자본시장에서의 변동성 파생상품의 역할과 활용전략

2014 추계 심포지엄
한국파생상품학회

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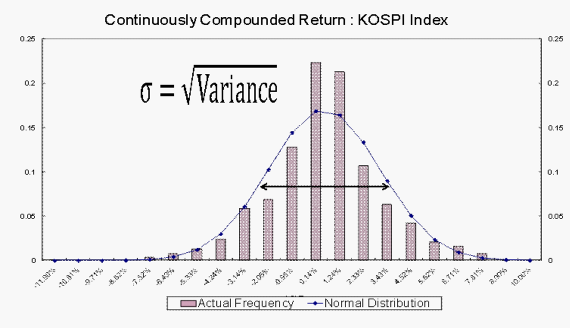
엄영호(연세대학교)

Outline

Current Section

What is Volatility?



- Volatility: the degree to which the price of an asset tends to fluctuate over time
- Realized volatility: The volatility of the underlying contract over some period of time
- Implied volatility: The marketplace's consensus forecast of future volatility derived from the prices of options in the marketplace

Realized Variance

- Let $t = t_0 < t_1 < \cdots < t_n = t + \tau$ denote the trading days over a given time period, then the realized variance (RV) could be defined as

$$RV_{t,t+\tau} = \frac{A}{k} \sum_{i=1}^{k} \left(\ln \frac{S_{t_i}}{S_{t_{i-1}}} \right)^2$$

  where A is the annualizing factor (e.g. 252)

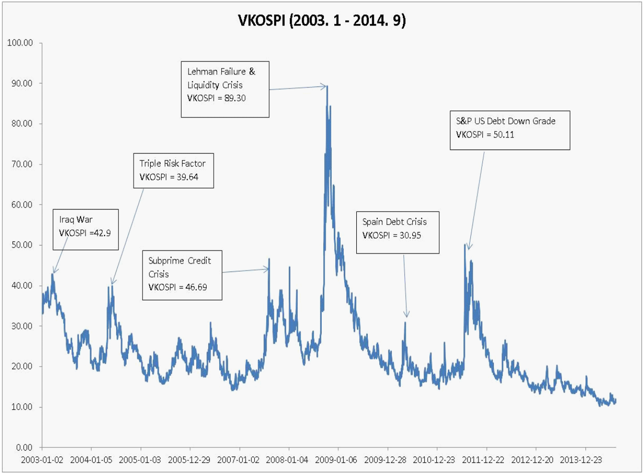
- The realized variance converges in probability to the annualized quadratic variation of the log-price:

$$RV_{t,t+\tau} = \frac{A}{k} \sum_{i=1}^{k} \left(\ln \frac{S_{t_i}}{S_{t_{i-1}}} \right)^2 \longrightarrow QV_{t,t+\tau} = \frac{1}{\tau} \int_t^{t+\tau} v_s ds + \frac{1}{\tau} \sum_{u=N_t}^{N_{t+\tau}} J_u^2$$

- Note that, if the spot variance includes a jump component, the convergence above still holds and such jumps in variance are accommodated in the time integral of v_s.

Implied Volatility in Korea

- The financial press, which has covered the VIX since its inception, often refer to it as the "fear gauge."



Fear Gauge

- 90 Trillion won have lost for last three weeks in the Korean Stock market : Maeil Economic Newspaper May 17, 2004

"Fear is the worst factor in current market. Since no one can predict the bottom of the market and can do anything, gloomy atmosphere dominates the market. There has been no sign of any momentum which release the fear, though all negative factors have been shown up. The market has been like entangled knots for last two weeks , negative news tailed backed by negative news. Park Yoon Soo, the managing director of LG Investment Securities said, 'the current stock market is consumed by fear."'

- 거래소 시가총액 3주새 90兆 증발 : 매일경제 2004-05-17

"지금 증권시장의 최대 악재는 '공포감' 이다.어디까지 빠질지 아무도 예측 할 수 없는, 그래서 지금 아무 것도 할 수 없는 그런 우울한 분위기가 시장을 지배하고 있다. 시장을 짓누르는 모든 악재는 다 드러난 상태지만 이 '공포' 를 해소해 줄 어떤 모멘텀도 나타날 기미조차 보이지 않는다. 악재가 악재를 물고 나가는, 실타래가 뒤엉킨 것 같은 이런 분위기는 벌써 2주일째 이어지고 있다. 박윤수 LG투자증권 상무는 이를 "공포가 지배하는 시장" 이라고 표현했다.

Why do Volatility Derivatives Exist?

- Proponents of derivatives have argued that they benefit market welfare, by facilitating risk management, asset allocation, and price discovery.
- Utilizing volatility derivatives, market participants can benefit from
  - New investment opportunities to capture variance risk premium
  - Creating new products
  - Relative value trading opportunities
  - Tail risk hedging
  - Optimal asset allocation
  - Vega hedging

Who has risk exposures to volatility

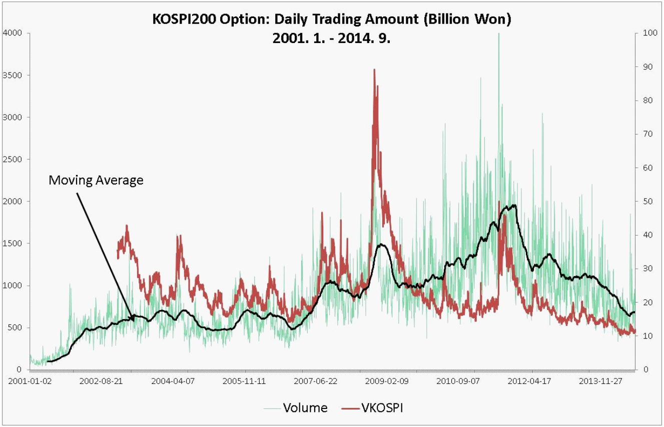
- Equity investors such as investment funds and pension funds are probably short volatility due to the negative correlation between index level and volatility. Correlation risk makes portfolio diversification very difficult.
- Portfolio strategies such as protective put and covered call have exposures to volatility risk (long or short).
- Option trading firms may have significant volatility risk (long or short). They also typically benefit from higher volatility since it correlated to increased trading activity.
- Securities firms selling structured products with long vega position may have significant exposures to decreases in volatility.
- As volatility is a key input for risk management and capital adequacy models, such as the VaR, banks are exposed to shifts in volatility and correlation during stress market conditions.

Who has risk exposures to volatility

- Many insurance companies issued the structured products that have embedded options. Typically, these products allow the insurance buyer to invest in a policy or annuity that can provide a guaranteed floor (GMAB, GMDB). These products leave the insurance company short a lot of volatility. Due to the long-dated nature of the options, this is mostly vega risk.
- The amount of VA products is roughly 90 trillion won in Korea. The vega amount is estimated to be about 130 billion won. Insurance companies may have to hedge their vega risk by buying forward-starting variance swaps.
- Bloom (2009) shows that uncertainty shocks measured by VIX lead to a rapid drop and rebound in aggregate output and employment. Under stressed economic conditions, firms have difficulties in funding investment projects and may face the increased cost of capital.

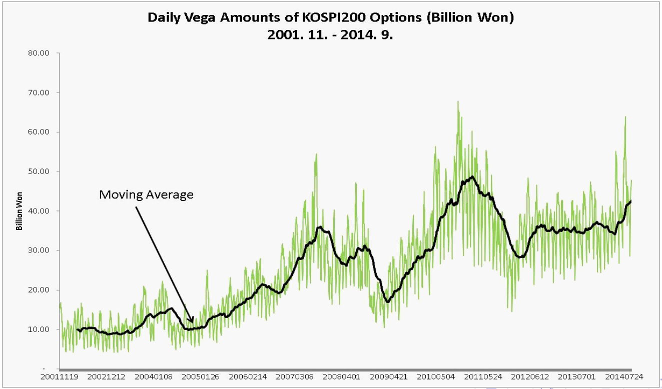
KOSPI 200 Option: Trading Amount

- The demand of KOSPI 200 options has been peaked at the time of US downgrade.



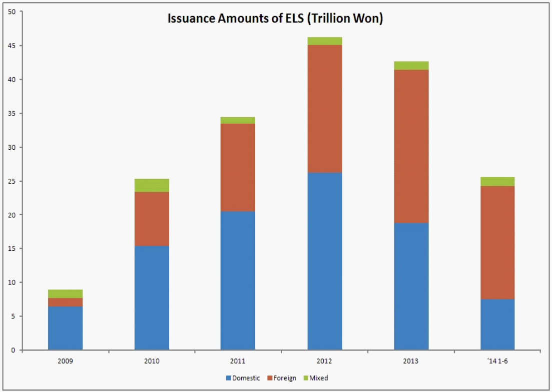
KOSPI 200 Option: Vega Amount

- The daily vega amount of KOSPI 200 options peaked at 67.9 billon won during sovereign debt crisis.
- As of Aug. 29, 2014, the vega amount is 40.4 billon won, which means that option value will change by 404 billion won if volatility changes by 10 points.



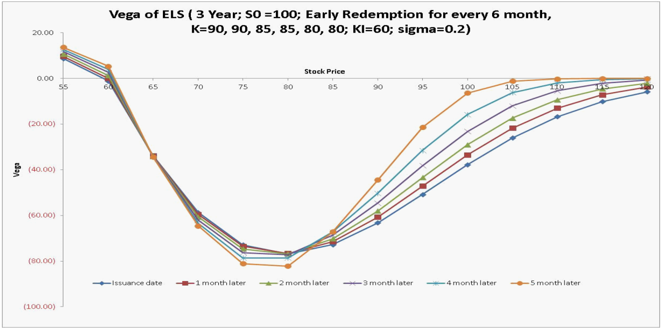
Structured Products Vega Exposure

- ELS market in Korea have grown rapidly over the past decade. The current balance of issuance amount is about 52 trillion won, and the number of annual new issues reached to more than 16 thousands in 2013.



