Expectation-Driven Term Structure of Equity and Bond Yields*

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Abstract

Recent findings on the term structure of equity and bond yields pose serious challenges to existing equilibrium asset pricing models. This paper presents a new equilibrium model of subjective expectations to explain the joint historical dynamics of equity and bond yields (and their yield spreads). Equity/bond yields movements are mainly driven by subjective dividend/GDP growth expectations. Yields on short-term dividend claims are more volatile because short-term dividend growth expectation mean-reverts to its less volatile long-run counterpart. Procyclical slope of equity yields is due to the counter-cyclical slope of dividend growth expectations. The correlation between equity returns/yields and nominal bond returns/yields switched from positive to negative after the late 1990s, mainly owing to a stronger correlation between real GDP growth and real dividend growth expectations, and only partially due to procyclical inflation. Dividend strip returns are predictable and the strength of predictability decreases with maturity due to predictable dividend forecast errors and revisions. The model is also consistent with the data in generating persistent and volatile price-dividend ratios, and excess return volatility.

Keywords: Subjective expectations, over-reaction, under-reaction, term structure, equity yields, stock-bond correlation, return predictability, forecast error, forecast revision

JEL Classification: G00, G12, E43.

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1 Introduction

The fundamental question in asset pricing research is what drives the equity and bond price movements. To see the link between them, researchers can express equity price at time t as its discounted cash flows:

$$P_t = \sum_{n \geq 0} \frac{E_t(D_{t+n})}{R_{t:t+n}} = \sum_{n \geq 0} \frac{E_t(D_{t+n})}{\exp\left(n\left(y_t^{(n)} + \theta_t^{(n)}\right)\right)},$$

where $E_t(D_{t+n})$ is the expected nominal dividend and $R_{t:t+n}$ is the required return. $R_{t:t+n}$ can be decomposed further into nominal bond yield $y_t^{(n)}$ and dividend risk premium $\theta_t^{(n)}$. This equation holds under marginal investors' expectations, which could be rational or irrational. Motivated by the finding that prices are too volatile than expected dividend and the finding that future returns are predictable by the price-dividend ratio under the rational expectation (Shiller, 1981; Campbell and Shiller, 1988b; Cochrane, 2008, 2011), researchers have proposed several asset pricing models based on time-varying dividend risk premium $\theta_t^{(n)}$. Leading examples include the habit formation model (Campbell and Cochrane, 1999), the long-run risk model (Bansal and Yaron, 2004), and the disaster risk model (Barro, 2006; Gabaix, 2012; Gourio, 2012).

Recent empirical findings pose new challenges to existing equilibrium asset pricing models from different dimensions. (1) Survey-based evidence indicates weak time-variations in expected returns (Greenwood and Shleifer, 2014) instead of the strong variations implied by many existing models. (2) De La O and Myers (2021) show that most aggregate stock price movements are caused by cash flow growth expectations rather than by subjective return expectations, and Bordalo et al. (2020b) show that long-term earnings growth expectations over-react to news, leading to stock return predictability and excess price volatility. (3) Short-term equity yields are more volatile than long-term equity yields, and both are driven mainly by dividend growth expectations (Van Binsbergen et al., 2013) rather than by dividend risk premium. (4) The slope of equity yields (long-term minus short-term yields) is procyclical. (5) Dividend strip returns are predictable, but the strength of predictability decreases with maturity (Van Binsbergen et al., 2012). (6) The correlation between aggregate stock returns and long-term nominal bond returns has switched from positive to negative since the late 1990s (Li, 2002; Fleming et al., 2003; Campbell et al., 2017). Although the change in inflation cyclicality is the standard explanation in the literature, Duffee (2021) provides empirical evidence that the correlation between stock returns and *real* bond returns plays a significant role in explaining this fact.

This paper contributes to the literature by proposing an equilibrium model that explains the joint dynamics of the term structure of equity and bond yields and is consistent with the above empirical findings. In our model, variations in equity (bond) yields are due to subjective dividend growth (GDP growth) expectations instead of dividend risk premium (bond risk premium). We show that the model can match the historical dynamics of the term structure of equity and bond yields and their comovements. Yields on short-term dividend claims are more volatile because the short-term dividend growth expectation mean-reverts to the less volatile levered long-run GDP growth expectations. The negative slope of equity term structure during recessions reflects the countercyclical slope of dividend growth expectations. Long-term Treasury bonds have switched from risky assets to safe assets since the late 1990s driven mainly by a stronger correlation between real GDP and real dividend growth expectations and only partially by the procyclical inflation. Finally, the ex-post realized dividend strip returns are predictable because of predictable dividend expectation errors and revisions, and prices are volatile because of volatile subjective beliefs.

To model subjective beliefs, we depart from rational expectation by assuming that the agent has the "belief in the law of small numbers" as labeled by Tversky and Kahneman (1971). Under this assumption, the agent perceives small samples to represent their population equally as well as large samples. Such cognitive bias implies that the agent produces forecasts for the future by extrapolating fundamentals, i.e., the recent realizations of dividend growth, GDP growth, and inflation. Meanwhile, despite a theory of overreaction, the formulated subjective belief dynamics can also be used to parsimoniously describe underreaction to news, depending on the parameter of cognitive bias. Therefore, our model of expectation may accommodate rich dynamics of subjective beliefs as observed in the survey data.

We assume that the data generating processes (DGPs) for real dividend growth, real GDP growth, and inflation all contain latent states. The agent forms subjective expectations on these latent states using the learning framework discussed above. Specifically, the aggregate log dividend is decomposed into two components: (1) long-duration dividend d_t^l and (2) share of long-duration dividend in total dividend d_t^s. The aggregate endowment risk is embedded in the long-duration dividend d_t^l, which is assumed to be levered on log real GDP. The share of long-duration dividend d_t^s carries no aggregate risks and follows a stationary process.[1] Similarly, the real GDP growth (as endowment growth) and inflation are each decomposed into one stable and one transitory/volatile component. The stable component varies less with the business cycle and is assumed to contain a random-walk state variable (capturing trend growth and inflation), while the transitory component varies

[1] We find that the separation of long-duration dividend from total dividend can help the model best match dividend growth expectations in survey data, and is also consistent with the recent finding that the cross-section of cash-flow duration contains useful information for the aggregate market.

greatly during economic contractions and expansions and is assumed to contain a stationary state variable (measuring short-run deviations from the trend).

We estimate the subjective learning gains by matching model-implied subjective expectations with consensus forecasts in the survey. The model implies that, consistent with Coibion and Gorodnichenko (2015), the agent under-reacts to news when forming the subjective expectations on real GDP growth and inflation. However, the agent overreacts to news of real dividend growth, consistent with Bordalo et al. (2019, 2020b).[2] Furthermore, we find that the subjective expectation of dividend growth from the model closely tracks the full time-series of consensus forecasts for the aggregate dividend growth, with the correlation of 0.80 for both 1-year and 2-year growth forecasts.

Our model assumes that, in addition to subjective beliefs, the representative agent has a constant relative risk aversion (CRRA) utility when pricing assets, which implies a constant subjective risk premium. We show that our model can capture the entire time-series dynamics of the term structure of equity yield over the past three decades. Using equity yield data from Giglio et al. (2021), Figure 2 displays some salient features in the data: (1) more volatile short-term equity yields, (2) a secular decline in equity yields since the late 1980s followed by an upward trend after 2000, (3) a sharp increase in yields during recessions, and (4) a procyclical equity yield slope. The constant subjective dividend risk premium suggests that the equity yields implied by the model are driven by subjective dividend growth expectations. The short-term subjective dividend growth expectation is more volatile and mean-reverts to the less volatile long-run growth expectation, thus the long-term equity yields are more stable. The subjective dividend growth expectations experienced an upward trend starting from the late 1980s and decreased steadily after 2000, which causes the equity yields to have the opposite trend movements. During recessions, growth expectations are low, with the short-term expectation being much lower than its long-run counterpart. Therefore, we observe sharp increases in equity yields and in the procyclical equity yield slope. We also show that the ex-post realized returns generated from the model align well with their empirical counterparts. The implied 2-year (10-year) dividend strip returns have a correlation of 0.63 (0.52) with the data. Regarding the bond market, since our model block for bond pricing closely follows the setup of Zhao (2020), it inherits the explanatory power for several essential facts

[2]The finding of under-reaction (for growth and inflation) to news is potentially related to the Fed's dual mandates. The agent chooses (optimally) to put less weights on inflation and growth shocks for belief updating if she/he expects that the Fed will stabilize future growth and inflation. For other variables that are not directly related to the Fed's mandates, for example, dividend and earnings growth, the cognitive bias of "belief in the law of small numbers" applies and the agent tends to over-react to news.

in the bond yield dynamics. Within our sample, we find that the 1-year and 10-year nominal bond yields that the model implies have a correlation of 0.92 and 0.95 with the data counterpart, suggesting that the model is also successful in pricing nominal bonds.

We next investigate the comovements between equity and bond markets. A well-known stylized fact is that the long-term nominal bonds switched from risky to safe assets after the late 1990s; that is, the correlation between bond and stock returns changed from positive to negative. The same pattern is observed for the correlation between nominal bond yields and real equity yields: the "Fed model". Changes in inflation cyclicality (from countercyclical to procyclical) can help inflation risk premium in equity returns switch signs (from negative to positive) and hence can potentially explain these facts.[3] However, Duffee (2021) finds that changes in the *real* bond and stock return correlation, rather than changes in inflation cyclicality, play a significant role in explaining this fact. We reconcile these empirical findings using subjective expectations of real GDP growth, inflation, and real dividend growth (rather than inflation risk premium). While the real effect of inflation, defined as the covariance between subjective inflation and subjective real growth, explains approximately 30% of the total changes in bond-stock covariance, we find that changes in the *nominal* bond-stock covariance indeed are driven mainly by changes in the *real* bond-stock covariance. More explicitly, such changes are due to stronger co-movements between *real* GDP and *real* dividend growth expectations after 2000, which accounts for around 90% of total changes in nominal bond-stock covariance. That is, a new channel driving the bond-stock correlation is that the real bonds provide a better hedge to aggregate real dividend risks after 2000.

Our model also sheds new light on why equity returns are predictable (e.g., Campbell and Shiller, 1988b) and why the strength of predictability declines from short- to long-maturity dividend claims (e.g., Van Binsbergen et al., 2012). With constant risk premium, strip returns can be decomposed to bond excess returns matched by maturity, dividend forecast errors, and dividend forecast revisions. We find that (1) bond return predictability has a small contribution to the total strip return predictability, especially for short maturities, (2) forecast errors and forecast revisions have opposite predictability, and (3) the downward-sloping term structure of return predictability is driven by the weaker predictability of forecast error as maturity increases, because the current news has a smaller impact on longer-term dividend expectations. The reason why forecast errors and revisions are predictable is that the agent overreacts to dividend news and the dividend level mean-reverts.

[3] In equilibrium models, inflation risk premium in equity returns can arise from, for example, "money illusion" (David and Veronesi, 2013), time varying risk aversion (Campbell et al., 2020), long-run risk (Piazzesi and Schneider, 2007; Bansal and Shaliastovich, 2013; Song, 2017), or time-varying ambiguity Zhao (2017).

Bad news (higher equity yield) triggers excessively lower subjective dividend growth, which is associated with positive subsequent forecast errors on average. As the dividend level mean-reverts, the forecast revisions that follow lower.

Finally, the model reconciles major aggregate stock market puzzles. The time-series of aggregate dividend yields that the model implies are close to and as persistent as the data. The unconditional mean and volatility of model-implied dividend yields and market returns are comparable to the data. The model also replicates the positive (negative) correlation between the market returns and the long-term Treasury bond returns before (after) 2000.

In the Internet Appendix, we extend the model to further take into account the agent's fear over model misspecification. We show that when the agent is ambiguous about real GDP and dividend growth, the extended model implies time-series of equity yields and returns that are closer to the data.

Related literature

This paper is motivated by some new evidence in the empirical asset pricing literature: for example, the importance of subjective expectation in equity markets (Bordalo et al., 2020b; De La O and Myers, 2021) and in bond markets (Froot, 1989; Piazzesi et al., 2015; Cieslak, 2018; Duffee, 2018), the term structure of equity yields (Van Binsbergen et al., 2012, 2013; Van Binsbergen and Koijen, 2017; Van Binsbergen, 2021; Giglio et al., 2021), and the relationship between stock and bond markets (Li, 2002; Fleming et al., 2003; Campbell et al., 2017; Duffee, 2021). To the best of our knowledge, this is the first article that proposes an asset pricing model with subjective beliefs to jointly explain the historical dynamics of the term structure of equity and bond yields and is consistent with the above empirical findings. In fact, echoing Brunnermeier et al. (2021), who suggest that "research focus should be on motivating, building, calibrating, and estimating models with non-RE beliefs ... we need structural models of belief dynamics that can compete with RE models in explaining asset prices and empirically observed beliefs," this paper builds the subjective expectation on a psychology foundation and carefully estimates it using survey data.

This paper is also related to an extensive equilibrium asset pricing literature focusing on (1) rational expectation and aggregate stock market puzzles (Campbell and Cochrane, 1999; Bansal and Yaron, 2004; Barro, 2006; Gabaix, 2012; Gourio, 2012; Albuquerque et al., 2016), (2) rational expectation and bond markets (Piazzesi and Schneider, 2007; Wachter, 2006; Bansal and Shaliastovich, 2013), (3) the link between stock and bond markets (David and Veronesi, 2013; Campbell et al., 2020; Song, 2017; Zhao, 2017), (4) risk premium and the term structure of equity returns (Hasler and Marfe, 2016; Bansal et al., 2021; Breugem et al., 2020; Gonçalves, 2021a; Li and Xu, 2020), and (5) subjective beliefs in equity and bond markets (Barberis et al., 2015; Adam et al., 2016; Nagel and Xu, 2021; Zhao , 2020). We extend the literature by providing a unified framework of bond and equity pricing under subjective expectations, and the model matches many stylized facts in these two markets. In particular, while Zhao (2020) shows how subjective expectation may resolve some puzzles in bond markets, our paper addresses leading puzzles in equity markets and bond-stock comovements.

The paper continues as follows. Section 2 outlines the framework for expectation formation and asset pricing. Section 3 describes the data and steps of model estimation and calibration. Section 4 shows the empirical results. Section 5 provides some robustness analysis, and Section 6 concludes.

2 Expectation Formation and Asset Prices

In this section we describe how the agent forms subjective beliefs over real dividend growth, real endowment growth, and inflation, and how these beliefs are associated with agents' cognitive biases. Then, assuming the CRRA utility, we derive the equilibrium bond and equity prices that are consistent with those subjective beliefs.

2.1 Subjective expectation

We first introduce an illustrative framework to understand how the agent forms subjective expectations, and we will apply the same idea to specific economic forecasts in later subsections. The agent has the following state-space model in mind when predicting future economic outcomes y:

$$y_t = x_t + \sigma_\epsilon \epsilon_t \tag{1}$$

$$x_{t+1} = \rho x_t + \sigma_u u_{t+1}, \tag{2}$$

with $i.i.d.$ ϵ_t and u_t following standard normal distribution and x_t the latent state. The rational belief updating is defined as the Bayesian updating:

$$p(x_t|I_t) \propto p(y_t|x_t) \times p(x_t|I_{t-1}), \tag{3}$$

which implies the following dynamics for the posterior belief over x_t (i.e., the Kalman filter):

$$E_t x_t = \rho E_{t-1} x_{t-1} + K(y_t - \rho E_{t-1} x_{t-1}), \tag{4}$$

with the steady-state Kalman gain $K = \frac{P}{P+\sigma_\epsilon^2} > 0$ and P the steady-state variance of the predictive distribution for the latent state.

