disclosure of "biased" stress test results affects diversification.

Since social welfare is defined as the sum of all investors' expected utilities, the regulator's welfare maximization problem is:

$$\max_{\{\alpha(b) \in [0,1] \,:\, b \in [0,\infty)\}} \int_0^\infty U(g, \alpha(b), b) dF(b), \quad (21)$$

subject to (20). As a benchmark, it is useful to consider an unconstrained program in which the regulator maximizes (21) ignoring the constraint (20), i.e. she can simply dictate that measure $g \in (0,1)$ of banks choose type γ. In this relaxed optimization problem, the regulator would choose $g = p$ since full diversification is always socially desirable. Moreover, we know from Theorem 3 that the regulator can always weakly maximize the ex-post social welfare by fully disclosing her information. Therefore, an unconstrained-optimal disclosure policy would be $\alpha(b) = 1$ for every $b \geq 0$. The presence of the constraint (20), however, makes such an allocation unimplementable. Indeed, the following observation reveals that the regulator's information always reduces diversification in the financial system.

Lemma 1. *If a disclosure policy $\alpha(\cdot)$ yields $g = p$, then we must have $\alpha(b) = 0$ with probability 1. If $\alpha(b) > 0$ with positive probability, then we must have $g > p$. Under optimal disclosure policy, $g > p$.*

Proof. See Appendix A.2. *Q.E.D.*

The regulator's stress tests are biased against type δ banks: they correctly identify type γ banks as "good" ones, but do not distinguish between δ banks and bad banks.[11] Disclosure of such biased stress test results inevitably reduces diversification ($g > p$) relative to the first-best outcome ($g = p$). It is *feasible* to keep diversification at its first-best level, but this can only be implemented by completely refraining from disclosure (i.e., $\alpha(b) = 0$ with probability 1) only. Obviously, such a policy is strictly suboptimal because the regulator forgoes an opportunity to alleviate ex-post adverse selection. Instead, a constrained-efficient policy balances the benefit and cost of disclosing stress test results. In what follows, we describe in more details the structure of optimal disclosure policies which manage this tradeoff arising from the inherent bias in the regulator's stress test model. Throughout, we denote an ex-ante optimal disclosure policy by $\alpha^{**}(b)$ for every $b \geq 0$.

4.1 Optimal Disclosure under Mild Adverse Selection

We first characterize the optimal disclosure policy $\alpha(\cdot)$ for low values of $b \leq \frac{1-p}{x} - (1-g)$. When b is sufficiently low, banks are never rationed at the capital market even if they are labelled as "not good" (i.e., $\sigma = 0$). That is, we have $\phi_0^* = \phi_1^* = \phi^* = 1$ no matter what the

[11] Our model therefore has the empirical implication that disclosure of stress test results should cause banks to choose more similar portfolios. This is consistent with Bräuning and Fillat (2019), who find that stress-tested banks became more likely to have similar loan portfolios after the implementation of DFAST in 2011 (while this trend was not present for banks not subject to DFAST). In particular, banks with poor stress test results adjust their portfolios so that they become similar to the portfolios of the banks which performed well in the tests.

level of disclosure $\alpha \in [0,1]$ is. Consequently, disclosure in these states does not directly affect social welfare, i.e., $U(g,\alpha,b) = U(g,0,b)$ for every $\alpha > 0$. Hence, optimal disclosure policy will concentrate on minimizing the (negative) impact of the stress tests on diversification by adjusting $\Delta(g,\alpha,b)$. Since $\Delta(g,\alpha,b)$ is decreasing in g, the regulator's objective is to minimize $\int_0^\infty \Delta(g,\alpha(b),b)dF(b)$ in order to raise $(1-g)$, the total measure of type δ banks. The following lemma states how optimal disclosure policy accomplishes this goal.

Lemma 2. *There exists a* $\underline{b}^{**} := \frac{g}{p} - 1$ *such that any optimal disclosure policy* $\alpha^{**}(\cdot)$ *must have* $\alpha^{**}(b) = 1$ *for all* $b \leq \underline{b}^{**}$ *and* $\alpha(b) = 0$ *for all* $b \in \left(\underline{b}^{**}, \frac{1-p}{x} - (1-g)\right]$.

Proof. See Appendix A.2. *Q.E.D.*

Underlying Lemma 2 is the fact that $\Delta(g,\alpha,b)$ is strictly decreasing in α if $b < \underline{b}^{**}$ but strictly increasing if $b \geq \underline{b}^{**}$. Hence, the regulator can optimally mitigate the under-diversification problem by following a cutoff rule: full disclosure when the adverse selection problem is sufficiently mild (i.e., $b < \underline{b}^{**}$), and zero disclosure otherwise.

To understand the intuition behind Lemma 2, first recall from Lemma 1 that disclosure excessively reduces the supply of bonds issued by type δ banks (i.e., $1-g < 1-p$). Investors, who need to hedge the risk associated with state D, strongly prefer purchasing these scarce bonds issued by type δ banks even by paying a high premium. Thus, where possible, the regulator can mitigate the under-diversification problem by identifying type δ banks to the investors so that these banks can enjoy a *scarcity* premium in the capital market.

When b is sufficiently small, one way to identify δ banks to investors, at least partially, is to reveal all of type γ banks to investors. In this case, the remaining banks who are *not* identified as type γ are likely to be "good" δ banks, rather than "bad" β banks. Hence, full disclosure ($\alpha = 1$) maximizes the likelihood that these "undisclosed" banks have type δ, and thus maximizes the scarcity premium increasing the price of type δ bonds. From an ex-ante perspective, this improvement in the expected average funding conditions of type δ banks encourages good banks to choose project δ. However, if the fraction of bad banks b is not very small, identifying type γ banks implies that the remaining "undisclosed" banks are likely to be bad banks. In this case, full disclosure would worsen adverse selection for the "undisclosed" banks – in particular type δ banks – reducing the incentive to choose a δ project from an ex-ante perspective. Thus, for relatively high values of b, the regulator should adopt a zero disclosure policy to minimize the adverse effect on diversification.

4.2 Optimal Disclosure under Severe Adverse Selection

Next, we characterize optimal disclosure policy when $b > \frac{1-p}{x} - (1-g)$, i.e., adverse selection is relatively severe. In this case, credit rationing may take place at the $\sigma = 0$ submarket for banks with $\sigma = 0$ — i.e., banks who "fail" the stress test. For expositional convenience, we throughout work with the Lagrangian of the regulator's optimization problem (21) subject to (20), which is expressed as

$$\mathcal{L} = \int [U(g, \alpha(b), b) - \lambda \Delta(g, \alpha(b), b)] dF(b).$$

Note that the Lagrangian multiplier λ is strictly positive: under log utility, the constraint (20) is always binding for some interior values of $g \in (0,1)$. For each b, we next define the net welfare gains from a disclosure policy $\alpha > 0$ as follows:

$$W(g, \alpha, b) := [U(g, \alpha, b) - \lambda \Delta(g, \alpha, b)] - [U(g, 0, b) - \lambda \Delta(g, 0, b)]. \tag{22}$$

For example, a disclosure policy α increases welfare if the positive effect on the alleviation of adverse selection is large relative to the negative impact on the ex-ante diversification. Note that if a disclosure policy $\alpha^{**}(\cdot)$ is optimal, we must have $W(g, \alpha^{**}(b), b) > 0$ for any b such that $\alpha^{**}(b) > 0$.

In our model, the net benefit from revealing stress test results varies continuously with α, i.e. the proportion of type γ banks whose type is revealed to investors. However, this does not necessarily mean that an optimal disclosure policy $\alpha^{**}(b)$ must have interior values for every degree of adverse selection b. Instead – surprisingly – it is optimal for the regulator to disclose *all* type γ banks for some realizations of b, and none of type γ banks for all other realizations of b.

Lemma 3. $\alpha^{**}(b) \in \{0, 1\}$ *for* $b > \frac{1-p}{x} - (1-g)$.

Proof. See Appendix A.2. *Q.E.D.*

To understand the intuition for Lemma 3, we first show that choosing a small value of $\alpha > 0$ close to zero cannot be optimal. Formally, there exists a threshold $\hat{\alpha} \geq 0$ such that any disclosure policy $\alpha \in (0, \hat{\alpha}]$ is always strictly suboptimal. Under such a disclosure policy, even the banks "failing" the stress test do not suffer credit rationing. Consequently, disclosure does not directly increase social welfare by rationing credit to bad banks (i.e.,

$U(g,\alpha,b) = U(g,0,b)$), but does increase good banks' ex-ante incentive to select project γ. When $b > \frac{1-p}{x} - (1-g)$, it follows from Lemma 2 that a small $\alpha > 0$ raises $\Delta(g,\alpha,b)$ above $\Delta(g,0,b)$, increases the incentive to choose project γ, and worsens diversification. Since a small value of $\alpha > 0$ worsens diversification without increasing welfare, it cannot be optimal. For this reason, we henceforth focus on relatively large values of $\alpha \in (\hat{\alpha}, 1]$.[12]

We next analyze how severely each disclosure policy α worsens diversification relative to the welfare benefits from alleviating ex-post adverse selection. We first consider a sufficiently small value of α, close to $\hat{\alpha}$. Under severe adverse selection, bonds issued by good banks (of either type γ or δ) are relatively scarce. Thus, if the regulator reveals the type of only a small fraction α of type γ banks, their bonds – claims on a bank which is known to be "good" – will be extremely scarce, and outside investors will be willing to pay a high price for them. This makes it very valuable to pass the stress test, strongly discouraging good banks from choosing type δ (which gives zero chance of passing the stress test), and significantly lowering diversification. On the other hand, we know from Theorem 3 that disclosing a small fraction α of type γ banks provides little information to outside investors, and yields little direct welfare benefit. Consequently, disclosing a small fraction of type γ banks decreases social welfare on net, and therefore, cannot be optimal.

However, the benefit from information disclosure can outweigh its cost if the regulator discloses a large fraction of type γ banks, i.e. chooses α close to 1. If the regulator reveals the type of a large fraction of type γ banks, their bonds will not be very scarce, and their borrowing costs will not be extremely low. This makes the benefit from passing the stress test – while still positive – smaller than it would be for a lower value of α, mitigating the adverse effect of disclosure on diversification. At the same time, Theorem 3 implies that the ex-post benefit from disclosure is relatively high when the regulator chooses a large α.

Mathematically, the marginal effect of an increase in α on good banks' incentive to choose project γ is decreasing in α, which tends to push the regulator towards corner solutions ($\alpha = 0$ or $\alpha = 1$). For $\alpha > \hat{\alpha}$, we have

$$\Delta(g,\alpha,b) = \alpha \left[\frac{p}{g} \left\{ \frac{1}{\alpha + (1-\alpha)\phi_0^*} \right\} - x \right]. \tag{23}$$

The partial derivative of $\Delta(g,\alpha,b)$ with respect to α can be interpreted as the marginal effect

[12] In the proof of Lemma 3 in Appendix A.2, we formally show that $\hat{\alpha}$ is monotone decreasing in b. Intuitively, as the adverse selection problem grows more severe, even a disclosure policy revealing only a small fraction of type γ banks can improve ex-post social welfare.

of higher α on the banks' incentive to choose project δ. We have $\frac{\partial^2 \Delta}{\partial \alpha^2} < 0$, suggesting that this marginal adverse impact of disclosure on diversification is increasing in α. At the same time, the marginal welfare benefit from alleviating adverse selection for type γ banks is increasing in α. Since $\frac{d\phi_0^*}{d\alpha} < 0$ from (18), type γ banks that fail the stress test suffer more severe credit rationing as α increases, so the marginal benefit from saving these banks from being rationed is also higher. To sum up, both the diminishing marginal cost and the increasing marginal benefit of disclosure push the regulator towards corner solutions: if the regulator discloses her information, it is always optimal to choose full disclosure ($\alpha^{**} = 1$).

The following lemma provides one final result allowing us to characterize optimal disclosure policy.

Lemma 4. *$\frac{d}{db} W(g, 1, b) > 0$ for every $b > \frac{1-p}{x} - (1 - g)$.*

Proof. See Appendix A.2. *Q.E.D.*

In Theorem 3, we have already seen that the direct gains from disclosure captured by $U(g, 1, b) - U(g, 0, b)$ is stictly increasing in b for every $b > \frac{1-p}{x} - (1 - g)$. That is, disclosure is more valuable when the adverse selection problems become more severe.

What remains to be shown is how $\Delta(g, 1, b) - \Delta(g, 0, b)$, capturing the welfare loss from under-diversification under full disclosure, changes with b, i.e., the severity of adverse selection. From (23), we have $\Delta(g, 1, b) = \frac{p}{g} - x$: under full disclosure, the banks' ex-ante incentive to choose project γ does not depend on the severity of adverse selection. This follows from the fact that the regulator's information, when fully disclosed, rations credit for for the failing banks no matter how severe the adverse selection problem is.

Next, by plugging $\alpha = 1$ into (19), we have $\Delta(g, 0, b) = \left(\frac{p}{g} - \frac{1-p}{1-g}\right) \max\{1 - R_0^* x, 0\}$. Then it can be seen that $\Delta(g, 0, b)$ is non-positive and weakly increasing in b: by Lemma 1, there is always under-diversification in equilibrium under optimal policy ($g > p$); the borrowing term R_0^* for the failing banks weakly increases with the degree of adverse selection, following from (11). Intuitively, $\Delta(g, 0, b) \leq 0$ means that absent the disclosure of stress test results, type δ banks are more valuable to equity holders than type γ banks since they pay off in state D, when consumption is relatively low. However, $\frac{d}{db}\Delta(g, 0, b) \geq 0$, implying that the benefit from choosing project δ eventually vanishes as the adverse selection problem becomes worse, which raises the cost of borrowing. In sum, the incentive cost of the ex-ante project selection is weakly decreasing in the severity of adverse selection. Consequently, $W(g, 1, b)$ – the net welfare gains from full disclosure – is strictly increasing in b.

A key feature from Lemma 4 is that the optimal disclosure policy must have the single-crossing property for $b > \frac{1-p}{x} - (1-g)$: the regulator should optimally choose full disclosure if b exceeds a threshold $\overline{b}^{**} \geq \frac{1-p}{x} - (1-g)$ but zero disclosure otherwise. By combining this feature with the others derived from Lemma 1 – 3, we can characterize the structure of the optimal disclosure policy as follows.

Theorem 4. *An optimal disclosure policy $\alpha^{**}(\cdot)$ is characterized by two cutoffs $0 < \underline{b}^{**} < \overline{b}^{**} < \infty$ such that $\alpha^{**}(b) = 1$ if $b \in [0, \underline{b}^{**}] \cup [\overline{b}^{**}, \infty)$ and $\alpha^{**}(b) = 0$ otherwise. Any optimal disclosure policy admits excessive supply of bonds issued by type γ banks, i.e., $g > p$.*

Proof. See Appendix A.2. *Q.E.D.*

Figure 2 graphically illustrates how optimal policy is shaped by two cutoffs. The lower threshold $\underline{b}^{**}$ is the lowest value of b such that $\Delta(g,1,b) - \Delta(g,0,b) \leq 0$. When $b < \underline{b}^{**}$, disclosure of the regulator's information increases welfare by *improving* diversification because banks identified as either type δ or β are very likely to be δ, and therefore disclosure allows type δ banks to obtain cheaper funding. For intermediate values of b ($\underline{b}^{**} < b \leq \frac{1-p}{x} - (1-g)$), disclosure worsens diversification ($\Delta(g,1,b) - \Delta(g,0,b) > 0$), whereas it does not address adverse selection problems at all ($U(g,1,b) - U(g,0,b) = 0$). Hence, the regulator should refrain from disclosing any information.

For high values of b ($b > \frac{1-p}{x} - (1-g)$), it can be seen that disclosure alleviates adverse selection problems ($U(g,1,b) - U(g,0,b) > 0$), and this welfare benefit increases with the severity of adverse selection ($\frac{d}{db}[U(g,1,b) - U(g,0,b)] > 0$). Furthermore, it can be seen that $\Delta(g,1,b) - \Delta(g,0,b) > 0$ for every $b > \frac{1-p}{x} - (1-g)$: disclosure is costly due to the associated under-diversification. However, this cost of disclosure decreases with the severity of adverse selection ($\frac{d}{db}[\Delta(g,1,b) - \Delta(g,0,b)] < 0$). Hence, the net welfare gain from disclosure increases with the severity of adverse selection. Therefore, to maximize welfare, the regulator should disclose its information if and only if the adverse selection problems are sufficiently severe, i.e., $b > \overline{b}^{**}$.

Another interpretation of the optimal disclosure policy is that the regulator releases her information when it is relatively accurate.[13] To see this, note from (3) that the posterior belief about an individual bank labelled $\sigma = 0$ is

$$Pr(\theta = \delta | \sigma = 0) = \frac{1-g}{1-g+b}.$$

[13]The authors specially thank Youngwoo Koh for this helpful comment.