Structured Products Vega Exposure

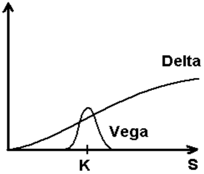
- The vega of ELS for investor is negative, because the investor buys digital calls and sells barrier put option, which has longer maturities. The vega amount of a typical step-down auto-callable ELS around at-the-money is 80 million won for the issuance size of 10 billion won.
- Thus vega amount of ELS could be more than roughly 100 billion won.



Current Section

Vanilla Options

- Vanilla options are impure: they provide exposure to both direction of the stock price and its volatility.
- Call option has :
  - Sensitivity to S : Δ
  - Sensitivity to σ : Vega
- These sensitivities vary through time and spot, and vol.



- To have pure exposure,
  - Need of constant sensitivity to vol
  - Ideally achieved by a log profile : variance swaps

What is Variance Swap?

- A variance swap is a forward contract on the annualized realized variance.
- Example)

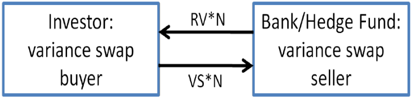


Figure: Cash-flow of a variance swap at maturity

- Example: 1-year variance swap with notional $N = 1000$
- Volatility swap rate $= 20\%$
- Realized volatility $= 25\%$
- P&L $= 1000 * (0.25^2 - 0.2^2) = 22.5$

- Because swap rates are typically set with reference to implied volatility in option markets, the return on a variance swap will also reflect any difference between implied and realized volatility over time.

Advantage of Variance Swap

- According to Carr & Lee (2009), the first volatility derivative appears to have been a variance swap dealt in 1993 at the UBS.
- By 1998, variance swaps on stock indices started to take off, due to the historically high implied volatilities experienced in that year.
  - Sellers: investment banks, hedge funds, . . . (exposure to volatility risk)
  - Buyers: mutual funds, portfolio managers . . . (as insurance against rising volatility)
- Traditional volatility trading requires daily delta-hedging, but it involves significant transaction costs and time. Its P/L is path dependent. These features make pure volatility trading messy.
- Variance swaps offered pure exposure to equity volatility .
- Due to characteristic of no path dependency, user base of variance swaps was broadened to non-volatility-focused hedge funds, insurance companies, pension funds.

Variance Swap Pricing and VIX

- A major break-through in variance swap pricing was achieved by Neuberger (1994), Dupire (1993), and Carr and Madan (1999).
- They showed that, in theory, the payoff on a continuously monitored variance swap was perfectly replicated by combining static positions in a continuum of options on price with a continuously rebalanced position in the underlying.
- Jiang and Tian (2005), and Carr and Wu (2009) show that, under very general settings, an approximation of the variance swap rate is

$$\begin{aligned} VS_{t,t+\tau} &= \frac{1}{\tau}\mathbb{E}_t^{\mathbb{Q}}\left[QV_{t,t+\tau}\right] \\ &= \underbrace{\frac{2e^{r\tau}}{\tau}\int_0^\infty \frac{O_t\left(K,T\right)}{K^2}dK}_{\text{VIX Valuation Formula}} + \epsilon \end{aligned}$$

  where $O_t\left(K,T\right)$ is out-of-the-money option price and ϵ is error term (positive, since returns are negatively skewed.
- Thus, the VIX approximates the variance swap rate on SPX.

VIX

- The concept behind the VIX index was developed by Whaley (1993).
- Although the VIX index was launched in 1993, the current form was introduced in 2003 for the following reasons.
  - Tradability: VIX^2 can be replicated by a static portfolio of S&P 500 options. It provides the critical link to develop tradable products to hedge and arbitrage OTC volatility derivatives.
  - Liquidity: use the options on S&P 500, the most liquid contracts.
  - Model free: unlike implied vol from Black-Scholes model, VIX is calculated from option prices without assuming a specific model

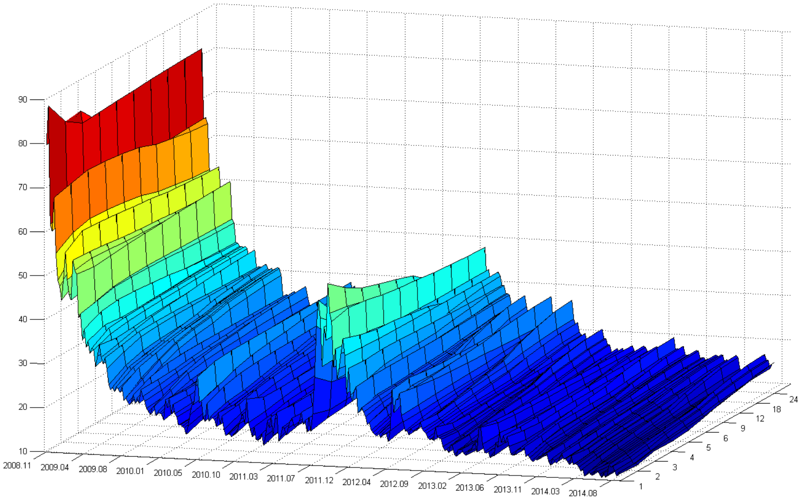
Volatility Indices

Index	Exchange	Underlying	
VIX	CBOE	S&P 500	Futures, Options Mini-futures,Variance Futures
DJIA Volatility Index	CBOE	DJIA	Futures delisted
NADSAQ-100 Volatility Index	CBOE	NASDAQ 100	Futures
Russell 2000 Volatility Index	CBOE	Russell 2000	Futures delisted
S&P 500 3-Month Volatility Index	CBOE	S&P 500	
VDAX-new	Deutsche Börse	DAX 30	Futures delisted
VSTOXX	Eurex	Dow Jones EURO STOXX 50	Mini-Futures, Options
VSMI	SWX Swiss Exchange	SMI	Futures delisted
Nikkei 225 Volatility Index	OSE	Nikkei 225	Futures
HSI Volatility Index	HKSE	Hang Seng	Futures
KOSPI200 Volatility Index	KRX	KOSPI200	

- There are volatility indices on alternative assets such as interest rates, gold, oil and ETFs, some of which have futures listed.

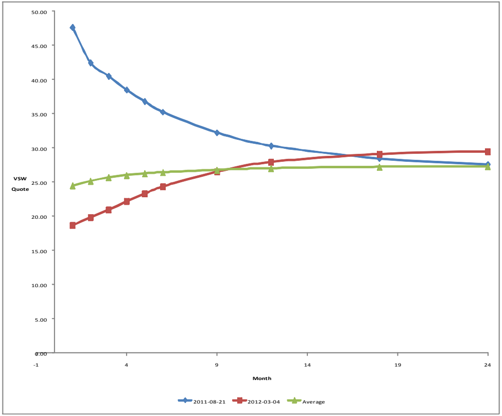
Variance Swap Rate on KOSPI 200 Index

- On average, variance swap curve is upward sloping. (Period: 2008.11 - 2014.9, Source: Bloomberg)



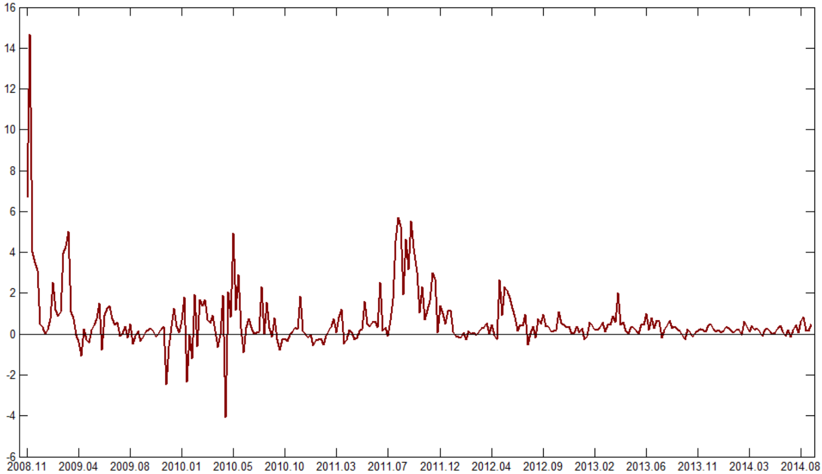
Variance Swap Term Structure

- According to Alexander and Korovilas (2012), the US VIX term structure is contango in 75% and backwardation in 25%.
- For variance term structure in Korea, 85% is in contango and 15% in backwardation.



1M Variance Swap & VKOSPI

- The difference is, on average, 0.6299% (t-value = 7.74).

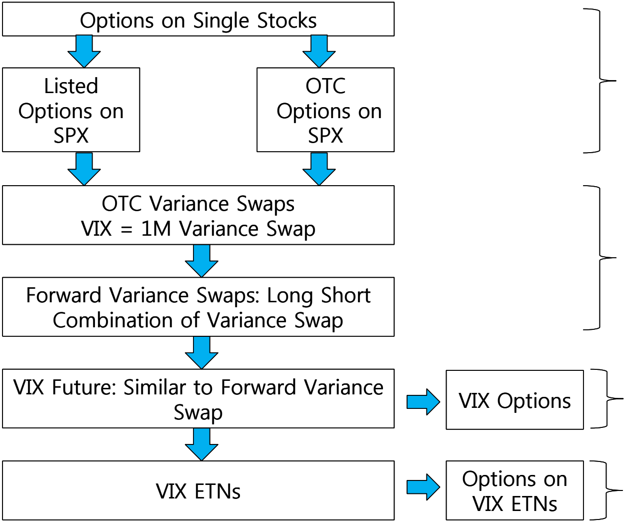


Current Section

Major Volatility Derivatives

- Volatility futures: Volatility futures are forward contracts that settle to implied volatility at expiration date.
- Volatility options : Volatility options also settle to implied volatility at expiration date.
- Variance swaps: Variance swaps are contracts that settle based on the difference between realized variance over the contract term and the agreed swap rate.
- Forward variance swaps: The payoff on forward variance swaps reflects the difference between an agreed swap rate at which a variance swap may be purchased in the future, and the actual swap rate at the time of expiry.
- There are other derivatives such as volatility swap, conditional variance corridor variance swap and options on realized volatility/variance at OTC market, and variance futures, weekly options, digital options and options on ETNs.