To model subjective belief, we depart from the above rational learning by assuming that the agent has the "belief in the law of small numbers" as labeled by Tversky and Kahneman (1971). Under this assumption, the agent perceives small samples to represent their population equally well as large samples, and such cognitive bias has been widely researched in the literature (see e.g., Rabin, 2002). In our setting given by (1) and (2), Santosh (2021) shows that the law of small number can be embodied by exaggerating the likelihood part of (3):

$$p(x_t|I_t) \propto p(y_t|x_t)^{1+\theta} \times p(x_t|I_{t-1}), \tag{5}$$

with θ capturing the magnitude of cognitive bias. The belief dynamics is then:

$$\tilde{E}_t x_t = \rho \tilde{E}_{t-1} x_{t-1} + \nu(y_t - \rho \tilde{E}_{t-1} x_{t-1}), \tag{6}$$

where we label $\tilde{E}(\cdot)$ as subjective expectation and define the subjective learning gain as $\nu = \frac{(1+\theta)\tilde{P}}{(1+\theta)\tilde{P}+\sigma_\epsilon^2}$.[4] To ease notation, we write the subjective posterior mean as $\tilde{E}_t x_t$ as $\tilde{x}_t$.

Although the proposed expectation formation has a different psychological foundation than the diagnostic expectation studied by e.g., Bordalo et al. (2019, 2020a,b),[5] they could both imply agents' overreaction to news when updating beliefs. To see this, we derive the following relation between expectation wedge and news:

$$\tilde{E}_t x_{t+1} - E_t x_{t+1} = \frac{\nu - K}{K} \underbrace{(E_t x_{t+1} - E_{t-1} x_{t+1})}_{news} + \rho(1-\nu)(\tilde{E}_{t-1} x_t - E_{t-1} x_t). \tag{7}$$

When the subjective learning gain is higher than the Kalman gain ($\nu > K$),[6] the agent becomes excessively

[4] We note that $\tilde{P}$ is the steady-state variance of the *subjective* predictive distribution for the latent state, and it may differ from P in the Kalman filter. See their explicit formula in Appendix A.1.

[5] The diagnostic expectation is based on another similar bias, the representativeness heuristic (Kahneman and Tversky, 1972). Santosh (2021) provides more detailed analysis on their difference.

[6] This can be achieved when θ is large enough such that $\theta > \frac{P-\tilde{P}}{\tilde{P}-\sigma_u^2}$.

optimistic (pessimistic) after good (bad) news about the latent state, relative to the rational benchmark. It shall be noted that to clarify interpretation, our definition of news refers to the innovations to rational beliefs obtained from the Kalman filter (so that two terms on the right-hand side of (7) are uncorrelated). In contrast, Bordalo et al. (2020a) define the news as innovations to subjective beliefs. We show that the two notions of news are highly correlated (see simulation results in Table IA.2). Moreover, our empirical analysis in the following section confirms the predictions by using subjective news.

Despite a theory of overreaction, the formulated subjective belief dynamics (6) can also be used to parsimoniously capture underreaction to news when θ is small enough. In fact, both over- and under-reaction have been documented empirically in recent macroeconomic and finance literature. Coibion and Gorodnichenko (2015); Bordalo et al. (2020a) find that for inflation and growth, agents on average display underreactions. In contrast, Bordalo et al. (2019, 2020b) document overreaction to news of stock-level and aggregate cash-flow growth. Our model of expectation thus may accommodate rich dynamics of subjective beliefs observed in the survey data.

2.2 Subjective beliefs on real dividend growth

In this subsection, we describe how the agent forms beliefs over aggregate real dividend growth with the "belief in the law of small numbers". We begin with a two-component dividend model and let the agent learn separately these two components when forming expectation. Specifically, the logarithm of aggregate real dividend can be decomposed as:

$$d_t = d_t^l + d_t^s, \tag{8}$$

where $d_t = \log D_t$ is the log total real dividend and $d_t^l = \log D_t^l$ measures the log real dividend from the sector of long-duration stocks. As a result, $d_t^s = d_t - d_t^l$ quantifies the share of the long-duration dividend in the aggregate dividend. The decomposition (8) extracts useful information from the cross-section of cash-flow duration when predicting the aggregate market, similar to the recent literature (see e.g., Kelly and Pruitt, 2013; Li and Wang, 2018; Gormsen and Lazarus, 2020). Indeed, while most literature on dividend learning relies on observations of aggregate dividend or endowment series (see e.g., Johannes et al., 2016; Nagel and Xu, 2021), incorporating other informative series into the learning framework can be a promising avenue.[7] We

[7] For example, Jagannathan and Liu (2019); De La O and Myers (2021) incorporate earnings information into their model when predicting aggregate dividend growth.

follow this direction and consider two dividend components in our learning model.

We first assume that the total dividend from long-duration stocks d_t^l is more tied to the aggregate economy and linked to the total endowment y_t through a leverage parameter λ, with the following state-space model:

$$d_t^l - \lambda y_t = \mu_{d,t} + \sigma_d^l \epsilon_{d,t}^l \tag{9}$$

$$\mu_{d,t+1} = \mu_{d,t} + \sigma_d^\mu \epsilon_{d,t+1}^\mu, \tag{10}$$

where $\epsilon_{d,t}^l$ and $\epsilon_{d,t}^\mu$ are $i.i.d.$ shocks following standard normal distribution. Then, following Menzly et al. (2004); Cochrane et al. (2008), we assume that the dividend share is stationary and follows the state-space model:

$$d_t^s = x_{d,t} + \sigma_d^s \epsilon_{d,t}^s \tag{11}$$

$$x_{d,t+1} = \rho_d x_{d,t} + \sigma_d^x \epsilon_{d,t+1}^x, \tag{12}$$

where $\epsilon_{d,t}^s$ and $\epsilon_{d,t}^x$ are $i.i.d.$ shocks following standard normal distribution. The specification captures the idea that dividends from long-duration stocks cannot deviate permanently from the aggregate dividend and that information from such deviation d_t^s is helpful to infer future dividend growth. When the share of long-duration dividend is temporarily higher, the aggregate dividend will have to increase more.

With specified DGPs, we assume that the agent forms subjective expectation following the rule discussed in Section 2.1. Then the agent's posterior beliefs over latent states $\tilde{\mu}_{d,t}$ and $\tilde{x}_{d,t}$ evolve according to:

$$\tilde{\mu}_{d,t+1} = \tilde{\mu}_{d,t} + \nu_d^l (d_{t+1}^l - \lambda y_{t+1} - \tilde{\mu}_{d,t}) \tag{13}$$

$$\tilde{x}_{d,t+1} = \rho_d \tilde{x}_{d,t} + \nu_d^s (d_{t+1}^s - \rho_d \tilde{x}_{d,t}). \tag{14}$$

We note that the subjective learning gains ν_d^l, ν_d^s can be different for two dividend components. The subjective dividend growth then can be obtained:

$$\tilde{E}_t \Delta d_{t+1} = \lambda \tilde{E}_t \Delta g_{t+1} + (\rho_d - 1)\tilde{x}_{d,t} + (\nu_d^s - 1)(d_t^s - \rho_d \tilde{x}_{d,t-1}) + (\nu_d^l - 1)(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}), \tag{15}$$

where $\Delta g_{t+1} = y_{t+1} - y_t$ is the growth rate of real output. We will specify how its subjective expectation is formed in the next subsection.

Similar to Equation (7), the agent's forecast for future dividend growth reacts to news of two dividend components according to:

$$(\tilde{E}_t - E_t)(\Delta d_{t+1} - \lambda \Delta g_{t+1}) = \frac{K_d^l - \nu_d^l}{1 - K_d^l}(E_t - E_{t-1})(\Delta d_{t+1}^l - \lambda \Delta g_{t+1}) + \rho_d \frac{K_d^s - \nu_d^s}{1 - \rho_d K_d^s}(E_t - E_{t-1})\Delta d_{t+1}^s + F_{t-1}, \quad (16)$$

where F_{t-1} depends on information only up to $t-1$; its expression is given in Appendix A.1. By comparing (7) and (16), we note that underreaction to news about dividend level translates into overreaction to news about dividend growth. In Section 3.2, we show that the subjective learning gains ν_d^l, ν_d^s estimated from the survey data are smaller than the corresponding Kalman gains. Thus, the agent overreacts to news when updating their forecasts for future dividend growth, consistent with the evidence in Bordalo et al. (2019, 2020b).

Our model differs from recent attempts to model beliefs in stock markets. Jagannathan and Liu (2019), De La O and Myers (2021) and Nagel and Xu (2021) incorporate earnings information when modeling the beliefs over dividend growth, in addition to the standard aggregate dividend or growth data. While this is an intuitive strategy following e.g., Campbell and Shiller (1988a), how to incorporate earnings information remains controversial (see e.g., Boudoukh et al., 2007). Importantly, our methodology exploits information from dividends in different sectors, so we follow a very different path. In contrast, Bordalo et al. (2020b) and Guo and Wachter (2021) assess which kind of beliefs may be empirically consistent with leading asset pricing puzzles. We model a new expectation formation that has a clear psychological foundation and empirical predictions, and we will show how it quantitatively explains many asset pricing puzzles in bond and stock markets.[8]

2.3 Subjective beliefs on endowment growth and inflation

We use the component models of real GDP growth and inflation following Zhao (2020), who shows that the model reconciles many stylized facts in the bond market. The four components of GDP – investment spending, net exports, government spending, and consumption (PCE) – do not move in lockstep with each other. In fact, their volatility differs greatly. PCE is very stable and varies less with the business cycle. In contrast, the other three components vary greatly across economic contractions and expansions. Similarly for inflation, the core

[8] A body of research in behavioral finance, including Greenwood and Shleifer (2014); Barberis et al. (2015); Cassella and Gulen (2018), suggests that investors may extrapolate past stock returns or fundamentals, and these irrational beliefs are consistent with several asset pricing anomalies. However, to the best of our knowledge, none of the existing papers can reconcile all the asset pricing puzzles discussed in Section 4 in a unified framework.

inflation is much more stable than other inflation components. With this in mind, we can decompose output growth and inflation via the following accounting identity:

$$\Delta g_t = \Delta g_t^* + Gap_t^g$$
$$\pi_t = \pi_t^* + Gap_t^\pi,$$

where π_t is inflation. The stable components Δg_t^* and π_t^* are PCE growth and core inflation, and Gap_t^g and Gap_t^π are the volatile components (GDP growth excluding the PCE and GDP deflator excluding the core inflation).

The real consumption growth and core inflation follow:

$$\Delta g_t^* = \mu_{g,t} + \sigma_g \varepsilon_{g,t}^* \tag{17}$$
$$\pi_t^* = \mu_{\pi,t} + \sigma_\pi \varepsilon_{\pi,t}^*, \tag{18}$$

where $\varepsilon_{g,t+1}^*$ and $\varepsilon_{\pi,t+1}^*$ are $i.i.d.$ standard normal shocks. The latent states are assumed to follow the unit-root processes:

$$\mu_{g,t+1} = \mu_{g,t} + \sigma_g^\mu \varepsilon_{g,t+1}^\mu \tag{19}$$
$$\mu_{\pi,t+1} = \mu_{\pi,t} + \sigma_\pi^\mu \varepsilon_{\pi,t+1}^\mu. \tag{20}$$

The two gap components are assumed to contain latent stationary states:

$$Gap_t^i = x_{i,t} + \sigma_i^{gap} \varepsilon_{i,t}^{gap} \tag{21}$$
$$x_{i,t+1} = \rho_i x_{i,t} + \sigma_i^x \varepsilon_{i,t+1}^x, \tag{22}$$

where $i = g, \pi$, and $\varepsilon_{i,t+1}^{gap}$, $\varepsilon_{i,t+1}^x$ are $i.i.d.$ standard normal shocks.

The agent forms beliefs based on the same learning scheme in Section 2.1:

$$\begin{aligned}
\tilde{\mu}_{g,t} &= \tilde{\mu}_{g,t-1} + \upsilon_g^* \left(\Delta g_t^* - \tilde{\mu}_{g,t-1}\right) \\
\tilde{\mu}_{\pi,t} &= \tilde{\mu}_{\pi,t-1} + \upsilon_\pi^* \left(\pi_t^* - \tilde{\mu}_{\pi,t-1}\right) \\
\tilde{x}_{g,t} &= \rho_g \tilde{x}_{g,t-1} + \upsilon_g^{gap} \left(Gap_t^g - \rho_g \tilde{x}_{g,t-1}\right) \\
\tilde{x}_{\pi,t} &= \rho_\pi \tilde{x}_{\pi,t-1} + \upsilon_\pi^{gap} \left(Gap_t^\pi - \rho_\pi \tilde{x}_{\pi,t-1}\right),
\end{aligned}$$

where υ_g^*, υ_π^*, υ_g^{gap} and υ_π^{gap} are subjective learning gains linked to each component. In Section 3.2, we show that most of them are higher than the corresponding Kalman gains. Thus, the agent underreacts to news when updating their forecasts for inflation and real GDP growth, consistent with the evidence in Coibion and Gorodnichenko (2015).

2.4 Asset prices

We assume that the representative agent has the standard CRRA utility $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$, where β stands for the agent's subjective discount factor and γ is the risk-aversion coefficient. The log nominal pricing kernel implied from the CRRA utility is:

$$m_{t+1}^{\$} = \log\beta - \gamma\Delta g_{t+1} - \pi_{t+1}. \tag{23}$$

The equity *spot* yield for the $n-$period dividend strip (claim to the $n-$period ahead aggregate nominal dividend) is defined as:

$$ey_t^{(n)} = \frac{1}{n}(d_t^{\$} - p_t^{(n)}), \tag{24}$$

where $p_t^{(n)}$ is the log strip price and $d_t^{\$}$ is the log nominal aggregate dividend. Beginning from $n = 1$, the time-t equilibrium price of one-period dividend strip is:

$$P_t^{(1)} = \tilde{E}_t[M_{t+1}^{\$} D_{t+1}^{\$}], \tag{25}$$

where the conditional expectation is taken under the worst-case belief. Similarly, the price of $n-$period dividend strip is:

$$P_t^{(n)} = \tilde{E}_t[M_{t+1}^{\$} P_{t+1}^{(n-1)}]. \tag{26}$$

Solving the iterations forward, for the $n-$period equity spot yield we obtain:

$$\begin{aligned} ey_t^{(n)} = & \frac{A_e^{(n)}}{n} - (\lambda-\gamma)(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g \tilde{x}_{g,t}) + \frac{1-\rho_d^n}{n}\tilde{x}_{d,t} - \frac{\nu_d^l - 1}{n}(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) \\ & - \frac{\nu_d^s - 1}{n}(d_t^s - \rho_d \tilde{x}_{d,t-1}) + \frac{1-\rho_{ag}^n}{n(1-\rho_{ag})}(\lambda-\gamma) a_{g,t} + \frac{1-\rho_{ad}^n}{n(1-\rho_{ad})} a_{d,t}, \end{aligned} \tag{27}$$

with expression of $A_e^{(n)}$ given in the Appendix.