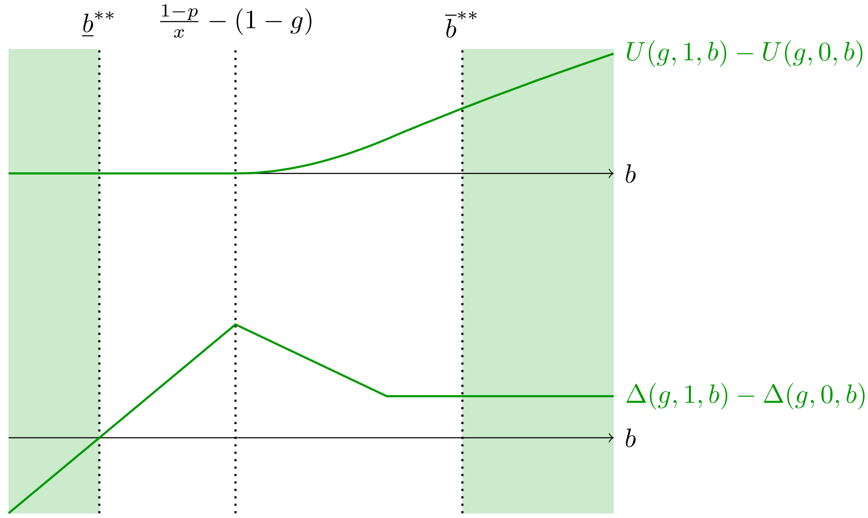


Figure 2 – The structure of the ex-ante optimal disclosure policy

Since $Pr(\theta = \delta|\sigma = 0) \to 1$ as $b \to 0$ and $Pr(\theta = \beta|\sigma = 0) \to 0$ as $b \to \infty$, the regulator's signal becomes more accurate as b converges to either zero or infinity. Information disclosure can be costly when it is inaccurate, mistakenly classifying type δ banks as bad banks. To maximize welfare, the regulator only releases her information when it accurately identifies a bank's type, i.e. when b is either very low or very high.

Throughout, we have assumed that the private and social benefits from diversification coincide. As we have shown, even when the outside investors and equity holders fully internalize the social benefits from a diversified financial system, disclosing imperfect stress test results can inefficiently reduce diversification if banks are able to game the stress tests. This cost would be even more severe to the extent that private individuals do not fully internalize the benefits from diversification: this would only amplify our main result. Indeed, a key justification of stress tests (and financial regulation more broadly) in practice is that in a crisis, low levels of bank capital have negative effects on the wider economy, which are not internalized by individual banks. To the extent that such externalities are present, policy should try even harder to induce banks to diversify their portfolios. Absent other instruments, this can only be achieved by releasing stress test results even less frequently.[14]

Finally, one should not necessarily conclude from our analysis that releasing stress test

[14]Earlier versions of this paper modeled this formally by studying an extension in which when banks' projects fail, they are forced to sell productive assets to repay depositors. This introduces a pecuniary externality, because an individual bank does not internalize that its asset sales reduce the prices that other banks receive for their assets. Asset sales reduce welfare because banks are more productive managers of assets. In this case, the optimal level of diversification is higher than in the model without externalities (the optimal g is lower).

results always reduces diversification. Information disclosure is only costly to the extent that banks can anticipate which projects the regulator will deem as safe, which is a maintained assumption in the model. If instead banks do not know at date 0 whether type γ or δ project will "fail" at date 1, committing to unconditional disclosure may not reduce diversification, and would be possibly optimal.[15] Thus, our model would suggest that regulators should attempt to conceal their models from subject banks, if possible. Recently, the Federal Reserve has discussed whether to disclose additional details about its supervisory models; some commentators have argued that the models should be fully disclosed.[16] Our analysis lends support to concerns regarding the disclosure of supervisory risk models: increasing predictability of the supervisory stress testing could allow banks to manipulate their stress test results without prudently reducing actual risks, leading to model monoculture. Our analysis also suggests the empirical prediction the that disclosing additional details about the models used in supervisory stress tests would cause an even larger increase in portfolio similarity among tested banks.

5 Extensions

Our baseline model makes a number of simplifying assumptions for tractability. We now investigate whether (and how) relaxing these assumptions changes our main results.

5.1 More General CRRA Preferences

Disclosure can be costly because it leads to an inefficiently under-diversified financial system. In our model, under-diversification is inefficient because investors are risk-averse. It is therefore natural to ask how the optimal policy changes as the investors become more risk-averse, and the value of diversification increases. To answer this question, we modify our main model by assuming that investors' preferences over their date-2 consumption are described by a CRRA utility function $u(c) = \frac{c^{1-\rho}}{1-\rho}$ with $\rho > 1$. In Appendix A.3, we show that the optimal disclosure policy has the same structure as that with $\rho = 1$ (i.e., $B^{**} := [0, \underline{b}^{**}] \cup [\overline{b}^{**}, \infty)$ for some $0 \leq \underline{b}^{**} \leq \overline{b}^{**} < \infty$). Furthermore, as in our baseline model, the optimal disclosure policy features a larger fraction of type γ banks than in the unconstrained efficient allocation.

15 In Section 5.2, we formalize this idea and numerically analyze an optimal level of unpredictability in the regulator's stress test models.

16 Go to https://www.gpo.gov/fdsys/pkg/FR-2017-12-15/pdf/2017-26856.pdf for more details.

Theorem 5. *When investors have CRRA preferences with $\rho > 1$, optimal disclosure policy $\alpha^{**}(\cdot)$ is characterized by two cutoffs $0 < \underline{b}^{**} < \overline{b}^{**}$ such that $\alpha^{**}(b) = 1$ if $b \in [0, \underline{b}^{**}] \cup [\overline{b}^{**}, \infty)$ and $\alpha^{**}(b) = 0$ otherwise. Any optimal disclosure policy admits excessive supply of bonds issued by type γ banks, i.e., $g^{\rho} > p$.*

Proof. See Appendix A.3. *Q.E.D.*

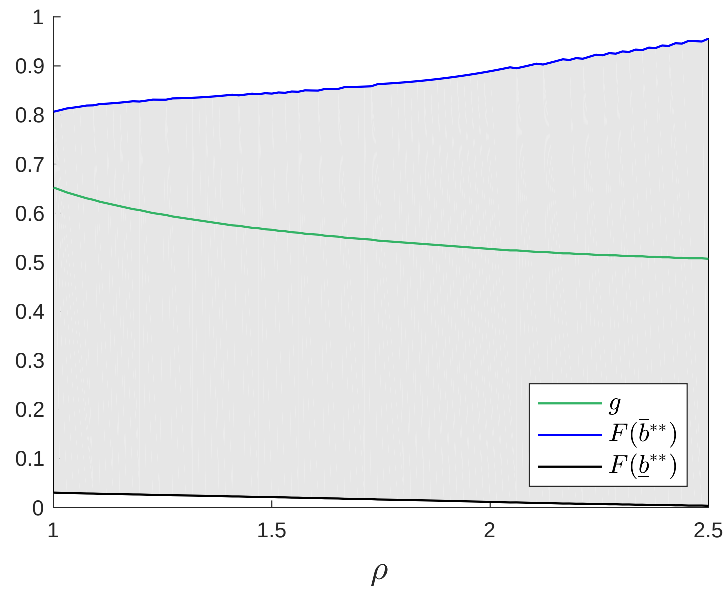


Figure 3 – Comparative statics of the optimal disclosure policy with respect to ρ

To see how the optimal disclosure policy changes with the social value of diversification, we conduct a comparative statics exercise with respect to the degree of the investors' risk aversion $\rho > 1$. Specifically, we consider a numerical example in which $x = 0.2$, $p = 0.5$, and b – the population of type-β banks on the market – is uniformly distributed over $[0, 10]$. Figure 3 graphically illustrates how the optimal policy varies with ρ.

There are a couple of interesting observations. First, the equilibrium fraction g of type γ banks induced by the optimal disclosure policy converges downward to the first-best level $g = p^{\frac{1}{\rho}}$ as ρ increases. In part, this reflects investors' private incentives: as equity holders become more risk averse, banks will choose a more diversified mix of projects, for the same disclosure policy, in order to maximize the equity holders' expected utility. In addition, the regulator takes into account the increased social value of diversification and discloses stress test results less often as the investors become more risk-averse, implementing an even higher level of portfolio diversification than would arise given the same disclosure policy. Indeed, Figure 3 shows that the total probability of disclosure $F(\underline{b}^{**}) + (1 - F(\overline{b}^{**}))$ is decreasing in ρ. Specifically, one can see that the upper threshold $\overline{b}^{**}$ increases with ρ, whereas the lower threshold $\underline{b}^{**}$ decreases with ρ, increasing the size of the non-disclosure region (shaded gray).

5.2 Predictable Bias of Supervisory Stress Testing

In the last paragraph of Section 4, we remarked that model monoculture arises from the *predictability* of the bias in supervisory stress test models. Indeed, subject banks are able to game the system because these banks know precisely which type of risks the regulator focuses on in its supervisory stress tests. We also conjectured that full disclosure of stress test results would maximize welfare if banks were unable to predict the bias of the regulator's stress test models.

In this section, we extend our baseline model to prove our conjecture. Suppose that the regulator tests for project γ with probability μ (that is, distinguishes between γ on the one hand and δ and β on the other hand, as in our baseline model), while the regulator tests for project δ with probability $1 - \mu$. To simplify the analysis, we throughout assume $p = \frac{1}{2}$. If the regulator can costlessly choose any value of μ, it will be obviously optimal to set $\mu = \frac{1}{2}$ and $\alpha(b) = 1$ for all b. Under $\mu = \frac{1}{2}$, we have $\Delta(\frac{1}{2}, \alpha, b) = 0$ for all α, b, so full diversification ($g = \frac{1}{2}$) will obtain for any disclosure policy $\alpha(b)$.

Suppose then that $\mu \in (\frac{1}{2}, 1)$ is exogenous. With probability μ, the regulator learns the identities of all type γ banks (but not type δ banks), and can decide whether or not to release that information, as in our baseline model. Recall that one interpretation is that the regulator is able to study how banks will perform in a "stress scenario" corresponding to state G, but cannot study another risk corresponding to state D. The regulator then commits to a binary disclosure policy $\alpha_\gamma(b) \in \{0, 1\}$: $\alpha_\gamma(b) = 1$ iff the regulator reveals the test results under scenario G when the fraction of bad banks equals b. Similarly, with probability $1 - \mu$, the regulator learns the identities of all type δ banks, but not type γ banks (i.e. she can study a scenario based on state D, but not state G), and then commits to a binary disclosure policy $\alpha_\delta(b) \in \{0, 1\}$. That is, the disclosure policy is contingent on the scenarios that the regulator tests for.

Given this uncertainty about which stress scenario the regulator will be able to test for, her optimization problem becomes:

$$\max_{g,\alpha_\gamma(\cdot),\alpha_{\delta(\cdot)}} \mu \int [\alpha_\gamma(b)U_\gamma(g,b) + (1-\alpha_\gamma(b))U_n(g,b)]dF(b) \\ + (1-\mu)\int[\alpha_\delta(b)U_\delta(g,b) + (1-\alpha_\delta(b))U_n(g,b)]dF(b), \tag{24}$$

subject to

$$\mu \int [\alpha_\gamma(b)\Delta_\gamma(g,b) + (1-\alpha_\gamma(b))\Delta_n(g,b)]dF(b) \\ + (1-\mu)\int[\alpha_\delta(b)\Delta_\delta(g,b) + (1-\alpha_\delta(b))\Delta_n(g,b)]dF(b) = 0. \tag{25}$$

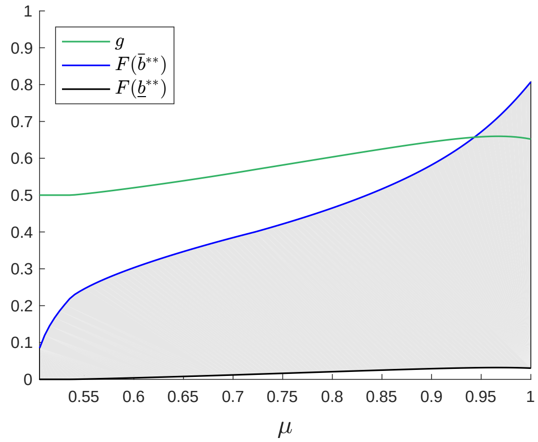
Here $U_n(g,b) := U(g,0,b)$ and $\Delta_n(g,b) := \Delta(g,0,b)$ denote welfare and the net benefit to equity holders from project γ when the regulator does not reveal any information, respectively. Similarly, $U_\gamma(g,b) := U(g,1,b)$ and $\Delta_\gamma(g,b) := \Delta(g,1,b)$ denote welfare and the net benefit to equity holders from project γ when the regulator discloses the result of a test based on scenario G, respectively. These functions are described by the same expressions as in our baseline model with $\mu = 1$. Furthermore, $U_\delta(g,b)$ and $\Delta_\delta(g,b)$ denote welfare and the net benefit to shareholders from a γ project when the regulator discloses the result of a test based on scenario D. These functions can be written as

$$U_\delta = p\log g + (1-p)\log(1-g) + p\log\phi_\delta - (1-g+\phi_\delta(g+b))x,$$
$$\Delta_\delta(g,b) = \frac{p}{g}\max\left\{1-\frac{g+b}{p}x, 0\right\} - \frac{1-p}{1-g}\left(1-\frac{1-g}{1-p}x\right),$$

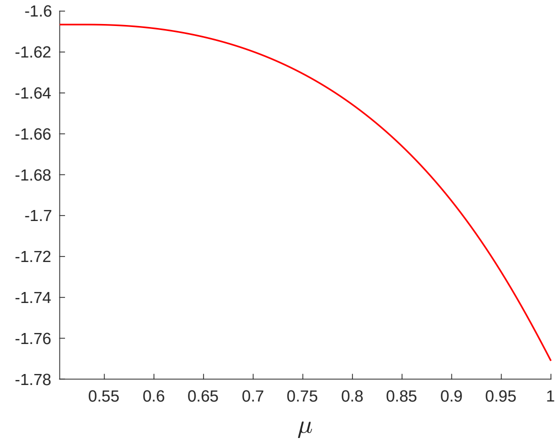
where $\phi_\delta = \min\left\{\frac{p}{(g+b)x}, 1\right\}$ denotes the fraction of γ and β banks who are rationed when the results of a test based on scenario D are disclosed.

The possibility of testing for project δ makes it difficult to analytically find an optimal disclosure policy $(\alpha_\gamma^{**}(\cdot), \alpha_\delta^{**}(\cdot))$ in general. Nevertheless, we can numerically find the optimal policy for each $\mu \in \left(\frac{1}{2}, 1\right)$ for a given distribution function F. For example, Figure 4 depicts the numerical results with $x = 0.2$ and $b \sim U[0,10]$.[17] The left panel depicts the optimal disclosure policy when the regulator can test for scenario G, $\alpha_\gamma^{**}(\cdot)$, as a correspondence of $\mu \in (\frac{1}{2}, 1)$. When the regulator can test for scenario D, disclosure is always optimal ($\alpha_\delta^{**}(b) = 1$ for every $b \geq 0$), so we do not plot this policy. In addition, the right panel illustrates how

[17] We stress that the qualitative results depicted by Figure 4 generally hold for $x < p = \frac{1}{2}$ and $b \sim U[0,\bar{b}]$ with $\bar{b} > 1$.



(a) Optimal policy ($\alpha_\gamma^{**}(\cdot)$)



(b) Welfare

Figure 4 – Comparative statics of the optimal disclosure policy with respect to μ

welfare under the optimal policy varies with μ. Higher values of μ mean that the regulator's tests are more predictably biased towards type γ banks. In this context, our baseline model is viewed as the most predictable case at the rightmost point $\mu = 1$.

This numerical example provides some features worth discussing. First, since the regulator's model is, albeit stochastically, biased against state D ($\mu > \frac{1}{2}$), the optimal disclosure policy still admits under-diversification against state D (i.e., $g > p$). Hence, the optimal policy $\alpha_\gamma^{**}(\cdot)$ when the regulator tests for state G must have the similar two-cutoff structure to the baseline analysis. Furthermore, the regulator should unconditionally disclose its information when the regulator's model tests for state D. Specifically, releasing the information does not only perfectly identify type δ banks so that they can cheaply finance their project with a scarcity premium, but it also addresses the ex-post adverse selection for type δ banks.

Second, the probability that the regulator discloses her information – represented by the unshaded region in the left panel of Figure 4 – increases as the average bias of the stress tests falls towards $\mu = 0.5$. That is, the model monoculture problem becomes less important in the design of optimal disclosure policy when the bias of the stress tests becomes more unpredictable. This can also be seen by the fact that g, the equilibrium fraction of type γ banks under the optimal policy, converges to the unconstrained optimal level ($g = 0.5$), and welfare increases, as μ approaches $\frac{1}{2}$. That is, a low predictable bias against type δ banks allows the regulator to disclose tests results more often without inevitably worsening diversification. This all resonates with our conjecture in Section 4.2 that the model monoculture problem is associated with the *predictable* bias in the regulator's stress tests.

5.3 Alternative Timing

In our baseline model, we assume that the regulator first commits to a disclosure policy $\alpha(b)$, then good banks choose project γ or δ, then the measure of bad banks b is realized, and finally the regulator discloses a fraction $\alpha(b)$ of good banks. In this setup, when each good bank chooses its project, it takes into account expected profits across all possible realizations of b. One might wonder whether it would have been better to assume that b is realized first, the regulator chooses a disclosure probability $\alpha(b)$ next, and finally good banks choose projects γ and δ. In this alternative setup, good banks would be able to adjust their financial projects in real time in response to both the degree of adverse selection in the capital market, and the regulator's decision about whether to disclose stress test results.

We view our baseline timing assumption as more realistic than this alternative setup, since it captures the idea that a bank's choice of portfolio cannot be adjusted rapidly in response to the announcement that a particular stress test will be conducted and the results disclosed, but can be adjusted over time in response to a systematic change in policy. For example, the SCAP was announced in February 2009, and the results released in May 2009. Many aspects of the stress test were based on banks' balance sheets as of December 31 2008.[18] Thus, it would not have been possible for a bank to change its portfolio in order to improve its test results in response to the February 2009 announcement.