Developments in US



Options on stocks started trading in 1973, while index options started trading from 1983

OTC variance swaps started trading from early 90s.

CBOE introduced VIX in 1993, The current VIX was created in 2003.

VIX futures first listed in 2004. VIX options in 2006.

First VIX ETNs listed in 2009 and option on VIX ETNs in May 2010.

Volatility as an Asset Class?

- Do investors benefit from adding exposure to volatility (or variance) to their portfolio?
- BlackRock: Volatility is an Asset (July 6, 2013, Barron's)
  - *BlackRock, the world's largest asset manager, is advising clients to change old ideas about portfolio management by blending volatility strategies into investment portfolios. BlackRock likes selling equity volatility - not buying it - because of the persistent demand among many investors buying derivatives to hedge portfolio. BlackRock's embrace of volatility reinforces the view of major investment banks that volatility is an asset class. When a firm that manages about $4 trillion puts its money where its mouth is, people listen, and some will even follow.*
- The general character of long volatility exposure is that it generates negative returns on average, but hedges equity risk. The negative returns, variance risk premium, go hand-in-hand with the fact that volatility exposures has negative equity beta.

Volatility as an Asset Class

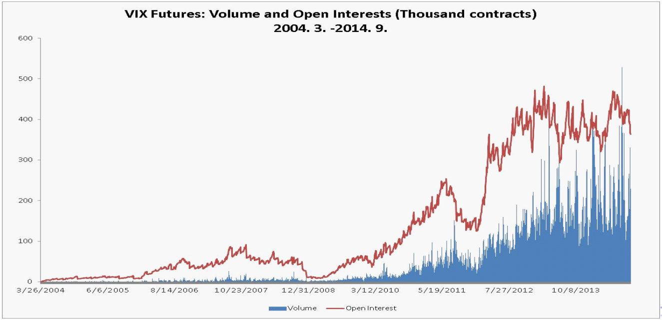
- VIX Index is not directly investable. The replicating portfolio of options will give the value of VIX^2, not the VIX.
- Compared to exchange-traded options and futures on volatility index, variance swaps are traded on the OTC market, therefore they offer flexible notional value and maturity. In addition, the choice of the underlying asset is not limited to a volatility index.
- The drawbacks of variance swaps are their lack of liquidity and absence of secondary market, and the counterparty risk. This also makes them quite expensive, as the market maker claims a premium for providing the liquidity.
- How can investors get exposure to the VIX Index? A variety of volatility instruments are available.
  - VIX Futures
  - VIX Options
  - VIX ETNs

VIX Futures

- Intuitively, VIX futures price reflects today's expectation on what the VIX will be worth on the futures maturity date, as VIX on maturity date will represent the 30-day implied volatility conveyed by Index option prices.
- Currently the most readily tradable volatility futures are available on the VIX, which measures the implied volatility for S&P 500 index options of 1-month to expiry. VIX futures have been traded on the Chicago Futures Exchange since March 2004.
- There are usually 8 listed maturities for VIX futures, one for each of the first eight following months.
- The future contract size is $1,000.
- VIX futures have become the most liquid volatility instruments for up to 3 month maturity. It is as large as the vega exposure traded on the SPX options listed market.

VIX Futures: Volume and Open Interests

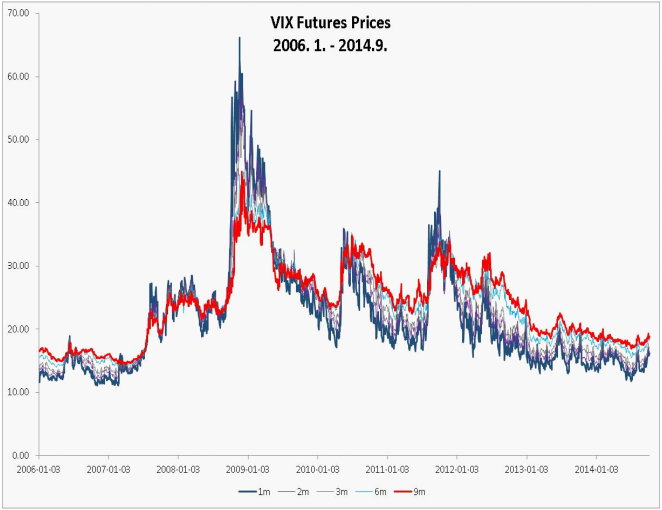
- From trading just a few hundred contracts a day shortly after its initial introduction in 2004, to trading four hundred thousand contracts a day today, volume and open interest in VIX futures has grown unabated.
- The history of the growth in volume for VIX futures has reflected the broadening of the trader base over time to include a diverse group including investors, traders, hedgers and market-makers.



Common Strategies using VIX Futures

- Long VIX future to hedge a long equity portfolio (beware of the roll cost)
- Short VIX future on volatility spikes.
- Long VIX future to protect a short realized volatility strategy.
- Term structure : Short front month future / Long long term future.
- Relative value arbitrage :
  - VIX futures vs OTC variance swaps
  - VIX futures vs listed options

VIX Futures Prices

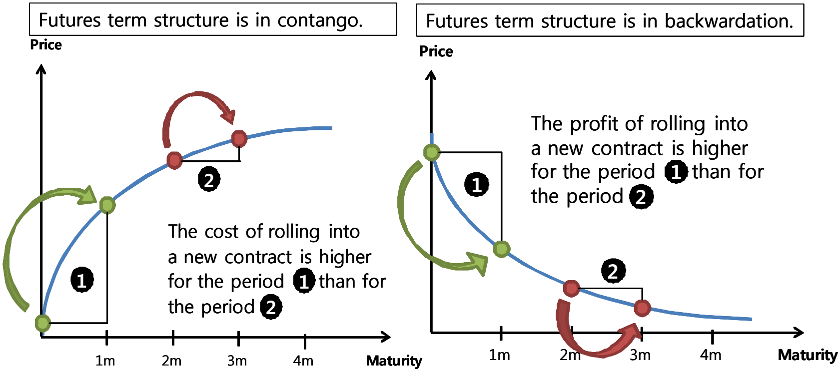


VIX Futures Prices

- Term structure of vol based on VIX futures prices is on average upward sloping (contango).
- As in term structure of interest rates, the term structure of VIX futures prices reflects the expectation of future vol and variance risk premium. Variance risk premium tends to be lower for longer-dated futures, whose payoffs tend to be a function of vol expectation.
- Term structure of vol of vol is on average downward sloping.

	1m	2m	3m	4m	5m	6m	7m	8m	9m
Average	21.62	22.28	22.68	22.96	23.22	23.44	23.62	23.74	23.84
Stdev	9.03	8.17	7.55	7.11	6.80	6.56	6.40	6.24	6.05

The Roll Cost



- Usually, this curve is in contango and is steep in the first few months, leveling off for the longer term maturities. This results in a significant decay ("roll cost") of short term VIX futures.
- During times of market uncertainty the term structure curve temporarily inverts into backwardation

VIX Options

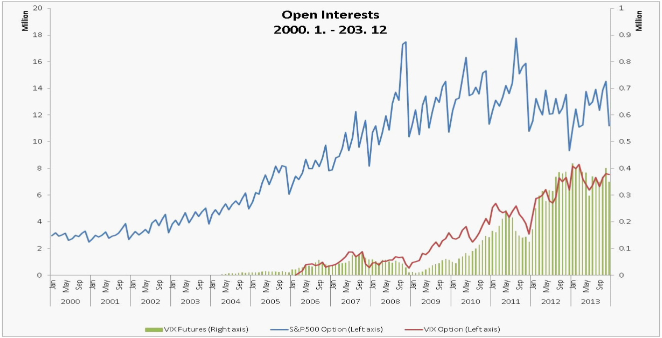
- Options over the VIX have been traded on the CBOE since February 2006. They are European options and cash-settled.
- There are usually 6 listed maturities for VIX options, one for each of the first six following months.
- VIX options expire on the same dates as VIX Futures and with the same settlement value.
- Contract size is $100.
- Delta hedging can be done through VIX futures.
- Good liquidity on the first 3 expiries.

Common Strategies using VIX Options

- Buying out of the money calls to protect a long equity portfolio against a market crash.
- Selling at the money calls to monetize high implied volatility levels
- Buying long dated out of the money puts to monetize a steep contango
- Selling at the money straddle to monetize the high level of vol of vol.

VIX Options: Trading Volume

- According to Rhoads (2011), VIX option trading accounts for about 25 percent of the daily index option at CBOE, while SPX option accounts for 50 percent of index trading volume.
- The high open interest and growth in volume for VIX options is a direct result of ability to use VIX options for hedging on the direction of the overall stock market.