Meanwhile, the time-t price of $n-$period nominal discount bond satisfies the recursion:

$$P_{b,t}^{(n)} = \tilde{E}_t[M_{t+1}^{\$} P_{b,t+1}^{(n-1)}]. \tag{28}$$

Conjecturing that the log nominal bond price is linear in states, we solve out the $n-$period nominal bond yield as:

$$y_t^{(n)} = \frac{A_b^{(n)}}{n} + \gamma(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g \tilde{x}_{g,t}) + (\tilde{\mu}_{\pi,t} + \frac{1-\rho_\pi^n}{n(1-\rho_\pi)}\rho_\pi \tilde{x}_{\pi,t}). \tag{29}$$

To better connect equity and nominal bond yields, we can write (27) as:

$$ey_t^{(n)} = \frac{A_e^{(n)} - A_b^{(n)}}{n} + y_t^{(n)} - \frac{1}{n}\tilde{E}_t \Delta d_{t+1:t+n}^{\$} = \theta_t^{(n)} + y_t^{(n)} - g_t^{(n)}, \tag{30}$$

where $\frac{1}{n}\tilde{E}_t \Delta d_{t+1:t+n}^{\$}$ is the subjective belief over the life-time nominal dividend growth for the $n-$period dividend strip. The second equality follows Equation (4) in Van Binsbergen et al. (2013) by disentangling the equity yield into the risk premium component ($\theta_t^{(n)}$), nominal bond yield ($y_t^{(n)}$), and growth component ($g_t^{(n)}$). The risk premium component in our model is constant, thus the time-variations in equity yields are entirely driven by the subjective beliefs over real GDP and dividend growth.

3 Data, Estimation and Calibration

3.1 Data

We collect the full term structure data of dividend strip yields from Giglio et al. (2021). Using a large cross-section of US stock returns, they estimate an affine model of equity prices and derive the strip yields for the aggregate market. Their method not only accurately replicates the dividend futures data used in recent studies such as Van Binsbergen et al. (2013); Van Binsbergen and Koijen (2017); Bansal et al. (2021), but also extends the length of data substantially.[9] A longer sample creates an ideal laboratory for us to study the dynamics of the equity term structure over the business cycles. As for the bond term structure, we use the end-of-quarter zero-coupon nominal bond yields from Gürkaynak et al. (2007).

To construct the dividend series used for learning, we obtain firm-level quarterly dividends from the CRSP/Compustat Merged Database for all firms listed on NYSE, NASDAQ, and AMEX. Following De La O and Myers (2021) and Giglio et al. (2021), we focus on ordinary cash dividends. To implement our two-component dividend model (8), we consider firm-level long-term earnings growth median forecasts (LTG) as the benchmark measure of the equity duration, with the data available from the IBES unadjusted summary file.[10] La Porta (1996) and Gormsen and Lazarus (2020) show that such a model-free measure can be interpreted as the equity duration. Since equity duration is defined as the weighted sum of time with the weights given by expected cash-flows, higher long-term expected cash-flows relative to today naturally translate into higher duration.[11] Then, we calculate the dividend from the long-duration sector as the following. At the end of each quarter, we assign all dividend-paying firms into either of the two groups based on firms' LTG forecasts in the previous quarter. If a firm's LTG is above or equal to the cross-sectional median of the LTG of all dividend paying firms, then it is assigned to the long-duration group. Otherwise, it is allocated to the short-duration group. Within each quarter, we then sum all dividends from the long-duration and short-duration sectors respectively. We deflate the obtained two nominal dividend series using the GDP deflator and take a four-quarter trailing summation to remove their seasonality, following the usual practice. By construction, the sum of two dividend series will be the real aggregate dividend. The equity duration data is available from 1981Q3; hence the deseasonalized dividend series ranges from 1982Q4 to 2019Q4. We use the initial 5-year training period for agent's learning, and we start our empirical analysis from 1987Q4.

Regarding the data for subjective dividend growth that will be used to estimate the dividend learning gains, we extend the 1-year aggregate expected dividend growth data constructed by De La O and Myers

[9]Dividend futures data usually starts from 2003 and hence is not suitable for our study. Van Binsbergen et al. (2012) use option returns to assess the equity term structure, with data going back to 1996. However, option-based data is only available for short maturities up to two-year, and Boguth et al. (2019) find that noises from highly levered option positions may significantly contaminate the inference from option prices.

[10]While IBES data is available at the monthly frequency, we transform it to quarterly frequency by taking the end-of-quarter readings. Results from using the within-quarter average are almost identical.

[11]Existing measures of equity duration (see e.g., Dechow et al., 2004; Weber, 2018; Gonçalves, 2021b) require formal econometric modeling and estimation. We do not take a stand on such modeling issues and prefer to use the model-free duration. In Section 5.1 we run robustness checks by using these measures, and results are quantitatively similar.

(2021) to 2019Q4 using the same empirical steps.[12] Similarly, to estimate learning gains for the real GDP growth and the inflation, we rely on the consensus forecasts for 1-year real GDP growth and inflation from the Survey of Professional Forecasters (SPF). The data ranges from 1981Q3 to 2019Q4. Finally, we collect the data on real output growth and GDP deflator from the Bureau of Economic Analysis (BEA). The real personal consumption expenditure (PCE) and core inflation, i.e., the stable components of GDP and total inflation, are also obtained from the BEA. Since the learning gains on growth and inflation may be very small (see e.g., Malmendier and Nagel, 2016; Nagel and Xu, 2021), we need a long training sample to form reasonable beliefs. Thus we allow the agent to learn these quantities using the data back to 1959Q1.

3.2 Parameter estimation and calibration

Our analysis assumes that the representative agent knows all parameters of state-space models of real dividends, real GDP growth, and inflation. While we will estimate most parameters using maximum likelihood with the Kalman filter (Appendix B provides estimation details), we calibrate a key parameter that is hard to identify from the data. As widely discussed in the literature (e.g., Bansal and Yaron, 2004; Schorfheide et al., 2018), it is challenging to estimate the persistent component $x_{g,t}$ from the endowment growth series, but its persistence is crucial for our model to match dynamics of bond yield spread (Zhao , 2020). Therefore, given other parameters, we calibrate ρ_g to maximize the correlation between the model-implied nominal bond yield spread (10-year minus 1-year) and the data counterpart. Panel A of Table 1 lists the estimated parameter values for state-space models under the physical measure. In Panel B, we list values for some standard parameters. The leverage parameter λ is set to 3, the risk aversion coefficient γ is set to 4, and the subjective discount factor β is calibrated to match the average level of the 10-year equity yields in the data.

[12] Their original data spans the period from 2003Q1 to 2015Q3. We find that their data and our replicated series have a correlation coefficient of 0.92 over the same sample.

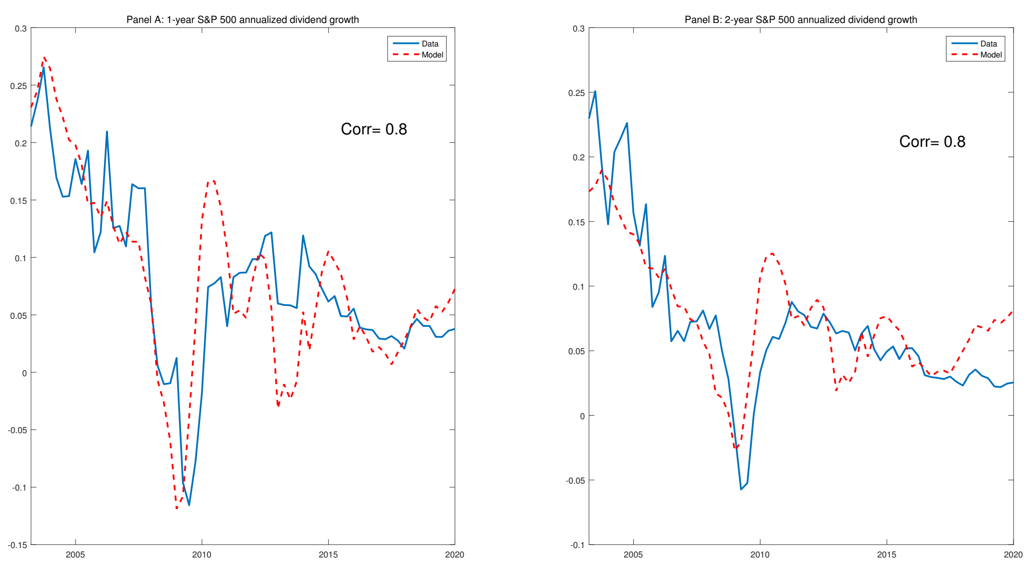
Table 1: Model parameters

The table reports model parameter values. Panel A displays parameter estimates for state-space models discussed in Section 2. Panel B reports the leverage parameter λ, risk-aversion coefficient γ, and subjective discount factor β. Panel C reports the learning gains for real dividend growth, real GDP growth, and inflation, all estimated from the corresponding 1-year consensus forecast data. All volatility parameters are reported in percentage.

Panel A: Parameters for physical dynamics										
Real GDP and inflation										
	σ_g	σ_g^{μ}	σ_g^{gap}	σ_g^{x}	ρ_g	σ_π	σ_π^{μ}	σ_π^{gap}	σ_π^{x}	ρ_π
	0.33	0.12	0.63	0.01	0.941	0.08	0.09	0.14	0.10	0.932
Real dividend										
	σ_d^{l}	σ_d^{μ}	σ_d^{s}	σ_d^{x}	ρ_d					
	8.54	6.93	6.16	7.42	0.94					
Panel B: Leverage and preference parameters										
	λ	γ	β							
	3	4	1.0065							
Panel C: Subjective learning gains										
	ν_d^{l}	ν_d^{s}	ν_g^{*}	ν_g^{gap}	ν_π^{*}	ν_π^{gap}				
	0.166	0.458	0.012	0.065	0.049	0.228				

Figure 1: Fit of real dividend growth expectations

The figure plots the model-implied 1-year and 2-year aggregate dividend growth expectations together with the data. The sample period is from 2003Q1 to 2019Q4.



The final set of parameters to be determined are the subjective learning gains. Following e.g., Branch and Evans (2006); Cieslak and Povala (2015), we estimate them from the 1-year consensus forecasts for real GDP growth, inflation, and aggregate dividend growth by minimizing the root mean square errors (RMSE) between data and model. Panel C of Table 1 reports the estimated gains. In this paper, we are particularly interested in whether our model can match the full time-series of dividend expectation data. Figure 1 shows that the model-implied 1-year subjective dividend growth tracks its empirical counterpart well, and the unconditional correlation reaches 0.8. In the right plot, even though we do not use 2-year survey dividend growth in the estimation, the model-implied quantity also closely matches the data. Hence, the estimation results support our model in capturing salient features of subjective dividend growth.

Recent literature documents that relative to the rational forecasts, survey forecasts display under- or over-reaction to news, depending on the studied economic series (e.g., Coibion and Gorodnichenko, 2015; Bordalo et al., 2020a). In Section 2.1, we have shown that our model can generate either under- or over-reaction, depending on the relative magnitude of subjective and Kalman learning gains. Panel A of Table 2 lists such comparison for real dividend growth, real GDP growth, and inflation. Although the subjective learning gains are smaller than Kalman gains for dividend components, we have shown in Section 2.2 that underreaction to dividend level news translates into overreaction to dividend growth news. In contrast, we find that the agent exhibits underreaction to news for real GDP growth and inflation.

As a formal empirical test, within the model we run the rationality test following Coibion and Gorodnichenko (2015) by regressing realized forecast errors on lagged forecast revisions. Intuitively, when the agent displays overreaction, she will excessively adjust the forecast upward after good news. The overly optimistic forecast is likely to be disappointed in the future, leading to a negative slope coefficient from the regression. Panel B of Table 2 reports the regression results, where we consider both short horizon (1-year) and long horizon (5-year) forecasts. Consistent with Bordalo et al. (2019, 2020b), we find that the agent displays overreaction to stock cash-flow news and the slope coefficients are highly significant. Meanwhile, in line with Coibion and Gorodnichenko (2015), the agent underreacts to news regarding the real GDP growth and inflation. The reason for such underreaction is potentially related to the Fed's dual mandates. The agent chooses (optimally) to put less weight on inflation and growth shocks for belief updating if she expects that the Fed will stabilize future growth and inflation. For other variables that are not directly related to the Fed's mandates, such as the earnings growth, the cognitive bias of "belief in the law of small numbers" applies and the agent tends to over-react to news.

Table 2: Under- and over-reaction of subjective beliefs

The table compares subjective beliefs from our model of expectation formation with rational beliefs from the Kalman filter. We display results for beliefs over real dividend growth, real GDP growth, and inflation. Panel A lists the estimated subjective and Kalman gains for each component. Panel B runs the rationality test of subjective beliefs following Coibion and Gorodnichenko (2015) (CG(2015)). For each quantity of interest x, the realized forecast errors are regressed on lagged subjective forecast revisions:

$$x_{t+n} - \tilde{E}_t(x_{t+n}) = \alpha + \beta[\tilde{E}_t x_{t+n} - \tilde{E}_{t-1} x_{t+n}] + \epsilon_{t+n},$$

with n chosen to be 1-year or 5-year. $\tilde{E}_t x_{t+n}$ denotes the subjective forecast implied from the agent's learning discussed in Section 2.2 and 2.3. The Newey-West t-statistics are reported in parentheses. The sample period is from 1987Q4 to 2019Q4.

Panel A: Subjective learning gains and Kalman gains						
	d_t^l	d_t^s	Δg_t^*	Gap_t^g	π_t^*	Gap_t^π
ν	0.19	0.49	0.01	0.07	0.05	0.23
K	0.52	0.67	0.29	0.01	0.67	0.48
Panel B: Rationality test of subjective beliefs following CG(2015)						
	Real dividend growth		real GDP growth		Inflation	
	1Y	5Y	1Y	5Y	1Y	5Y
β	-0.69	-0.97	2.08	0.01	1.01	0.72
(t)	(-3.63)	(-3.22)	(1.86)	(0.01)	(2.02)	(0.87)

4 Empirical Results

This section explores whether our general equilibrium model accounts for leading asset pricing puzzles in equity and bond markets. Since the model nests Zhao (2020), it naturally accounts for key stylized facts in the bond markets. We thus focus more on the equity market by first studying the model implications for the equity term structure. We discuss whether the model generates time-varying bond-stock correlation and return predictability, as usually observed in the data. Finally, we assess model performance in explaining well-known puzzles in the aggregate stock market.