Notwithstanding these concerns, adopting the alternative timing assumption would amplify the adverse effect of disclosure on diversification. In fact, under the alternative timing it would *never* be optimal for the regulator to disclose stress test results.

Theorem 6. *$\alpha^{**}(b) = 0$ for every $b \geq 0$ is weakly optimal when the fraction of bad banks is realized first, the regulator commits to a disclosure policy next, and good banks choose their projects last.*

Proof. See Appendix A.4. *Q.E.D.*

Under the alternative timing, the regulator chooses α, and thus implements g state by state to maximize $U(g, \alpha, b)$ subject to the constraint $\Delta(g, \alpha, b) = 0$. If b and $\alpha(b)$ are high enough that failing banks are rationed, the constraint $\Delta(g, \alpha, b) = 0$ implies that g must converge to 1: banks know that project δ will make zero profits, so all of them will choose γ. Such an allocation of financial projects is never optimal since $g = 1$ implies $U(g, \alpha, b) = -\infty$.

[18] See https://www.federalreserve.gov/bankinforeg/bcreg20090424a1.pdf

Thus, the regulator will refrain from any policy that causes credit rationing to type δ banks in any degree. This in turn implies that social welfare equals $U(g, \alpha, b) = p \log g + (1-p) \log(1-g) + (1+b)x$, which is maximized by setting $g = p$. The regulator can implement this allocation by never disclosing, i.e., $\alpha = 0$.

6 Related Literature

A growing theoretical literature discusses the benefits and costs of disclosing stress test results or other information held by regulators. As surveyed by Goldstein and Sapra (2013) and Leitner (2014), there are a number of arguments against unconditional disclosure of stress test results: full disclosure of stress test results may reduce risk-sharing *à la* Hirshleifer (1971) (Goldstein and Leitner, 2018); forcing firms to disclose their financial status too often may encourage short-termism (Gigler et al., 2014); the public information provided by stress tests may crowd out private information held by individual creditors (Morris and Shin, 2002; Bond and Goldstein, 2015); and releasing the public information may cause coordination failure among banks or market participants (Bouvard, Chaigneau and Motta, 2015; Inostroza and Pavan, 2017). In particular, Williams (2017) presents a model in which even though a regulator has perfect information about banks, it is optimal to only release just enough information to prevent a bank run.[19] We complement this literature by focusing on another potential cost of stress tests, the impacts of disclosure on banks' investments and risk assessments. In particular, we study how the disclosure of stress test results may distort diversification in the entire financial system when stress tests can misclassify some financially sound banks as risky.

Another recent paper that argues that limited disclosure of public information may be optimal is Quigley and Walther (2018). They consider an environment in which market "insiders" can choose to release verifiable private information at a cost. Although the disclosure of "outside" information by the public authority can crowd out "inside" information, such crowding out can be socially desirable because disclosure of inside information can separate high-quality banks from the rest, causing inefficient bank runs. We instead focus on the way in which information disclosure affects banks' investment decisions rather than their communication with investors (which is ruled out in our setting).

[19] The recent literature also emphasizes that the effectiveness and optimal design of stress tests depends on what other instruments are available to policymakers. Spargoli (2013) argues that disclosure of negative stress test results either reduces lending or requires costly bailouts. Similarly, Faria-e Castro, Martinez and Philippon (2016) argue that disclosure can create inefficient bank runs unless there is a fiscal backstop.

Leitner and Yilmaz (2019) also consider a similar environment in which banks choose whether to privately produce information with their own risk models, given that regulators decide whether to monitor the regulated banks' information. It may be (constrained) optimal not to monitor the banks' models because monitoring would reduce the banks' incentives to collect information *ex ante*. While we abstract from the *production* of information either by banks or by regulators, our main result resonates with Leitner and Yilmaz (2019): full disclosure of the regulators' (imperfect) information may adversely influence banks' choices of investments, which are socially valuable but not easily recognized as such. However, an important difference is that their model is applicable to settings in which regulators rely on the banks' models (e.g. the internal ratings-based approach to calculate regulatory capital). We instead assume regulators use their own model to regulate banks, and banks do not have the option of using their own models instead. This assumption is more applicable to the CCAR and DFAST in which the Federal Reserve conducts supervisory stress tests using its own scenarios and models (though these stress tests do use data supplied by the firms).[20]

Our results suggest that regulators should keep their models secret; otherwise, banks can game the regulators' model, reducing diversification. Leitner and Williams (2017) argue that secrecy may not, in fact, be optimal because it can deter banks from making risky investments. We abstract from this possibility since banks always make some investment, whether in good or diversifying projects. However, we highlight an additional cost that arises when banks can game the regulator's model. Banks do not only invest in excessively risky projects; rather, they also invest in particular types of projects that are difficult to identify as such, reducing diversification in the overall economy.

Our framework is consistent with the role that stress tests played during the 2007 – 09 Global Financial Crisis. A major goal of SCAP was to alleviate widespread adverse selection problems in financial markets during the crisis (Geithner, 2015, pp. 286-7). The academic literature (e.g. Tirole (2012)) also points to adverse selection as a key reason for the lending freeze observed during the crisis. Flannery, Kwan and Nimalendran (2013) document empirically that adverse selection problems worsened during the crisis.

While our result that releasing public information reduces diversification, may seem reminiscent of Morris and Shin (2002), the mechanism in our paper is different. In Morris and

[20] While banks are *also* required to conduct their own company-run stress tests under the DFAST and CCAR, these tests cannot be used as a substitute for the supervisory stress tests. The Fed can object to banks' capital plans based on whether they fail the CCAR stress tests using the Fed: subject banks cannot use their own test results to avoid being required to take these corrective actions.

Shin (2002), public information reduces diversification because agents' actions are strategic complements, and they coordinate on the public signal. In our paper, banks have no intrinsic coordination motive. If anything, they have an intrinsic motive for diversification because risk-averse households reward banks for investing in assets that pay off when consumption is low. However, the disclosure of public information causes outside investors to rationally punish banks for investing differently from the herd because those banks may be risky.

Our paper relates to a growing theoretical literature that studies the causes and consequences of increased similarity in the portfolios of financial institutions. In Farhi and Tirole (2012), imperfectly targeted policy interventions make banks' leverage choices strategic complements: banks only wish to be exposed to a common shock if other institutions are also exposed, implying that policymakers will bail them out in a crisis. Kopytov (2018) presents a dynamic model in which banks' portfolios become more similar at the end of credit expansions, as these portfolios become more diversified at the individual level (see also Khorrami (2019) for a model in which an increase in diversification at the level of an individual intermediary can increase aggregate risk). We focus on a different potential cause of correlated portfolios across banks, namely the disclosure of stress test results.

Bräuning and Fillat (2019) provide the most relevant empirical evidence supporting our theoretical result that stress test disclosure can increase portfolio similarity. They find that the portfolios of U.S. banks who became subject to Dodd-Frank Act Stress Testing (DFAST) starting in 2011 became more similar since then, while the portfolios of banks not subject to DFAST remained essentially unchanged. In particular, banks with poor stress test results tend to have dissimilar portfolios before the test; after failing the test, they adjust their portfolios to become closer to the portfolios of the banks with good DFAST results. This is consistent with our theoretical finding that supervisory stress testing may create a "model monoculture" by inducing banks to choose similar business and risk models in order to avoid failing the tests, inefficiently increasing the financial system's exposure to the same aggregate risks.

7 Conclusion

Stress tests have moved from an exceptional measure of crisis management to a routine part of financial regulation. In a crisis, stress tests can reduce uncertainty about banks' financial soundness and prevent market failure caused by financial instability. However, routine stress tests may also lead to model monoculture, in which banks mimic the regulators' models in order to pass the tests while ignoring their own measures of risk. This can leave the financial system less diversified and vulnerable to those risks that the regulators' model ignores. We presented a simple model to understand the tradeoff facing financial regulators that use stress tests as a routine policy tool. When stress test results are predictably biased against banks holding a certain class of financial assets, our analysis suggests that the stress test results should be released to the public only selectively, either in a severe crisis or when stress test scenarios accurately evaluate the financial health of tested banks.

Our paper mainly focused on the direct impacts of releasing stress test results on portfolio adjustment decisions by subject banks. One should not hastily generalize these results to the the *overall* effect of supervisory stress tests before carefully examining their potential indirect effects. Disclosure of stress test results may improve financial stability through indirect channels which are not captured by our paper. In particular, the subject banks' strategic response to the stress testing can endogenously influence businesses of the other banks not subject to the supervisory stress testing. For instance, while supervisory stress testing increases the cost of lending to risky businesses for subject banks, this can create a positive selection effect where only banks with expertise in assessing the financial risks or sufficiently capitalized banks maintain or increase lending to the risky businesses (Cortés et al., 2019). Exploring such tradeoffs is beyond the scope of our paper.

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Appendix

A Proofs

A.1 Proofs for Section 3

Proof of Theorem 1. First, fix any $b \leq \frac{1}{x} - 1$. Suppose there exists an equilibrium in which the investors place a buy order $(\phi', R') \neq (\phi^*, R^*)$. If $\phi' \leq \phi^* = 1$ but $R' \geq R^*$, we have $\frac{R'}{\phi'} - (1+b) > 0$ from (6). Hence, an investor can get a higher payoff by placing a buy order $(1+\varepsilon, R' - \varepsilon')$ for some $\varepsilon, \varepsilon' > 0$. If $\phi' = \phi^* = 1$ but $R' < R^*$, we have $\frac{R'}{\phi'} - (1+b) < 0$. Thus, an investor can get a higher payoff by offering $(\phi, R) = \left(\frac{R'}{1+b}, R'\right)$. If $\phi' < \phi^* = 1$ and $R' < R^*$, some banks with measure $(1-\phi')(1+b) > 0$ are not funded at the market. Thus, an investor can get a higher payoff by placing a buy order $(\phi, R) = (1, R' + \varepsilon)$ for some $\varepsilon > 0$.

Second, fix any $b > \frac{1}{x} - 1$ and suppose there exists an equilibrium in which the investors place the buy order $(\phi', R') \neq (\phi^*, R^*)$. If $\phi' = \phi^*$ but $R' < R^*$, then we have $\frac{R'}{\phi'} - (1+b) < 0$, and thus an investor has an incentive to deviate by offering $(\phi, R) = \left(\frac{R'}{1+b}, R'\right)$. If $\phi' < \phi^*$ and $R' = R^*$, then we have $\frac{R'}{\phi'} - (1+b) > 0$, and thus an investor has an incentive to deviate by placing an order $(\phi, R) = (\phi' + \varepsilon, R')$ for some $\varepsilon > 0$. If $\phi' < \phi^*$ and $R' < R^*$, an investor can get a higher payoff by placing a buy order $(\phi, R) = (\phi' + \varepsilon, R' + \varepsilon')$ for some $\varepsilon, \varepsilon' > 0$. Putting all these results together, if there exists an equilibrium, then all investors place the buy order (ϕ^*, R^*) in such equilibrium. Conversely, as shown in Appendix B, it is optimal for an investor to place the same buy order given that the other investors place (ϕ^*, R^*). *Q.E.D.*

Proof of Theorem 2. We omit the formal proof because of its logical similarity to the proof of Theorem 1. *Q.E.D.*

Proof of Theorem 3. First, suppose $b \leq \frac{1-p}{x} - (1-g)$. Then we have $U(g, \alpha, b) = U(g, 0, b)$ for every $\alpha \in [0, 1]$, which implies that $\alpha = 1$ is weakly optimal.

Next, suppose $b > \frac{1-p}{x} - (1-g)$. From (11), there exists a $\hat{\alpha} \in [0, 1]$. defined as

$$\hat{\alpha} = 0 \vee \left[\left(1 - \frac{\frac{1-p}{x} - [(1-g) + b]}{\left(\frac{p}{x} - g\right)} \right) \wedge 1 \right], \tag{26}$$

such that $\phi_0^* = 1$ if $\alpha \leq \hat{\alpha}$ and $\phi_0^* < 1$ otherwise. For every $\alpha \leq \hat{\alpha}$, we have $U(g, \alpha, b) = U(g, 0, b)$. For every $\alpha > \hat{\alpha}$, we have $\frac{d}{d\alpha}[U(g, \alpha, b) - U(g, 0, b)] > 0$ from (18) since $\phi_0^* < 1$.

Furthermore, we have $\lim_{\alpha\to\hat{\alpha}}[U(g,\alpha,b)-U(g,0,b)]=0$ since $\phi_0^*\to 1$ as $\alpha\to\hat{\alpha}$. Therefore, we have $[U(g,\alpha,b)-U(g,0,b)]>0$ for every $\alpha>\hat{\alpha}$. In particular, $[U(g,\alpha,b)-U(g,0,b)]$ is maximzed at $\alpha=1$ since $\frac{d}{d\alpha}[U(g,\alpha,b)-U(g,0,b)]>0$ for every $\alpha>\hat{\alpha}$. To sum up, it is optimal for the regulator to $\alpha=1$ for every $b\geq 0$. *Q.E.D.*

A.2 Proofs for Section 4

We first discuss a mathematical property of $\hat{\alpha}$ in (26). Recall from (11) that $\phi_0^*=1$ if $\alpha\leq\hat{\alpha}$ and $\phi_0^*<1$ otherwise. By construction, the following properties are immediate.

Lemma A.1.

(i) $\hat{\alpha}=1$ *at* $b=\frac{1-p}{x}-(1-g)$ *and* $\hat{\alpha}=0$ *at* $b=\frac{1}{x}-1$;

(ii) $\hat{\alpha}$ *is strictly decreasing in* b *for all* $b\in\left[\frac{1-p}{x}-(1-g),\frac{1}{x}-1\right]$.

Proof of Lemma 1. We first show that $g\geq p$ under any feasible policy. Suppose to the contrary that some optimal disclosure policy induces $g<p$. Recall that $\Delta(g,\alpha,b)$ has different forms, depending on whether $\phi_0^*=1$ or $\phi_0^*<1$. Consider $\phi_0^*=1$ first. Recall that $\phi_0^*=1$ if either $b\leq\frac{1-p}{x}-(1-g)$ or $b\in\left(\frac{1-p}{x}-(1-g),\frac{1}{x}-1\right]$ and $\alpha\leq\hat{\alpha}$. By plugging (11) into (20), we have

$$\Delta(g,\alpha,b)=\left[\left(\frac{p}{g}-\frac{1-p}{1-g}\right)-\left(1+\frac{(1-g)+b}{g}\right)x\right]+\frac{x(1-p)}{p(1-g)}\left(1+\frac{\frac{(1-g)+b}{g}-\frac{1-p}{p}}{(1-\alpha)+\frac{1-p}{p}}\right).$$

$\Delta(g,\alpha,b)$ expressed as above is increasing in α if and only if $\frac{(1-g)+b}{g}-\frac{1-p}{p}>0$, or equivalently, $b>\frac{g}{p}-1$. Since $g<p$, we have $\frac{g}{p}<1$. Hence, $\Delta(g,\alpha,b)$ is increasing in α for every $b\leq\frac{1}{x}-1$ and $\alpha\leq\hat{\alpha}$. Furthermore, $g<p$ implies $\Delta(g,\alpha,0)>0$ for every $\alpha\in[0,1]$. Lastly, $\Delta(g,0,b)=\left(\frac{p}{g}-\frac{1-p}{1-g}\right)[1-(1+b)x]\geq 0$ for every $b\leq\frac{1}{x}-1$, where the equality holds only for the case $b=\frac{1}{x}-1$. From all the observations above, we have $\Delta(g,\alpha,b)\geq 0$ for every (α,b) that yields $\phi_0^*=1$, where the equality holds only for the case $b=\frac{1}{x}-1$ and $\alpha=0$.

Consider the case $\phi_0^*<1$ next. Since $R_0^*=\frac{1}{x}$, we have $\Delta(g,\alpha,b)=\alpha\left[\frac{1}{\alpha+(1-\alpha)\phi_0^*}\left(\frac{p}{g}\right)-x\right]$. Since $p-x>0$, we have $\Delta(g,\alpha,b)>0$ for every $\alpha>0$ and $\Delta(g,0,b)=0$ at $\alpha=0$. Combining all the observations together, we have $\int_0^\infty\Delta(g,\alpha(b),b)dF(b)>0$ for any disclosure policy $\alpha(b)$, which contradicts (20).

Next, suppose $g = p$. By applying the same logic used in the previous case $g < p$, one can find that $\Delta(g, \alpha, b) > 0$ for every $b \geq 0$ if $\alpha > 0$, whereas $\Delta(g, 0, b) = 0$ for every $b \geq 0$. Hence, to support $g = p$, we must have $\alpha(b) = 0$ with probability 1.