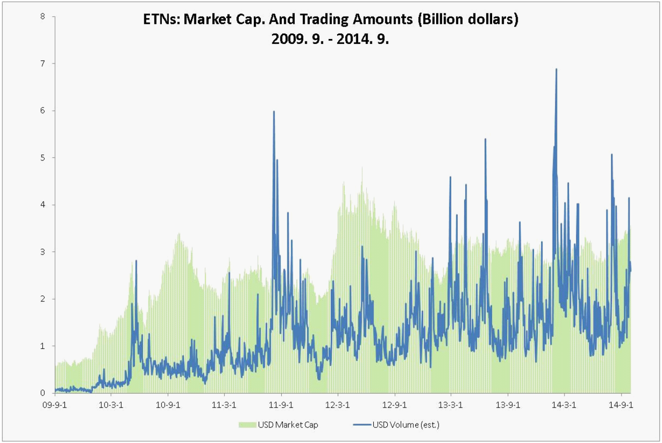


Volatility Related ETNs

- An exchange-traded note (ETN) is a debt security that is backed by the credit rating of the issuer. The goal of an ETN is to replicate an investment strategy or performance of another investment vehicle. ETNs trade in the same manner at exchange traded funds (ETF).
- The iShares division of Baclays Bank introduced VIX related equity-like ETNs, VXX and VXZ. The goal of the VXX ETN is to mirror an investment in the S&P constant-maturity VIX futures indices, S&P 500 Short-Term Futures Index. This index is a measure of returns from investing in long position consisting of the first and second month VIX futures contracts.
- At the end of May 2010 the CBOE introduced options on the VXX. VXX options quickly gained acceptance as a trading vehicle, becoming one of the more actively traded options series at CBOE.

ETN : Trading Volume & Market Cap

- An average daily volume surged to just over 40 million shares a day, when the market event termed the 'flash crash' occurred.



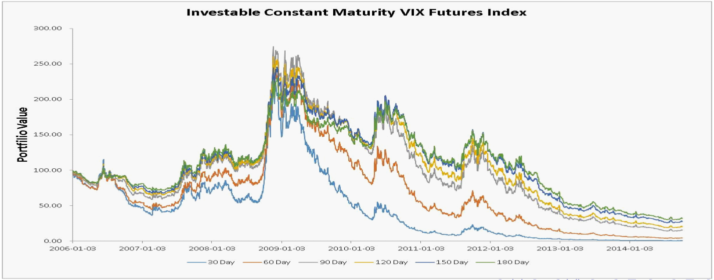
ETN Chronology

- Rising demand for protection against stock declines has made the VXX the fifth-most active U.S. exchange-traded security. (Bloomberg, Aug., 9, 2013)

Code	Full NAME	Inception Date	30D Avg Volume*	Dollar Volume*	Market Cap*	Leverage	Fees
VXX	IPATH S&P 500 VIX S/T FU ETN	2009-01-29	32,577,668	1,015,739,111	1,358,632,813	N	89bp
TVIX	VELOCITYSHARES 2X VIX SH-TRM	2010-11-29	15,481,421	50,314,618	263,241,608	Y	165bp
XIV	VELOCITYSHARES INV VIX SH-TM	2010-11-29	9,631,506	368,405,105	592,105,530	Y	135bp
UVXY	PROSHARES ULTRA VIX ST FUTUR	2011-10-04	9,256,516	274,270,569	282,080,536	Y	95bp
HVU	HORIZONS BETAPRO S&P 500 VIX	2010-12-16	1,795,003	26,123,104	28,701,253	Y	163bp
VIXY	PROSHARES VIX SHORT-TERM FUT	2011-01-04	1,142,668	23,744,631	137,663,605	N	85bp
1552	KOKUSAI S&P500 VIX SHORT ETF	2010-12-20	996,018	7,831,096	154,838,112	N	36bp
VXZ	IPATH S&P 500 VIX M/T FU ETN	2009-01-29	896,715	11,764,894	70,264,030	N	89bp
SVXY	PROSHARES SHORT VIX ST FUTUR	2011-10-04	873,708	65,475,668	277,281,006	Y	95bp
2030	IPATH S&P 500 VIX S/T FU JDR	2011-09-21	148,071	185,948	6,498,772	N	89bp
VIXS	SOURCE S&P 500 VIX FUTURES U	2010-06-24	121,350	931,966	12,938,972	N	60bp
ZIV	VELOCITYSHARES INV VIX MEDIU	2010-11-29	79,373	3,490,029	139,384,903	Y	135bp
VIXM	PROSHARES VIX MID-TERM FUT	2011-01-04	46,810	747,559	45,514,580	N	85bp
VIIX	VELOCITYSHARES VIX SHORT-TRM	2010-11-29	34,624	1,458,699	8,129,658	N	89bp
XVZ	IPATH S&P 500 DYN VIX ETN	2011-08-17	12,058	371,241	23,112,747	N	95bp
VXIS	IPATH S&P 500 VIX S/T FU ETN	2009-12-09	10,762	13,035	8,181,505	N	N/A
VXIS	IPATH S&P 500 VIX S/T FU ETN	2010-04-29	7,337	890,997	8,202,904	N	N/A
LVO	LYXOR S&P500 VIX ENH ROLL	2013-01-29	3,552	94,429	49,954,563	N	60bp
TVIZ	VELOCITYSHARES 2X VIX MED-TM	2010-11-29	2,948	67,511	973,067	Y	165bp
IVOP	IPATH INVERSE S&P 500 VIX II	2011-09-16	1,746	66,688	1,909,495	Y	89bp
XVIX	ETRACS DAILY LONG/SHORT VIX	2010-11-30	1,205	19,770	12,299,999	N	85bp
DLVO	LYXOR DYN LONG VIX FUTURES	2013-04-05	344	27,639	5,380,583	N	75bp
XXV	IPATH INVERSE S&P 500 VIX SH	2010-07-16	302	11,573	3,751,608	Y	89bp
VIIZ	VELOCITYSHARES VIX MED-TERM	2010-11-29	278	5,037	1,633,500	N	89bp
	(* based on average during September, 2014)						

ETN : Investable Constant Maturity Indices

- Simple buy-and-hold investments in VIX ETPs, especially for short-term VIX ETNs, are not suitable for long-term investors due to roll cost.
- Alexander and Korovilas (2012) show that certain portfolios of VIX futures, or their ETNs, which typically take a short position on short-term VIX futures and a long position on longer-term VIX futures, can offer attractive risk and return characteristics to long-term investors.



ETN : Dark side

- The terms and conditions for early redemption initiated by the investor present problems for both ETN issuers and regulators.
- ETN issuers can hedge their exposure to early redemption perfectly only when they trade VIX futures at the daily closing price. However, the large-scale front-running of issuers' hedging activity has made issuer's profits difficult to secure.
- In February 2012, Credit Suisse stopped the issue of TVIX (sending its traded price to a 90% premium over indicative value at one point, such is the speculative demand on this product) out of concerns that demand for the security would start to have an undue influence on the price of VIX futures, rather than tracking that market.
- Credit Suisse re-opened it only on the condition that the hedging risk be passed on to the market makers. Still, such large positions must be taken on VIX futures for hedging certain ETNs. The ETN market may leads the VIX futures that they are supposed to track.

Current Section

Variance Risk Premiums

- It is well-recognized that a long exposure to volatility has generated negative returns on average. The academic literature has come to talk of these negative returns as a volatility (or variance) risk premium.
- As noted by Carr and Wu (2009), the variance swap (VS) rate and the realized variance (RV) are related with a pricing kernel $M_{t,T}$:

$$VS_{t,T} = \mathbb{E}_t^{\mathbb{Q}}[RV_{t,T}] = \frac{\mathbb{E}_t^{\mathbb{P}}[M_{t,T} RV_{t,T}]}{\mathbb{E}_t^{\mathbb{P}}[M_{t,T}]} = \mathbb{E}_t^{\mathbb{P}}[m_{t,T} RV_{t,T}]$$

$$\text{where} \quad m_{t,T} = \frac{M_{t,T}}{\mathbb{E}_t^{\mathbb{P}}[M_{t,T}]}$$

which can be decomposed into the following two terms:

$$\begin{aligned} VS_{t,T} &= \mathbb{E}_t^{\mathbb{Q}}[RV_{t,T}] = \mathbb{E}_t^{\mathbb{P}}[m_{t,T} RV_{t,T}] \\ &= \mathbb{E}_t^{\mathbb{P}}[RV_{t,T}] + Cov\left(m_{t,T}\ ,\ RV_{t,T}\right) \end{aligned}$$

The covariance term captures the conditional covariance between the normalized pricing kernel and the realized variance.

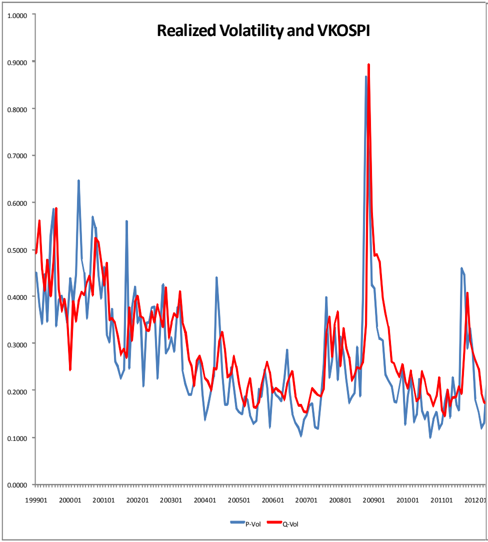
Variance Risk Premiums

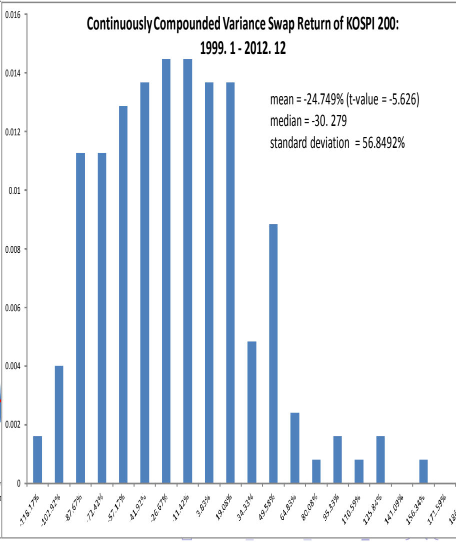
- The **negative of this covariance,** $-Cov\left(m_{t,T}\ ,\ RV_{t,T}\right)$, defines the **return variance risk premium**. Thus, a direct estimate of the average variance risk premium is the sample average of the difference between the variance swap rate and the realized variance,

$$\text{Variance Risk Premium}_{t,T} \equiv RV_{t,T} - VS_{t,T}$$

  This difference also measures the terminal profit and loss from a long variance swap contract and holding it to maturity.
- The negative sign on the variance risk premium indicates that **variance buyers are willing to accept a negative average excess return to hedge away upward movements in stock market volatility**.