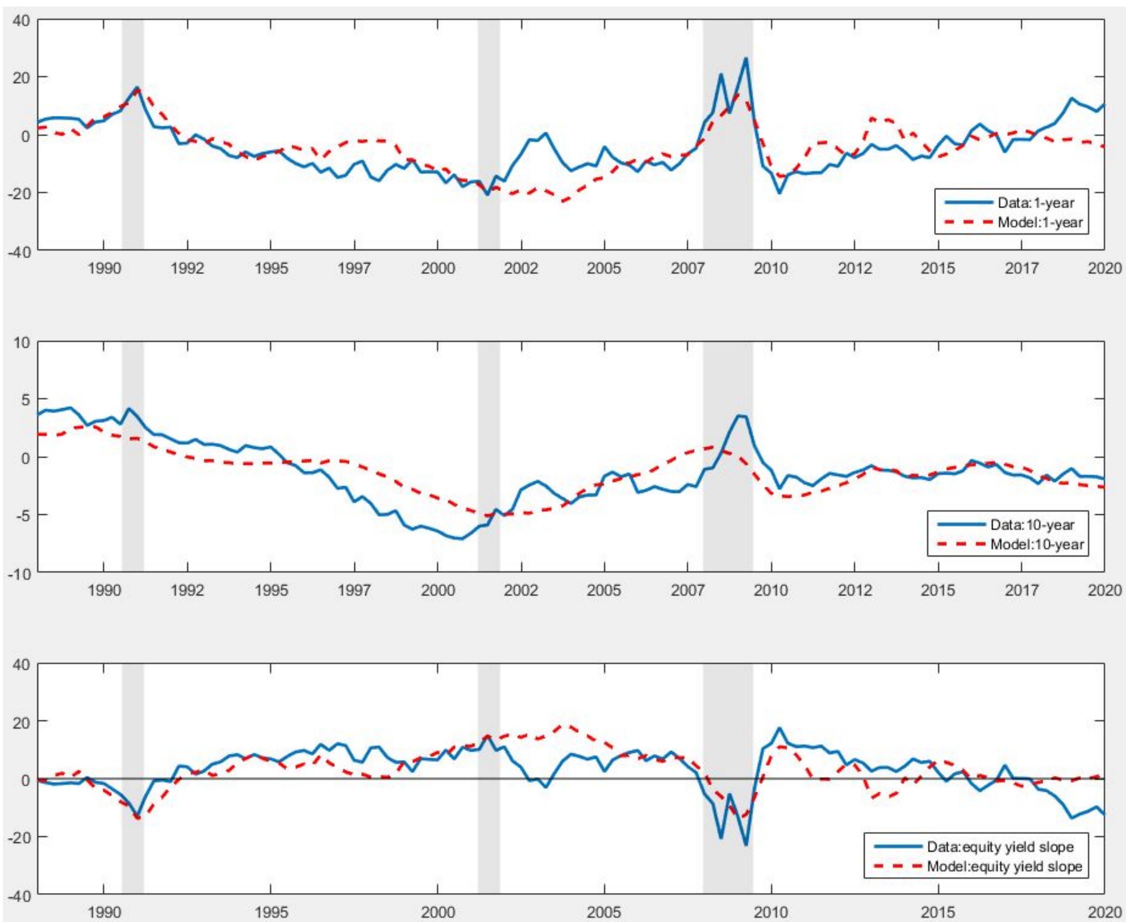
4.1 Term structure of equity yields and returns

4.1.1 Historical dynamics of equity term structure

We start with the business cycle dynamics of the equity term structure. Figure 2 plots the equity yields defined in (27) together with the data from Giglio et al. (2021). The model-implied yields closely track the movements in the data over the entire sample. Comparing the results for 1-year and 10-year, our model can match both the volatile 1-year equity yields and the less volatile 10-year yields. The model also generates a secular decline in equity yields since the late 1980s, followed by an upward trend post-2000, and replicates the equity yield spikes during the recession periods in 1990s and around 2008. The last row of Figure 2 plots the slopes of equity term structure, defined as the difference between 10-year and 1-year yields. The time-series plot suggests that model-implied slopes co-move tightly with the data.

Figure 2: Term structure of equity yields

The figures compare the model-implied term structure of equity yields with the data from Giglio et al. (2021). The last row plots the spread between 10-year and 1-year equity yields. Shaded areas correspond to NBER recessions. The sample period is from 1987Q4 to 2019Q4, and all numbers are in annualized percentage terms.



Panel A of Table 3 reports their unconditional mean, volatility, and correlation. We find that the model-implied moments are close to the empirical counterparts and that the correlation is also high. For example, the 1-year (10-year) model-implied equity yields have a correlation coefficient of 0.68 (0.79) with the data, and the correlation between slopes is 0.59. Overall, the evidence favors the model in terms of fitting the term structure of equity yields.

Table 3: Summary statistics of equity yields

The first part of Panel A reports the unconditional mean and standard deviation of equity yields from both our model and data, and their correlation coefficients. Also in Panel A, we report moments of equity yields during expansion and recession periods, identified via the NBER business cycle dating. Panel B reports the decomposition of average slope of equity yields into the components described in Equation (31), i.e., components related to the constant, real GDP growth, and dividend-specific growth. All numbers are in annualized percentage terms, and the sample period is from 1987Q4 to 2019Q4.

Panel A: Moments		Data				Model		
		1Y	10Y	10Y-1Y		1Y	10Y	10Y-1Y
Unconditional	Mean	-4.39	-1.34	3.05		-4.59	-1.34	3.24
	Volatility	8.89	2.70	7.22		8.22	1.89	6.83
	Corr with data					0.68	0.79	0.59
Expansion	Mean	-5.44	-1.51	3.93		-5.35	-1.39	3.96
	Volatility	7.29	2.54	5.95		7.10	1.81	5.81
Recession	Mean	5.82	0.33	-5.50		2.84	-0.92	-3.77
	Volatility	15.25	3.65	12.10		13.74	2.61	11.29
Panel B: Slope decomposition		Const	RGDP	Div-spec	Total			
Expansion		1.44	0.04	2.48	3.96			
Recession		1.44	0.18	-5.38	-3.77			

4.1.2 Procyclical equity slope

The slope of equity yield is found to be procyclical (see e.g., Van Binsbergen et al., 2013; Bansal et al., 2021), that is, during the recession the slope is deeply negative while in normal times it can be positive. We evaluate whether the conditional moments of model-implied equity yields exhibit similar patterns in Panel A of Table 3. We find that the equity term structure is upward sloping during the expansion period, yet it becomes negative during the recession, with an average equity slope of –5.50%. Our model successfully generates such sign reversal, with an average slope of 3.96% (–3.77%) during the expansion (recession). The model-implied yields also display higher volatilities during the recession, in line with the data.

Remarkably, our model captures the cyclicality of equity yield slopes in a way different from the previous literature. While most prior studies rely on the procyclical term structure of risk premia to reconcile this evidence (see e.g., Hasler and Marfe, 2016; Breugem et al., 2020; Gonçalves, 2021a; Li and Xu, 2020), their channels may not be coherent with recent survey-based evidence on the importance of cash-flow variations.[13]

[13] In fact, Table 5 in Van Binsbergen et al. (2013) does find that the dividend growth expectation accounts for a substantial

In contrast, the CRRA utility implies a negligible dividend risk premium in our model, and equity yield movements are driven mostly by subjective dividend growth expectations. During recessions, growth expectations are exceptionally lower, with short-term expectation being much lower than its long-run counterpart; therefore, we observe sharp increases in equity yields and procyclical equity yield slopes.

Panel B of Table 3 further explores which factors contribute to the sign reversal of equity slopes by disentangling two economic forces in the equity yield:

$$ey_t^{(n)} = Const^{(n)} \underbrace{-(\lambda-\gamma)(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g\tilde{x}_{g,t})}_{RGDP_t^{(n)}} + \underbrace{\frac{1-\rho_d^n}{n}\tilde{x}_{d,t} - \frac{\nu_d^l - 1}{n}(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) - \frac{\nu_d^s - 1}{n}(d_t^s - \rho_d \tilde{x}_{d,t-1})}_{Div-specific_t^{(n)}}. \tag{31}$$

The average term structure of equity yields can be decomposed to (the negative of) the term structure of subjective real GDP growth and real dividend-specific growth. Our results show that during the recession, short-maturity equity yield is higher mainly because the agent perceives lower real dividend-specific growth in the short-run compared to the long-run.

4.1.3 Returns on dividend claims

We then evaluate whether the model also generates reasonable dynamics for returns on dividend claims.[14] Following Van Binsbergen and Koijen (2017), we study the $h-$period realized futures return of the dividend strip with $n-$period maturity, which can be computed as:

$$r_{F,t+1:t+h}^{(n)} = \Delta d_{t+1:t+h}^{\$} + ney_t^{(n)} - (n-h)ey_{t+h}^{(n-h)} - r_{B,t+1:t+h}^{(n)}, \tag{32}$$

with $r_{B,t+1:t+h}^{(n)}$ as the $h-$period realized return of the nominal bond with $n-$period maturity. Under constant dividend risk premium, the time-varying futures returns are then driven by time-varying subjective dividend

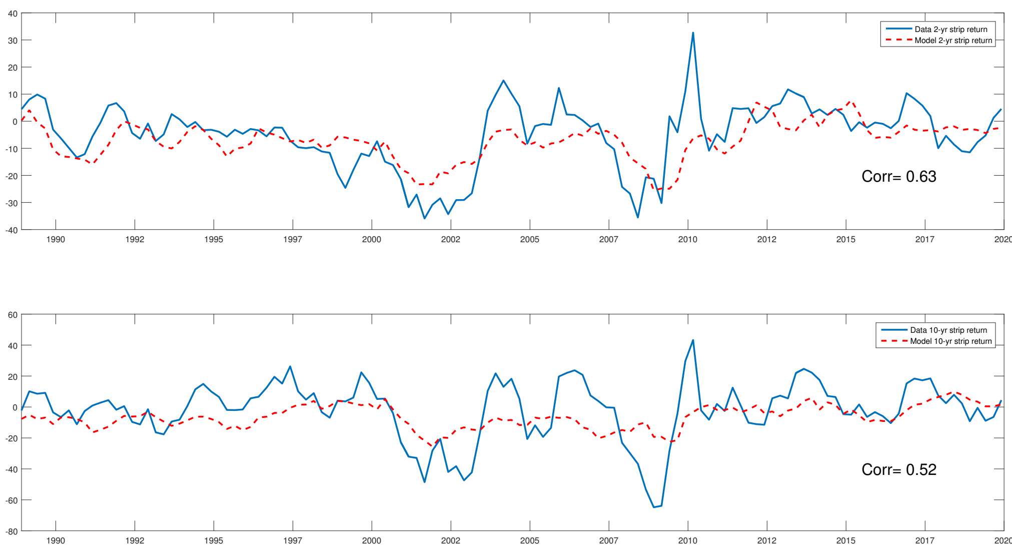
share of equity yield variations. Cassella et al. (2021) document that the term structure of biased beliefs over cash-flows may be empirically consistent with the equity term structure dynamics.

[14]Our analysis is silent on the expected equity returns and its term structure because of latent states under the statistical measure, but yield dynamics from the model still allow for inferring the properties of ex-post realized returns. In fact, there is an ongoing debate on the term structure of equity risk premia. Canonical asset pricing models imply a flat or upward-sloping term structure, and recent evidence suggests that it may be downward sloping unconditionally based on options data (Van Binsbergen et al., 2012) or strip futures returns (Van Binsbergen and Koijen, 2017). However, Boguth et al. (2019) find no evidence of downward-sloping term structure once accounting for noises from highly levered option positions. Bansal et al. (2021) document procyclical term structure and show that it can be upward sloping unconditionally after matching reasonable business cycle frequencies. Gormsen (2021) finds a counter-cyclical term structure.

growth. Figure 3 suggests that our model does reasonably well in capturing ex-post returns in the data. The correlation of 2-year realized strip returns reaches 0.63, and that of 10-year strip is still at a high level of 0.52. An important caveat is that the fit of equity yields in Figure 2 does not necessarily translate into the close fit of returns in Figure 3, as the latter further requires the model to capture well the equity yield *changes*.

Figure 3: Strip futures returns: data vs. model

The figure compares the model-implied futures returns of dividend strips with the data calculated from Giglio et al. (2021). We display results for 2-year and 10-year strip returns, and the sample period is from 1988Q4 to 2019Q4. All numbers are in annualized percentage terms.



4.2 Bond-stock comovement

This subsection addresses the puzzling behavior of bond-stock correlation. A stylized finding from the literature is that the nominal long-term Treasury bonds switched from risky to safe assets around 2000; that is, the correlation between bond returns and stock returns changed from positive to negative. Similar comovements are also observed between equity yields and long-term nominal Treasury yields, the "Fed model."[15] To clarify the mechanism behind these facts, we first study the comovements between long-term nominal bonds and dividend strips, as they have analytical solutions from the model. Corresponding results between bond returns

[15]See discussions on bond-stock return correlations in Baele et al. (2010); David and Veronesi (2013); Campbell et al. (2017, 2020); Kozak (2019); Li et al. (2020), and on the Fed-model in Asness (2003); Campbell and Vuolteenaho (2004); Bekaert and Engstrom (2010); Burkhardt and Hasseltoft (2012).

and aggregate stock returns, as typically studied in the literature, will be presented in Section 4.4.

4.2.1 Return covariance

The $h-$period return of a nominal bond and a dividend strip with $n-$period maturity are defined as:

$$r^{(n)}_{B,t+1:t+h} = ny^{(n)}_t - (n-h)y^{(n-h)}_{t+h}, \tag{33}$$

$$r^{(n)}_{S,t+1:t+h} = ney^{(n)}_t - (n-h)ey^{(n-h)}_{t+h} + \Delta d^{\$}_{t+1:t+h}. \tag{34}$$

Equation (30) implies that the bond and dividend strip returns are linked:

$$r^{(n)}_{S,t+1:t+h} = r^{(n)}_{B,t+1:t+h} + (\tilde{E}_{t+h} - \tilde{E}_t)\Delta d^{\$}_{t+1:t+n} + n\theta^{(n)}_t - (n-h)\theta^{(n-h)}_{t+h}. \tag{35}$$

Prior literature entertains different models of $\theta^{(n)}_t$ to reconcile the bond-stock return correlation. For instance, David and Veronesi (2013) rely on belief changes and money illusion to generate negative bond-stock correlation after 2000; Song (2017) embeds procyclical inflation into the long-run risk model; and Campbell et al. (2020) connect inflation with time-varying risk-aversion. We take an orthogonal route because our model implies constant $\theta^{(n)}_t$. Thus, the model generates sign-reversal in bond-stock correlation via changes in correlation between expected cash-flows instead of via risk premium variations.

Our model further allows for detailed decomposition of bond-stock covariance. Following Duffee (2018, 2021), we extract nominal and real components from the equity strip return and long-term nominal bond return (with $N-$period maturity):

$$r^{(N)}_{B,t+1:t+h} = INFL^{(N)}_B + RGDP^{(N)}, \tag{36}$$

$$r^{(n)}_{S,t+1:t+h} = INFL^{(n)}_S + RGDP^{(n)} + RDIV^{(n)}, \tag{37}$$

with expressions for each term given in Appendix A. Absent risk premium variations, the bond and equity returns reflect variations in underlying real cash-flow and inflation expectations. Then, we can decompose the return covariance as:

$$
\begin{aligned}
Cov(r^{(N)}_{B,t+1:t+h}, r^{(n)}_{S,t+1:t+h}) = & \underbrace{Cov(INFL^{(N)}_{B}, RGDP^{(n)} + RDIV^{(n)}) + Cov(INFL^{(n)}_{S}, RGDP^{(N)})}_{Inflation\ real\ effect} \\
& + \underbrace{Cov(INFL^{(N)}_{B}, INFL^{(n)}_{S}) + Cov(RGDP^{(N)}, RGDP^{(n)})}_{Inflation\,\&\, real\ growth\ volatility} \\
& + \underbrace{Cov(RGDP^{(N)}, RDIV^{(n)})}_{real\ growth\ corr}. \qquad (38)
\end{aligned}
$$

The three components represent: (1) inflation real effect, which we define as the correlation between shocks to expected inflation and real growth;[16] (2) volatility of shocks to expected inflation and real growth; and (3) correlation between shocks to expected real GDP growth and real dividend growth. The sign-reversal we observe for the bond-stock correlation must stem from these three components. Note that the last component reflects whether the real bonds provide a hedge to aggregate real dividend risks or not and is a channel that has not been examined by prior literature.

Panel A of Table 4 reports the bond-stock return correlation computed for various long-term dividend strips, with the annual holding period ($h = 4$). The analysis is implemented under two subsamples separated by 2000Q1, and we follow the literature by setting $N = 10$ years. Results show that the bond-stock correlations indeed turn to negative after 2000, in both the data and the model.[17] In Panel B, we report the contribution of each component in (38) to the total bond-stock covariance *changes* across two subsamples. The decomposition reveals three interesting facts. First, although the inflation real effect, as widely discussed in the previous literature, has non-negligible impact, this channel explains only 30% of total bond-stock covariance changes. Thus our results are more consistent with recent findings documenting a modest impact of inflation on the bond-stock correlation (Duffee 2018, 2021; Gomez-Cram and Yaron 2021). Second, the negative contribution from the volatility of shocks to expected inflation and real growth reflects more volatile beliefs during the latter sample, mostly resulting from the global financial turmoil around 2008.[18] Third, and most importantly, we find that the covariance of expected *real* growth accounts for around 90% of the total bond-stock covariance

[16] Piazzesi and Schneider (2007); Bansal and Shaliastovich (2013); Song (2017) incorporate the real effect of inflation via the predictive relation under the *statistical* measure. Zhao (2017) builds a link between inflation and ambiguity over real growth. Our definition is different, as it concerns the correlation between subjective beliefs.