Finally, suppose by contradiction that $g = p$ under an *optimal* disclosure policy, implying that $\alpha(b) = 0$ with probability 1. Consider a deviation in which $\alpha(b) = 1$ for $b \in \left(\frac{1}{x} - 1, \frac{1}{x} - 1 + \varepsilon\right)$ for $\varepsilon > 0$ sufficiently small. For given ε, $g(\varepsilon)$ is determined as a differentiable function of ε by (20), which becomes

$$\int_0^\infty \Delta(g(\varepsilon), 0, b)dF(b) + \int_{\frac{1}{x}-1}^{\frac{1}{x}-1+\varepsilon} [\Delta(g(\varepsilon), 1, b) - \Delta(g(\varepsilon), 0, b)]dF(b) = 0$$

Welfare under this deviation is

$$\int_0^\infty U(g(\varepsilon), 0, b)dF(b) + \int_{\frac{1}{x}-1}^{\frac{1}{x}-1+\varepsilon} [U(g(\varepsilon), 1, b) - U(g(\varepsilon), 0, b)]dF(b) = 0$$

The derivative of welfare with respect to ε, evaluated at $\varepsilon = 0$, equals

$$\left[U\left(p, 1, \frac{1}{x} - 1\right) - U\left(p, 0, \frac{1}{x} - 1\right)\right] f\left(\frac{1}{x} - 1\right) > 0$$

This implies that the deviation increases welfare for small enough $\varepsilon > 0$, which contradicts the original allocation being optimal. *Q.E.D.*

Proof of Lemma 2. We first derive some useful mathematical properties. First, we show $\frac{d\Delta}{dg} < 0$. If $\phi_0^* = 1$, $\Delta(g, \alpha, b)$ is rewritten as

$$\Delta(g, \alpha, b) = \left(\frac{p}{g} - \frac{1-p}{1-g}\right)(1 - R_0^* x) + \alpha x(R_0^* - 1).$$

From (11), we have $\frac{dR_0^*}{dg} < 0$. Furthermore, we have $g > p$ by Lemma A.1. In sum, we have $\frac{d\Delta}{dg} < 0$. If $\phi_0^* < 1$, we have

$$\Delta(g, \alpha, b) = \alpha\left(\frac{p}{g}\left\{\frac{1}{\alpha + (1-\alpha)\phi_0^*}\right\} - x\right).$$

By applying the implicit function theorem to (12), we have $\frac{d\phi_0^*}{dg} > 0$ whenever $\phi_0^* < 1$. Hence we have $\frac{d\Delta}{dg} < 0$. Second, we show $\frac{dU}{dg} \leq 0$. If $\phi_0^* = 1$, we have $U(g, \alpha, b) = -(1 + b)x$, and therefore, $\frac{dU}{dg} = 0$. If $\phi_0^* < 1$, we have $\frac{dU}{dg} = \frac{\partial U}{\partial g} = -\alpha(1 - \phi_0^*)x < 0$, where the first equality follows from the envelope theorem.

Thoroughout, fix an optimal disclosure policy $\alpha(\cdot)$ such that $\mathbb{E}[\alpha(b)] > 0$. By Lemma 1, we have $g > p$, which implies $\frac{g}{p} - 1 > 0$. Furthermore, by Lemma A.1, we have

$$\Delta(g, \alpha, b) = \left[\left(\frac{p}{g} - \frac{1-p}{1-g}\right) - \left(1 + \frac{(1-g)+b}{g}\right)x\right] + \frac{x(1-p)}{p(1-g)}\left(1 + \frac{\frac{(1-g)+b}{g} - \frac{1-p}{p}}{(1-\alpha) + \frac{1-p}{p}}\right)$$

for each $b \leq \frac{1-p}{x} - (1-g)$. As was seen in the proof of Lemma 1, $\Delta(g, \alpha, b)$ is decreasing in α if $b \leq \frac{g}{p} - 1$ and increasing in α if $b \in \left(\frac{g}{p} - 1, \frac{1-p}{x} - (1-g)\right]$. Lastly, $U(g, \alpha, b) - U(g, 0, b) = 0$ for all $b \leq \frac{1-p}{x} - (1-g)$ and $\alpha \in (0, 1]$.

To show optimality of $\alpha(b) = 1$ for all $b \leq \frac{g}{p} - 1$, suppose to the contrary that $\alpha(b) < 1$ for a positive measure of $b \leq \frac{g}{p} - 1$. Since g, the equilibrium fraction of type-γ banks, is determined by (20). Since $\frac{d\Delta}{dg} < 0$ and $\frac{dU}{dg} \leq 0$, the regulator can increase welfare by making g closer to p by switching the disclosure policy to a $\alpha'(b) = (\alpha(b) + \varepsilon) \wedge 1$ for a small $\varepsilon > 0$ and every $b \leq \frac{g}{p} - 1$, while the change in α does not directly affect welfare since $U(g, \alpha', b) = U(g, \alpha, b)$ for every $b \leq \frac{g}{p} - 1$.

Next, to show optimality of $\alpha(b) = 0$ for all $b \in \left(\frac{g}{p} - 1, \frac{1-p}{x} - (1-g)\right]$, suppose to the contrary that $\alpha(b) > 0$ for a positive measure of $b \in \left(\frac{g}{p} - 1, \frac{1-p}{x} - (1-g)\right]$. Then, by changing the disclosure policy to $\alpha'(b) = (\alpha(b) - \varepsilon) \vee 0$ for a small $\varepsilon > 0$ and every $b \in \left(\frac{g}{p} - 1, \frac{1-p}{x} - (1-g)\right]$, the regulator can indirectly increase welfare by bringing g closer to p, while not directly changing welfare since $U(g, \alpha', b) = U(g, \alpha, b)$ for every $b \in \left(\frac{g}{p} - 1, \frac{1-p}{x} - (1-g)\right]$. Putting all these observations together, any optimal disclosure policy must satisfy $\alpha(b) = 1$ if $b \leq \frac{g}{p} - 1$ and $\alpha(b) = 0$ if $b \in \left(\frac{g}{p} - 1, \frac{1-p}{x} - (1-g)\right]$. *Q.E.D.*

Proof of Lemma 3. We first prove that any $\alpha \leq \hat{\alpha}$ cannot be optimal for every $\frac{1-p}{x} - (1-g) < b < \frac{1}{x} - 1$. For any $b \in \left(\frac{1-p}{x} - (1-g), \frac{1}{x} - 1\right]$, we have $U(g, \alpha, b) - U(g, 0, b) = 0$ but $\Delta(g, \alpha, b) > 0$, and thus $W(g, \alpha, b) < 0$ for every $\alpha \leq \hat{\alpha}$. Hence, the regulator never chooses $\alpha \leq \hat{\alpha}$. Hence, we focus on the disclosure policies $\alpha \in (\hat{\alpha}, 1]$ throughout in the proof.

Next, we introduce some new notations used throughout the proof. For $b > \frac{1-p}{x} - (1-g)$, we define $\overline{\phi}_0^* := \sup_{\alpha \geq \hat{\alpha}} \phi_0^*$ and $\underline{\phi}_0^* := \inf_{\alpha \geq \hat{\alpha}} \phi_0^*$. By (13), we have $\overline{\phi}_0^* = \lim_{\alpha \to \hat{\alpha}} \phi_0^* (\leq 1)$ and $\underline{\phi}_0^* = \lim_{\alpha \to 1} \phi_0^* = \frac{((1-g)+b)x}{(1-p)}$. Moreover, since $\Delta(g, \alpha, b) = \alpha\left(\frac{p}{g}\left\{\frac{1}{\alpha + (1-\alpha)\phi_0^*}\right\} - x\right)$, $W(g, \alpha, b)$

is expressed for every $\alpha \geq \hat{\alpha}$ and $b > \frac{1-p}{x} - (1-g)$ as follows:

$$\begin{aligned} W(g,\alpha,b) &= [p\log(\alpha+(1-\alpha)\phi_0^*) - gx(\alpha+(1-\alpha)\phi_0^*)] + [(1-p)\log\phi_0^* - ((1-g)+b)x\phi_0^*] \\ &\quad - \lambda\alpha\left(\frac{1}{\alpha+(1-\alpha)\phi_0^*}\frac{p}{g} - x\right) - [p\log\overline{\phi}_0^* + (1-p)\log\overline{\phi}_0^* - (1+b)x\overline{\phi}_0^*] \\ &= \left[p\log(\alpha+(1-\alpha)\phi_0^*) - gx(\alpha+(1-\alpha)\phi_0^*) - \lambda\alpha\left(\frac{1}{\alpha+(1-\alpha)\phi_0^*}\frac{p}{g} - x\right)\right] \\ &\quad + [(1-p)\log\phi_0^* - ((1-g)+b)x\phi_0^*] - [p\log\overline{\phi}_0^* + (1-p)\log\overline{\phi}_0^* - (1+b)x\overline{\phi}_0^*]. \end{aligned}$$

In the last expression, the first bracketed term is increasing in ϕ_0^* since $p > gx$, and thus, maximized at $\phi_0^* = \overline{\phi}_0^*$. Furthermore, the second bracketed term is concave in ϕ_0^*, and maximized at the unique extreme point $\phi_0^* = \underline{\phi}_0^*$.

Using these properties, define $\overline{W}(g,\alpha,b)$ as follows:

$$\begin{aligned} \overline{W}(g,\alpha,b) &= \left[p\log(\alpha+(1-\alpha)\overline{\phi}_0^*) - gx(\alpha+(1-\alpha)\overline{\phi}_0^*) - \lambda\alpha\left(\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}\frac{p}{g} - x\right)\right] \\ &\quad + [(1-p)\log\underline{\phi}_0^* - ((1-g)+b)x\underline{\phi}_0^*] - [p\log\overline{\phi}_0^* + (1-p)\log\overline{\phi}_0^* - (1+b)x\overline{\phi}_0^*]. \end{aligned} \tag{27}$$

By construction, $\overline{W}(g,\alpha,b) \geq W(g,\alpha,b)$ for every $\alpha \in (\hat{\alpha},1]$, where the inequality is strict for all $\alpha \geq \hat{\alpha}$ but $\alpha = 1$. Lastly, for every $\phi_\gamma, \phi_\delta \in [\underline{\phi}_0^*, \overline{\phi}_0^*]$, define

$$\begin{aligned} \tilde{W}(\phi_\gamma,\phi_\delta,g,\alpha,b) &:= \left[p\log(\alpha+(1-\alpha)\phi_\gamma) - (\alpha+(1-\alpha)\phi_\gamma)gx - \frac{\lambda}{g}\alpha\left(p\frac{1}{\alpha+(1-\alpha)\phi_\gamma} - gx\right)\right] \\ &\quad + [(1-p)\log\phi_\delta - \phi_\delta((1-g)+b)x] - [p\log\overline{\phi}_0^* + (1-p)\log\overline{\phi}_0^* - (1+b)x\overline{\phi}_0^*]. \end{aligned} \tag{28}$$

$\tilde{W}$ is increasing in ϕ_γ and decreasing in ϕ_δ for any $\phi_\gamma, \phi_\delta \in [\underline{\phi}_0^*, \overline{\phi}_0^*]$, respectively.

Now, the proof proceeds in several claims.

Claim A.1. *Both $\overline{W}(g,\alpha,b)$ and $\tilde{W}(\phi_\gamma,\phi_\delta,g,\alpha,b)$ are quasi-convex in α for all $\alpha \geq \hat{\alpha}$.*

Proof. We prove quasi-convexity of $\overline{W}$ only, because the proof of quasi-convexity of $\tilde{W}$ is similar. Differentiating $\overline{W}$ with respect to α, we have

$$\begin{aligned}
\frac{d\overline{W}}{d\alpha} &= (1-\overline{\phi}_0^*)\left(p\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}-gx\right)-\frac{\lambda}{g}\left(p\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}-gx\right)+\lambda\frac{p}{g}\frac{\alpha(1-\overline{\phi}_0^*)}{(\alpha+(1-\alpha)\overline{\phi}_0^*)^2}\\
&= (1-\overline{\phi}_0^*)\left(p\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}-gx\right)+\lambda\frac{p}{g}\left(\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}\right)\left(\frac{\alpha(1-\overline{\phi}_0^*)}{\alpha+(1-\alpha)\overline{\phi}_0^*}-1\right)+\lambda x\\
&= (1-\overline{\phi}_0^*)\left(p\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}-gx\right)+\lambda\frac{p}{g}\left(\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}\right)\left(-\frac{\overline{\phi}_0^*}{\alpha+(1-\alpha)\overline{\phi}_0^*}\right)+\lambda x\\
&= \frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}\cdot\frac{p}{g}\left[(1-\overline{\phi}_0^*)g-\lambda\frac{\overline{\phi}_0^*}{\alpha+(1-\alpha)\overline{\phi}_0^*}\right]-((1-\overline{\phi}_0^*)g-\lambda)x.
\end{aligned}$$

Since $\frac{\overline{\phi}_0^*}{\alpha+(1-\alpha)\overline{\phi}_0^*}\leq 1$, $\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}\geq 1$, and $p-gx>0$, we have

$$\left(\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}\right)\frac{p}{g}>x$$

and

$$(1-\overline{\phi}_0^*)g-\lambda\frac{\overline{\phi}_0^*}{\alpha+(1-\alpha)\overline{\phi}_0^*}\geq((1-\overline{\phi}_0^*)g-\lambda).$$

If $(1-\overline{\phi}_0^*)g-\lambda\frac{\overline{\phi}_0^*}{\alpha+(1-\alpha)\overline{\phi}_0^*}\geq 0$, then we always have $\frac{d\overline{W}}{d\alpha}>0$. Consider $(1-\overline{\phi}_0^*)g-\lambda\frac{\overline{\phi}_0^*}{\alpha+(1-\alpha)\overline{\phi}_0^*}<0$ next. If $\frac{d\overline{W}}{d\alpha}=0$, we have

$$\frac{d\overline{W}}{d\alpha}=\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}\cdot\frac{p}{g}\left[(1-\overline{\phi}_0^*)g-\lambda\frac{\overline{\phi}_0^*}{\alpha+(1-\alpha)\overline{\phi}_0^*}\right]-((1-\overline{\phi}_0^*)g-\lambda)x=0.$$

Since $(1-\overline{\phi}_0^*)g-\lambda\frac{\overline{\phi}_0^*}{\alpha+(1-\alpha)\overline{\phi}_0^*}<0$, we have

$$\begin{aligned}
\frac{d^2\overline{W}}{d\alpha^2} &= \frac{d}{d\alpha}\left[\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}\cdot\frac{p}{g}\left((1-\overline{\phi}_0^*)g-\lambda\frac{\overline{\phi}_0^*}{\alpha+(1-\alpha)\overline{\phi}_0^*}\right)\right]\\
&= \frac{p}{g}\left[-\frac{(1-\overline{\phi}_0^*)}{(\alpha+(1-\alpha)\overline{\phi}_0^*)^2}\left((1-\overline{\phi}_0^*)g-\lambda\frac{\overline{\phi}_0^*}{\alpha+(1-\alpha)\overline{\phi}_0^*}\right)+\frac{1}{\alpha+(1-\alpha)\overline{\phi}_0^*}\left(\frac{\lambda\overline{\phi}_0^*(1-\overline{\phi}_0^*)}{(\alpha+(1-\alpha)\overline{\phi}_0^*)^2}\right)\right]\\
&> 0
\end{aligned}$$

if $\frac{d\overline{W}}{d\alpha}=0$, which implies the desired result. *Q.E.D.*

Claim A.2. *If $\overline{W}(g, \hat{\alpha}, b) > \overline{W}(g, 1, b)$, there exists a $\overline{\alpha} \in (\hat{\alpha}, 1]$ such that $\overline{W}(g, \alpha, b) > \overline{W}(g, 1, b) \vee 0$ if and only if $\alpha \in (\hat{\alpha}, \overline{\alpha})$.*

Proof. The proof is straightforward from Claim A.1. *Q.E.D.*

Claim A.3. *$\tilde{W}(\phi, \phi, g, \hat{\alpha}, b) < W(g, \hat{\alpha}, b)(\leq 0)$ for all $\phi < \overline{\phi}_0^*$.*

Proof. Since $\tilde{W}(\phi, \phi, g, \hat{\alpha}, b)$ is a linear sum of two strictly concave functions of ϕ, $\tilde{W}(\phi, \phi, g, \hat{\alpha}, b)$ is a globally concave function of ϕ. Furthermore, $\tilde{W}(\phi, \phi, g, \hat{\alpha}, b)$ is uniquely maximized at $\phi = \overline{\phi}_0^*$, which implies $\tilde{W}(\phi, \phi, g, \hat{\alpha}, b) < W(\overline{\phi}_0^*, \overline{\phi}_0^*, g, \hat{\alpha}, b) = W(g, \hat{\alpha}, b) \leq 0$. *Q.E.D.*

Claim A.4. *Suppose $\tilde{W}(\phi_\gamma, \phi_\delta, g, \hat{\alpha}, b) = W(g, 1, b) \vee 0$ for some $\phi_\gamma \in [\underline{\phi}_0^*, \overline{\phi}_0^*)$ and $\phi_\delta \geq \underline{\phi}_0^*$. Then, $\tilde{W}(\phi_\gamma, \phi_\delta, g, \alpha, b) < W(g, 1, b) \vee 0$ for all $\alpha \in [\hat{\alpha}, \overline{\alpha}]$.*

Proof. Recall that $\overline{W}(g, 1, b) = W(g, 1, b)$. Suppose to the contrary that there exists a $\tilde{\alpha} \in [\hat{\alpha}, \overline{\alpha}]$ such that $\tilde{W}(\phi_\gamma, \phi_\delta, g, \tilde{\alpha}, b) \geq W(g, 1, b) \vee 0 = \tilde{W}(\phi_\gamma, \phi_\delta, g, \hat{\alpha}, b)$. Since $\tilde{W}(\phi_\gamma, \phi_\delta, g, \alpha, b)$ is quasi-convex in α and $\tilde{W}(\phi_\gamma, \phi_\delta, g, \hat{\alpha}, b) = W(g, 1, b) \vee 0$ by assumption, $\tilde{W}(\phi_\gamma, \phi_\delta, g, \alpha, b)$ must be monotone increasing in α for all $\alpha \geq \tilde{\alpha}$.