Variance Risk Premium in Korea





VRP and Risk Factors

- Variance risk premium is only partly explained by known risk factors. For instance, excess returns are found to prevail after accounting for exposure to market beta (i.e. CAPM), the Fama-French three-factor model, its 4-factor extension including momentum, and after allowing for loading on the term structure and the default spread (see Bondarenko, 2007; Hafner and Wallmeier, 2007; Carr and Wu, 2009).
- Using the monthly data of Korean stock market for the period from 1999.1 to 2012. 9, we can find the similar results.

	Intercept	Market Excess Return	SMB	HML	Momentum
Coefficient	-21.63	-3.14	-1.44	-0.52	-0.78
t-value	-4.98	-5.63	-2.25	-0.57	-1.48

Is Volatility Priced Risk?

- Eom and Jang (2013) estimate the two factor variance process with jumps in asset return process, using variance swap data for KOSPI200.
- Under risk-neutral measure

$$\begin{aligned} d\ln S_t &= \left((r-\delta) - \frac{1}{2}v_t - g^{\mathbb{Q}}\lambda_t\right)dt + \sqrt{1-\rho^2}\sqrt{v_t}dW_{1t}^{\mathbb{Q}} + \rho\sqrt{v_t}d \\ &\quad + J_t^{\mathbb{Q}}dN_t^{\mathbb{Q}} \\ dv_t &= \kappa_v^{\mathbb{Q}}\left[m_t - v_t\right]dt + \sigma_v\sqrt{v_t}dW_{2t}^{\mathbb{Q}} \\ dm_t &= \kappa_m^{\mathbb{Q}}\left(\theta_m^{\mathbb{Q}} - m_t\right)dt + \sigma_m\sqrt{m_t}dW_{3t}^{\mathbb{Q}} \end{aligned}$$

where $dW_{it}^{\mathbb{Q}}$, $i = 1, 2, 3$, are uncorrelated. $J_t^{\mathbb{Q}} \sim N\left[\mu_j^{\mathbb{Q}}, \sigma_j^2\right]$, $N_t^{\mathbb{Q}}$ is the jump process with intensity λ_t, $g^{\mathbb{Q}} = \exp\left(\mu_j^{\mathbb{Q}} + \frac{1}{2}\sigma_j^2\right) - 1$.

- The jump intensity is assumed to be time-varying:

$$\lambda_t = \lambda_0 + \lambda_1 v_t, \quad \text{where} \quad \lambda_0, \ \lambda_1 > 0$$

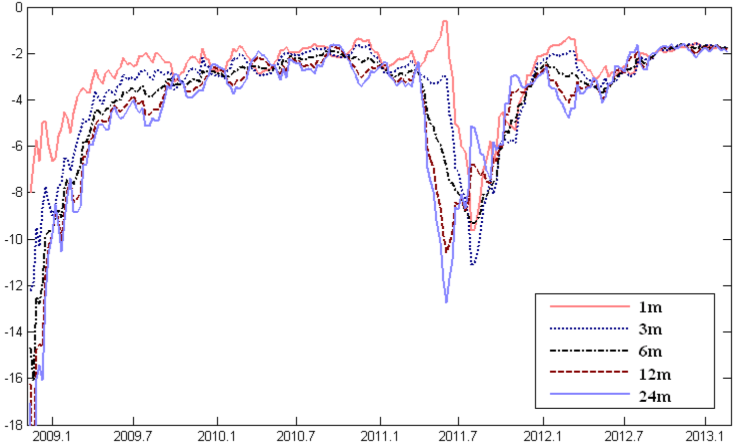
Parameter Estimates

	SV2Js	SV2Jc	SV2	SV1Js	SV1Jc	SV1
κ_v^Q	5.6827	6.1125	8.1309	0.1873	0.1888	0.2182
	(0.0036)	(0.0012)	(0.0005)	(0.0000)	(0.0545)	(0.0865)
θ_v^Q				0.0280	0.0989	0.1350
				(0.0000)	(0.0188)	(0.0199)
σ_v	1.4703	1.3172	1.4398	0.4840	0.9188	0.8035
	(0.0025)	(0.0036)	(0.0007)	(0.0003)	(0.0806)	(0.0429)
γ_v	-3.4522	-3.0191	-5.8910	-1.7825	-1.3392	-1.6838
	(0.0033)	(0.001)	(0.0006)	(0.0039)	(0.065)	(0.0378)
κ_m^Q	0.0068	0.0296	0.0132			
	(0.0025)	(0.0008)	(0.0007)			
θ_m^Q	0.0943	0.0960	0.1407			
	(0.0023)	(0.0011)	(0.002)			
σ_m	1.3446	1.1478	0.7758			
	(0.0011)	(0.0016)	(0.0025)			
γ_m	0.0045	0.0240	0.0157			
	(0.0017)	(0.001)	(0.0003)			
ζ^Q		0.0379			0.0354	
		(0.0006)			(0.0013)	
ζ^P		0.0162			0.0160	
		(0.0002)			(0.0008)	
$E^Q[J^2]$	0.6421			1.2331		
	(0.0013)			(0.0000)		
$E^P[J^2]$	0.3086			0.5929		
	(0.0034)			(0.0000)		
λ_0	0.0566			0.0284		
	(0.0006)			(0.0000)		
λ_1	0.2593			1.9506		
	(0.0008)			(0.0000)		
Log Likelihood	7585.551	7467.32	7400.35	6481.23	6443.63	6281.35

Term Structure of Variance Risk Premium

- Integrated variance risk premium (IVRP)

$$IVRP_{t,t+\tau} = \mathbb{E}_t^{\mathbb{P}}\left[QV_{t,t+\tau}\right] - \mathbb{E}_t^{\mathbb{Q}}\left[QV_{t,t+\tau}\right]$$



Current Section

Pricing Volatility Futures: Model free approach

- Carr and Wu (2006) derive valuation bounds on VIX futures. They hey show that

$$\mathbb{E}_t^{\mathbb{Q}} \sqrt{RV_{T,T+\tau_0}} \leq F_t\left(T\right) \leq \sqrt{\mathbb{E}_t^{\mathbb{Q}} RV_{T,T+\tau_0}}$$

  where $\tau_0 = 30/365$. The lower bound is the forward volatility swap rate. The upper bound is the forward-starting variance swap rate quoted in volatility percentage points.
- Duprie (2006) states that forward variance subtracting the concavity adjustment can derive the fair value of VIX futures.

$$F_t\left(T\right) = \mathbb{E}_t^{\mathbb{Q}}\left[VIX_T\right] = \sqrt{\mathbb{E}_t^{\mathbb{Q}}\left(VIX_T^2\right) - Var_t^{\mathbb{Q}}\left(VIX_T\right)}$$

- In US, VVIX, which is a proxy for the standard deviation of the VIX at expiration date and is obtained from VIX options, can be used for an estimate of the risk-neutral variance of the VIX future

$$Var_t^{\mathbb{Q}}\left(VIX_T\right) \approx F_t\left(T\right)^2 \frac{T}{365}\left(\frac{VVIX_T}{100}\right)^2$$

Pricing Volatility Futures: Two approaches

- Concerning the modeling perspective of the underlying volatility process, there are at least two approaches.
- Stand-alone approach models the dynamics of the volatility index, effectively decoupling it from the underlying return process. The main advantage of this approach is tractability, since closed-form solutions are often available. In this approach, volatility risk is modelled through observable volatility indicators, such as VIX and/or forward variance swap rates
- However, the return dynamics and the VIX are related. While SPX options are written on the S&P 500 index, a portfolio of SPX options resembles the VIX. Directly modeling the dynamics of the VIX does not guarantee that the portfolio of SPX options replicates the volatility index.

Pricing Volatility Futures: Two approaches

- A consistent modeling of VIX related derivatives with the underlying SPX option prices introduces an additional layer of complexity. The volatility index needs to be derived from the assumed return dynamics. Afterwards, this quantity can be used as underlying for derivatives.
- From a theoretical perspective, this modeling approach is superior as inconsistencies in the pricing of related financial instruments are avoided.
- In practice, however, modelling joint dynamics can be highly intractable.

Pricing Volatility Futures: Consistent modelling approach

- Suppose the asset price, denoted as S_t, follows diffusion process with stochastic instantaneous variance, V_t (Heston, 1993),

$$dS_t = \mu S_t dt + \sqrt{V_t} S_t dB_t^{\mathbb{P}}$$

$$dV_t = \kappa \left(\theta - V_t\right) dt + \sigma_V \sqrt{V_t} dZ_t^{\mathbb{P}}$$

where $dB_t^{\mathbb{P}} dZ_t^{\mathbb{P}} = \rho dt$.

- With the change of probability measure, we are able to describe the SPX in a risk-neutral measure by following

$$dS_t = rS_t dt + \sqrt{V_t} S_t dB_t^{\mathbb{Q}}$$

$$dV_t = \kappa^{\mathbb{Q}} \left(\theta^{\mathbb{Q}} - V_t\right) dt + \sigma_V \sqrt{V_t} dZ_t^{\mathbb{Q}}$$

where $\kappa^{\mathbb{Q}} = (\kappa + \lambda)$, and $\theta^{\mathbb{Q}} = \kappa\theta / (\kappa + \lambda)$.