[17] Results are robust to other choices of breakpoints, such as 2001Q2, used by Campbell et al. (2020).

[18] Excluding the period of global financial crisis, we actually obtain less volatile beliefs and hence a positive explanatory share for the volatility of shocks to expected inflation and real growth.

changes; that is, the real bonds provide a better hedge to aggregate real dividend risks after 2000. The quantitative effect from this real bond hedging channel is large and in line with the evidence in Duffee (2021).

Table 4: Bond-stock return correlation and covariance decomposition

Panel A reports the bond-stock return correlation under each subsample. Panel B reports the decomposition results of bond-stock return covariance (in percentage) based on (38). n denotes the maturity of the corresponding dividend strip, and we use the 10-year nominal bond throughout the analysis. Results are computed based on the sample before 2000Q1 (1988Q4 to 1999Q4) and after 2000Q1 (2000Q1 to 2019Q4).

n		5Y	7Y	10Y
Panel A: Return correlation				
Data	Before 2000	0.46	0.45	0.43
	After 2000	-0.49	-0.49	-0.48
Model	Before 2000	0.14	0.24	0.29
	After 2000	-0.56	-0.42	-0.28
Panel B: Decomposition				
Infl. real effect		27.3%	29.3%	31.1%
Infl. & real growth vol.		-16.4%	-21.4%	-25.9%
Real growth corr.		89.1%	92.1%	94.8%

4.2.2 Fed model

Does the same story also hold for the correlation between equity and nominal bond yield *levels*, the so-called Fed model? Similar to (36) and (37), the nominal bond yields and equity yields can be written as:

$$y_t^{(N)} = INFL_B^{(N)} + RGDP^{(N)} \tag{39}$$

$$ey_t^{(n)} = RGDP^{(n)} + RDIV^{(n)}. \tag{40}$$

Note that the equity yield is in real terms and thus has no inflation component. The yield covariance can be decomposed as:

$$\begin{aligned} Cov(y_t^{(N)}, ey_t^{(n)}) = & \underbrace{Cov(INFL_B^{(N)}, RGDP^{(n)} + RDIV^{(n)})}_{Inflation\ real\ effect} + \underbrace{Cov(RGDP^{(N)}, RGDP^{(n)})}_{real\ growth\ volatility} \\ & + \underbrace{Cov(RGDP^{(N)}, RDIV^{(n)})}_{real\ growth\ corr}. \end{aligned} \tag{41}$$

Panel A of Table 5 reports the yield correlation, and we find a similar sign-reversal from positive to negative after 2000. The decomposition results in Panel B show that a dominant force driving the Fed model still appears to be the comovements between expected real GDP growth and real dividend growth, which contributes to over 70% of the changes in bond-stock yield covariance after 2000. Interestingly, the explanatory share by inflation real effect now increases to around 40%. The key reason is that the persistent expected inflation, though it cannot move enough at high-frequency to explain return correlation, in fact correlates strongly with the level of expected real GDP growth. Overall, our decomposition results uncover qualitatively similar but quantitatively different mechanisms behind the bond-stock return correlation and the Fed model.

Table 5: Bond-stock yield correlation and covariance decomposition

Panel A reports the bond-stock yield correlation under each subsample. Panel B reports the decomposition results of bond-stock yield covariance (in percentage) based on (41). n denotes the maturity of corresponding dividend strip, and we use the 10-year nominal bond throughout the analysis. Results are computed based on the sample before 2000Q1 (1987Q4 to 1999Q4) and after 2000Q1 (2000Q1 to 2019Q4).

n		5Y	7Y	10Y
Panel A: Yield correlation				
Data	Before 2000	0.85	0.85	0.84
	After 2000	-0.60	-0.62	-0.62
Model	Before 2000	0.85	0.86	0.87
	After 2000	-0.41	-0.36	-0.31
Panel B: Decomposition				
Infl. real effect		41.2%	42.5%	43.9%
Vol. of growth		-13.6%	-17.6%	-23.9%
Real growth corr.		72.4%	75.1%	80.1%

4.3 Return predictability

We now study the return predictability puzzle. There are two important stylized facts: equity market returns are predictable by lagged dividend-price ratios (e.g., Campbell and Shiller, 1988b; Cochrane, 2011), and the strength of predictability decreases from short-term dividend claims to long-term claims (Van Binsbergen et al., 2012). To evaluate whether our model is consistent with them, we run two predictive regressions using model-implied quantities. The first regression is on predicting the market excess returns using the market log dividend-price ratio:

$$r_{M,t+1:t+h} - y_t^{(h)} = \alpha + \beta(d_t^{\$} - p_t) + \epsilon_{t+1:t+h}, \tag{42}$$

and the second concerns predicting the strip excess returns using its own lagged equity yields:

$$r^{(n)}_{S,t+1:t+h} - y^{(h)}_t = \alpha + \beta(d^{\$}_t - p^{(n)}_t) + \epsilon_{t+1:t+h}. \tag{43}$$

The holding period h is set to one year and the risk-free rate is the h-period nominal bond yield $y^{(h)}_t$.

Panel A of Table 6 reports the results. During the period from 1988Q4 to 2019Q4, annual market excess returns are positively predicted by the lagged log dividend-price ratio in the data, with a $t-$statistic of 2.86 and R^2 of 10%. Our model regression generates similar R^2, and the slope coefficient is significant. Meanwhile, our model produces the downward-sloping strength of strip return predictability, as in Van Binsbergen et al. (2012). For instance, in the data, 5-year strip excess returns are strongly predictable with R^2 of 20.2%, yet the R^2 decreases to 14.4% for the 10-year strip. The model-implied term structure of predictability is also downward sloping, with the R^2 decreases from 14.2% to 10.4%.

What is the source of return predictability? To clarify the mechanism, we write the excess return of the $n-$period dividend strip as:

$$r^{(n)}_{S,t+1:t+h} - y^{(h)}_t = Cte + \underbrace{rx^{(n)}_{B,t+1:t+h}}_{Bond} + \underbrace{\Delta d^{\$}_{t+1:t+h} - \tilde{E}_t \Delta d^{\$}_{t+1:t+h}}_{FE} + \underbrace{(\tilde{E}_{t+h} - \tilde{E}_t)\Delta d^{\$}_{t+h+1:t+n}}_{FR}. \tag{44}$$

Realized strip excess return consists of three components: (1) the maturity-matched realized bond excess return (Bond), (2) the forecast error of dividend growth within the holding period (FE), and (3) the forecast revision regarding the dividend growth after the holding period (FR). Equity yields that predict strip returns must predict some (or all) of these components. Panel B of Table 6 evaluates their predictability, and we document different patterns for short-term and long-term strips. For short maturities, bond predictability contributes little while the forecast error and forecast revision predictability dominates. Since the agent displays overreaction in our model, bad news (higher yields) triggers excessively lower subjective dividend growth, which is associated with higher subsequent forecast errors. As dividend mean-reverts, higher subsequent dividend realizations will likely drive down the forecast for dividend growth thereafter (see Equation (15)), leading to lower forecast revisions in the future. When n becomes larger, the current news has smaller impact on long-term dividend expectations, leaving lower shares of predictability to forecast errors and revisions. Taken together, these effects generate the downward-sloping term structure of return predictability.

Recent literature also emphasizes the role of biased beliefs and predictable forecast errors in reconciling return predictability (e.g., Bordalo et al., 2020b; Nagel and Xu, 2021). Our explanations have two important difference. First, we document predictability of dividend forecast revisions as a new channel for return predictability. Second, we show that predictable forecast errors also help generate the downward-sloping term structure of return predictability.

Table 6: Return predictability

Panel A reports the results of predictive regressions (42) and (43). Panel B reports the decomposition results of predictive regression (43) via (44). In brackets, we report the Newey-West t-statistics. The sample period is from 1988Q4 to 2019Q4.

Panel A: Return predictability					
		MKT	5Y	7Y	10Y
Data	β	0.17	0.31	0.26	0.22
	(t)	(2.86)	(4.21)	(3.89)	(3.62)
	$R^2(\%)$	10.29	20.19	17.14	14.48
Model	β	0.14	0.11	0.10	0.09
	(t)	(3.18)	(3.30)	(3.45)	(3.20)
	$R^2(\%)$	9.02	14.20	12.90	10.37
Panel B: Decomposition					
	$Bond$		0.05	0.05	0.07
	(t)		(1.79)	(1.89)	(2.08)
	$R^2(\%)$		6.66	7.40	8.78
	FE		0.35	0.29	0.26
	(t)		(5.56)	(5.20)	(4.80)
	$R^2(\%)$		31.48	29.17	26.58
	FR		-0.28	-0.25	-0.23
	(t)		(-7.85)	(-6.96)	(-6.22)
	$R^2(\%)$		35.63	30.19	26.50

4.4 Puzzles about the aggregate stock market

In this subsection, we revisit several aggregate stock market puzzles via our equilibrium model. We model aggregate market portfolio as the portfolio of dividend strips by writing the ex-dividend aggregate stock price as:

$$P_t = \sum_{n=1}^{H_t} P_t^{(n)}, \tag{45}$$

where $P_t^{(n)}$ is the price of the $n-$period dividend strip and stochastic H_t may be interpreted as the life expectancy of the aggregate portfolio. Since the focus in this subsection is simply on whether previous stories explaining the equity yields also translate to the aggregate market, we do not manually set values for H_t, nor clarify the mechanisms for its time-variations (e.g., Fama and French, 2004; Chen, 2011). Instead, we adopt a simple approach by modeling it as a reduced-form function of our equity duration measures for the aggregate market:

$$H_t = a + bLTG_t, \tag{46}$$

where LTG_t is calculated as the value-weighted average of long-term growth forecasts over all firms. The specification captures the time-varying cash-flow duration of the aggregate stock market in a parsimonious way. Based on (45), we pin down parameters a and b by minimizing the RMSE between the data and the model. It should be noted that although we use the data of the dividend-price ratio when estimating a and b, the time-variations of the model-implied aggregate dividend-price ratio are driven entirely by the strip yield variations and exogenous movements in aggregate equity duration.[19]

Figure 4: Aggregate dividend-price ratio: data vs. model

The figure compares the model-implied aggregate dividend-price ratio with the data. The model-implied quantitiy is obtained following the method in Section 4.4. The correlation coefficient between model and data is reported in the plot. The sample period is from 1987Q4 to 2019Q4 and the numbers are in annualized percentage terms.

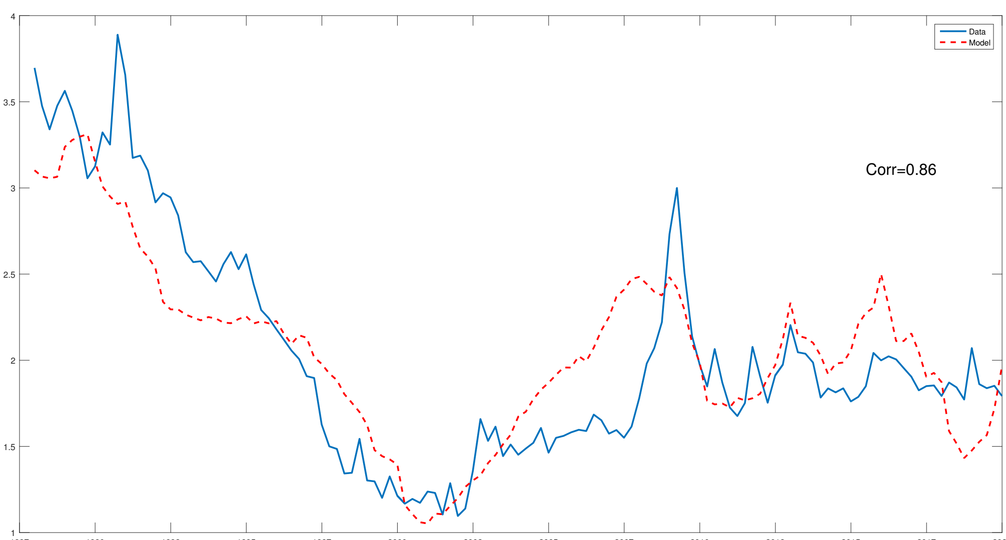


Figure 4 demonstrates that the model-implied aggregate dividend-price ratio is close to the data, with a

[19]Model performance is similar if we use a fixed cut-off level when calculating (45), following e.g., Van Binsbergen (2021).

correlation coefficient of 0.86. In Table 7, we find that they are equally persistent, with the AR(1) coefficients around 0.95. Meanwhile, the market log dividend-price ratio implied by the model has an annualized volatility of 26% and is close to 30% in the data. With the time-series of aggregate dividend-price ratio, we can also calculate the implied market returns. We find that our model generates an average market return of around 9%, replicating the high equity return (8%) in the data. Market returns are also volatile in our model, with an annualized volatility of 12%. Finally, we replicate sign switches of the correlation between long-term nominal bond returns and aggregate stock returns after 2000, with the correlation coefficient changing from 0.13 to -0.46.

Table 7: Moments for aggregate market

The table reports the moments of the aggregate stock market, including the annualized mean and volatility of market returns, volatility and AR(1) coefficient of market log dividend-price ratio, and the correlation between 10-year nominal bond returns and aggregate stock returns. We also report the correlation between the model and the data regarding the log dividend-price ratio and market returns. The sample period is from 1987Q4 to 2019Q4.

	$E(r_M)$	$\sigma(r_M)$	$\sigma(d-p)$	$\rho(d-p)$	$Corr(r_M, r_B \mid t < 2000)$	$Corr(r_M, r_B \mid t \geq 2000)$
Data	0.08	0.16	0.30	0.95	0.39	-0.64
Model	0.09	0.12	0.26	0.96	0.13	-0.46
$Corr(dp^{data}, dp^{model})$	0.86					

5 Robustness

5.1 Duration measures and dividend components

The baseline measure of equity duration is the analyst forecast for long-term earnings growth. As a robustness check, we experiment with alternative duration measures as proposed in the literature, including those discussed in Dechow et al. (2004); Weber (2018); Gonçalves (2021b). In addition, we consider the book-to-market ratio as a duration measure following Lettau and Wachter (2007). After constructing the dividend series sorted over these duration measures, Table IA.1 reports the moments of model-implied equity yields and their correlation with the data. Even if we use different measures of equity duration, the results show that the model is successful in replicating key moments of the data, and the time-series correlation coefficients are also high. In a related exercise, while still using LTG as the duration measure, we change the construction of long-duration dividends by using the 40th or 60th cross-sectional percentile of LTG as the breakpoint. Results remain similar, as found in last two rows of Table IA.1.

6 Conclusion

Motivated by the finding that future returns but not future cash flows are predictable by current price-dividend ratios, research in macro finance has, over the past three decades and within the rational expectations framework, been trying to come up with a force that moves prices but not expected future cash flows. This principle has guided equilibrium asset pricing literature and has given rise to model of time-varying risk attitude (habit formation) or time-varying risks (long-run risk or disaster risk).