There are two possible cases, $\phi_\delta > \underline{\phi}_0^*$ or $\phi_\delta = \underline{\phi}_0^*$. If $\phi_\delta > \underline{\phi}_0^*$, we must have $\tilde{W}(\phi_\gamma, \phi_\delta, g, 1, b) < W(g, 1, b) \vee 0$ since $\phi_\gamma < \overline{\phi}_0^*$. Furthermore, we have $\tilde{W}(\phi_\gamma, \phi_\delta, g, \tilde{\alpha}, b) \geq W(g, 1, b) > \tilde{W}(\phi_\gamma, \phi_\delta, g, 1, b)$. Since $\tilde{\alpha} < 1$, $\tilde{W}(\phi_\gamma, \phi_\delta, g, \alpha, b)$ must decrease in α for some $\alpha \geq \tilde{\alpha}$, which contradicts quasi-convexity of $\tilde{W}(\phi_\gamma, \phi_\delta, g, \alpha, b)$.

If $\phi_\delta = \underline{\phi}_0^*$, we must have

$$\tilde{W}(\phi_\gamma, \phi_\delta, g, \overline{\alpha}, b) \geq \tilde{W}(\phi_\gamma, \phi_\delta, g, \tilde{\alpha}, b) \geq W(g, 1, b) \vee 0 = \overline{W}(g, \overline{\alpha}, b),$$

where the first inequality follows from the observation that $\tilde{W}(\phi_\gamma, \phi_\delta, g, \alpha, b)$ is monotone increasing in α for all $\alpha \geq \tilde{\alpha}$, and the equality follows from the definition of $\overline{\alpha}$. However, since $\phi_\gamma < \overline{\phi}_0^*$, we must have

$$\tilde{W}(\phi_\gamma, \phi_\delta, g, \overline{\alpha}, b) < \tilde{W}(\overline{\phi}_0^*, \underline{\phi}_0^*, g, \overline{\alpha}, b) = \overline{W}(g, \overline{\alpha}, b),$$

where the strict inequality follows from $\frac{\partial \tilde{W}}{\partial \phi_\gamma} > 0$, and the equality follows from the definition of $\overline{W}(g, a, b)$. Combining the observations above together, we have $\tilde{W}(\phi_\gamma, \phi_\delta, g, \overline{\alpha}, b) \geq \overline{W}(g, \overline{\alpha}, b) > \tilde{W}(\phi_\gamma, \phi_\delta, g, \overline{\alpha}, b)$, a contradiction. *Q.E.D.*

Claim A.5. *Suppose* $\overline{W}(g,\hat{\alpha},b) > \overline{W}(g,1,b) \vee 0$. *Then* $W(g,\alpha,b) < W(g,1,b)$ *for all* $\alpha \in [0,1)$.

Proof. Recall that $\overline{W}(g,1,b) = W(g,1,b)$ and $W(g,\hat{\alpha},b) = \tilde{W}(\overline{\phi}_0^*, \overline{\phi}_0^*, g, \hat{\alpha}, b) \leq 0$. By Claim A.2 and quasi-convexity of $\overline{W}$, we know

$$W(g,\alpha,b) < \overline{W}(g,\alpha,b) \leq \overline{W}(g,1,b) \vee 0 = W(g,1,b) \vee 0$$

for all $\alpha \in (\overline{\alpha}, 1]$. Hence, $W(g,\alpha,b) < W(g,1,b) \vee 0$ for all $\alpha \in (\hat{\alpha}, 1)$. Furthermore, by **??**, we have $W(g,\alpha,b) < 0$ for every $\alpha \leq \hat{\alpha}$.

What remains to prove is $W(g,\alpha,b) < \overline{W}(g,1,b) \vee 0$ for all $\alpha \in (\hat{\alpha}, \overline{\alpha}]$. Suppose $\overline{W}(g,\hat{\alpha},b) > \overline{W}(g,1,b) \vee 0$. We first prove for the case $W(g,1,b) > 0$. Since $\tilde{W}(\phi_\gamma, \phi_\delta, g, \alpha, b)$ is decreasing in ϕ_δ and $\phi_0^*(\alpha)$ is decreasing in α, there exists a $\alpha_1 \in (\hat{\alpha}, 1)$ (and thus $\underline{\phi}_0^* < \phi_0^*(\alpha_1) < \overline{\phi}_0^*$) such that $\tilde{W}(\overline{\phi}_0^*, \phi_0^*(\alpha_1), g, \hat{\alpha}, b) = W(g,1,b)$.

We first show $\tilde{W}(\overline{\phi}_0^*, \phi_0^*(\alpha_1), g, \alpha, b) \leq W(g,1,b)$ for every $\alpha \leq \overline{\alpha} \vee \alpha_1$. Suppose to the contrary that $\tilde{W}(\overline{\phi}_0^*, \phi_0^*(\alpha_1), g, \alpha', b) > W(g,1,b)$ for some $\alpha' \leq \overline{\alpha} \vee \alpha_1$. By quasi-convexity of $\tilde{W}$, we must have $\tilde{W}(\overline{\phi}_0^*, \phi_0^*(\alpha_1), g, \alpha, b) > W(g,1,b)$ for all $\alpha > \alpha'$. Since $\overline{W}(g,\alpha,b)$ is decreasing in α for all $\alpha \leq \overline{\alpha}$ by definition of $\overline{\alpha}$, we must have $\tilde{W}(\overline{\phi}_0^*, \phi_0^*(\alpha_1), g, \alpha, b) > \overline{W}(g,\alpha,b)$ for some $\alpha \in \alpha'$, which contradicts $\tilde{W} \leq \overline{W}$ for every α. From this observation, we have

$$W(g,\alpha,b) = \tilde{W}(\phi_0^*(\alpha), \phi_0^*(\alpha), g, \alpha, b) < \tilde{W}(\overline{\phi}_0^*, \phi_0^*(\alpha_1), g, \alpha, b) \leq W(g,1,b) \tag{29}$$

for all $\alpha \leq \alpha_1$, where the strict inequality follows from $\phi_0^*(\alpha_1) < \phi_0^*(\alpha) \leq \overline{\phi}_0^*$ for all $\hat{\alpha} \leq \alpha < \alpha_1$, $\frac{\partial \tilde{W}}{\partial \phi_\gamma} > 0$, and $\frac{\partial \tilde{W}}{\partial \phi_\delta} < 0$. If $\alpha_1 \geq \overline{\alpha}$, the proof is done since $W(g,\alpha,b) < W(g,1,b)$ for all $\alpha \leq \overline{\alpha}$ by (29). Hence we restrict our focus on $\alpha_1 < \overline{\alpha}$.

Since $\tilde{W}(\phi_0^*(\alpha_1), \phi_\delta, g, \hat{\alpha}, b)$ is decreasing in ϕ_δ for all $\phi_\delta \in [\underline{\phi}_0^*, \overline{\phi}_0^*]$, there are two possible cases: *(i)* $\tilde{W}(\phi_0^*(\alpha_1), \phi_0^*(\overline{\alpha}), g, \hat{\alpha}, b) \leq W(g,1,b)$; *(ii)* there exists a $\alpha_2 > \alpha_1$ such that $\tilde{W}(\phi_0^*(\alpha_1), \phi_0^*(\alpha_2), g, \hat{\alpha}, b) = W(g,1,b)$. Consider the former case. By Claim A.4, we have $\tilde{W}(\phi_0^*(\alpha_1), \phi_0^*(\overline{\alpha}), g, \alpha, b) \leq W(g,1,b)$ for all $\alpha \in (\alpha_1, \overline{\alpha}]$, which implies

$$W(g,\alpha,b) < \tilde{W}(\phi_0^*(\alpha_1), \phi_0^*(\overline{\alpha}), g, \alpha, b) \leq W(g,1,b)$$

for every $\alpha \in (\alpha_1, \overline{\alpha}]$, so the proof is done. In the latter case, we know $\phi_0^*(\alpha_1) \in [\underline{\phi}_0^*, \overline{\phi}_0^*)$ and

$\phi_0^*(\alpha_2) \geq \underline{\phi}_0^*$. Hence, by Claim A.4, we have

$$\tilde{W}(\phi_0^*(\alpha_1), \phi_0^*(\alpha_2), g, \alpha, b) < W(g, 1, b) = \tilde{W}(\phi_0^*(\alpha_1), \phi_0^*(\alpha_2), g, \hat{\alpha}, b)$$

for all $\alpha \leq \overline{\alpha}$. Furthermore, since $\phi_0^*(\alpha_1) > \phi_0^*(\alpha) > \phi_0^*(\alpha_2)$ for all $\alpha \in (\alpha_1, \alpha_2)$, we have

$$W(g, \alpha, b) = \tilde{W}(\phi_0^*(\alpha), \phi_0^*(\alpha), g, \alpha, b) < \tilde{W}(\phi_0^*(\alpha_1), \phi_0^*(\alpha_2), g, \alpha, b) < W(g, 1, b)$$

for all $\alpha \in (\alpha_1, \alpha_2]$, where the first inequality follows from $\frac{\partial}{\partial \phi_\gamma}\tilde{W}(\phi_\gamma, \phi_\delta, g, \alpha, b) > 0$ and $\frac{\partial}{\partial \phi_\delta}\tilde{W}(\phi_\gamma, \phi_\delta, g, \alpha, b) < 0$. If $\alpha_2 \geq \overline{\alpha}$, then the proof is complete. If not, construct a sequence $\{\alpha_3, \alpha_4, \ldots\}$ inductively such that $\tilde{W}(\phi_0^*(\alpha_n), \phi_0^*(\alpha_{n+1}), g, \hat{\alpha}, b) = W(g, 1, b)$. Then, for every (α_n, α_{n+1}) in this sequence, we have

$$\tilde{W}(\phi_0^*(\alpha_n), \phi_0^*(\alpha_n), g, \hat{\alpha}, b) < W(g, \hat{\alpha}, b) \leq 0 < \tilde{W}(\phi_0^*(\alpha_n), \phi_0^*(\alpha_{n+1}), g, \hat{\alpha}, b) = W(g, 1, b),$$

where the first strict inequality follows from Claim A.3 and the second strict inequality holds by construction of the sequence (α_n). This implies the sequence (α_n) should not have a limit point at some $\alpha' \leq \overline{\alpha}$. Hence, there must exist a $\alpha_N > \overline{\alpha}$ for a finitely large number N, which implies $W(g, \alpha, b) < W(g, 1, b)$ for all $\alpha \leq \overline{\alpha}$.

We next prove for the remaining case $W(g, 1, b) \leq 0$. To this end, take an arbitrary positive number $\varepsilon \in (0, \overline{W}(g, \hat{\alpha}, b))$. Then, by applying the same logic above used for the case $W(g, 1, b) > 0$, one can find that $W(g, \alpha, b) < \varepsilon$ for every $\alpha \in (\hat{\alpha}, \overline{\alpha}]$. Since ε is an arbitrary number, we have that $W(g, \alpha, b) \leq 0$ for every $\alpha \in (\hat{\alpha}, \overline{\alpha}]$. Therefore, we find that $W(g, \alpha, b) < W(g, 1, b) \vee 0$ for every $\alpha \in (\hat{\alpha}, \overline{\alpha}]$, which completes the proof. *Q.E.D.*

From Claim A.1 – A.5, we have that $W(g, \alpha, b) \leq W(g, 1, b) \vee 0$, where the equality holds if and only if $W(g, 1, b) > 0$ and $\alpha = 1$. This means $\alpha^{**}(b) \in \{0, 1\}$ for every $b > \frac{1-p}{x} - (1 - g)$. *Q.E.D.*

Proof of Lemma 4. For every b that yields $\phi_0^* < 1$ and $R_0^* = \frac{1}{x}$ when $\alpha = 1$, we have

$$W(g, 1, b) = [U(g, 1, b) - U(g, 0, b)] - \lambda \left[\left(\frac{p}{g} - x \right) - \left(\frac{p}{g} - \frac{1-p}{1-g} \right) \max\{1 - (1 + b)x, 0\} \right].$$

Since $g > p$ by Lemma 1 and $\frac{d}{db}[U(g, 1, b) - U(g, 0, b)] > 0$ from (17), we have $\frac{d}{db}W(g, 1, b) > 0$, which is the desired result. *Q.E.D.*

Proof of Theorem 4. Fix an equilibrium fraction $g > p$ induced by an optimal disclosure policy $\alpha^{**}(\cdot)$. By Lemma 2, we have $\alpha^{**}(b) = 1$ if $b \leq \frac{g}{p} - 1$ and $\alpha^{**}(b) = 0$ if $\frac{g}{p} - 1 < b \leq \frac{1-p}{x} - (1-g)$. It therefore remains to show that there exists an upper threshold $\overline{b}^{**} \geq \frac{1-p}{x} - (1-g)$ such that $\alpha^{**}(b) = 0$ if $\frac{1-p}{x} - (1-g) < b < \overline{b}^{**}$ and $\alpha^{**}(b) = 1$ if $b > \overline{b}^{**}$. By Lemma 3, the optimal disclosure rule must be either $\alpha^{**}(b) = 0$ or $\alpha^{**}(b) = 1$ for each $b > \frac{1-p}{x} - (1-g)$. Since $\alpha^{**}(b)$ maximizes the Lagrangian, we must have $\alpha^{**}(b) = 1$ for $b > \frac{1-p}{x} - (1-g)$ if and only if $W(g, 1, b) > 0$. Moreover, since $W(g, 1, b)$ is increasing in b by Lemma 4, there exists a unique $\overline{b}^{**} \geq \frac{1-p}{x} - (1-g)$ such that $W(g, 1, \overline{b}^{**}) = 0$. Then, the optimal disclosure policy must have $\alpha^{**}(b) = 0$ if $b \in \left(\frac{1-p}{x} - (1-g), \overline{b}^{**}\right)$ and $\alpha^{**}(b) = 1$ if $b \geq \overline{b}^{**}$. *Q.E.D.*

A.3 Proofs for Section 5.1

In this section, we show that the main results of Theorem 3 and 4 are qualitatively unchanged if we modify our model by assuming that the date-2 utility function of each investor is represented by $u(x) = \frac{1}{1-\rho}x^{1-\rho}$ for some $\rho > 1$.

A.3.1 Date-1 Equilibrium

Consider an individual investor at the capital market, who makes purchase offers (R_1^*, ϕ_1^*) to the banks labeled $s = 1$ and (R_0^*, ϕ_0^*) to the banks labeled $s = 0$, respectively. If the regulator reveals $s = 1$ with probability $\alpha \in [0, 1]$, the investor's expected utility is

$$\begin{aligned} &\frac{p}{1-\rho}(y_G + \alpha\phi_1^* g R_1^* x + (1-\alpha)\phi_0^* g R_0^* x)^{1-\rho} + \frac{1-p}{1-\rho}(y_D + \phi_0^*(1-g)R_0^* x)^{1-\rho} \\ &\quad - (\alpha\phi_1^* + (1-\alpha)\phi_0^*)gx - \phi_0^*((1-g) + b)x, \end{aligned} \tag{30}$$

where y_G and y_D are dividends equally distributed to every equity-holder. By differentiating with respect to ϕ_1^*, we have the following first order condition:

$$p\frac{\alpha g R_1^* x}{[(\alpha\phi_1^* + (1-\alpha)\phi_0^*)g]^\rho} - agx \geq 0.$$

Since $p - x > 0$, we have

$$(R_1^*, \phi_1^*) = \left(\frac{1}{p}[(\alpha\phi_1^* + (1-\alpha)\phi_0^*)g]^\rho, 1\right) \text{ for all } b \geq 0. \tag{31}$$

Furthermore, by differentiating with respect to ϕ_0^*, we have the following first order condition:

$$\left[p\frac{(1-\alpha)g}{[(\alpha\phi_1^*+(1-\alpha)\phi_0^*)g]^\rho}+(1-p)\frac{(1-g)}{[(1-g)\phi_0^*]^\rho}\right]R_0^*\geq((1-\alpha)g+(1-g)+b).$$

Therefore, we have

$$(R_0^*,\phi_0^*)=\left(\frac{(1-\alpha)g+(1-g)+b}{\frac{p}{g^{\rho-1}}(1-\alpha)+\frac{1-p}{(1-g)^{\rho-1}}},1\right)\text{ if }\frac{(1-\alpha)g+(1-g)+b}{\frac{p}{g^{\rho-1}}(1-\alpha)+\frac{1-p}{(1-g)^{\rho-1}}}\leq\frac{1}{x}.\tag{32}$$

However, if $\frac{(1-\alpha)g+(1-g)+b}{\frac{p}{g^{\rho-1}}(1-\alpha)+\frac{1-p}{(1-g)^{\rho-1}}}>\frac{1}{x}$, and thus $R_0^*=\frac{1}{x}$, then ϕ_0^* is uniquely determined by the following equation:

$$\left[p\frac{(1-\alpha)g}{[(\alpha\phi_1^*+(1-\alpha)\phi_0^*)g]^\rho}+(1-p)\frac{(1-g)}{[(1-g)\phi_0^*]^\rho}\right]=((1-\alpha)g+(1-g)+b)x.\tag{33}$$

By applying the Implicit Function Theorem, one can easily find that $\frac{d\phi_0^*}{d\alpha}<0$ and $\frac{d\phi_0^*}{db}<0$. From this result, we can construct the investors' expected utility function $U(g,\alpha,b)$ at $t=1$ as follows:

$$\begin{aligned}U(g,\alpha,b):=&\frac{p}{1-\rho}[(\alpha+(1-\alpha)\phi_0^*)g]^{1-\rho}-(\alpha+(1-\alpha)\phi_0^*)gx\\&+\frac{1-p}{1-\rho}[\phi_0^*(1-g)]^{1-\rho}-\phi_0^*((1-g)+b)x.\end{aligned}\tag{34}$$

In particular, the expected utility function without the regulator's information is $U(g,0,b)$, which is written as

$$U(g,0,b)=\frac{p}{1-\rho}(\phi^*g)^{1-\rho}+\frac{1-p}{1-\rho}(\phi^*(1-g))^{1-\rho}-\phi^*(1+b)x,\tag{35}$$

where $\phi^*=\left[\frac{pg^{1-\rho}+(1-p)(1-g)^{1-\rho}}{(1+b)x}\right]^{\frac{1}{\rho}}\wedge 1$.