Pricing Volatility Futures: Consistent modelling approach

- Under these assumptions, the VIX squared is the conditional expectation in the risk-neutral measure

$$VIX_t^2 = E_t^{\mathbb{O}}\left[\frac{1}{\tau_0}\int_t^{t+\tau_0} V_s ds\right] = A + BV_t$$

where $A = \theta^{\mathbb{O}}\left[1 - \frac{1-e^{-\kappa^{\mathbb{O}}\tau_0}}{\kappa^{\mathbb{O}}\tau_0}\right]$, $B = \frac{1-e^{-\kappa^{\mathbb{O}}\tau_0}}{\kappa^{\mathbb{O}}\tau_0}$, $\tau_0 = 30/365$.

- Zhang and Zhu (2007) show that the price of VIX futures with maturity T is given by following formula

$$F_t^T = E_t^{\mathbb{O}}[VIX_T] = \int_0^\infty \sqrt{A + BV_T} f^{\mathbb{O}}(V_T | V_t)\, dV_T$$

where $f^{\mathbb{O}}(V_T|V_t)$ is the noncentral chi-square, $\chi^2(2v; 2q+2, 2u)$, with $2q+2$ degrees of freedom and parameter of noncentrality $2u$ proportional to the current variance, V_t, and $u = cV_t e^{-(\kappa+\lambda)(T-t)}$, $v = cV_T$, $q = \frac{2\kappa\theta}{\sigma_V^2} - 1$, $c = \frac{2(\kappa+\lambda)}{\sigma_V^2\left(1-e^{-(\kappa+\lambda)(T-t)}\right)}$.

Pricing Volatility Futures: Consistent modelling approach

- Or, from the approximation of Lin (2007), who use the second-order Taylor expansion, the current VIX futures is approximately

$$
\begin{aligned}
F_t^{VIX}(T) &= E_t^{\mathbb{Q}}(VIX_T) \\
&\approx \sqrt{E_t^{\mathbb{Q}}(VIX_T^2)} - \frac{Var_t^{\mathbb{Q}}(VIX_T^2)}{8\left[E_t^{\mathbb{Q}}(VIX_T^2)\right]^{3/2}}
\end{aligned}
$$

Thus to calculate the VIX futures one needs both $E_t^{\mathbb{Q}}(VIX_T^2)$ and $Var_t^{\mathbb{Q}}(VIX_T^2)$.

$$
E_t^{\mathbb{Q}}\left[VIX_T^2\right] = B\exp\left(-\kappa^{\mathbb{Q}}(T-t)\right)V_t + B\theta^{\mathbb{Q}}\left[1-\exp\left(-\kappa^{\mathbb{Q}}(T-t)\right)\right] + A
$$

$$
Var_t^{\mathbb{Q}}(VIX_T^2) = B^2\left[\begin{array}{c} \frac{\sigma_V^2}{\kappa^{\mathbb{Q}}}\left[e^{-\kappa^{\mathbb{Q}}(T-t)} - e^{-2\kappa^{\mathbb{Q}}(T-t)}\right]V_t \\ +\frac{\sigma_V^2}{2\kappa^{\mathbb{Q}}}\theta^{\mathbb{Q}}\left[1-e^{-\kappa^{\mathbb{Q}}(T-t)}\right]^2 \end{array}\right]
$$

Pricing Volatility Futures: Stand-alone approach

- Denote F_t^T the price of a futures contract at time t with maturity T. The the pice of VIX futures contract at time t with maturity T is given by
$$F_t^T = E_t^{\mathbb{Q}}[VIX_T]$$
where $\mathbb{Q}$ is the risk neutral probability measure and VIX_T is the forward VIX. Since VIX is not a tradable asset, the fair value of VIX futures cannot be derived by the cost-of-carry relationship.
- Instead of modelling instantaneous variance process, stand-alone approach model the VIX process itself to develop a pricing model for volatility futures and volatility options .
- For instance, a mean-reverting squared-root volatility process for VIX is considered in Grűnbichler and Longstaff (1996). Detemple and Osakwe (2000) adopt the log-normal OS process. Mancia and Sentana (2013) suggest that a process for the log of the VIX combining central tendency and stochastic volatility reliably prices VIX derivatives.

Stand-alone approach: VKOSPI process

- We consider the following non-linear drift CEV process

$$dx_t = \left(c_1 + \frac{c_2}{x_t} + c_3 x_t \ln x_t + c_4 x_t + c_5 {x_t}^2 \right) dt + k {x_t}^{\gamma} dZ$$

where $x_t = VKOSPI/100$.

- We estimate the parameters using the corresponding discrete-time econometric specification:

$$x_{t+\Delta t} - x_t = \left(c_1 + \frac{c_2}{x_t} + c_3 x_t \ln x_t + c_4 x_t + c_5 x_t^2 \right) \Delta t + \varepsilon_{t+1}$$

$$E\left[\varepsilon_{t+1}\right] = 0$$

$$E\left[\varepsilon_{t+1}^2\right] = k^2 x_t^{2\gamma} \Delta t$$

Stand-alone approach: VKOSPI process

- We let θ be the parameter vector with elements $c_1, c_2, c_3, c_4, c_5, k$ and γ define the vector

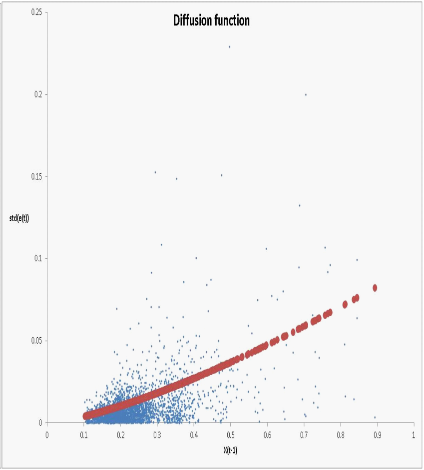
$$f_t(\theta) = \begin{bmatrix} \varepsilon_{t+1} \otimes \left(1, x_t, x_t^{-1}, x_t \ln x_t, x_t^2\right) \\ \left(\varepsilon_{t+1}^2 - k^2 x_t^{2\gamma} \Delta t\right) \otimes (1, x_t) \end{bmatrix}$$

  The GMM technique replaces $E(f_t(\theta))$ with its sample counterpart $m(\theta) = \frac{1}{T}\sum_{t=1}^{T} f_t(\theta)$ using T observations where $m(\theta) = \frac{1}{T}\sum_{t=1}^{T} f_t(\theta)$ and then estimates the parameters in the vector θ which minimize the quadratic form $q(\theta) = m(\theta)^T Wm(\theta)$.
- We estimate the model using the daily observations of VKOSPI for the period from 2003. 1. to 2014. 9.

VKOSPI process: estimation results

	Full	MRP	SQR	MRLP	3/2 with Quad Drift	3/2 with Linear Drift	MRGP	GBMP
const	65.60 **	0.42 ***	0.55 ***	0.78 **		0.42 ***	0.57 ***	
	(32.35)	(0.1398)	(0.13)	(0.37)		(0.14)	(0.14)	
1/V	-2.61 **							
	(1.33)							
Vln(V)	244.71 **			2.23 *				
	(115.28)			(1.27)				
V	186.10 **	-1.75 **	-2.52 ***		2.52 ***	-1.76 **	-2.66 ***	0.49 **
	(80.97)	(0.78)	(0.76)		(0.59)	(0.78)	(0.76)	(0.21)
V^2	-273.59 **				(9.86) ***			
	(116.3171)				(2.72)			
k	1.56 ***	1.78 ***	0.39 ***	0.88 ***	1.71 ***	1.73 ***	0.17 ***	0.89 ***
	(0.24)	(0.21)	(0.02)	(0.04)	(0.08)	(0.08)	(0.01)	(0.04)
gamma	1.38 ***	1.52 ***	0.5	1	1.5	1.5	0	1
	(0.11)	(0.09)						
J-test	NA	4.79	20.20	15.62	3.29	4.79	23.46	25.60
(P-value)	NA	0.19	0.00	0.00	0.51	0.31	0.00	0.00

VKOSPI process: estimation results



Stand-alone approach: MRLP process

- Suppose that the VKOSPI process under the risk neutral probability measure $\mathbb{O}$ is given by:

$$d\left(\ln VIX_t\right) = \kappa^{\mathbb{O}}\left(\theta^{\mathbb{O}} - \ln VIX_t\right)dt + \sigma dZ_t^{\mathbb{O}} + ydq_t$$

  where the volatility risk premium is proportional to the logarithm of the current volatility level, i.e., $\xi_t = \xi \ln VIX_t$, $\kappa^{\mathbb{O}} = (\kappa + \xi)$, $\theta^{\mathbb{O}} = \kappa\theta / \left(\kappa + \xi\right)$.
- Suppose that dq_t are compound Poisson processes with a constant arrival rate λ, and y is the jump size. The jump size is drawn from exponential distribution with density:

$$f\left(y\right) = \eta e^{-\eta y} 1_{\{y \geq 0\}}$$

  where $1/\eta$ is the mean of the upward jump.