However, new empirical findings on subjective expectations, the term structure of bond and equity yields, and the correlation of stocks and bonds all pose serious challenges to existing rational models. Subjective expectations of cash flows are found to be the most important drivers of equity and bond prices, while subjective return expectations are not as important as predicted by the rational models. Meanwhile, dividend risk and bond risk premiums in the rational model cannot explain equity and bond yield spread movements in data. Furthermore, using the inflation risk premium to explain the change in stock-bond correlation implies too much inflation risk in equity returns.

We provide a unified framework of bond and equity pricing that is consistent with these empirical findings. The movements of equity/bond yields are driven by subjective dividend/GDP growth expectation, and subjective risk premium is negligible. The model-implied long- and short-yields of dividend strips and bonds and their spreads are close to the data (both time-series dynamics and moments). Long-term Treasury bonds switched from risky assets to safe assets after the late 1990s mainly due to a stronger correlation between real GDP and real dividend growth expectations, and only partially driven by the procyclical inflation. Equity returns are predictable, but the strength of predictability decreases from short-term to long-term claims as a result of subjective forecast errors and forecast revisions for future dividend growth. Our framework also quantitatively matches several major aggregate stock market puzzles, such as the persistent and volatile price-dividend ratios, and excess volatility of stock returns.

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Internet Appendix for "Expectation-Driven Term Structure of Equity and Bond Yields"

A Model Solution Details

A.1 Derivation of rational and subjective beliefs

For the steady-state Kalman filter, the posterior (or the filtered) distribution for x_t after observing y_t is:

$$p(x_t|I_t) \propto \exp(-\frac{(y_t - x_t)^2}{2\sigma_\epsilon^2}) \times \exp(-\frac{(x_t - \rho E_{t-1}x_{t-1})^2}{2P}),$$

with P the steady-state conditional variance of the predictive distribution under the Kalman filter. We then calculate the posterior mean and variance for x_t:

$$\begin{aligned} E_t x_t &= \rho E_{t-1}x_{t-1} + \frac{P}{P+\sigma_\epsilon^2}(y_t - \rho E_{t-1}x_{t-1}) \\ Var_t x_t &= \frac{\sigma_\epsilon^2 P}{P+\sigma_\epsilon^2}. \end{aligned}$$

Note that by definition, P solves:

$$P = \frac{\rho^2\sigma_\epsilon^2 P}{P+\sigma_\epsilon^2} + \sigma_u^2.$$

Then we solve for the *subjective* posterior distribution:

$$p(x_t|I_t) \propto \exp(-\frac{(1+\theta)(y_t - x_t)^2}{2\sigma_\epsilon^2}) \times \exp(-\frac{(x_t - \rho \tilde{E}_{t-1}x_{t-1})^2}{2\tilde{P}}),$$

with $\tilde{P}$ the steady-state conditional variance of the predictive distribution under the subjective learning. We then calculate the subjective posterior mean and variance for x_t:

$$\begin{aligned} \tilde{E}_t x_t &= \rho \tilde{E}_{t-1}x_{t-1} + \underbrace{\frac{(1+\theta)\tilde{P}}{(1+\theta)\tilde{P}+\sigma_\epsilon^2}}_{\nu}(y_t - \rho \tilde{E}_{t-1}x_{t-1}) \\ \widetilde{Var}_t x_t &= \frac{\sigma_\epsilon^2 \tilde{P}}{(1+\theta)\tilde{P}+\sigma_\epsilon^2}. \end{aligned}$$

Note that by definition, $\tilde{P}$ solves:

$$\tilde{P} = \frac{\rho^2\sigma_\epsilon^2\tilde{P}}{(1+\theta)\tilde{P}+\sigma_\epsilon^2} + \sigma_u^2. \tag{IA.1}$$

In our empirical analysis, we estimate out ν from the survey expectation data. Then, combining with above definition of $\tilde{P}$ we can obtain values for θ and $\tilde{P}$.

To derive (7), from the Kalman filter we can write y_t as:

$$y_t = \frac{E_t x_{t+1} - \rho E_{t-1}x_t}{K} + \rho E_{t-1}x_t = \frac{E_t x_{t+1} - E_{t-1}x_{t+1}}{K} + E_{t-1}x_{t+1},$$

which is replaced into (6). After subtracting both sides by $E_t x_{t+1}$, we can obtain (7).

Then, to derive (16), we first note:

$$(E_t - E_{t-1})(\Delta d_{t+1}^l - \lambda\Delta g_{t+1}) = (K_d^l - 1)(d_t^l - \lambda y_t - \hat{\mu}_{d,t-1}), \tag{IA.2}$$

where $\hat{\mu}_{d,t-1}$ is the posterior mean for the latent state produced by the Kalman filter. Without loss of generality, suppose that the initial mean μ_0 is zero, iterating $\hat{\mu}_{d,t-1}$ backward, and (IA.2) can be expanded as:

$$(E_t - E_{t-1})(\Delta d_{t+1}^l - \lambda\Delta g_{t+1}) = (K_d^l - 1)(d_t^l - \lambda y_t) + K_d^l \sum_{j=1}^{t-1}(1-K_d^l)^j(d_{t-j}^l - \lambda y_{t-j}).$$

Similarly, for d_t^s we can obtain:

$$(E_t - E_{t-1})\Delta d_{t+1}^s = (\rho_d K_d^s - 1)d_t^s + K_d^s(1-\rho_d K_d^s)\sum_{j=1}^{t-1}(1-K_d^s)^{j-1}\rho_d^j d_{t-j}^s.$$

In contrast, the expectation wedges are solved out as:

$$(\tilde{E}_t - E_t)(\Delta d_{t+1}^l - \lambda\Delta g_{t+1}) = (\nu_d^l - K_d^l)(d_t^l - \lambda y_t) + \sum_{j=1}^{t-1}[\nu_d^l(1-\nu_d^l)^j - K_d^l(1-K_d^l)^j](d_{t-j}^l - \lambda y_{t-j}) \tag{IA.3}$$

$$(\tilde{E}_t - E_t)\Delta d_{t+1}^s = \rho_d(\nu_d^s - K_d^s)d_t^s + \sum_{j=1}^{t-1}[\nu_d^s(1-\nu_d^s)^j - K_d^s(1-K_d^s)^j]\rho_d^{j+1}d_{t-j}^s. \tag{IA.4}$$

Combining the above four equations leads to:

$$(\tilde{E}_t - E_t)(\Delta d^l_{t+1} - \lambda \Delta g_{t+1}) = \frac{K^l_d - \nu^l_d}{1 - K^l_d}(E_t - E_{t-1})(\Delta d^l_{t+1} - \lambda \Delta g_{t+1}) + \sum_{j=1}^{t-1}[\nu^l_d(1-\nu^l_d)^j - \frac{1-\nu^l_d}{1-K^l_d}K^l_d(1-K^l_d)^j](d^l_{t-j} - \lambda y_{t-j}) \tag{IA.5}$$

$$(\tilde{E}_t - E_t)\Delta d^s_{t+1} = \rho_d \frac{K^s_d - \nu^s_d}{1 - \rho_d K^s_d}(E_t - E_{t-1})\Delta d^s_{t+1} + \sum_{j=1}^{t-1}[\nu^s_d(1-\nu^s_d)^j - \frac{1-\nu^s_d}{1-K^s_d}K^s_d(1-K^s_d)^j]\rho_d^{j+1} d^s_{t-j}. \tag{IA.6}$$

Adding up the two equations yields (16).

A.2 Equilibrium prices and returns

A.2.1 Bond prices

We first derive the equilibrium nominal bond prices. Conjecture that the log price of $n-$period nominal bond follows:

$$p^{(n)}_{b,t} = -A^{(n)}_b - B^{(n)}_b \widetilde{\mu}_{g,t} - C^{(n)}_b \widetilde{x}_{g,t} - D^{(n)}_b \widetilde{\mu}_{\pi,t} - E^{(n)}_b \widetilde{x}_{\pi,t}.$$

The pricing of one-period bond implies:

$$p^{(1)}_{b,t} = \tilde{E}_t m^{\$}_{t+1} + \frac{1}{2}\widetilde{Var}_t m^{\$}_{t+1},$$

from which we can solve out the coefficients:

$$\begin{aligned}
A^{(1)}_b &= -\log\beta - \frac{1}{2}\gamma^2(\tilde{P}_{\mu g} + \sigma_g^2) - \frac{1}{2}\gamma^2(\tilde{P}_{xg} + (\sigma_g^{gap})^2) - \frac{1}{2}(\tilde{P}_{\mu\pi} + \sigma_\pi^2) - \frac{1}{2}(\tilde{P}_{x\pi} + (\sigma_\pi^{gap})^2) \\
B^{(1)}_b &= \gamma \\
C^{(1)}_b &= \gamma\rho_g \\
D^{(1)}_b &= 1 \\
E^{(1)}_b &= \rho_\pi.
\end{aligned}$$

Four steady-state conditional variances, $\tilde{P}_{\mu g}, \tilde{P}_{xg}, \tilde{P}_{\mu\pi}, \tilde{P}_{x\pi}$, can be solved out from the volatility parameters and subjective learning gains based on (IA.1).

Then, from the pricing of $n-$period nominal bond:

$$p_{b,t}^{(n)} = \tilde{E}_t(m_{t+1}^{\$} + p_{b,t+1}^{(n-1)}) + \frac{1}{2}\widetilde{Var}_t(m_{t+1}^{\$} + p_{b,t+1}^{(n-1)}),$$

we can solve out the iteration for coefficients

$$\begin{aligned}
A_b^{(n)} &= A_b^{(n-1)} - \log\beta - \frac{1}{2}(B_b^{(n-1)}\nu_g^* + \gamma)^2(\tilde{P}_{\mu g} + \sigma_g^2) - \frac{1}{2}(C_b^{(n-1)}\nu_g^{gap} + \gamma)^2(\tilde{P}_{xg} + (\sigma_g^{gap})^2) \\
&\quad - \frac{1}{2}(D_b^{(n-1)}\nu_\pi^* + 1)^2(\tilde{P}_{\mu\pi} + \sigma_\pi^2) - \frac{1}{2}(E_b^{(n-1)}\nu_\pi^{gap} + 1)^2(\tilde{P}_{x\pi} + (\sigma_\pi^{gap})^2) \\
B_b^{(n)} &= B_b^{(n-1)} + \gamma \\
C_b^{(n)} &= C_b^{(n-1)}\rho_g + \gamma\rho_g \\
D_b^{(n)} &= D_b^{(n-1)} + 1 \\
E_b^{(n)} &= E_b^{(n-1)}\rho_\pi + \rho_\pi.
\end{aligned}$$

In fact, we have the following explicit formula for coefficients:

$$B_b^{(n)} = n\gamma, C_b^{(n)} = \frac{\rho_g(1-\rho_g^n)}{1-\rho_g}\gamma, D_b^{(n)} = n, E_b^{(n)} = \frac{\rho_\pi(1-\rho_\pi^n)}{1-\rho_\pi}$$

A.2.2 Dividend strip prices

Then we solve for the equilibrium dividend strip yield (log price-dividend ratio). Since it is in real terms, we conjecture that $p_{e,t}^{(n)} - d_t^{\$}$ has the following functional form:

$$p_{e,t}^{(n)} - d_t^{\$} = -A_e^{(n)} - B_e^{(n)}\widetilde{\mu}_{g,t} - C_e^{(n)}\widetilde{x}_{g,t} - D_e^{(n)}\widetilde{x}_{d,t} - E_e^{(n)}(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) - F_e^{(n)}(d_t^s - \rho_d\tilde{x}_{d,t-1}).$$

For the 1-period strip, its log price-dividend ratio follows:

$$p_{e,t}^{(1)} - d_t^{\$} = \tilde{E}_t(m_{t+1} + \Delta d_{t+1}) + \frac{1}{2}\widetilde{Var}_t(m_{t+1} + \Delta d_{t+1}),$$

from which we can solve out the coefficients:

$$
\begin{aligned}
A_e^{(1)} &= -\log\beta - \frac{1}{2}(\lambda-\gamma)^2(\tilde{P}_{\mu g}+\sigma_g^2) - \frac{1}{2}(\lambda-\gamma)^2(\tilde{P}_{xg}+(\sigma_g^{gap})^2) \\
&\quad - \frac{1}{2}(\tilde{P}_{\mu d}+\sigma_d^{l2}) - \frac{1}{2}(\tilde{P}_{xd}+\sigma_d^{s2}) \\
B_e^{(1)} &= \gamma-\lambda \\
C_e^{(1)} &= (\gamma-\lambda)\rho_g \\
D_e^{(1)} &= 1-\rho_d \\
E_e^{(1)} &= 1-\nu_d^l \\
F_e^{(1)} &= 1-\nu_d^s.
\end{aligned}
$$

Similarly, the $n-$period strip price solves:

$$
p_{e,t}^{(n)} - d_t^{\$} = \tilde{E}_t(m_{t+1}+\Delta d_{t+1}+p_{e,t+1}^{(n-1)}-d_{t+1}^{\$}) + \frac{1}{2}\widetilde{Var}_t(m_{t+1}+\Delta d_{t+1}+p_{e,t+1}^{(n-1)}-d_{t+1}^{\$}),
$$

$$
\begin{aligned}
A_e^{(n)} &= A_e^{(n-1)} - \log\beta - \frac{1}{2}(\lambda-\gamma-B_e^{(n-1)}\nu_g^*)^2(\tilde{P}_{\mu g}+\sigma_g^2) - \frac{1}{2}(\lambda-\gamma-C_e^{(n-1)}\nu_g^{gap})^2(\tilde{P}_{xg}+(\sigma_g^{gap})^2) \\
&\quad - \frac{1}{2}(\nu_d^l)^2(\tilde{P}_{\mu d}+(\sigma_d^l)^2) - \frac{1}{2}(\nu_d^s)^2(1-D_e^{(n-1)})^2(\tilde{P}_{xd}+(\sigma_d^s)^2) \\
B_e^{(n)} &= B_e^{(n-1)}+\gamma-\lambda \\
C_e^{(n)} &= C_e^{(n-1)}\rho_g+(\gamma-\lambda)\rho_g \\
D_e^{(n)} &= D_e^{(n-1)}\rho_d+1-\rho_d \\
E_e^{(n)} &= 1-\nu_d^l \\
F_e^{(n)} &= 1-\nu_d^s.
\end{aligned}
$$

We thus have the following explicit formula for coefficients:

$$
B_e^{(n)} = n(\gamma-\lambda), C_e^{(n)} = \frac{\rho_g(1-\rho_g^n)}{1-\rho_g}(\gamma-\lambda), D_e^{(n)} = 1-\rho_d^n, E_e^{(n)} = 1-\nu_d^l, F_e^{(n)} = 1-\nu_d^s.
$$

Finally, to decompose bond and equity strip return as in (36) and (37), from the return definitions we have (ignoring the constant):

$$r^{(N)}_{B,t+1:t+h} = \underbrace{\tilde{E}_t\pi_{t+1:t+N} - \tilde{E}_{t+h}\pi_{t+h+1:t+N}}_{INFL^{(N)}_B} + \underbrace{\gamma(\tilde{E}_t\Delta g_{t+1:t+N} - \tilde{E}_{t+h}\Delta g_{t+h+1:t+N})}_{RGDP^{(N)}}, \tag{IA.7}$$

$$r^{(n)}_{S,t+1:t+h} = \underbrace{\pi_{t+1:t+h}}_{INFL^{(n)}_S} + \underbrace{\gamma(\tilde{E}_t\Delta g_{t+1:t+n} - \tilde{E}_{t+h}\Delta g_{t+h+1:t+n})}_{RGDP^{(n)}} + \underbrace{(\tilde{E}_{t+h} - \tilde{E}_t)\Delta d_{t+1:t+n}}_{RDIV^{(n)}}. \tag{IA.8}$$