Lastly, the ex-post net gains from information disclosure is $U(g,\alpha,b)-U(g,0,b)$. By applying the Envelope Theorem, we have

$$\frac{d}{d\alpha}[U(g,\alpha,b)-U(g,0,b)]=(1-\phi_0^*)\left[\frac{p}{[(\alpha+(1-\alpha)\phi_0^*)g]^\rho}-gx\right]\geq 0,\tag{36}$$

where the inequality strictly holds if and only if $\phi_0^*<1$. That is, the regulator can improve the investors' ex-post welfare by revealing more information on banks' types to the market.

Similarly, we have

$$\frac{d}{db}[U(g,\alpha,b) - U(g,0,b)] = (\phi^* - \phi_0^*)x \geq 0, \tag{37}$$

where the inequality is strict if and only if $\phi^* > \phi_0^*$, i.e., the adverse selection is relatively severe.

A.3.2 Date-0 Project Choice

To analyze how the disclosure policy α influences the banks' project choices, we next consider the date-0 equilibrium. Then, each bank will take into account the following equity-holders' expected surplus:

$$\frac{p}{1-\rho}\left[\alpha g(1-R_1^*x) + (1-\alpha)g\phi_0^*(1-R_0^*x) + z_G\right]^{1-\rho} + \frac{1-p}{1-\rho}\left[(1-g)\phi_0^*(1-R_0^*x) + z_D\right]^{1-\rho} - C^*.$$

If a bank chooses project type γ rather than δ, the marginal surplus will be equal to

$$\begin{aligned}\Delta(g,\alpha,b) := {} & p\frac{\alpha}{[(\alpha+(1-\alpha)\phi_0^*)g]^\rho}(1-R_1^*x) \\ & + \left[p\frac{1-\alpha}{[(\alpha+(1-\alpha)\phi_0^*)g]^\rho} - (1-p)\frac{1}{[(1-g)\phi_0^*]^\rho}\right]\phi_0^*(1-R_0^*x).\end{aligned}$$

Given a disclosure policy $\{\alpha(b)\}_{b\geq 0}$, the fraction g must be determined to make $\int \Delta(g,\alpha,b)dF(b) = 0$ in equilibrium.

If $\phi_0^* = 1$, we have

$$\Delta(g,\alpha,b) = \left(\frac{p}{g^\rho} - \frac{1-p}{(1-g)^\rho}\right) - \left[\alpha x + R_0^* x\left(\frac{p}{g^\rho}(1-\alpha) - \frac{1-p}{(1-g)^\rho}\right)\right]. \tag{38}$$

By plugging $R_0^* = \frac{(1-\alpha)g+(1-g)+b}{\frac{p}{g^{\rho-1}}(1-\alpha)+\frac{1-p}{(1-g)^{\rho-1}}}$ into (38), $\Delta(g,\alpha,b)$ is rewritten as

$$\Delta(g,\alpha,b) = \left(\frac{p}{g^\rho} - \frac{1-p}{(1-g)^\rho}\right) - \left(1 + \frac{(1-g)+b}{g}\right)x + \left(1 + \frac{\frac{(1-g)+b}{g} - \frac{(1-p)g^{\rho-1}}{p(1-g)^{\rho-1}}}{(1-\alpha) + \frac{(1-p)g^{\rho-1}}{p(1-g)^{\rho-1}}}\right)\frac{(1-p)g^{1-\rho}}{p(1-g)^\rho}x. \tag{39}$$

Hence, $\Delta(g,\alpha,b)$ is decreasing in α if and only if $\frac{(1-g)+b}{g} - \frac{(1-p)g^{\rho-1}}{p(1-g)^{\rho-1}} < 0$, or equivalently,

$$b < (1-g)\left[\frac{1-p}{p}\left(\frac{g}{1-g}\right)^{\rho} - 1\right]. \tag{40}$$

The right hand side of (40) is positive if and only if $g \in (0,1)$ and $g^{\rho} > p$.

If $\phi_0^* < 1$, we have

$$\Delta(g,\alpha,b) = \alpha\left[\frac{p}{[(\alpha+(1-\alpha)\phi_0^*)g]^{\rho}} - x\right]. \tag{41}$$

Lastly, since $\frac{d\phi^*}{d\alpha} < 0$, we can find $\hat{\alpha} \in [0,1]$ for every $b \geq 0$ such that $\Delta(g,\alpha,b)$ is equal to (39) if $\alpha \leq \hat{\alpha}$ but equal to (41) if $\alpha > \hat{\alpha}$. Furthermore, by applying $\frac{d\phi^*}{d\alpha} < 0$, we have that *(i)* $\hat{\alpha} = 1$ for every $b \leq \frac{1-p}{x(1-g)^{\rho-1}} - (1-g)$; *(ii)* $\hat{\alpha} \in (0,1)$ and $\hat{\alpha}$ is decreasing in b if $b \in \left(\frac{1-p}{x(1-g)^{\rho-1}} - (1-g), \frac{1}{x}\left[\frac{p}{g^{\rho-1}} + \frac{1-p}{(1-g)^{\rho-1}}\right] - 1\right]$; *(iii)* $\hat{\alpha} = 0$ if $b \geq \frac{1}{x}\left[\frac{p}{g^{\rho-1}} + \frac{1-p}{(1-g)^{\rho-1}}\right] - 1$.

A.3.3 The Ex-Ante Optimal Disclosure Policy

Proof of Theorem 5. First, consider $b \leq \frac{1-p}{x(1-g)^{\rho-1}} - (1-g)$. In this case, we have $\phi^* = \phi_0^* = \phi_1^* = 1$. Therefore, we have $U(g,\alpha,b) - U(g,0,b) = 0$. Furthermore, we know from (39) that $\Delta(g,\alpha,b)$ is decreasing in α if and only if $\frac{(1-g)+b}{g} - \frac{(1-p)g^{\rho-1}}{p(1-g)^{\rho-1}} < 0$. Since the mathematical properties of $U(g,\alpha,b)$ and $\Delta(g,\alpha,b)$ are similar to those in the baseline model, we have the following lemmas, which correspond to Lemma 1 and 2, respectively.

Lemma A.2. *Any optimal disclosure policy α^{**} must yield $g^{\rho} > p$.*

Lemma A.3. *For every $b \leq \frac{1-p}{x(1-g)^{\rho-1}} - (1-g)$, an optimal disclosure policy α^{**} is characterized by a threshold $\underline{b}^{**} := (1-g)\left[\frac{1-p}{p}\left(\frac{g}{1-g}\right)^{\rho} - 1\right]$ such that $\alpha^{**}(b) = 1$ if $b \leq \underline{b}^{**}$ and $\alpha^{**}(b) = 0$ if $b \in \left(\underline{b}^{**}, \frac{1-p}{x(1-g)^{\rho-1}} - (1-g)\right]$.*

Next, consider high $b > \frac{1-p}{x(1-g)^{\rho-1}} - (1-g)$. Like in the main model with $\rho = 1$, we know $U(g,\alpha,b) - U(g,0,b) = 0$ for all $\alpha \leq \hat{\alpha}$, where $\hat{\alpha} > 0$. However, $\Delta(g,\alpha,b) - \Delta(g,0,b) > 0$ since $\Delta(g,\alpha,b)$ is increasing in b for all $b > (1-g)\left[\frac{1-p}{p}\left(\frac{g}{1-g}\right)^{\rho} - 1\right]$ and $\frac{1-p}{x(1-g)^{\rho-1}} - (1-g) > (1-g)\left[\frac{1-p}{p}\left(\frac{g}{1-g}\right)^{\rho} - 1\right]$. These results imply that the regulator should not choose $\alpha \leq \hat{\theta}$ for any b.

Lemma A.4. *$\alpha^{**}(b) \geq \hat{\alpha}(b)$ for every $b > \frac{1-p}{x(1-g)^{\rho-1}} - (1-g)$ such that $\hat{\alpha}(b) > 0$.*

Thus, like we did in the main model, consider the Lagrangian for a fixed $g > (p)^{\frac{1}{\rho}}$ corresponding to the regulator's ex-ante optimization problem:

$$\mathcal{L} := \int_{\frac{1-p}{x(1-g)^{\rho-1}}-(1-g)}^{\infty} [U(g,\alpha,b) - \lambda\Delta(g,\alpha,b)]\, dF(b), \tag{42}$$

where

$$\begin{aligned}[U - \lambda\Delta]\,(g,\alpha,b) = &\left[\frac{p}{1-\rho}[(\alpha+(1-\alpha)\phi_0^*)g]^{1-\rho} - (\alpha+(1-\alpha)\phi_0^*)gx - \lambda\alpha\left(\frac{p}{[(\alpha+(1-\alpha)\phi_0^*)g]^{\rho}} - x\right)\right] \\ &+ \left\{\frac{1-p}{1-\rho}[\phi_0^*(1-g)]^{1-\rho} - \phi_0^*((1-g)+b)x\right\}\end{aligned} \tag{43}$$

for every $\alpha \geq \hat{\alpha}$ and $b > \frac{1-p}{x(1-g)^{\rho-1}} - (1-g)$. Furthermore, define the net gains from information disclosure, denoted by $W(g,\alpha,b)$, as

$$\begin{aligned}W(g,\alpha,b) &:= [U(g,\alpha,b) - \lambda\Delta(g,\alpha,b)] - [U(g,0,b) - \lambda\Delta(g,0,b)] \\ &= [U(g,\alpha,b) - \lambda\Delta(g,\alpha,b)] - U(g,0,b).\end{aligned}$$

From (43), we can define $\overline{W}(g,\alpha,b)$ and $\tilde{W}(g,\alpha,b)$, modified forms of $W(g,\alpha,b)$, as follows:

$$\begin{aligned}\overline{W}(g,\alpha,b) = &\left[\frac{p}{1-\rho}[(\alpha+(1-\alpha)\overline{\phi}_0^*)g]^{1-\rho} - (\alpha+(1-\alpha)\overline{\phi}_0^*)gx - \lambda\alpha\left(\frac{p}{[(\alpha+(1-\alpha)\overline{\phi}_0^*)g]^{\rho}} - x\right)\right] \\ &+ \left\{\frac{1-p}{1-\rho}[\underline{\phi}_0^*(1-g)]^{1-\rho} - \underline{\phi}_0^*((1-g)+b)x\right\} - U(g,0,b),\end{aligned} \tag{44}$$

and

$$\begin{aligned}\tilde{W}(\phi_\gamma,\phi_\delta,g,\alpha,b) = &\left[\frac{p}{1-\rho}[(\alpha+(1-\alpha)\phi_\gamma)g]^{1-\rho} - (\alpha+(1-\alpha)\phi_\gamma)gx - \lambda\alpha\left(\frac{p}{[(\alpha+(1-\alpha)\phi_\gamma)g]^{\rho}} - x\right)\right] \\ &+ \left\{\frac{1-p}{1-\rho}[\phi_\delta(1-g)]^{1-\rho} - \phi_\delta((1-g)+b)x\right\} - U(g,0,b);\end{aligned} \tag{45}$$

One can find that Claim A.1 – A.5 hold true, which then implies that Lemma 3 holds true, too. Furthermore, $W(g,1,b)$ is increasing in b by (37), which implies that Lemma 4 holds true. By combining all the results together, we have that the optimal disclosure policy $\alpha^{**}(b)$ is characterized by two cutoffs $0 < \underline{b}^{**} < \overline{b}^{**} < \infty$ such that $\alpha^{**}(b) = 1$ if $b \in [0 < \underline{b}^{**}] \cup [\overline{b}^{**}, \infty)$ and $\alpha^{**}(b) = 0$ otherwise. *Q.E.D.*

A.4 Proofs for Section 5.3

Proof of Theorem 6. Fix any $b \geq 0$. If the regulator chooses no disclosure $\alpha = 0$, g is determined by

$$\Delta(g, 0, b) = \left(\frac{p}{g} - \frac{1-p}{1-g}\right) \max\{1 - (1+b)x, 0\} = 0.$$

If $b < \frac{1}{x} - 1$, no disclosure yields $g = p$ and the total welfare $U(p, 0, b)$. If $b \geq \frac{1}{x} - 1$, no disclosure yields any $g \in (0, 1)$ and the total welfare $U(g, 0, b)$.

If the regulator chooses $\alpha > 0$, there are two possible cases, either $R_0^* < \frac{1}{x}$ or $R_0^* = \frac{1}{x}$. First, suppose the regulator's policy $\alpha > 0$ yields $R_0^* < \frac{1}{x}$. Then we have

$$\Delta(g, \alpha, b) = \left[\left(\frac{p}{g} - \frac{1-p}{1-g}\right) - \left(1 + \frac{(1-g)+b}{g}\right)x\right] + \frac{x(1-p)}{p(1-g)}\left(1 + \frac{\frac{(1-g)+b}{g} - \frac{1-p}{p}}{(1-\alpha) + \frac{1-p}{p}}\right).$$

If $\Delta(p, \alpha, b) < 0$, the disclosure policy $\alpha > 0$ yields $g \neq p$ and the resulting total welfare $U(g, \alpha, b) = U(g, 0, b)$, where the equality follows from $\phi_1^* = \phi_0^* = \phi^* = 1$ when $R_0^* < \frac{1}{x}$. If $\Delta(p, \alpha, b) = 0$, the disclosure policy yields $g = p$ and the resulting total welfare $U(p, \alpha, b) = U(p, 0, b)$. Since $\max_{g \in [0,1]} U(g, 0, b) = U(p, 0, b)$, it is weakly optimal for the regulator to choose no disclosure ($\alpha = 0$).

Next, suppose $\alpha > 0$ yields $R_0^* = \frac{1}{x}$. Then, we have

$$\Delta(g, \alpha, b) = \alpha\left(\frac{p}{(\alpha + (1-\alpha)\phi_0^*)g} - x\right) > 0,$$

where the strict inequality follows from $\alpha > 0$, $p > x$, and $g \in [0, 1]$. Therefore, any disclosure $\alpha > 0$ yields $g = 1$ and the resulting welfare $U(1, \alpha, b) = -\infty$. Therefore, it is optimal for the regulator to choose $\alpha = 0$. To sum up, a disclosure policy $\alpha^{**}(b) = 0$ for every $b \geq 0$ is weakly optimal. *Q.E.D.*

Online Appendix

B Full Description of the Bidding Game

In this section we provide a more detailed description of the bidding game in which investors fund banks. There exists a continuum of banks with measure S who each inelastically seek x units of funding. There also exists a continuum of identical investors indexed by $i \in [0, 1]$ who supply funding. An investor's expected utility from providing Qx units of funding to banks with repayment rate R is

$$V(\mathbb{1}(R \leq 1/x)RQx) - Qx$$

where V is strictly increasing, concave, and differentiable. As described below, the function V and the measure of banks seeking funding in a particular market S will depend on both the state of the world (i.e. the measure of bad banks b seeking funding) and on the information released by the regulator. This general framework allows us to treat all these cases simultaneously. Note that an investor effectively gets no return from loans at a rate of return above $1/x$, because any bank promising to repay more than 1 will default for sure.

Each investor submits a buy order R_i, q_i. Trade takes place as follows.

1. (No rationing) If there exists R^* such that $\int_0^1 \mathbb{1}(R_i \leq R^*)q_i di = S$, then every investor who requested a rate of return less than or equal to the market clearing price R^* can buy q_i at the price they quoted. That is, $Q_i = \mathbb{1}(R_i \leq R^*)q_i$. Each bank is fully funded and borrows from a representative sample of investors.

2. (Banks rationed) If $\int_0^1 q_i di < S$, banks are probabilistically rationed. Each bank is fully funded with probability $\frac{\int_0^1 q_i di}{S}$. Each investor can buy q_i at the price they quoted: $Q_i = q_i$.

3. (Investors rationed) If there exists R^* such that

$$\int_0^1 \mathbb{1}(R_i < R^*)q_i di < S < \int_0^1 \mathbb{1}(R_i \leq R^*)q_i di$$

then all investors who posted a price $R_i < R^*$ can buy q_i at the price they quoted.

Investors who posted $R_i = R^*$ are stochastically rationed. With probability

$$\xi = \frac{S - \int_0^1 \mathbb{1}(R_i < R^*) q_i di}{\int_0^1 \mathbb{1}(R_i = R^*) q_i di}$$

they can buy q_i at the price R^*. With probability $1 - \xi$, they cannot transact at all, $Q_i = 0$.

From the investor's perspective, this can be summarized by a stochastic function $Q(q_i, \boldsymbol{q}, \boldsymbol{R})$ where $\boldsymbol{q}, \boldsymbol{R}$ refer to the whole profile of bids and prices submitted by other investors. An equilibrium of the bidding game is a profile $\boldsymbol{q}, \boldsymbol{R}$ such that, for each i,

$$(q_i, R_i) \in \arg\max V(\mathbb{1}(R_i \leq 1/x) R_i Q(q_i, \boldsymbol{q}, \boldsymbol{R})x) - Q(q_i, \boldsymbol{q}, \boldsymbol{R})x$$

We now characterize the equilibrium of the bidding game.