MRLP process: Calibration

- Then, the futures price is

$$
\begin{aligned}
& E_t^{\mathbb{Q}}\left[VIX_T\right] \\
= \; & \left(VIX_t\right)^{-\kappa^{\mathbb{Q}}(T-t)} e^{\theta^{\mathbb{Q}}\left(1-e^{-\kappa^{\mathbb{Q}}(T-t)}\right)+\frac{\left(1-e^{-2\kappa^{\mathbb{Q}}(T-t)}\right)}{4\kappa^{\mathbb{Q}}}\sigma^2+\frac{\lambda}{\kappa^{\mathbb{Q}}}\ln\left(\frac{\eta-e^{-\kappa^{\mathbb{Q}}(T-t)}}{\eta-1}\right)}
\end{aligned}
$$

- The 30-day forward variance swap, $FVS(t, T, T+\tau_0) = E_t^{\mathbb{Q}}\left[RV_T^{T+\tau_0}\right] = E_t^{\mathbb{Q}}\left[VIX_T^2\right] \; (\tau_0 = 30/365)$ can be explicitly solved as

$$
\begin{aligned}
& E_t^{\mathbb{Q}}\left[VIX_T^2\right] \\
= \; & \left(VIX_t^2\right)^{e^{-\kappa^{\mathbb{Q}}(T-t)}} e^{2\theta^{\mathbb{Q}}\left(1-e^{-\kappa^{\mathbb{Q}}(T-t)}\right)+\frac{\left(1-e^{-2\kappa^{\mathbb{Q}}(T-t)}\right)}{\kappa^{\mathbb{Q}}}\sigma^2+\frac{\lambda}{\kappa^{\mathbb{Q}}}\ln\left(\frac{\eta-2e^{-\kappa^{\mathbb{Q}}(T-t)}}{\eta-2}\right.}
\end{aligned}
$$

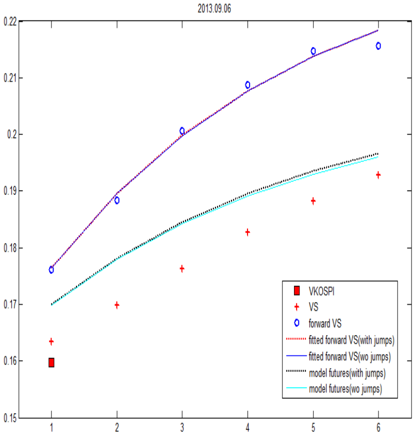
MRLP process: Calibration

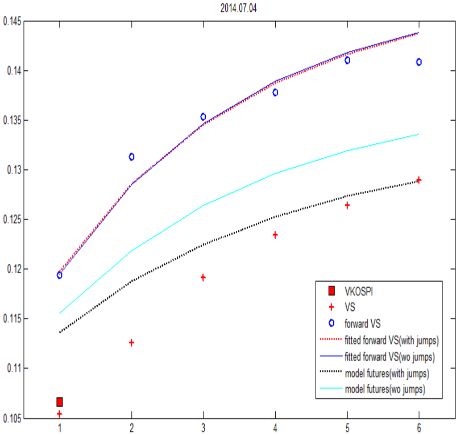
- Forward variance swap rate at date t for the period $[T_i, T_j]$ is given by

$$FVS(t, T_i, T_j) = \frac{(T_j - t)\, VS(t, T_j) - (T_i - t)\, VS(t, T_i)}{T_j - T_i}$$

- Thus, we can use the forward variance swap rates implied in variance swap rates to infer the parameters of the model.
- 2013-09-16:$\kappa^{\mathbb{O}} = 2.2375$, $\theta^{\mathbb{O}} = -2.2343$, $\sigma = 0.8625$, $\lambda = 12.7623$, $\eta = 10.1959$
- 2013-09-16:$\kappa^{\mathbb{O}} = 2.2067$, $\theta^{\mathbb{O}} = -1.6896$, $\sigma = 1.0360$
- 2014-07-04:$\kappa^{\mathbb{O}} = 2.9029$, $\theta^{\mathbb{O}} = -2.6056$, $\sigma = 0.2892$, $\lambda = 6.4930$, $\eta = 4.3227$
- 2014-07-04:$\kappa^{\mathbb{O}} = 3.5294$, $\theta^{\mathbb{O}} = -2.0560$, $\sigma = 1.0382$

Calibration Results





Current Section

Trading Strategies

- Exploiting a volatility view: VKOSPI futures can be used for taking a direct view on the volatility of KOSPI 200 without positions in KOSPI 200 options.
- Rolling short VKOSPI futures: Shorting VKOSPI futures can be used to capture the variance (volatility) risk premium.
- Trading the volatility term structure: VKOSPI futures can be used to trade the shape of the volatility term structure, analogous to the used of spreads and butterflies in fixed income market to trade the shape of the yield curve.
- Relative value trading: VKOSPI futures can be used to trade other variance related products such as variance swap, listed options and other volatility indices.
- Correlation trading: Trading VKOSPI futures against individual options provides exposure to equity correlation.

Volatility and Stock Returns

- Volatility is negatively correlated with market returns.
- Further, payoffs to volatility exposure are non-linear (convex). Gains from a long volatility exposure tend to be greater when the equity market is weaker, while losses are relatively modest when the equity market is rising.
- Thus, volatility exposure seems to act like a put option on equity markets or an insurance contract over equity market declines.
- A long volatility exposure has provided an equity market hedge. Volatility exposure has tended to deliver large positive returns when the market declines and modest negative returns in up-markets.

Mean-variance Asset Allocation

- The search for effective diversifiers has intensified since the banking crisis. International equities have become more highly correlated, as have international bonds, and the equity–bond correlation has also risen. Moreover, even commodities now offer little or no diversification gains.
- In this environment volatility products arises as a natural diversification choice because its negative correlation with equity increases exactly when diversification is needed most
- Several studies show that VIX futures could enhance the risk-adjusted performance of the portfolio in a mean-variance framework.

Tail Risk Hedging

- Volatility jumps when market crashes. A long position in variance swaps provides hedging instrument against market drops
- Lee (2012) shows that VIX volatility futures seems to be a more effective extreme downside hedge than traditional option rolling strategies with 5% and 10% out- of-the-money put options on the S&P 500 index. In particular, using 1-month rolling VIX futures and a reasonable hedging model presents a cost-effective choice as a hedging instrument for extreme downside risk protection as well as for upside preservation

Optimal Portfolio Choice under Expected Utility Maximization

- Given that initial wealth is W_0, and R_i is the return on asset $i \in \{E, 1, 5\}$ (where R_f is the riskless interest rate),

$$W_T = (1 + R_f + w_E (R_E - R_f) + w_1 (R_1 - R_f) + w_5 (R_5 - R_f)) W_0$$

  where w_E is the fraction of wealth invested in equity, w_1 is the fraction of wealth invested in 1 month VIX futures and w_5 is the fraction of wealth invested in 5 month VIX futures.
- Consider an investor who maximizes his/her expected utility

$$\max_{w_E, w_1, w_5} E\left[U(W_T)\right]$$

- We assume an investor with CRRA utility

$$U(W_T) = \frac{W_T^{1-\gamma}}{1-\gamma}, \ \gamma > 1$$

  where γ measures relative risk aversion.

Optimal Portfolio Choice: Estimation Method

- The first-order conditions for asset i are

$$E\left[U'\left(W_T\right)\left(R_i - R_f\right)W_0\right] = 0$$

  The optimal asset allocation weights can be estimated by utilizing GMM based on the sample moments of FOC's without imposing any parametric structure on the return dynamics and risk premia as in Brandt (1999) and Ait-Sahalia and Brandt (2001).

- We estimate the optimal portfolio weights for $\gamma = 2, 3, 5, 10$, and investment horizons $T = 1, 3, 6$ month.

Optimal Portfolio Choice: Data

- We use the investable constant-maturity indices constructed from VIX futures prices for the period from 2006. 1. to 2014. 9.
- Since return distributions are highly non-normal, the performance measures such as Sharpe ratio may be misleading

	30 Day	60 Day	90 Day	120 Day	150 Day	180 Day
Annualized Mean	-36.6%	-24.6%	-13.7%	-12.2%	-9.7%	-8.6%
Volatility	62.6%	46.8%	39.0%	34.3%	31.2%	29.4%
Sharpe Ratio	-0.585	-0.525	-0.352	-0.356	-0.310	-0.292
Skewness	0.869	0.656	0.607	0.615	0.595	0.623
Kurtosis	3.842	3.203	3.163	3.469	3.705	3.911
Total Return	-99.3%	-95.5%	-84.5%	-79.4%	-72.0%	-67.7%

Optimal Portfolio Choice: Estimation Results

- Investment Horizon: one-month (w_E: weight for equity, w_E: weight for 1 month futures, w_5: weight for 5 month futures)

	γ=2			γ=3			γ=5			γ=10		
	Estimate		t-value	Estimate		t-value	Estimate		t-value	Estimate		t-value
Stock + VIX Futures (1m)												
w_E	0.1143		0.2169	0.1241	**	2.4493	0.3308	***	14.2112	0.3300	***	27.8582
w_1	-0.3768		-0.1345	-0.3721	***	-34.6412	-0.3139	***	-51.3505	-0.3150	***	-98.9034
Stock + VIX Futures (5m)												
w_E	-0.4966		-0.1077	0.4349	***	10.0595	0.4628	***	17.9546	0.4601	***	4.2724
w_5	-0.5984		-0.0256	-0.2390	***	-19.1537	-0.2332	***	-30.2891	-0.2319	***	-9.6537
Stock + VIX Futures (1m, 5m)												
w_E	0.2561		0.3400	0.1547		1.4868	0.2747	**	2.4904	0.2700	***	4.5284
w_1	-0.4017		-0.1147	-0.3773	***	-13.7161	-0.2808	***	-10.2226	-0.2800	***	-19.0399
w_5	0.4894		0.4379	0.3874	***	6.6317	0.2259	***	7.8043	0.2200	***	14.9019

Optimal Portfolio Choice: Estimation Results

- Investment Horizon: three-month (w_E: weight for equity, w_E: weight for 1 month futures, w_5: weight for 5 month futures)