To derive (39) and (40), we have the following from the definition:

$$y^{(N)}_t = \underbrace{\frac{1}{N}\tilde{E}_t\pi_{t+1:t+N}}_{INFL^{(N)}_B} + \underbrace{\frac{\gamma}{N}\tilde{E}_t\Delta g_{t+1:t+N}}_{RGDP^{(N)}}, \tag{IA.9}$$

$$ey^{(n)}_t = \underbrace{\frac{\gamma}{n}\tilde{E}_t\Delta g_{t+1:t+n}}_{RGDP^{(n)}} - \underbrace{\frac{1}{n}\tilde{E}_t\Delta d_{t+1:t+n}}_{RDIV^{(n)}}. \tag{IA.10}$$

B Estimation of State-Space Models

With calibrated ρ_g, we then estimate the parameters in the state-space models from (17) to (22) using maximum likelihood (MLE) via the Kalman filter. However, the direct estimation of dividend state-space model (9) to (12) may be problematic because the models are in log dividend levels, but data of dividend levels are pre-processed via four-quarter trailing summation to remove seasonality. Hence, estimates of noise terms in (9) and (11) are downward biased because the trailing summation unintentionally smooths out the noise terms. Indeed, we find that the noise terms are very close to zero if we directly estimate (9) to (12) using the smoothed dividend series. To obtain reasonable parameter values for our state-space models, we fit a structural time-series model following e.g., Harvey and Shephard (1993) to explicitly estimate the seasonal and noise components of d^s_t and $d^l_t - \lambda y_t$ from unsmoothed data:

$$\begin{aligned}
d^l_t - \lambda y_t &= \mu_{d,t} + \gamma^l_t + \sigma^l_d \epsilon^*_{d,t} \\
\mu_{d,t+1} &= \mu_{d,t} + \sigma^\mu_d \epsilon_{d,t+1} \\
d^s_t &= x_{d,t} + \gamma^s_t + \sigma^s_d \epsilon^*_{x,t} \\
x_{d,t+1} &= \rho_d x_{d,t} + \sigma^x_d \epsilon_{x,t+1}.
\end{aligned}$$

The model is mostly identical to (9) to (12), except that this time we explicitly estimate seasonal components γ_t^l and γ_t^s (together with other parameters) from seasonally unadjusted dividend series. The seasonal components follow the process:

$$\gamma_t = -\sum_{j=1}^{3} \gamma_{t-j} + \omega_t,$$

with $i.i.d.\ \omega_t \sim N(0, \sigma_\omega^2)$. The whole system can be estimated using a standard MLE via Kalman filter, by treating seasonal components as other latent states.

C Additional Results

Table IA.1: Robustness: alternative decomposition of aggregate dividends

The table reports the unconditional mean and standard deviation of equity yields from both our model and data, and their correlation coefficients. For the model-implied quantities, we change our way of decomposing aggregate dividend in (8) based on different measures of equity duration. These include the measures proposed by Weber (2018), Gonçalves (2021a), and the book-to-market ratio in Lettau and Wachter (2007). Alternatively, when decomposing dividend using our baseline measure of duration (LTG), we change the breakpoint to the 40th or 60th cross-sectional percentile. The sample period is from 1987Q4 to 2019Q4.

		1Y	10Y	10Y-1Y
Data	Mean	-4.39	-1.34	3.05
	Volatility	8.89	2.70	7.22
Weber (2018)	Mean	-3.13	-1.33	1.79
	Volatility	10.35	2.68	8.09
	Corr.	0.59	0.82	0.43
Gonçalves (2020a)	Mean	-6.67	-1.34	5.34
	Volatility	10.73	1.84	9.00
	Corr.	0.62	0.71	0.50
Book-to-market	Mean	-4.02	-1.34	2.68
	Volatility	8.70	1.86	7.10
	Corr.	0.62	0.85	0.47
40th	Mean	-5.33	-1.34	3.98
	Volatility	7.79	1.49	6.54
	Corr.	0.64	0.71	0.55
60th	Mean	-4.95	-1.34	3.61
	Volatility	10.25	2.05	8.73
	Corr.	0.63	0.72	0.56

Table IA.2: Correlation between subjective and rational news

The table reports the average correlation between rational news defined in (7) and the subjective news when learning gain is ν instead of the Kalman gain K. For each path, we simulate both the rational news and subjective news under the specified learning gains in the table, with the sample length identical to our sample period from 1987Q4 to 2019Q4 (129 observations). The simulation is repeated 100,000 times, and reported numbers are the average correlation over all simulated paths.

Panel A: $\rho = 0.9$										
		Kalman gain K								
		0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Subjective gain ν	0.01	1	0.99	0.99	0.96	0.91	0.82	0.72	0.63	0.56
	0.05	0.99	1	0.99	0.98	0.94	0.87	0.78	0.70	0.64
	0.1	0.99	0.99	1	0.99	0.97	0.92	0.85	0.77	0.71
	0.2	0.96	0.98	0.99	1	0.99	0.97	0.93	0.87	0.82
	0.3	0.91	0.94	0.97	0.99	1	0.99	0.97	0.94	0.89
	0.4	0.82	0.87	0.92	0.97	0.99	1	0.99	0.97	0.94
	0.5	0.72	0.78	0.85	0.93	0.97	0.99	1	0.99	0.98
	0.6	0.63	0.70	0.77	0.87	0.94	0.97	0.99	1	0.99
	0.7	0.56	0.64	0.71	0.82	0.89	0.94	0.98	0.99	1

Panel B: $\rho = 1$										
		Kalman gain K								
		0.01	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Subjective gain ν	0.01	1.00	0.99	0.98	0.95	0.86	0.70	0.53	0.38	0.30
	0.05	0.99	1.00	0.99	0.97	0.93	0.83	0.69	0.56	0.47
	0.1	0.98	0.99	1.00	0.99	0.96	0.90	0.80	0.69	0.60
	0.2	0.95	0.97	0.99	1.00	0.99	0.97	0.92	0.84	0.77
	0.3	0.86	0.93	0.96	0.99	1.00	0.99	0.97	0.93	0.87
	0.4	0.70	0.83	0.90	0.97	0.99	1.00	0.99	0.97	0.94
	0.5	0.53	0.69	0.80	0.92	0.97	0.99	1.00	0.99	0.97
	0.6	0.38	0.56	0.69	0.84	0.93	0.97	0.99	1.00	0.99
	0.7	0.30	0.47	0.60	0.77	0.87	0.94	0.97	0.99	1.00

D Model with Ambiguity

D.1 Equilibrium bond and equity yields

We extend the analysis to consider the agent's fear over model misspecification of the real GDP growth and dividend growth. We show that the extended model better explains the dynamics of bond and equity yields relative to the benchmark model in Section 2. To begin with, we assume that the representative agent has a recursive multiple-priors preference (see e.g., Epstein and Schneider , 2003).

$$V_t(C_t) = \min_{p_t \in \mathscr{P}_t} \mathbb{E}^{p_t}[U(C_t) + \beta V_{t+1}(C_{t+1})], \tag{IA.11}$$

where $\mathscr{P}_t$ denotes the set of alternative models (probability measures) and the CRRA utility function $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$. The agent is ambiguous about real endowment growth and dividend growth. The set of alternative measures is generated by different mean growth rates around respective reference mean values. We assume that the reference model is the posterior distribution obtained from agent's learning over the real GDP growth and real dividend, as discussed in Subsection 2.2 and 2.3, but the agent evaluates future prospects under the *worst-case* measure. More explicitly, the agent will select the lowest real GDP growth and dividend growth forecasts when pricing assets.[1] Hence, the worst-case beliefs over the real GDP and dividend growth are:

$$\begin{aligned}
&\tilde{E}_t \Delta g_{t+1} = \tilde{\mu}_{g,t} + \rho_g \tilde{x}_{g,t} - a_{g,t} \\
&\tilde{E}_t \Delta d_{t+1} = \lambda(\tilde{\mu}_{g,t} + \rho_g \tilde{x}_{g,t} - a_{g,t}) + (\rho_d - 1)\tilde{x}_{d,t} + (\nu_d^s - 1)(d_t^s - \rho_d \tilde{x}_{d,t-1}) + (\nu_d^l - 1)(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) - a_{d,t},
\end{aligned} \tag{IA.12}$$

where $a_{g,t}$ denotes the ambiguity over real endowment growth. The ambiguity over total real dividend growth consists of two parts arising from distorting real endowment growth ($\lambda a_{g,t}$) and dividend-specific growth ($a_{d,t}$). We assume that they follow the standard AR(1) processes:

$$\begin{aligned}
a_{g,t+1} &= \mu_{ag} + \rho_{ag} a_{g,t} + \sigma_{ag} \epsilon_{ag,t+1} \\
a_{d,t+1} &= \mu_{ad} + \rho_{ad} a_{d,t} + \sigma_{ad} \epsilon_{ad,t+1},
\end{aligned} \tag{IA.13}$$

with $i.i.d.$ standard normal shocks $\varepsilon_{ag,t+1}$, $\varepsilon_{ad,t+1}$. The equilibrium $n-$period bond and equity yields are then solved out as:

$$y_t^{(n)} = \frac{A_b^{(n)}}{n} + \gamma(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g \tilde{x}_{g,t}) + (\tilde{\mu}_{\pi,t} + \frac{1-\rho_\pi^n}{n(1-\rho_\pi)}\rho_\pi \tilde{x}_{\pi,t}) - \frac{1-\rho_{ag}^n}{n(1-\rho_{ag})}\gamma a_{g,t}, \tag{IA.14}$$

$$\begin{aligned}
ey_t^{(n)} = &\frac{A_e^{(n)}}{n} - (\lambda-\gamma)(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g \tilde{x}_{g,t}) + \frac{1-\rho_d^n}{n}\tilde{x}_{d,t} - \frac{\nu_d^l - 1}{n}(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) \\
&- \frac{\nu_d^s - 1}{n}(d_t^s - \rho_d \tilde{x}_{d,t-1}) + \frac{1-\rho_{ag}^n}{n(1-\rho_{ag})}(\lambda-\gamma)a_{g,t} + \frac{1-\rho_{ad}^n}{n(1-\rho_{ad})}a_{d,t},
\end{aligned} \tag{IA.15}$$

[1] The worst-case distortion for dividend growth rests on the assumption that dividend shocks are positively correlated with endowment shocks. However, when learning from past data, we do not ask the agent to consider such correlation. This assumption greatly simplifies our analysis because it avoids additional parameters that are hard to pin down under correlated learning. Croce et al. (2015) consider a similar setting where the agent ignores some shock correlations when pricing assets.

with constant $A_b^{(n)}$ and $A_e^{(n)}$ given by iterations similar to those in Subsection A.2.

D.2 Ambiguity parameters

We use standard empirical measures for the ambiguity to calculate related parameters and equilibrium yields. First, the ambiguity over real GDP growth is constructed from the Survey of Professional Forecasters (SPF). For each quarter, we calculate the ambiguity by dividing the interquartile range of forecasts for the next year real GDP growth by two (see also Drechsler , 2013; Ilut and Schneider , 2014; Zhao , 2017). Second, a new variable that we need to obtain from the survey data is the dividend-specific ambiguity $a_{d,t}$. From Equation (IA.12), as long as the ambiguity over aggregate real dividend growth is empirically available, we can back out $a_{d,t}$ after removing the real GDP ambiguity part. To achieve this, we resort to firm-level earnings survey data from the IBES. Given that the IBES summary file does not provide the upper and lower quartiles of analyst forecasts for each firm, we retrieve them from the IBES unadjusted detail file.[2] For each firm and quarter, we collect individual analyst forecasts of future earnings per share (EPS) for multiple forecasting horizons. For each forecasting horizon, we obtain the upper and lower quartiles of analyst forecasts, and then we apply linear interpolations to obtain the forecasts at the 1-year horizon. After multiplying those interpolated forecasts with the shares outstanding in each quarter and aggregate over all stocks, we obtain the 25th and 75th percentiles of predicted 1-year-ahead earnings levels for the aggregate market. Ambiguity over aggregate cash-flows is then calculated as one-half of the log difference between these quartiles.

In spite of using earnings survey data when estimating dividend ambiguity, we show empirically that the obtained measure is sensible. Figure IA.1 displays reasonable time-variations in our ambiguity measure within the sample, with a correlation of 0.62 with the ambiguity over real GDP growth. Meanwhile, Ilut and Schneider (2014) suggest that valid empirical measure of ambiguity should not exceed twice the volatility of the forecasted time-series itself (see their Section III.B). In compliance with their ambiguity bound, we find that the sample average of ambiguity over annual real dividend growth is around 4% while the volatility of realized annual real dividend growth is around 7%. Finally, with the empirical measures in hand, we obtain ambiguity parameters by matching the simulated moments with the mean, volatility, and AR(1) coefficients from the ambiguity data. Table IA.3 displays the parameters.

[2] We do not use the dividend forecast in the IBES detail file when constructing the ambiguity measure, primarily because the dividend forecast is only available after 2003 and this will shorten the period for our analysis substantially. Also, the average number of analysts providing dividend estimates in the IBES detail file is much smaller than that for the earnings, which may yield inaccurate measure.

Figure IA.1: Ambiguity over aggregate dividend growth

The figure plots the annualized ambiguity over 1-year-ahead aggregate dividend growth. Shaded areas correspond to NBER recessions. The sample period is from 1987Q4 to 2019Q4.

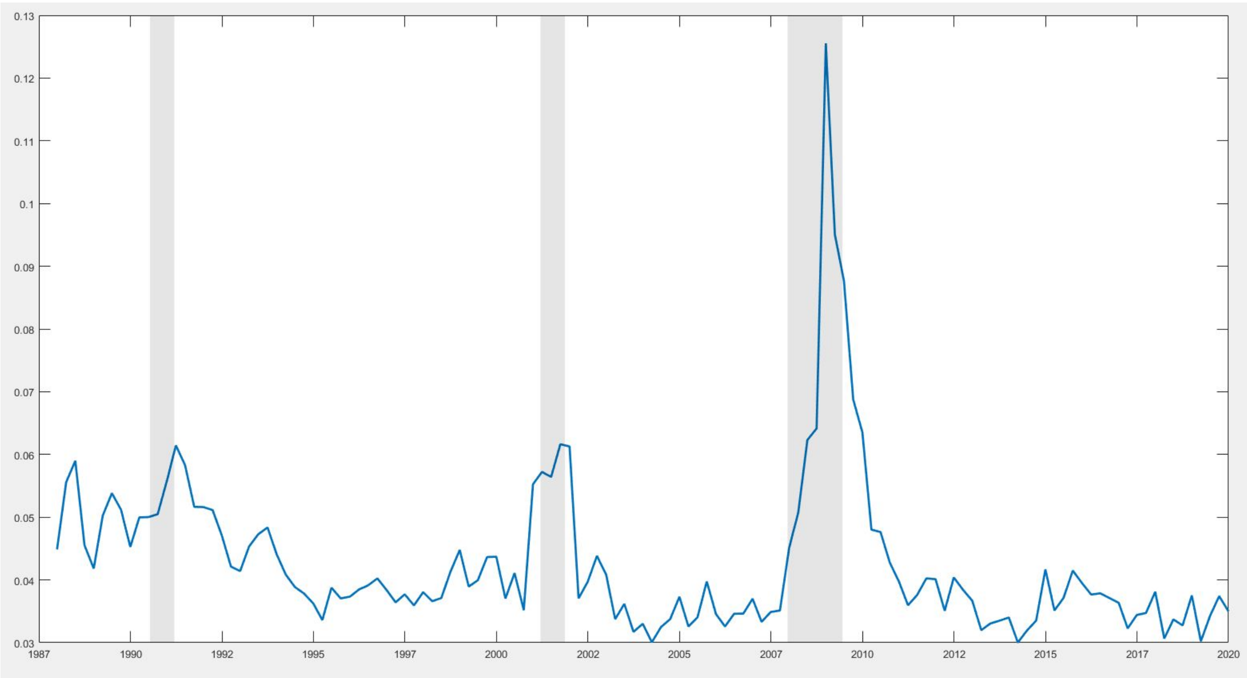


Table IA.3: Parameters for ambiguity processes

μ_{ag}	ρ_{ag}	σ_{ag}	μ_{ad}	ρ_{ad}	σ_{ad}
0.0004	0.65	0.05	0.0002	0.978	0.09

D.3 Asset pricing implications

We then discuss the asset pricing performance from the extended model. First, Table IA.4 reports the model-implied bond yield statistics, and they closely match the data. Second, the left panel of Figure IA.2 displays the time-series fit for the term structure of equity yields. Comparing this with Figure 2, we see that introducing the ambiguity helps improve the model's explanatory power for equity yields, especially during the 2008 global financial crisis. Table IA.5 summarizes the moments for data and model-implied quantities. The correlation coefficients between the data and the model indeed increase relative to the benchmark model. Furthermore, the right panel of Figure IA.2 plots the time-series fit for the term structure of equity *forward* yields, defined as the difference between equity and nominal bond yields. The model explains well both the level and variability of equity forward yields, as can be confirmed in the right panel of Table IA.5.