Lemma B.1. *In equilibrium, investors are not rationed.*

Proof. Suppose by contradiction that there exists some investor who posts a price R^* and can only execute her desired trade $q_i > 0$ with probability $\xi \in (0, 1)$. If $R^* > 1/x$, clearly it increases expected utility to deviate to $q_i = 0$. Suppose $R^* \leq 1/x$. Then i has expected utility

$$\xi V(R^* x q_i) + (1 - \xi)V(0) - \xi x q_i$$

Since V is concave, there exists $\varepsilon > 0$ such that

$$V(\xi(R^* - \varepsilon)xq_i)\xi x q_i > \xi V(R^* x q_i) + (1 - \xi)V(0) - \xi x q_i$$

Thus it is optimal for i to deviate to $(R^* - \varepsilon, \xi q_i)$, which contradicts the original strategy being optimal. *Q.E.D.*

Lemma B.2. *In any equilibrium, all investors post the same price, $R_i = R_j = R^*$ for all $R_i, R_j \in [0, 1]$, and $R^* \leq 1/x$.*

Proof. Suppose first that there is no rationing in equilibrium. If $R^* > 1/x$, any investor who posts $R_i > 1/x$ and $q_i > 0$ can increase expected utility by deviating to $q_i = 0$, so this cannot be an equilibrium. Suppose $R^* \leq 1/x$. Then any investor who posts $R_i < R^*$ can increase her expected utility by increasing R_i to R^*, since she still trades with probability 1. Thus in any equilibrium of this type, all investors post the same price. If banks are rationed in equilibrium, the same argument goes through. Take two investors who post $R_i < R_j$: the first investor can strictly increase her expected utility by increasing R_i to R_j. *Q.E.D.*

Lemma B.3. *Equilibria of the bidding game have the following structure.*

(i) If there exists $R \leq 1/x$ with $1 = RV'(RSx)$, then there exists an equilibrium in which all banks are funded and all investors post $(R_i, q_i) = (R, S)$.

(ii) If $V'(S) < x$, there exists an equilibrium in which banks are rationed and obtain funding with probability $\phi \in (0, 1)$, and all investors post $(R_i, q_i) = 1/x, \phi S$, where ϕ solves $V'(\phi S) = x$.

Proof. Proof of Lemma B.3-(i). Suppose there exists $R \leq 1/x$ with $1 = RV'(RSx)$. Then $q = S$ is optimal, given the price R, since it solves the investor's first order condition (which is sufficient for optimality since the objective is concave). Charging $R_i < R$ cannot be optimal, since it yields a strictly lower return and does not increase the probability of trade (which is already 1). Charging $R_i > R$ cannot be optimal, since it results in no trade and yields the same outcome as posting $(R, 0)$, which we have already seen is strictly inferior to (R, S). So this is indeed an equilibrium.

Proof of Lemma B.3-(ii). Suppose $V'(S) < x$. Setting a lower price than $1/x$ is clearly not optimal for an investor. Setting a higher price results in zero payoff at date 2 and is not optimal. Given the price $1/x$, $q = \phi S$ satisfies the investor's first order sufficient condition for optimality. *Q.E.D.*

We end this section by describing how $V(\cdot)$ and S are defined in the various scenarios discussed in the main text. In the baseline model with log utility, when the regulator does not release any information, we have $S = 1 + b$ and

$$V(z) = p \log \left(y_G + \frac{g}{1+b} z \right) + (1-p) \log \left(y_D + \frac{1-g}{1+b} z \right)$$

If the regulator does reveal her information, in the first market we have $S = g$ and

$$V(z) = p \log(y_G + z)$$

while in the second market we have $S = 1 - g + b$ and

$$V(z) = (1-p)\log\left(y_D + \frac{1-g}{1-g+b}z\right)$$

C An Alternative Model: Bonds Turn "Sour" Ex-Post

In this section, we discuss the robustness of our baseline model by conducting the same analysis in an alternative model with a different assumption on the source of adverse selection. Specifically, we assume that there are only "good" banks with measure 1 *ex ante*, which chooses project type γ or δ in $t = 0$. However, when the capital market opens at $t = 1$, each bank, regardless of its project selection, turns "sour" and change to a type-β bank with probability $b \in [0, 1]$. A key difference from the baseline model is that the population of "good" banks is now random (it equals $1 - b$) and strictly less than 1 almost surely.

For analytical tractability, we make the following assumptions. First, the outside investors' $t = 2$ utility function is $u(x) = \frac{1}{1-\rho}x^{1-\rho}$ for some $\rho \in (0, 1)$. Second, we assume that the regulator's disclosure policy is a binary choice variable, i.e., $\alpha : [0, 1] \to \{0, 1\}$ where $\alpha(b) = 1$ indicates that the regulator fully discloses its information when the fraction of type β banks is b and $\alpha(b) = 0$ means zero disclosure.

C.1 The Equilibrium Terms of External Financing

We first study how the terms of external financing vary with the regulator's information *ex post*, i.e., after the fractions of types γ, δ, and β banks are determined. First suppose the regulator does not disclose her information. Then, each (representative) investor faces the following utility maximization problem:

$$\max_{(\phi,R)} p\frac{1}{1-\rho}\left(y_G + \phi g(1-b)Rx\right)^{1-\rho} + (1-p)\frac{1}{1-\rho}\left(y_D + \phi(1-g)(1-b)Rx\right)^{1-\rho} - \phi x.$$

The first order condition with respect to ϕ yields

$$p\frac{g(1-b)Rx}{(\phi g(1-b))^{\rho}} + (1-p)\frac{(1-g)(1-b)Rx}{(\phi(1-g)(1-b))^{\rho}} - x \geq 0,$$

which is equivalent to

$$\phi^{-\rho}\left[p(g(1-b))^{1-\rho}+(1-p)((1-g)(1-b))^{1-\rho}\right]R\geq 1.$$

Thus, the equilibrium offer (ϕ^*, R^*) is determined by

$$(\phi^*, R^*) = \left(1, \frac{1}{p(g(1-b))^{1-\rho}+(1-p)((1-g)(1-b))^{1-\rho}}\right) \tag{46}$$

if $\frac{1}{[p(g(1-b))^{1-\rho}+(1-p)((1-g)(1-b))^{1-\rho}]} \leq \frac{1}{x}$, and

$$(\phi^*, R^*) = \left(\left[\frac{p(g(1-b))^{1-\rho}+(1-p)((1-g)(1-b))^{1-\rho}}{x}\right]^{\frac{1}{\rho}}, \frac{1}{x}\right) \tag{47}$$

otherwise.[21] Let $\check{b}_s$ denote the highest value of b that yields $\phi^* = 1$. Note from (47) that ϕ^* is decreasing in b if $b > \check{b}_s$: the banks are less likely to get funding as the adverse selection at the capital market grows more severe.

Next, suppose the regulator releases its information. For the outside investors funding type γ banks by offering (ϕ_1, R_1), the first order condition with respect to ϕ_1 is

$$p\frac{g(1-b)Rx}{(\phi_1 g(1-b))^{\rho}} - g(1-b)x \geq 0,$$

which is equivalent to

$$\phi_1^{-\rho}R_1\frac{p}{(g(1-b))^{\rho}} - 1 \geq 0.$$

Since $(g(1-b))^{\rho} < 1$ and $\phi_1 \leq 1$ by definition, type γ banks surely fund their projects at the capital market. Hence, the equilibrium offer (ϕ_1^*, R_1^*) is

$$(\phi_1^*, R_1^*) = \left(1, \frac{(g(1-b))^{\rho}}{p}\right). \tag{48}$$

Like in the baseline model, type γ banks are never rationed because the regulator's information fully eliminates adverse selection for these banks. Note that R_1^* is decreasing in b. That is, investors' willingness to pay for the bonds issued by type γ banks increases with the degree of adverse selection. This is because the bonds issued by "good" banks become very scarce

[21] Note that ϕ^* in (47) is strictly greater than 1 whenever $\rho \geq 1$, which suggests that ρ should be restricted to $\rho \in (0, 1)$ for the non-triviality of the main result.

as more banks turn into the bad ones, raising the premium for the banks fully certified as "good" ones by the regulator.

Next, for the investors funding the "not-type-γ" banks by offering (ϕ_0, R_0), we have the following first order condition with respect to ϕ_0:

$$(1-p)\frac{(1-g)(1-b)R_0 x}{(\phi_0(1-g)(1-b))^\rho} - (b+(1-g)(1-b))x \geq 0,$$

which is equivalent to

$$\phi_0^{-\rho}((1-g)(1-b))^{1-\rho}(1-p)R_0 - ((1-g)+gb) \geq 0.$$

Hence, the equilibrium offer (ϕ_0^*, R_0^*) is

$$(\phi_0^*, R_0^*) = \left(1, \frac{(1-g)+gb}{(1-p)((1-g)(1-b))^{1-\rho}}\right) \text{ if } \frac{(1-g)+gb}{(1-p)((1-g)(1-b))^{1-\rho}} \leq \frac{1}{x} \tag{49}$$

and

$$(\phi_0^*, R_0^*) = \left(\left(\frac{(1-p)((1-g)(1-b))^{1-\rho}}{((1-g)+gb)x}\right)^{\frac{1}{\rho}}, \frac{1}{x}\right) \text{ if } \frac{(1-g)+gb}{(1-p)((1-g)(1-b))^{1-\rho}} > \frac{1}{x}. \tag{50}$$

One can show that $\frac{(1-g)+gb}{(1-p)((1-g)(1-b))^{1-\rho}}$ is increasing in b. Let $\check{b}_s^0$ denote the highest value of b that yields $\frac{(1-g)+gb}{(1-p)((1-g)(1-b))^{1-\rho}} \leq \frac{1}{x}$. Then (ϕ_0^*, R_0^*) is determined by (49) if $b \leq \check{b}_s^0$ and determined by (50) if $b > \check{b}_s^0$. Note that ϕ_0^* is constant for small b's but decreasing in b for all $b > \check{b}_s^0$.

We derive some properties used for the following analysis.

Lemma C.1. *For any $b \leq \min\{\check{b}_s, \check{b}_s^0\}$, there exists a $\hat{b}$ such that $R_1^* > R^* > R_0^*$ if $b < \hat{b}$ and $R_1^* < R^* < R_0^*$ if $b > \hat{b}$. In particular, $\hat{b} = 0$ if and only if $\frac{p}{1-p} < \left(\frac{g}{1-g}\right)^\rho$.*

Proof. Fix any $b \leq \min\{\check{b}_s, \check{b}_s^0\}$, at which R^*, R_1^*, and R_0^* are determined by (46), (48), and (49), respectively.

We first compare R^* and R_1^*. From (46) and (48), we have

$$
\begin{aligned}
& R^* > R_1^*, \\
\iff & \frac{1}{[p(g(1-b))^{1-\rho} + (1-p)((1-g)(1-b))^{1-\rho}]} > \frac{(g(1-b))^\rho}{p}, \\
\iff & b > 1 - \frac{1}{g}\left[\frac{pg^{1-\rho}}{pg^{1-\rho} + (1-p)(1-g)^{1-\rho}}\right].
\end{aligned}
$$

We next compare R^* and R_0^*. From (46) and (49), we have

$$
\begin{aligned}
& R^* < R_0^*, \\
\iff & \frac{1}{[p(g(1-b))^{1-\rho} + (1-p)((1-g)(1-b))^{1-\rho}]} < \frac{(1-g)+gb}{(1-p)((1-g)(1-b))^{1-\rho}}, \\
\iff & b > 1 - \frac{1}{g}\left(\frac{pg^{1-\rho}}{pg^{1-\rho} + (1-p)(1-g)^{1-\rho}}\right).
\end{aligned}
$$

It follows from (46) that $R^* < \frac{1}{x}$ if and only if

$$
b < \check{b}_s := 1 - \left(\frac{x}{pg^{1-\rho} + (1-p)(1-g)^{1-\rho}}\right)^{\frac{1}{1-\rho}}.
$$

Thus, we have

$$
\check{b}_s > 1 - \frac{1}{g}\left[\frac{pg^{1-\rho}}{pg^{1-\rho} + (1-p)(1-g)^{1-\rho}}\right] \iff x < p\frac{1}{p^\rho g^{\rho(1-\rho)}}(pg^{1-\rho} + (1-p)(1-g)^{1-\rho})^\rho, \tag{51}
$$

where the last inequality indeed holds given that $x < p$, $(pg^{1-\rho} + (1-p)(1-g)^{1-\rho})^\rho > 1$, and $\frac{1}{p^\rho g^{\rho(1-\rho)}} > 1$. The last inequality in (51) implies that $R^* = R_1^* = R_0^* < \frac{1}{x}$ at $b = 1 - \frac{1}{g}\left[\frac{pg^{1-\rho}}{pg^{1-\rho}+(1-p)(1-g)^{1-\rho}}\right]$. Combining these observations together, there exists a $\hat{b}$ defined as

$$
\hat{b} := 0 \vee \left[1 - \frac{1}{g}\left(\frac{pg^{1-\rho}}{pg^{1-\rho} + (1-p)(1-g)^{1-\rho}}\right)\right] \tag{52}
$$

such that $R_1^* > R^* > R_0^*$ if $b < \hat{b}$ and $R_1^* < R^* < R_0^*$ if $b > \hat{b}$. Lastly, it is straightforward that $1 - \frac{1}{g}\left(\frac{pg^{1-\rho}}{pg^{1-\rho}+(1-p)(1-g)^{1-\rho}}\right) > 0 \iff \frac{p}{1-p} < \left(\frac{g}{1-g}\right)^\rho$. *Q.E.D.*

The following property is immediate from Lemma C.1.

Lemma C.2. *$\check{b}_s^0 < \check{b}_s$ for any $g \in (0,1)$. Furthermore, $\phi^* = \phi_0^* = 1$ if $b \leq \check{b}_s^0$, $\phi^* = 1 > \phi_0^*$ if $b \in (\check{b}_s^0, \check{b}_s]$, and $1 > \phi^* > \phi_0^*$ if $b > \check{b}_s$.*

Proof. For all $b > \hat{b}$ such that $R_0^* < \frac{1}{x}$, we have $R^* < R_0^*$. Thus, we have $\lim_{b \to \check{b}_s^0} R^* < \lim_{b \to \check{b}_s^0} R^* = \frac{1}{x}$, which implies $\check{b}_s^0 < \check{b}_s$. Furthermore, for all $b \in (\check{b}_s^0, \check{b}_s]$, we have $R_0^* = \frac{1}{x} > R_0^*$, which is equivalent to $\phi_0^* < 1 = \phi^*$. Lastly, for all $b \in (\check{b}_s, 1]$, we have

$$\frac{(1-g)+gb}{(1-p)((1-g)(1-b))^{1-\rho}} > \frac{1}{[p(g(1-b))^{1-\rho} + (1-p)((1-g)(1-b))^{1-\rho}]} > \frac{1}{x},$$

which is equivalent to $1 > \phi^* > \phi_0^*$. *Q.E.D.*

C.2 The Ex-post Optimal Disclosure Policy

We next find an ex-post optimal disclosure policy. To this end, we first derive an ex-post welfare function based on the regulator's disclosure policy. First, the expected welfare function without disclosure, denoted by U_n, is

$$U_n(g,b) := \frac{p}{1-\rho}(\phi^* g(1-b))^{1-\rho} + \frac{1-p}{1-\rho}(\phi^*(1-g)(1-b))^{1-\rho} - \phi^* x, \tag{53}$$

where $\phi^* = 1$ if $b \leq \check{b}_s$ and $\phi^* < 1$ and has the form as in (47) if $b > \check{b}_s$. Similarly, the expected welfare function without disclosure, denoted by U_r, is

$$U_r(g,b) := \frac{p}{1-\rho}(g(1-b))^{1-\rho} + \frac{1-p}{1-\rho}(\phi_0^*(1-g)(1-b))^{1-\rho} - [g(1-b) + \phi_0^*(b + (1-g)(1-b))]x, \tag{54}$$

where $\phi_0^* = 1$ if $b \leq \check{b}_s^0$ and $\phi_0^* < 1$—which is shaped by (50)—if $b > \check{b}_s^0$. The welfare generated by the regulator's disclosure for the realized value of $b \in [0,1]$ is thus $U_r - U_n$.

Then we can find the ex-post impact of releasing the regulator's information.

Proposition C.1. *For every $b > 0$, we have $U_r - U_n = 0$ if $b \leq \check{b}_s^0$, $U_r - U_n > 0$ if $b > (\check{b}_s^0, 1)$, and $U_r - U_n = 0$ if $b = 1$.*

Proof. Define the following functions:

$$U^\gamma(\phi) := \frac{p}{1-\rho}[\phi g(1-b)]^{1-\rho} - \phi g(1-b)x \tag{55}$$

and

$$U^{\delta}(\phi) := \frac{1-p}{1-\rho}[\phi(1-g)(1-b)]^{1-\rho} - \phi[(1-g)(1-b)+b]x. \tag{56}$$

Then, we have $U_n = U^{\gamma}(\phi^*) + U^{\delta}(\phi^*)$ and $U_r = U^{\gamma}(1) + U^{\delta}(\phi_0^*)$.