	γ=2			γ=3			γ=5			γ=10		
	Estimate		t-value	Estimate		t-value	Estimate		t-value	Estimate		t-value
Stock + VIX Futures (1m)												
w_E	0.2391		0.4924	0.2175		0.6540	0.2850		1.6170	0.2850		1.1388
w_1	-0.1763		-3.1632	-0.1800	***	-4.1234	-0.2100	***	-9.4522	-0.2100	***	-17.1746
Stock + VIX Futures (5m)												
w_E	0.8628		0.0794	0.6277	***	3.0599	0.3923	**	2.5286	0.4125	***	4.9103
w_5	-0.2701		-0.0111	-0.3153	***	-3.8427	-0.3785	***	-7.8366	-0.3900	***	-16.4381
Stock + VIX Futures (1m, 5m)												
w_E	1.8349		0.0091	0.6970	**	2.1657	0.8117	***	3.4469	0.4420	**	2.3423
w_1	-0.5261		-0.0036	-0.2407	***	-3.1256	-0.2182	***	-3.9627	-0.0842	***	-3.7087
w_5	1.4062		0.0085	0.2956		1.3317	0.2893		1.5651	-0.2029	***	-4.6873

Optimal Portfolio Choice: Estimation Results

- Investment Horizon: six-month (w_E: weight for equity, w_E: weight for 1 month futures, w_5: weight for 5 month futures)

	γ=2			γ=3			γ=5			γ=10		
	Estimate		t-value	Estimate		t-value	Estimate		t-value	Estimate		t-value
Stock + VIX Futures (1m)												
w_E	2.3563		0.0186	1.2698	**	1.9969	0.4113		0.6488	0.4275		0.9116
w_1	-0.2134		-0.0049	-0.3766	**	-2.5350	-0.4324	***	-3.2644	-0.4200	***	-4.8784
Stock + VIX Futures (5m)												
w_E	0.6769		0.6872	0.4645		0.7051	0.0188		0.0496	-0.0900		-0.4695
w_5	0.1434		0.3574	0.0954		0.3550	-0.0459		-0.2967	0.0075		0.0908
Stock + VIX Futures (1m, 5m)												
w_E	0.1510		0.1177	0.1250		0.1476	0.1200		0.2386	0.1800		0.6552
w_1	-0.1821		-0.4016	-0.1700		-0.5561	-0.1500		-0.7865	-0.1800	*	-1.7149
w_5	0.2837		0.3818	0.2925		0.5853	0.2900		0.9189	0.2300		1.3335

Optimal Portfolio Choice: Estimation Results

- If the fraction of equity (w_E) is fixed at 0.7,

		γ=2	γ=3	γ=5	γ=10
		Stock + VIX Futures (1m)			
1 Month	w_1	-0.8383	-0.8217	-0.8079	-0.7970
3 Month	w_1	-2.0617	-0.9959	-0.9823	-0.9718
6 Month	w_1	-0.5121	-0.4555	-0.4099	-0.3757
		Stock + VIX Futures (5m)			
1 Month	w_5	-0.7139	-0.7190	-0.7238	-1.5509
3 Month	w_5	0.3127	0.2874	0.2639	0.2708
6 Month	w_5	0.3524	-0.7133	-0.6821	-0.6568
		Stock + VIX Futures (1m, 5m)			
1 Month	w_1	-0.3278	-0.3361	-0.3382	-0.3338
	w_5	0.3172	0.3497	0.3537	0.3350
3 Month	w_1	-0.2125	-0.2521	-0.2120	-0.2091
	w_5	0.1890	0.3288	0.2394	0.2328
6 Month	w_1	-1.1785	-3.3090	-0.5516	-0.5250
	w_5	1.6323	4.1141	0.3186	0.2788

Optimal Portfolio for Investor Types

- Optimal portfolio choices vary with risk aversion and investment horizon.
- Investors who are best shorting (short-term) futures to capture the volatility risk premium are those
  - with lower risk aversion,
  - with longer investment horizons,
  - with a secure funding base
- Investors who might consider long (longer-dated) futures for hedging purposes are those
  - with higher risk aversion
  - with a focus on the total risk of equity-dominated portfolios
- Spread strategy with long position in longer-dated futures and short position in short-term futures seems to be a dominating strategy regardless of risk-aversion level.

Option Vega Hedging

- The explosive growth of the trading volume and open interests of futures and options on VIX in recent years clearly reflects a demand for a tradable vehicle which can be used to hedge or to implement a view on volatility.
- Traditionally, volatility could be traded via at-the-money straddles. But straddles have the disadvantage of creating both market and volatility exposure.
- In contrast, volatility derivatives allow pure volatility exposure by design. Whaley (2003) argues that using volatility derivatives to manage market volatility risk offers at least two advantages over index option contracts. First, the hedge is simpler to implement. Second, volatility derivatives are less expensive. Using volatility futures, the hedge costs nothing. Using volatility options, the hedge costs only a small fraction of a similar hedge created using index options

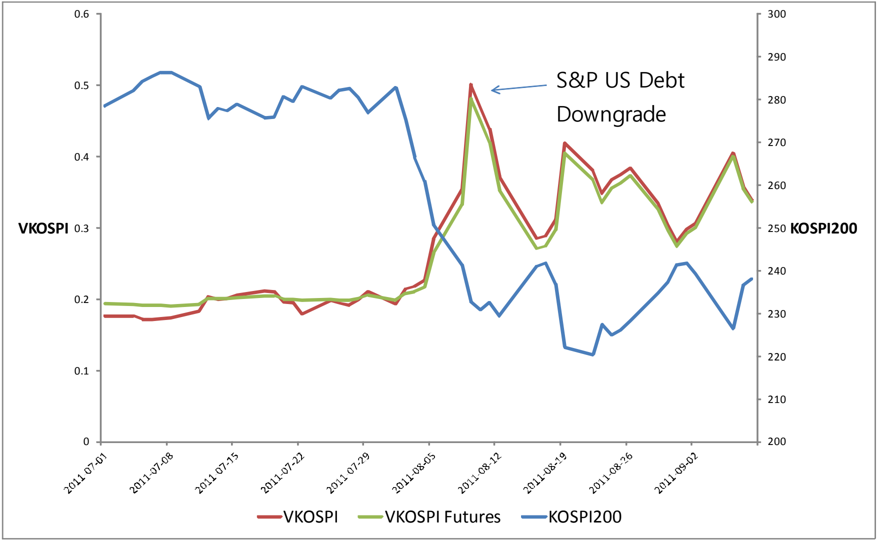
Option Hedging Performance

- Jiang and Oomen (2001) examine the hedging effectiveness of volatility futures versus plain-vanilla options; they found that the latter perform better than the former. Psychoyios and Skiadopoulos (2006) find that volatility options are not superior hedging vehicles to standard European options.
- However, An, Assaf & Yang (2007) find that volatility options are especially useful for hedging options with a exotic feature and there is no significant difference between the performances of volatility index and straddle options.
- Furthermore, Lin & Lin (2011) examines alternate models within a delta-vega neutral strategy. They find that VIX futures are found to outperform vanilla options in hedging a short position on S&P 500 futures call options.

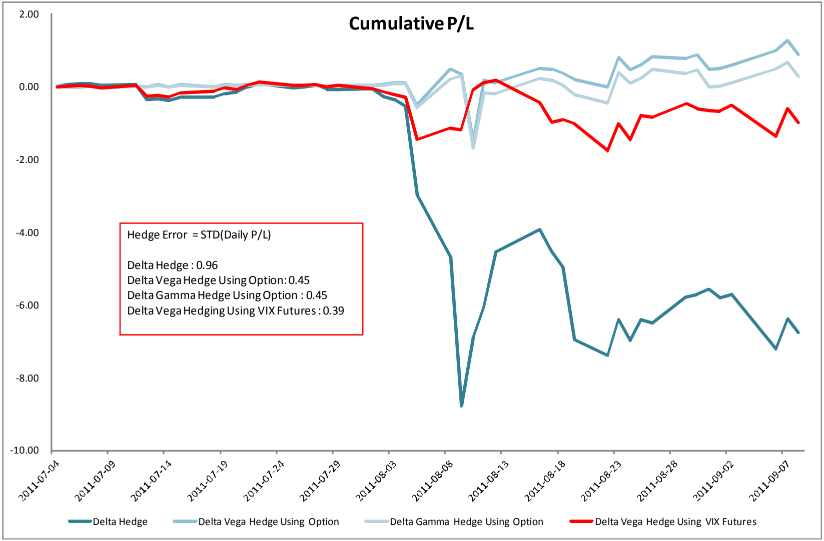
VKOSPI Futures Hedging Performance

- Volatility derivatives may be extremely useful when hedging instruments are not available or illiquid.
- Example) Target Option: KOSPI 200 Put with strike 250 maturing 2011. 9. 8
- Hedging Period: 2011. 7. 1 – 2011. 9. 8
- Hedging Instruments
  - Option: KOSPI 200 Put with strike 255 maturing 2011. 9. 8
  - VKOSPI Futures maturing 2011. 9. 8
- Futures prices are theoretical prices based on SVJ model. Model parameters are estimated, using variance swap quotes.

VKOSPI Futures Prices



Hedging Results



Current Section

Final Remarks

- The variance risk premiums are strongly negative in Korea. The volatility risk is generated by an independent risk factor that the market prices.
- Empirical results indicate that volatility derivatives are not redundant assets.
- The introduction of volatility derivatives may benefit market participants in Korea ;
  - new investment opportunities and better diversification effect
  - the improvement of the risk-adjusted performance
  - cost-effective hedging tool for extreme downside risk
  - instruments for hedging volatility risk of medium and long term options.
- The success of volatility derivatives will rest on how ETPs related to volatility develop in the future.