Table IA.4: Term structure of nominal bond yields: data vs. model

The table reports the mean and standard deviation of nominal bond yields. These numbers are in annualized percentage terms. We report statistics from both our model and the data, and also their correlation coefficients. The sample period is from 1987Q4 to 2019Q4.

		1Y	2Y	5Y	7Y	10Y
Data	Mean	3.40	3.63	4.25	4.57	4.92
	Volatility	2.59	2.56	2.35	2.24	2.14
Model	Mean	4.84	4.89	4.95	4.94	4.91
	Volatility	1.77	1.71	1.62	1.58	1.56
	Corr.	0.89	0.92	0.94	0.95	0.95

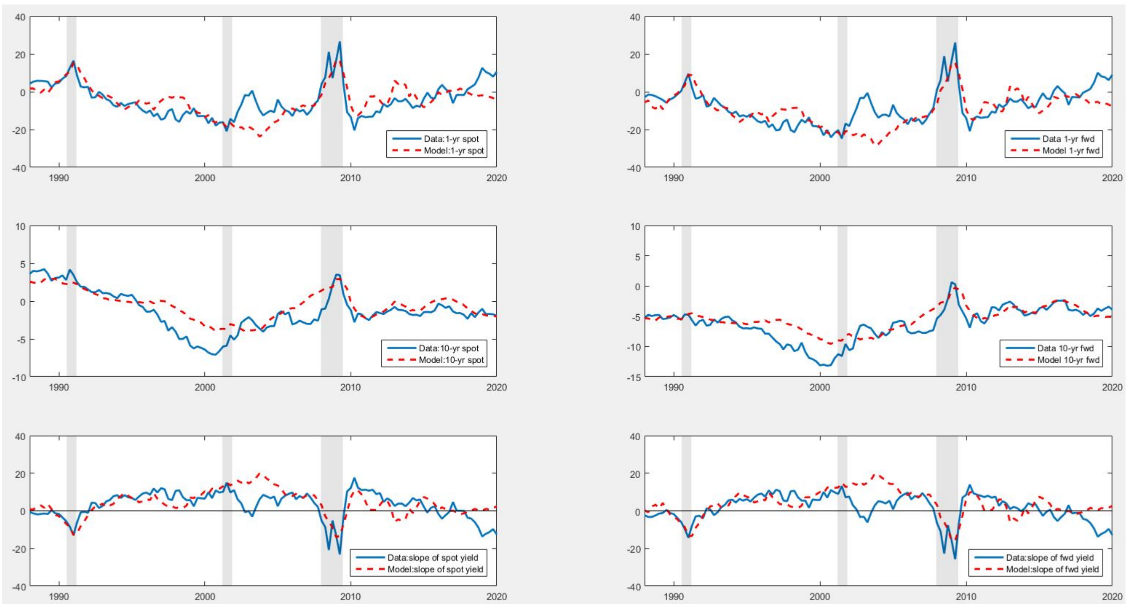
Table IA.5: Summary statistics of equity yields

The table reports the mean and standard deviation of spot and forward yields of dividend strips. These numbers are in annualized percentage terms. We report statistics from both our model and the data, and also their correlation coefficients. Sample period is from 1987Q4 to 2019Q4.

		Spot			Forward		
		1Y	10Y	10Y-1Y	1Y	10Y	10Y-1Y
Data	Mean	-4.39	-1.34	3.05	-7.79	-6.26	1.53
	Volatility	8.89	2.70	7.22	9.14	2.80	7.18
Model	Mean	-4.60	-0.55	4.05	-9.44	-5.46	3.97
	Volatility	8.39	1.89	6.92	8.65	1.96	7.09
	Corr	0.68	0.85	0.59	0.67	0.81	0.59

Figure IA.2: Term structure of equity spot and forward yields

The figure compares the model-implied spot (left panel) and forward yields (right panel) of dividend strips with the data from Giglio et al. (2021). The forward yields in the data are computed by subtracting the spot yields with the maturity-matched zero-coupon nominal Treasury bond yields. The last row plots the spread between 10-year and 1-year spot or forward yields. Shaded areas correspond to NBER recessions. The sample period is from 1987Q4 to 2019Q4, and all numbers are in annualized percentage terms.



Third, following Van Binsbergen et al. (2013), we run variance decomposition on forward equity yields to understand determinants of their time-variations. We can write the forward yields as:

$$
\begin{aligned}
ef_t^{(n)} = Const^{(n)} & \underbrace{-\lambda(\tilde{\mu}_{g,t} + \frac{1-\rho_g^n}{n(1-\rho_g)}\rho_g \tilde{x}_{g,t})}_{RGDP_t^{(n)}} + \underbrace{\frac{1-\rho_d^n}{n}\tilde{x}_{d,t} - \frac{\nu_d^l - 1}{n}(d_t^l - \lambda y_t - \tilde{\mu}_{d,t-1}) - \frac{\nu_d^s - 1}{n}(d_t^s - \rho_d \tilde{x}_{d,t-1})}_{Div-specific_t^{(n)}} \\
& \underbrace{-(\tilde{\mu}_{\pi,t} + \frac{1-\rho_\pi^n}{n(1-\rho_\pi)}\rho_\pi \tilde{x}_{\pi,t})}_{Infl_t^{(n)}} + \underbrace{\frac{1-\rho_{ag}^n}{n(1-\rho_{ag})}\lambda a_{g,t} + \frac{1-\rho_{ad}^n}{n(1-\rho_{ad})}a_{d,t}}_{Ambiguity_t^{(n)}},
\end{aligned}
\tag{IA.16}
$$

from which we obtain the following decomposition:

$$
\begin{aligned}
var(ef_t^{(n)}) = & \, cov(ef_t^{(n)}, RGDP_t^{(n)}) + cov(ef_t^{(n)}, Div-specific_t^{(n)}) \\
& + cov(ef_t^{(n)}, Infl_t^{(n)}) + cov(ef_t^{(n)}, Ambiguity_t^{(n)}).
\end{aligned}
\tag{IA.17}
$$

Table IA.6 illustrates the proportion of total forward yield variability explained by each component. Consistent with our mean decomposition results in Table 3, the subjective real dividend-specific growth contributes over 90% to the yield volatility at the 1-year horizon. Interestingly, the importance of subjective real GDP growth increases steadily with the horizon. For the 10-year forward yield, it explains 23% of total yield variance while the proportion of dividend-specific growth decreases to 59%. A similar pattern is observed for the ambiguity part, which explains around 10% of total variance at the 10-year horizon. Zooming in on different economic regimes, we find that the explanatory power of subjective real GDP growth is stronger during the expansion period, yet the ambiguity channel is more important during the recession. For instance, it explains 18% of the 10-year forward yield variance.

Table IA.6: Variance decomposition of forward equity yields

The table reports the model-based variance decomposition (IA.17), where forward yields are decomposed to the components related to the real GDP growth, dividend-specific growth, inflation, and ambiguity. The decomposition is run over the full sample from 1987Q4 to 2019Q4, or over expansion and recession periods identified via the NBER business cycle dating. The decomposition is done for the dividend strip with the maturity of 1-year, 5-year, 7-year, and 10-year.

		1Y	5Y	7Y	10Y
Unconditional	RGDP	0.04	0.11	0.15	0.23
	Div.-spec.	0.93	0.80	0.73	0.59
	Infl.	-0.01	-0.01	0.01	0.05
	Ambiguity	0.05	0.10	0.12	0.13
Expansion	RGDP	0.04	0.12	0.17	0.26
	Div.-spec.	0.95	0.84	0.76	0.60
	Infl.	-0.01	-0.01	0.01	0.06
	Ambiguity	0.01	0.05	0.06	0.08
Recession	RGDP	0.03	0.09	0.11	0.15
	Div.-spec.	0.94	0.78	0.71	0.61
	Infl.	-0.01	0.01	0.03	0.06
	Ambiguity	0.04	0.11	0.14	0.18

Incorporating ambiguity into the model also improves the fit for equity returns. Figure IA.3 shows that the correlation coefficients of 2-year (10-year) realized strip returns slightly increase, and the model matches better the significant return crash during the global financial crisis. The model further matches two stylized facts regarding return variations: (1) long-term dividend strips co-move more strongly with the market returns (Van Binsbergen and Koijen , 2017; Gonçalves , 2021b); (2) return volatilities of long-term dividend strips are higher than those of short-term strips (Lettau and Wachter , 2007; Van Binsbergen and Koijen , 2017).

Previous studies reconcile the facts via the idea that long duration assets have higher exposures to discount rate variations (see e.g., Campbell and Vuolteenaho , 2004; Brennan and Xia , 2006; Lettau and Wachter , 2007; Gonçalves , 2021b). Such an explanation may not be consistent with recent literature that casts doubt on the relevance of discount rate variations at both short and long horizons. Indeed, if discount rate variability per se does not contribute much to price volatility, we might expect it will explain little about the above patterns for return variations. Because our model imposes minimal discount rate variations, we have to use variations in beliefs over cash-flows to match these two stylized facts. Table IA.7 reports the results, where Panel A estimates CAPM betas of strip futures returns to gauge the magnitude of comovements and Panel B calculates volatilities of futures returns. Results imply that our model replicates well the upward-sloping term structure of both CAPM betas and volatilities, although the model-implied return volatilities are slightly smaller than the data.

Figure IA.3: Strip futures returns: data vs. model

The figure compares the model-implied futures returns of dividend strips with the data calculated from Giglio et al. (2021). We display results for 2-year and 10-year strip returns, and the sample period is from 1988Q4 to 2019Q4. All numbers are in annualized percentage terms.

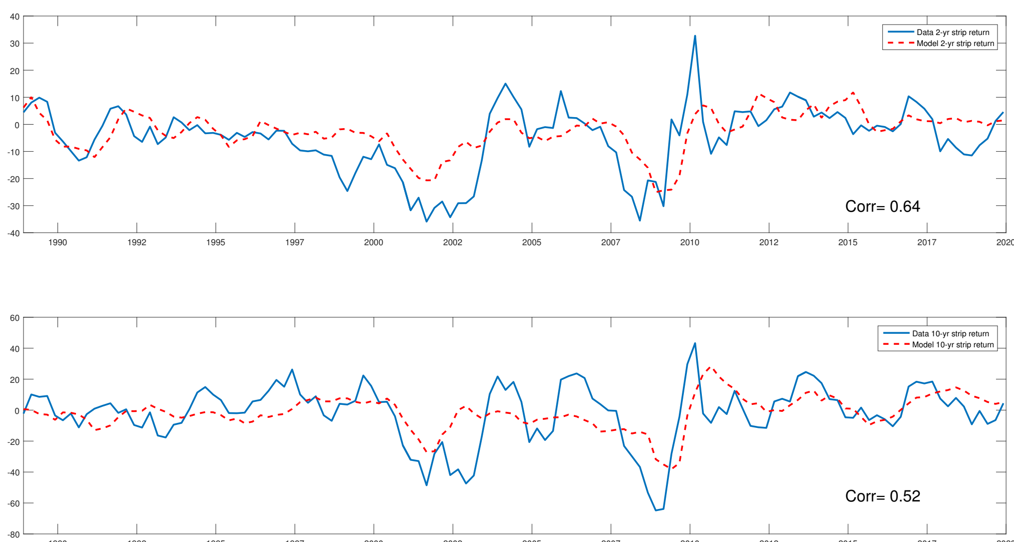


Table IA.7: Strip return comovements and volatilities

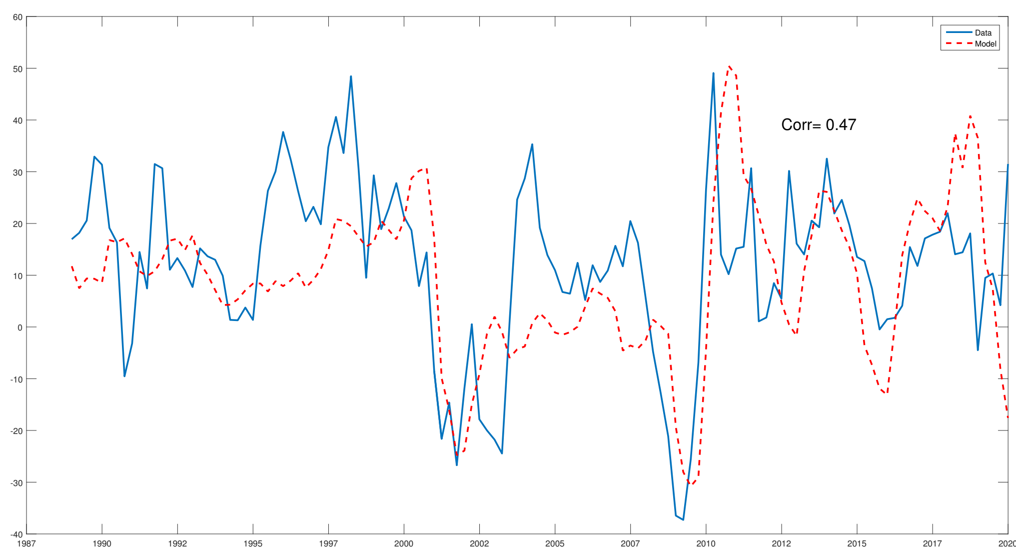
Panel A reports the model-implied CAPM betas of strip futures returns, and the Newey-West standard errors are in parentheses. Panel B reports the volatilities of strip futures returns in annualized percentage terms. The sample period is from 1988Q4 to 2019Q4.

	2Y	5Y	7Y	10Y
	Panel A: CAPM betas			
Data	0.47	0.80	0.88	0.93
	(0.10)	(0.10)	(0.10)	(0.11)
Model	0.25	0.44	0.53	0.61
	(0.06)	(0.06)	(0.06)	(0.08)
	Panel B: return volatilities			
Data	11.94	16.37	17.56	18.84
Model	7.33	8.30	9.40	10.81

Finally, Figure IA.4 plots the model-implied market returns together with the data. They are close with each other and the correlation coefficient is 0.47.

Figure IA.4: Aggregate stock returns: data vs. model

The figure compares the model-implied aggregate market returns with the data. The sample period is from 1987Q4 to 2019Q4. Plotted numbers are in annualized percentage terms.



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