We first show $U_r - U_n \geq 0$. To this end, consider $U^{\gamma}(\phi)$ first. The first order derivative of $U^{\gamma}(\phi)$ is

$$\frac{d}{d\phi}U^{\gamma} = p\phi^{-\rho}[g(1-b)]^{1-\rho} - [g(1-b)]x,$$

which is decreasing in ϕ, which implies U^{γ} is strictly concave in ϕ. Hence, the first order condition $\frac{d}{d\phi}U^{\gamma} = 0$ yields $\hat{\phi}^{\gamma}$ that maximizes $U^{\gamma}(\phi)$:

$$\hat{\phi}^{\gamma} = \frac{1}{g(1-b)}\left(\frac{p}{x}\right)^{\frac{1}{\rho}}.$$

Since $p > x$, $g \in (0,1)$, and $b \in [0,1]$, one can find $\hat{\phi}^{\gamma} > 1$. Since $\phi_1^* = 1$, $\phi^* = 1$ if $b \leq \check{b}_s^0$, and $\phi^* < 1$ if $b > \check{b}_s^0$, concavity of $U^{\gamma}(\phi)$ implies $U^{\gamma}(\phi_1^*) \geq U^{\gamma}(\phi^*)$ for every $b \in [0,1]$, where the inequality is strict if and only if $b > \check{b}_s^0$.

Next, consider $U^{\delta}(\phi)$. The first order derivative of $U^{\delta}(\phi)$ is

$$\frac{d}{d\phi}U^{\delta} = (1-p)\phi^{-\rho}[(1-g)(1-b)]^{1-\rho} - [(1-g)(1-b)+b]x,$$

which is decreasing in ϕ, which implies $U^{\delta}(\phi)$ is concave in ϕ. Therefore, there exists a $\hat{\phi}^{\delta}$ that maximizes $U^{\delta}(\phi)$:

$$\hat{\phi}^{\delta} = \left[\frac{(1-p)\{(1-g)(1-b)\}^{1-\rho}}{(b+(1-g)(1-b))x}\right]^{\frac{1}{\rho}}.$$

From (50), one can easily find that $\hat{\phi}^{\delta} > 1$ and $\phi_0^* = 1$ if $b \leq \check{b}_s$, and $\hat{\phi}^{\delta} = \phi_0^* < 1$ if $b > \check{b}_s$. Once again, by concavity of $U^{\delta}(\phi)$, we have $U^{\delta}(\phi_0^*) = U^{\delta}(\phi^*)$ if $b \leq \check{b}_s$ and $b \leq \check{b}_s^0$, and $U^{\delta}(\phi_0^*) > U^{\delta}(\phi^*)$ if $b > \check{b}_s$. Combining all these results, we have

$$U_r = U^{\gamma}(\phi_1^*) + U^{\delta}(\phi_0^*) \geq U^{\gamma}(\phi^*) + U^{\delta}(\phi^*) = U_n$$

for every $b \in [0,1]$, where the inequality is strict if $b \in (\check{b}_s^0, 1)$ by Lemma C.2. Lastly, we have $U_r = U_n = 0$ at $b = 1$. *Q.E.D.*

There are a couple of features worth further discussing. First, like in the baseline analysis, the regulator's information improves bond trade by alleviating the adverse selection problem for sufficiently high values of b. Second, more importantly, such a welfare improvement effect disappears ($U_r = U_n = 0$) as b approaches 1, which is not the case in our baseline model. This feature is associated with the assumption that almost every bank turns into a bad bank as b converges to 1: that is, there will be only few type γ banks that will benefit from the regulator's information.

C.3 The Ex-Ante Optimal Disclosure Policy

We next characterize the ex-ante optimal disclosure policy. To analyze how the corresponding equilibrium fraction g of type γ banks is determined, we need to derive Δ_r and Δ_n, where Δ_r (Δ_n) denotes the marginal net utility of equity-holders when each bank chooses project type γ rather than δ with (without) the regulator's information for each $b \in [0, 1]$.

We first derive Δ_n. The investors' expected utility when there is no disclosure is

$$p\frac{1}{1-\rho}\left(\phi^* g(1-b)(1-R^*x) + z_n^G\right)^{1-\rho} + (1-p)\frac{1}{1-\rho}\left(\phi^*(1-g)(1-b)(1-R^*x) + z_n^D\right)^{1-\rho} - C_n.$$

Differentiating with respect to g and substituting $z_G = \phi^* g(1-b)R^*x$ and $z_D = \phi^*(1-g)(1-b)R^*x$ yields

$$\Delta_n = (1-b)^{1-\rho}\left(\frac{p}{g^\rho} - \frac{1-p}{(1-g)^\rho}\right)\phi^{*1-\rho}(1-R^*x). \tag{57}$$

We next derive Δ_r. The investors' expected utility with disclosure is

$$p\frac{1}{1-\rho}\left(\phi_1^* g(1-b)(1-R_1^*x) + z_r^G\right)^{1-\rho} + (1-p)\frac{1}{1-\rho}\left(\phi_0^*(1-g)(1-b)(1-R_0^*x) + z_r^D\right)^{1-\rho} - C_r.$$

Since $\phi_1^* = 1$, differentiating with respect to g yields

$$\Delta_r = (1-b)^{1-\rho}\left[\frac{p}{g^\rho}(1-R_1^*x) - \frac{1-p}{(1-g)^\rho}\phi_0^{*1-\rho}(1-R_0^*x)\right]. \tag{58}$$

Then, $\Delta_r - \Delta_n$ indicates how the regulator's information release for a given b influences each bank's incentive to choose type γ. The mathematical expression of $\Delta_r - \Delta_n$ varies with

b. Specifically, if $b \leq \check{b}_s^0$, we have

$$\Delta_r - \Delta_n = (1-b)^{1-\rho} x \left[\frac{p}{g^\rho}(R^* - R_1^*) + \frac{(1-p)}{(1-g)^\rho}(R_0^* - R^*) \right], \tag{59}$$

where R^*, R_1^*, and R_0^* are represented by (46), (48), and (49), respectively. Furthermore, if $b \in (\check{b}_s^0, \check{b}_s]$, we have

$$\Delta_r - \Delta_n = (1-b)^{1-\rho} x \left[\frac{p}{g^\rho} \left(\frac{1}{x} - R_1^* \right) - \left(\frac{p}{g^\rho} - \frac{(1-p)}{(1-g)^\rho} \right) \left(\frac{1}{x} - R^* \right) \right], \tag{60}$$

where R^* and R_1^* are represented by (46) and (48), respectively. Lastly, if $b > \check{b}_s$, we have

$$\Delta_r - \Delta_n = (1-b)^{1-\rho} \frac{p}{g^\rho}(1 - R_1^* x), \tag{61}$$

where R_1^* is represented by (48). From (59) – (61), we can derive the following properties of $\Delta_r - \Delta_n$, which will be used for characterization of the ex-ante optimal disclosure policy.

Lemma C.3 (Properties of $\Delta_r - \Delta_n$)**.**

(i) $\lim_{b \to 0} [\Delta_r - \Delta_n] \gtreqless 0$ *if and only if* $\frac{p}{1-p} \gtreqless \left(\frac{g}{1-g}\right)^\rho$;

(ii) $\lim_{b \to 1} [\Delta_r - \Delta_n] = 0$;

(iii) $\frac{d}{db} [\Delta_r - \Delta_n] > 0$ *for any* $b \in (0, \check{b}_s^0]$;

(iv) $\Delta_r - \Delta_n > 0$ *and* $\frac{d^2}{db^2} [\Delta_r - \Delta_n] < 0$ *for any* $b \in (\check{b}_s^0, \check{b}_s]$;

(v) $\Delta_r - \Delta_n > 0$ *and* $\frac{d^2}{db^2} [\Delta_r - \Delta_n] < 0$ *for any* $b \in (\check{b}_s, 1]$;

Proof.

Proof of Part (i). Note that $\lim_{b \to 0} R^* = \frac{1}{pg^{1-\rho} + (1-p)(1-g)^{1-\rho}}$, $\lim_{b \to 0} R_1^* = \frac{g^\rho}{p}$, and $\lim_{b \to 0} R_0^* = \frac{(1-g)^\rho}{1-p}$. Since $\Delta_r - \Delta_n$ is equal to (59), we have

$$\lim_{b \to 0} (\Delta_r - \Delta_n) = x \left(\frac{p}{g^\rho} - \frac{1-p}{(1-g)^\rho} \right) \frac{1}{pg^{1-\rho} + (1-p)(1-g)^{1-\rho}}.$$

Since $\frac{1}{pg^{1-\rho} + (1-p)(1-g)^{1-\rho}} > 0$, $\lim_{b \to 0} (\Delta_r - \Delta_n) \gtreqless 0$ if and only if $\frac{p}{1-p} \gtreqless \left(\frac{g}{1-g}\right)^\rho$.

Proof of Part (ii). From (46) and (49), we have $\lim_{b\to 1} R^* = \lim_{b\to 1} R_0^* = \frac{1}{x}$. Therefore, $\lim_{b\to 1}(\Delta_r - \Delta_n)(g,b) = \lim_{b\to 1}(1-b)^{1-\rho}\frac{p}{g^\rho}(1-R_1^* x)$. Since $R_1^* = \frac{g^\rho(1-b)^\rho}{p}$ from (48), we have $\lim_{b\to 1}(1-b)^{1-\rho}\frac{p}{g^\rho}(1-R_1^* x) = 0$.

Proof of Part (iii). Fix any $b \leq \check{b}_s^0$. By plugging R^* in (46) and R_0^* in (49) into (59) respectively, we have

$$\Delta_r - \Delta_n = \left(\frac{p}{g^\rho} - \frac{1-p}{(1-g)^\rho}\right)\frac{x}{pg^{1-\rho} + (1-p)(1-g)^{1-\rho}} + \frac{[(1-g)+gb]x}{(1-g)} - (1-b)x,$$

which is strictly increasing in b.

Proof of Part (iv). Fix any $b \in (\check{b}_s^0, \check{b}_s]$. By Lemma C.1, we have $R_1^* < R^*$ for all $b > \check{b}_s^0$. Furthermore, it is easy to check $\frac{p}{g^\rho} > \frac{p}{g^\rho} - \frac{1-p}{(1-g)^\rho}$. Hence, we have

$$\frac{p}{g^\rho}\left(\frac{1}{x} - R_1^*\right) > \left(\frac{p}{g^\rho} - \frac{(1-p)}{(1-g)^\rho}\right)\left(\frac{1}{x} - R^*\right),$$

which implies $\Delta_r - \Delta_n > 0$.

Next, differentiating $\Delta_r - \Delta_n$ with respect to b after plugging R^* in (46) and R_1^* in (48) into (60), we have

$$\frac{d}{db}(\Delta_r - \Delta_n) = 1 - \frac{(1-\rho)(1-p)}{x(1-g)^\rho}(1-b)^{-\rho},$$

which is decreasing in b, and thus the desired result.

Proof of Part (v). Fix any $b \in (\check{b}_s, 1]$. Since it is immediate from (48) that $R_1^* < \frac{1}{x}$ for such a b, we have $\Delta_r - \Delta_n > 0$ when $\Delta_r - \Delta_n$ is expressed as (61). Furthermore, differentiating $\Delta_r - \Delta_n$ in (61) with respect to b yields

$$\frac{d}{db}(\Delta_r - \Delta_n) = 1 - \frac{(1-\rho)p}{xg^\rho}(1-b)^{-\rho},$$

which is decreasing in b, and therefore, is the desired result. *Q.E.D.*

From Lemma C.3, we find that $\Delta_r - \Delta_n$ has similar mathematical properties to the corresponding function in the original model. Specifically, we observe that $\Delta_r - \Delta_n < 0$ for sufficiently low values of b when there is an excessively large fraction of type γ banks, which will lead the optimal disclosure policy to reveal the information in the low states of b.

We now characterize an optimal disclosure policy that maximizes the following optimization problem facing the regulator *ex ante* (which is similar to the optimization problem in the original model):

$$\max_{B\subset[0,1]} \int_B (U_r - U_n)(g,b)dF(b) + \int_0^1 U_n(g,b)dF(b) \tag{62}$$

subject to

$$\int_B (\Delta_r - \Delta_n)(g,b)dF(b) + \int_0^1 \Delta_n(g,b)dF(b) = 0. \tag{63}$$

Proposition C.2. *Let $B_s^{**} \subset [0,1]$ be an optimal disclosure policy that solves (62) subject to (63). Then we have the following observations:*

(i) (Under-diversification) If $Pr(b \in B_s^{**}) > 0$, we must have $\frac{p}{1-p} < \left(\frac{g}{1-g}\right)^\rho$;

(ii) (Disclosure for low b's) there exists a $\underline{b}_s^{**} > 0$ such that $(0, \underline{b}_s^{**}] \subset B_s^{**}$;

(iii) (Disclosure for high b's) $Pr(b \in B_s^{**} \setminus [0, \underline{b}_s^{**}]) > 0$.

Proof.

Proof of Part (i). Suppose to the contrary that $\frac{p}{1-p} \geq \left(\frac{g}{1-g}\right)^\rho$ under an optimal policy B_s^{**} such that $Pr(b \in B_s^{**}) > 0$. From Lemma C.3, we have $\Delta_r - \Delta_n > 0$ for all $b \in (0,1)$ (since $\Delta_r - \Delta_n > 0$ for $b = 0$ and is increasing in b), which implies $\int_{B^{**}}(\Delta_r - \Delta_n)(g,b)dF(b) + \int_{(0,1)\setminus B^{**}} \Delta_n dF(b) > 0$, which contradicts (63).

Proof of Part (ii). From part (i) above, we have $\frac{p}{1-p} < \left(\frac{g}{1-g}\right)^\rho$, since a similar argument to the one in our baseline model shows that $Pr(b \in B^{*}{**}) = 0$ cannot be optimal. Recall from parts (i), (iii), and (iv) of Lemma C.3 that $\Delta_r - \Delta_n < 0$ for sufficiently low b's close to 0, $\Delta_r - \Delta_n > 0$ for every $b > \check{b}_s^0$, and $\Delta_r - \Delta_n$ is increasing in b for all $b \leq \check{b}_s^0$. Since $\Delta_r - \Delta_n$ is continuous in b, there exists a unique $\underline{b}_s^{**} \in (0, \check{b}_s^0)$ such that $\Delta_r - \Delta_n = 0$ at $b = \underline{b}_s^{**}$. Note that $\Delta_r - \Delta_n < 0$ if and only if $b < \underline{b}_s^{**}$.

To prove part (ii), suppose to the contrary that there exists a $\tilde{B} \subset (0, \underline{b}_s^{**}]$ such that $Pr(b \in \tilde{B}) > 0$ and $\tilde{B} \cap B_s^{**} = \emptyset$. Suppose the regulator takes a sufficiently small measurable interval $[\tilde{b}, \tilde{b}+\varepsilon] \subset \tilde{B}$ and changes the disclosure policy to $B' = [\tilde{b}, \tilde{b}+\varepsilon] \cup B_s^{**}$. Let g^{**} denote the fraction of type γ banks under B_s^{**}. Since $U_r = U_n$ for all $b \le \check{b}_s^0$, we have

$$\int_{B'} U_r(g^{**}, b)dF(b) + \int_{(0,1)\setminus B'} U_n(g^{**}, b)dF(b) = \int_{B_s^{**}} U_r(g^{**}, b)dF(b) + \int_{(0,1)\setminus B_s^{**}} U_n(g^{**}, b)dF(b).$$

However, since $\Delta_r - \Delta_n < 0$ for every $b \in \tilde{B}$, we have

$$\int_{B'} (\Delta_r - \Delta_n)(g^{**}, b)dF(b) + \int_0^1 \Delta_n(g^{**}, b)dF(b) < 0.$$

Hence, B' yields $g' < g^{**}$ to an extent that $\frac{p}{1-p} < \left(\frac{g'}{1-g'}\right)^\rho$. Since both U_r and U_n are maximized at g^* such that $\frac{p}{1-p} = \left(\frac{g^*}{1-g^*}\right)^\rho$, B' yields a strictly higher welfare than B_s^{**}, a contradiction.

Proof of Part (iii). Suppose to the contrary that $Pr(b \in B_s^{**} \setminus [0, \underline{b}_s^{**}]) = 0$. Then we have

$$\int_{B_s^{**}} (\Delta_r - \Delta_n)(g^{**}, b)dF(b) + \int_0^1 \Delta_n(g^{**}, b)dF(b) < 0,$$

which contradicts (63). *Q.E.D.*

There are some similarities between the main results in the alternative model and the baseline model. First, the bias of the regulator's information against state D significantly reduces diversification in the financial system. To address this under-diversification problem, the regulator should disclose her information when b is sufficiently low so that relatively scarce type δ banks can be indirectly identified, and therefore, fund their project cheaply. Second, the regulator should also optimally disclose the information to address the adverse selection problem when b is sufficiently high.

Nevertheless, we still prefer the baseline model for two reasons. First, the main result from the alternative model obtains only if ρ is relatively low, i.e., $\rho < 1$. In other words, the outside investors should be necessarily assumed to have relatively small degree of risk aversion. If $\rho \ge 1$, the investors will counterintuitively lend to "good" banks even by paying a hefty premium when b approaches 1 because there are too few good banks in the capital market. By contrast, as was seen in Section 5.1, the main result generally holds under the baseline model for $\rho \ge 1$. We believe that the baseline analysis is more relevant because the main result obtains for a wider range of ρ than the alternative model.

Second, the positive effect of the regulator's information on adverse selection vanishes to zero as b converges to 1. This is because almost every "good" bank turns into a bad bank as b goes to 1. This non-monotonicity of the welfare gains from disclosure adversely complicates the analysis, making it difficult to characterize optimal policy in a simple way. In the baseline model, however, we could characterize the optimal disclosure policy as a simple two-cutoff structure. This analytical tractability also makes the baseline model more favorable.